$\mathcal{L}_1$-Adaptive Controller Design for Lithium-Ion Batteries

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System Model Selection

State space representation of lumped thermal capacitance model of a battery :

$$\dot{x}(t) = Ax(t) + b\lambda(u(t) + f(x) + d)$$

$$y = cx$$

- $f(x)$: unstructured, unknown uncertainty
- $\lambda \in [0, 1]$: scaling uncertainty in the controller
- d : disturbance

$$f(x) = 5sin(2x_1(t)) + cos(4x_2(t)) + exp(-x_1(t)^2 - x_2(t)^2)$$

$$A = \begin{bmatrix} 0 & 1 \\ -0.01 & 0.02 \end{bmatrix},\ b = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ c = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -1.2 \end{bmatrix}, \quad \lambda = 0.75$$

Control Architecture

The control architecture and parameter bounds are shown below:

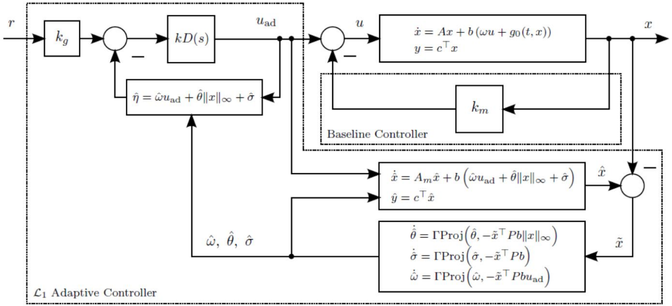

Figure 1: Closed loop adaptive system.

Parameter Bounds

$\Theta = [-5, 5]\ , \Sigma = [-10, 10]\ \&\ \Omega = [0.1, 2]$

$\mathcal{L}_1$-norm Condition

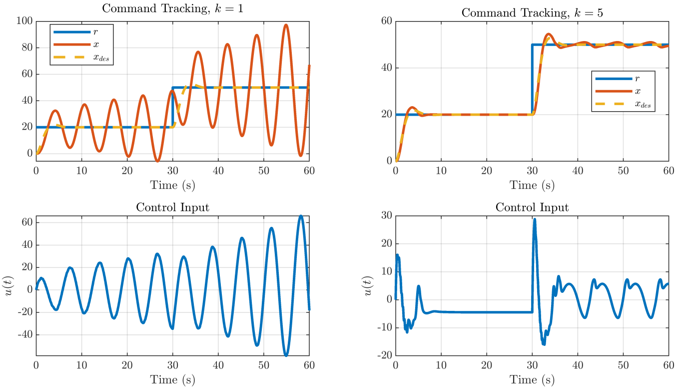

Figure 2: $\mathcal{L}_1$ Performance with $k = 1, 5$ & $\Gamma = 10000$

Notes & Observations

$k = 5$ is close to the minimum value that satisfies $\mathcal{L}_1$-norm condition $||G(s)||_{\mathcal{L}_1} L < 1$. Does not track when $k = 1$.

$\mathcal{L}_1$ vs. MRAC in Presence of d $= sin(\pi) + sin(5\pi/7)$

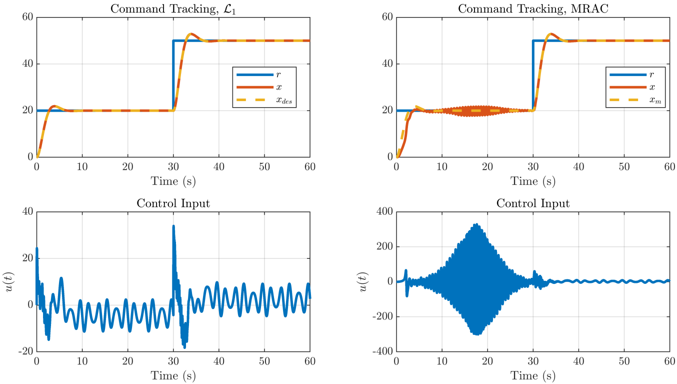

Figure 3: Tracking and input with disturbance for $k = 100$.

Observations

$\mathcal{L}_1$ compensates for disturbance by scaling control input, whereas MRAC does not.

Performance Comparison for $D(s) = \frac{1}{s}$ vs $D(s) = \frac{s+15}{s(s+60)}$

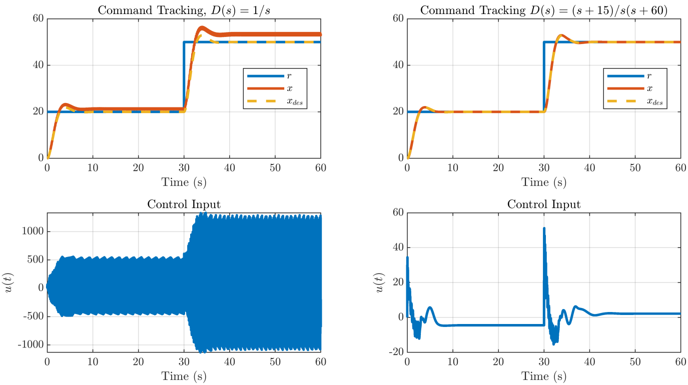

Figure 4: Performance with $t_d = 0.03$s & $k = 100$.

Observations

$D(s) = \frac{s+15}{s(s+60)}$ has larger time delay margin.

$\mathcal{L}_1$ Time Delay Margin Analysis

- $k = 5, 100, 500$ has TDM of $0.3, 0.025, 0.007(s)$, respectively

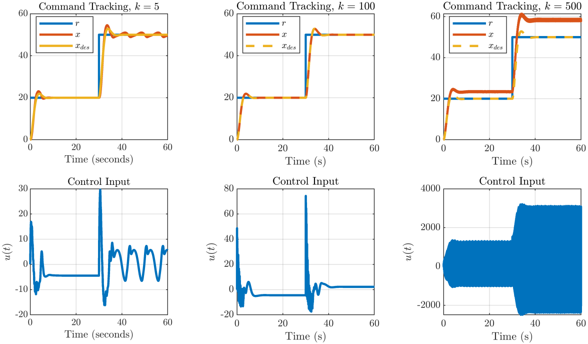

Figure 5: Responses with different k and $t_d = 0.02$

Observations

Higher k allows higher frequency signal, and has tracking performance. This comes at the cost of reduced time delay margin.

MRAC : Adaptation Parameter Tuning

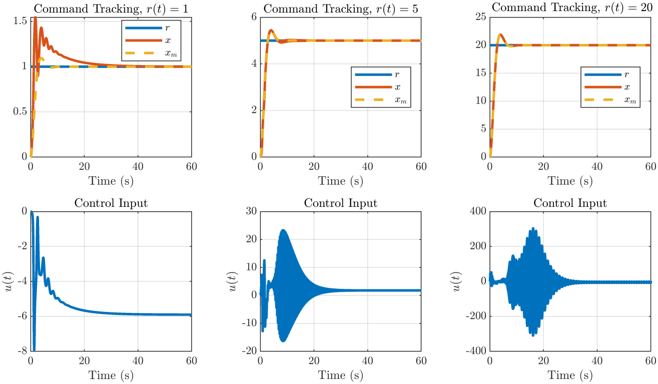

Figure 6: MRAC responses to different reference with $\Gamma = 1$

Observations

Re-tuning of adaptation parameters is required for different reference input.

$\mathcal{L}_1$: Adaptation Parameter Tuning

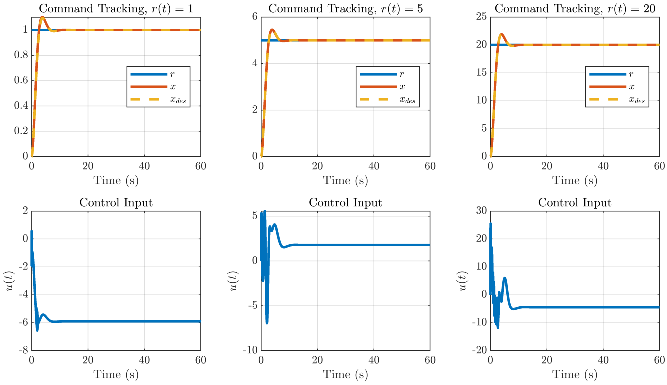

Figure 7: $\mathcal{L}_1$ with constant $\Gamma = 10000$ & $k = 100$

Observations

Without additional tuning, $\mathcal{L}_1$ shows good performance across different reference inputs.

- A is shifted from $\begin{bmatrix} 0 & 1 \\ -0.01 & 0.02 \end{bmatrix}$ to $\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ at $t = 20$

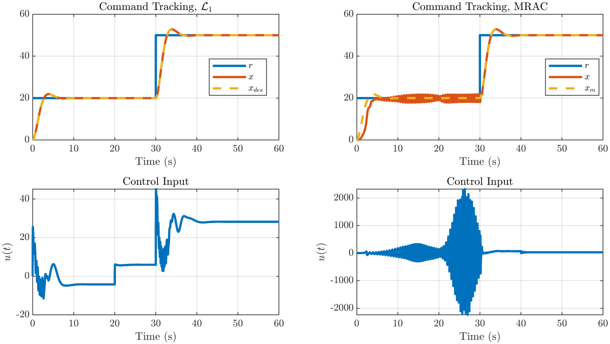

Figure 8: $\mathcal{L}_1$ and MRAC for shifting plant parameters

Observations

$\mathcal{L}_1$ can handle shifting plant parameters, but MRAC does not.

Summary

- $\mathcal{L}_1$ control framework provides **uniform and guaranteed performance**, even during the transient period.
- $\mathcal{L}_1$ **decouples adaptation from robustness** in contrast to the trade-off in MRAC.
- In the presence of disturbance, $\mathcal{L}_1$ guarantees **asymptotic convergence**, whereas MRAC only guarantees ultimate boundedness.
- For different reference inputs, MRAC requires re-tuning of adaptation parameters, while for $\mathcal{L}_1$ retuning is not required.
- $\mathcal{L}_1$ can handle shifting plant parameters, but MRAC cannot.

References

1. Hovakimyan, Naira, and Chengyu Cao. $\mathcal{L}_1$ adaptive control theory: Guaranteed robustness with fast adaptation. Society for Industrial and Applied Mathematics, 2010.
2. Nguyen, Nhan T. Model-Reference Adaptive Control. Springer, Cham, 2018. 83-123.