CS461 COMPUTER GRAPHICS

WRITTEN ASSIGNMENT-1

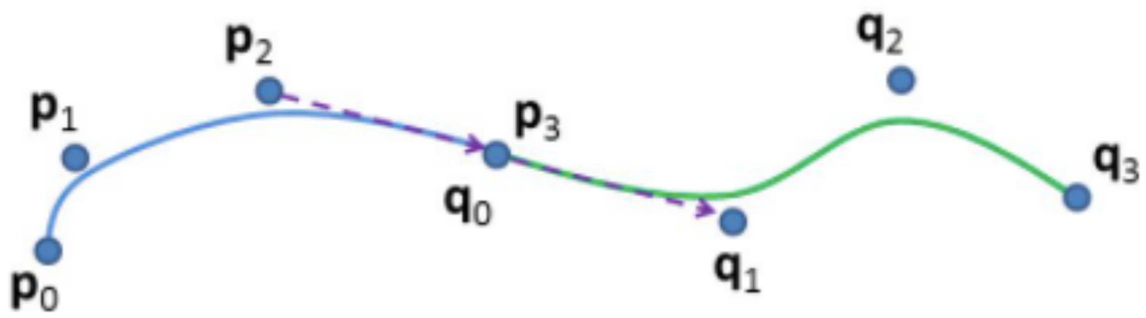
-MAYANK WADHWANI (170101038)

PROBLEM: How to ensure continuity when merging 2 cubic Bazier Curves?

Let our 2 Bazier curves have control points p0, p1, p2, p3 and q0, q1, q2, q3 respectively.

Now these curves are inherently independent of each other and is differentiable and continuous.

So we restrict our problem to ensure continuity on some conditions at the end points only.



POSITIONAL CONTINUITY

Needless to say, the first condition to ensure continuity between 2 curves is that the coordinates of the end point of the first curve and beginning point of the second curve are equal. Ie. these two points lie at the same place.

We can ensure this by the following condition:

$$p3 = q0$$

(or if given a general curve the condition can be written as pn = q0)

Where pn represents the last control point for the first curve.

TANGENTIAL CONTINUITY

Now that we know that the resultant curve is continuous (since the 2 end point are equal), we note that the curve should be tangentially continuous also. Ie. the tangents drawn at the two end points of join must be equal. This property comes from the fact that a Bazier curve is always differentiable. So the resultant of two Bazier curves should be differentiable

In short,

$$p'(3) = q'(0)$$

which can be also written as:

$$p3 – p2 = q1 – q0$$

$$q1 = q0 + p3 – p2$$

from positional continuity we know that p3 = q0, this implies

$$q1 = 2 * p3 - p2$$

Hence this is the tangential continuity condition.

CURVATURE CONDITION

Now that we have given the requirements for continuity and differentiability, we must also have rate of change of slopes equal at the two join end points. This also rises from the fact the Bazier curves follow this condition on all internal points and continuity and differentiability conditions do not imply this condition.

In simpler words,

$$p''(3) = q''(0)$$

Now,

p''(3) = rate of change of slope at p3

= p'(3) – p'(2)

= (p3 – p2) – (p2 – p1)

= p3 + p1 – 2 * p2

In a similar fashion, we can calculate the same for q''(0)

q''(0) = rate of change of slope at q0

= q'(1) – q'(0)

= (q2 – q1) – (q1 – q0)

= q2 + q0 – 2 * q1

Equating these equations, we get ->

q''(0) = p''(3)

$q2 + q0 – 2 * q1 = p3 + p1 – 2 * p2$

$q2 = p3 + p1 – 2 * p2 – q0 + 2 * q1$

since $q0 = p3$ (positional condition) and $q1 = 2 * p3 – p2$, we get

$q2 = p3 + p1 – 2 * p2 – p3 + 2 * (2* p3 – p2)$

$q2 = p3 + p1 – 2 * p2 – p3 + 4 * p3 – 2 * p2$

$q2 = p1 + 4 * p3 – 4 * p2$

$q2 = p1 + 4 * (p3 – p2)$

$q2 = p1 + 4 * (p3 – p2)$

CONCLUSION:

Summing up, we have found three conditions to ensure continuity to merge 2 given Bazier curves.

$p3= q0$ -> Positional Continuity

$q1 = 2 * p3 - p2$ -> Tangential Continuity

$q2 = p1 + 4 * (p3 – p2)$ -> Curvature Continuity