CS-461 COMPUTER GRAPHICS

ASSIGNMENT - 2

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PROBLEM STATEMENT

Can you define an implicit function given a set of points.

SOLUTION

INTUITION

To define a function that would represent an object from a given set of points, we try to redefine our problem statement mathematically. To draw the object, we can refer to a function called as an indicator function χ (chi operator).

This function returns a value of 1 if the point is inside the surface, 0 otherwise. This can be seen in figure 1. (However in practise we take positive ½ values inside the surface and negative ½ values outside so that the surface acts as the zero level set).
We have the oriented points with us. We know that the gradient of a scaler function tells us about the rate of change of values. Since the indicator function is 1 inside and 0 outside. The gradient outside and inside will be equal to 0. But on the surface it will be a vector pointing inwards. So given this information, we now restrict ourselves to finding the indicator function.

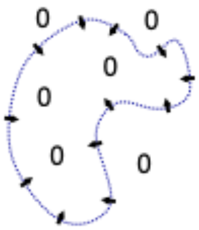

Figure 2 Gradient

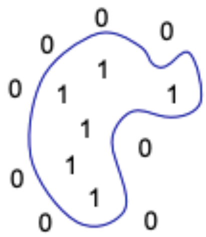

Figure 1 The indicator function

MATHEMATICAL BACKGROUND

The gradient of a function is defined as the sum of partial derivates with respect to different coordinates.

Or **Gradient(f) = $\partial f/\partial x + \partial f/\partial y + \partial f/\partial z$**
Just as a background, if we have to find a function f whose derivative g is known and if it is not easy to simply integrate it, then we make use of the mean square error here.
The mean square error = $\min(f)|\ df/dx - g\ |^2$
We apply a similar concept here, instead of the derivative, we have the gradient and instead of g we have the vector space V.
So we get-> Find $\min(\chi)\ |\ \nabla \chi - V\ |\ ^2$.

We know that the rate of change of a function is 0 at the minimum and maximum (extrema points).
So, applying the divergence operator here, this problem will get reduced to a Poisson problem.

Or $\nabla.\nabla\ \chi = \nabla V$ which is same as $\Delta \chi = \nabla V$.
Here ∇ = Laplacian(f) = **∂^2 f / ∂^2 x + ∂^2 f / ∂^2 y + ∂^2 f / ∂^2 z**

Laplacian(χ) = Gradient(V) => Laplacian(χ) = some known function f
This is analogous to the well-known Poisson equation.

Poisson's Equation

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

Figure 3 Poisson's Equation for Electric Field

Which can be read as, find a function chi so that its Laplacian is equal to the gradient of the vector space V. Once we get the Poisson equation, we can solve it to get the required values of χ. A reference for solving Poisson equations has been provided: https://bit.ly/3imnCLx

HOW IS IT WORKING

The main idea behind the solution was to come up with an indicator function that can be used to draw the surface, ie. we would give coordinates of a point as an input and we would come to know if it is inside the surface or not. We did this by trying to reduce the mean square error of the gradient of this unknown function and the vector space V. In order to minimize this function, we partial differentiate this again and get an equation of the form of the well-known Poisson equation. This can be easily solved to get the indicator function. A practical example has been shown in figure 3.

And hence in this manner, we will be able to represent our implicit function given a set of points.

Figure 4 Practical example of this approach

END OF ASSIGNMENT