Admissible Strategies in Safety Games

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AGENDA

01 INTRODUCTION AND PRELIMINARIES

02 SAFETY GAMES AND WINNING STRATEGIES

03 ADMISSIBLE STRATEGIES

04 RESULTS AND FUTURE WORK

01

INTRODUCTION AND PRELIMINARIES

Introduction and Motivation

1. Controller Synthesis

- System and Environment where system trying to achieve its goal.

2. Graph Games and Safety Games

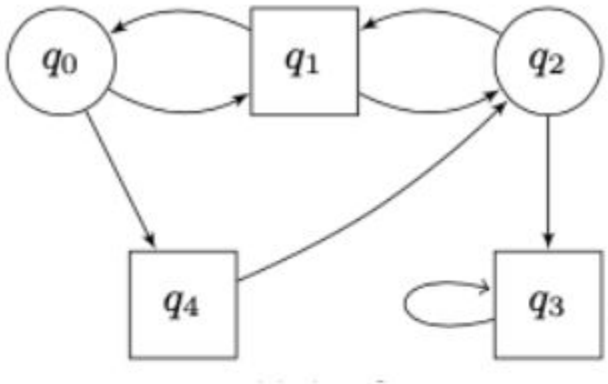
- Approach to attempt to solve controller synthesis problem.

3. Winning Strategies and Admissible Strategies

- Why do we need admissible strategies?

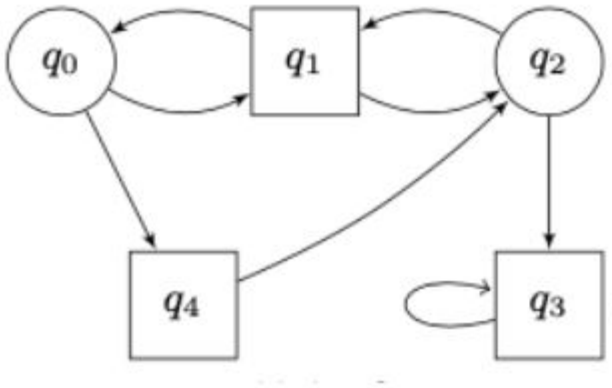
Preliminaries

- **Arena** → Directed graph with finite vertices, vertices divided between players, edges as moves



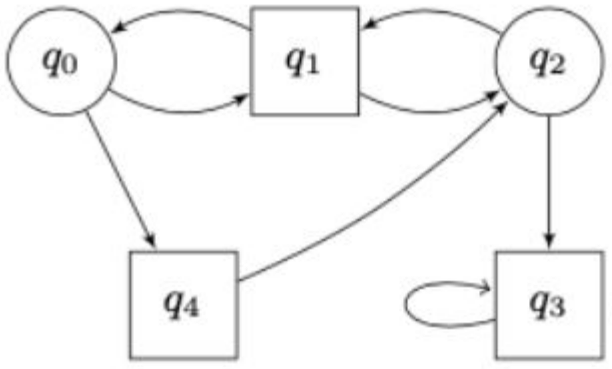
Preliminaries

- Arena
- **Winning Condition →** Some given constraints which the players has to abide by to win the game. (ex. Fencing in a park)



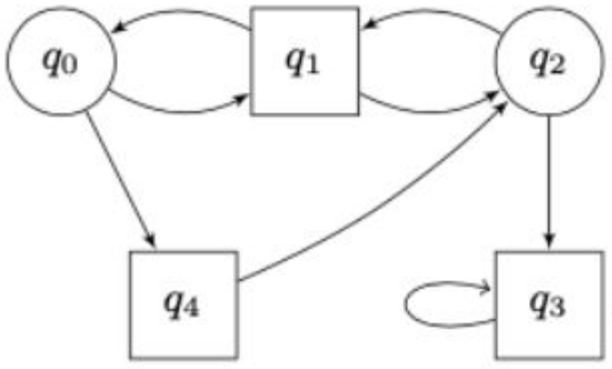
Preliminaries

- Arena
- Winning Condition
- **Multiplayer Game** → A game played on a given arena with many players trying to achieve some given conditions.



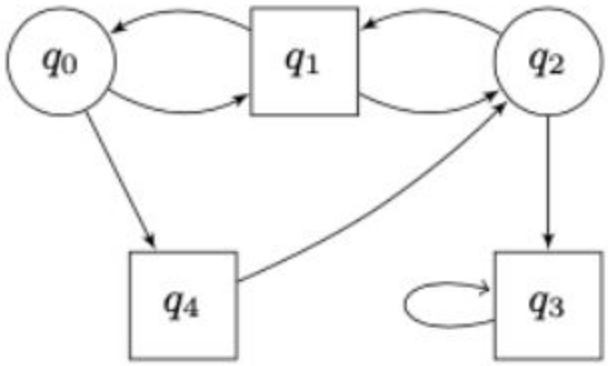
Preliminaries

- Arena
- Winning Condition
- Multiplayer Game
- **Strategy** → Given the past moves and the current state of a player, which state should it send the token next to.



Preliminaries

- Arena
- Winning Condition
- Multiplayer Game
- Strategy
- **Winning Strategy** → A strategy from a vertex which helps to satisfy the winning condition

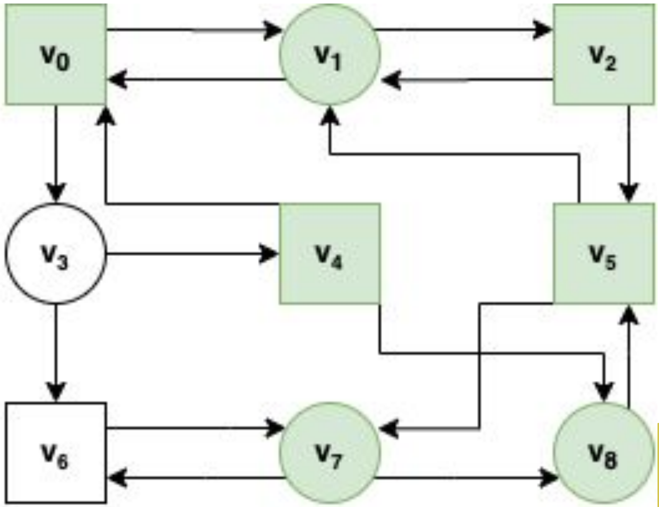


02

SAFETY GAMES AND WINNING STRATEGIES

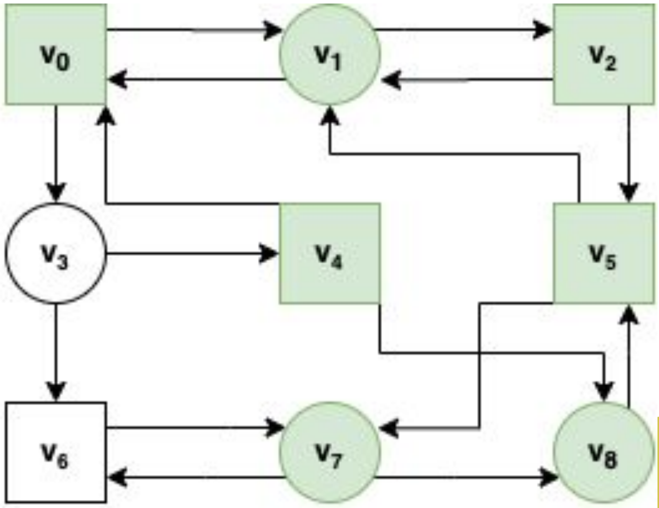
What is a Safety Game?

- A game with a defined winning condition.
- We have a set of **safe vertices** S.
- Bad vertices → All vertices except S.
- **Winning condition** → Do not transition to any "bad" vertex.



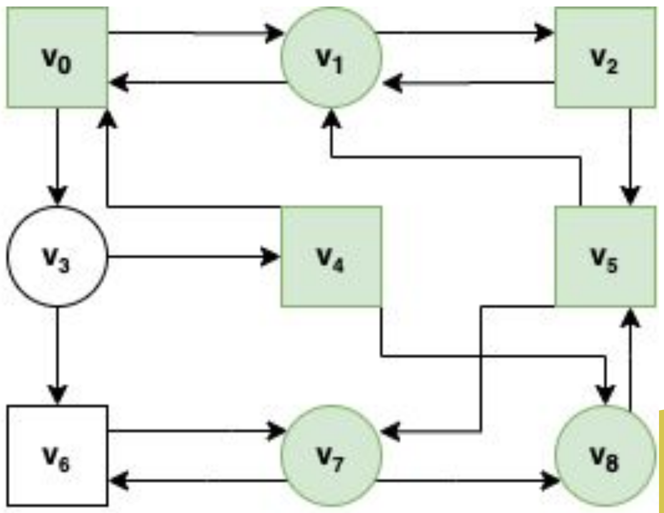
Brief Overview

- Assume we are player 0 → if we start from v_1, and we go to v_0, then player 0 can send token to v_3 which is unsafe and we lose.
- But if we play strategically, and go to v_2, no matter where player 1 sends the token, to v_5 and then to v_1 or v_7, its safe.



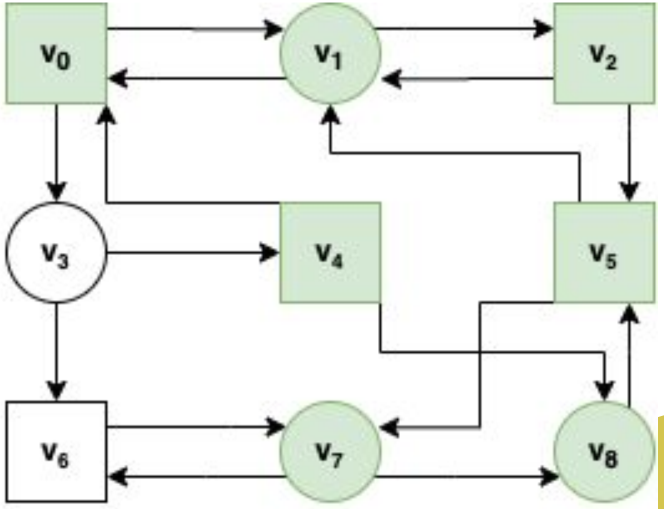
Finding if Winning strategy exist in a Safety Game?

- Implemented the **algorithm** from scratch to find the winning strategy.
- Used concepts from **Reachability Games** to solve.
- **Reachability Games** → Transition to a "good" vertex belonging to a set R. (Opposite of a safety game)

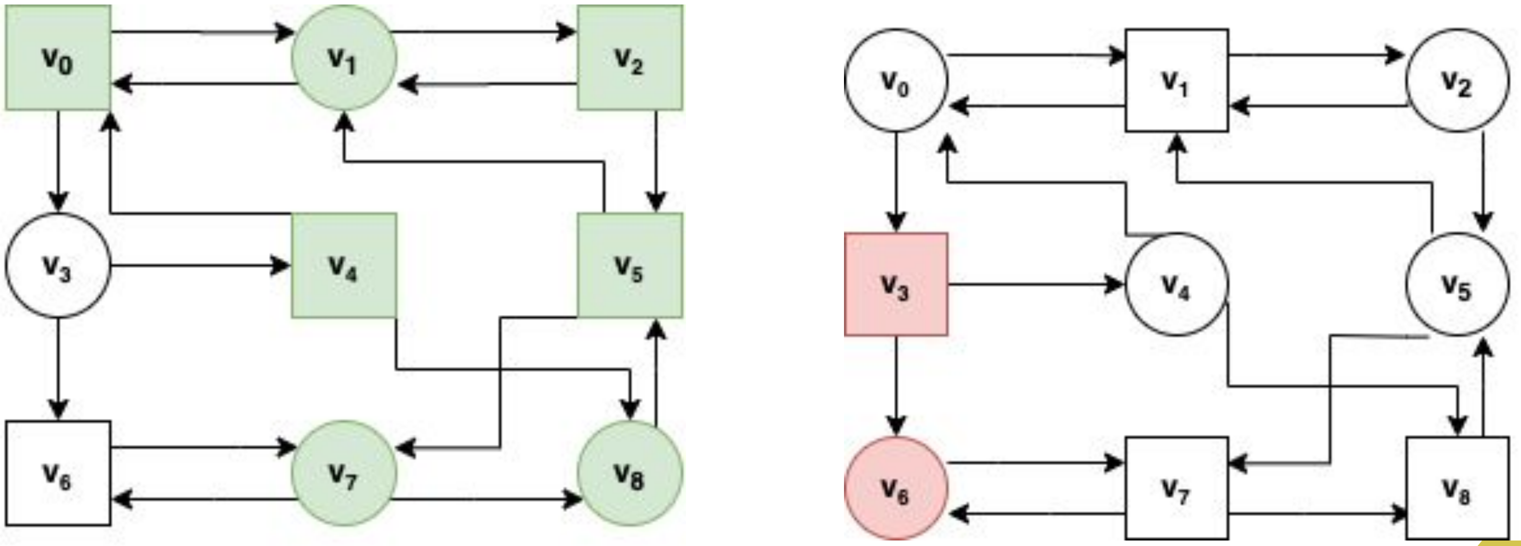


Finding if Winning strategy exist in a Safety Game?

- **Duality Theorem** → If player 0 wins in a safety game i.e. she is successful in keeping the token in S then player 1 wins the reachability game in the dual of the graph (with vertices swapped and R = complement of S)

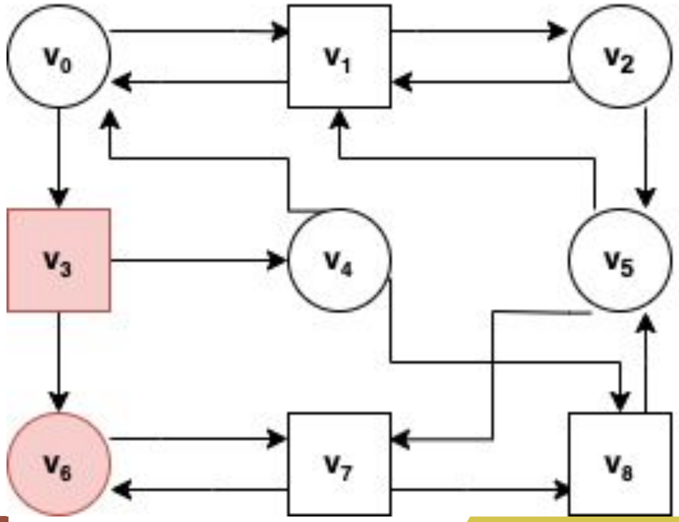


Dual

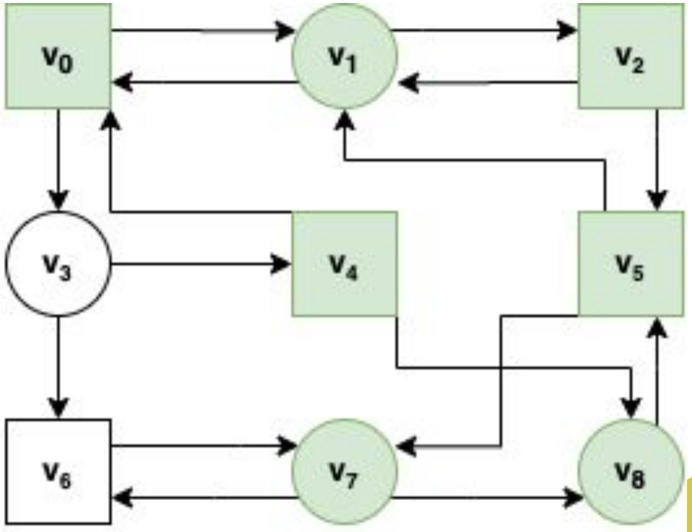


Finding a Winning strategy in a Reachability Game?

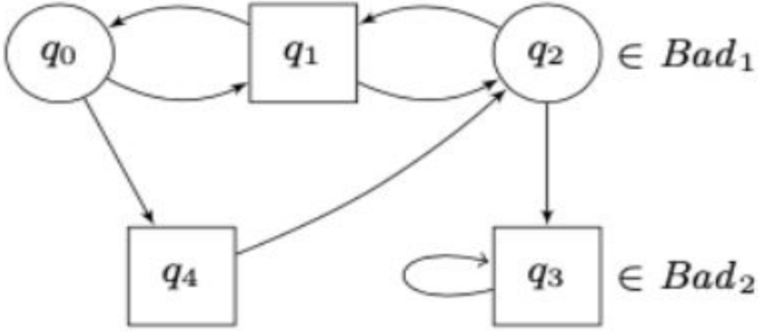
- **Intuition** → Start with set R and some player.
- Now expand R to include those states from where we are bound to reach R. **($CPre_i$)**
- Worst case → only one vertex added this time. Keep **iterating** and continue adding elements to R, **eventually will converge** since number of vertices finite. Will yield what is called the **attractor** for the region.



Code Demonstration to find if winning strategy exists



What if.....?
No winning strategy for Player 0 :(



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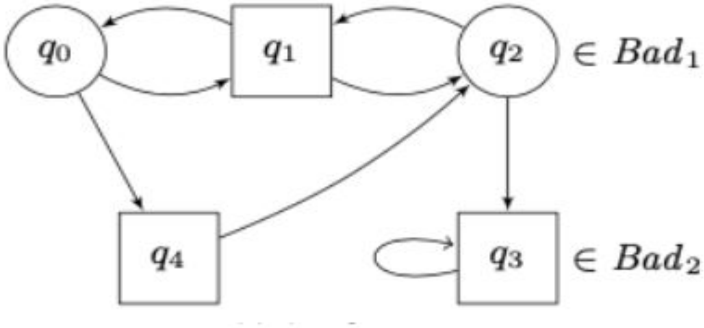
ADMISSIBLE STRATEGIES

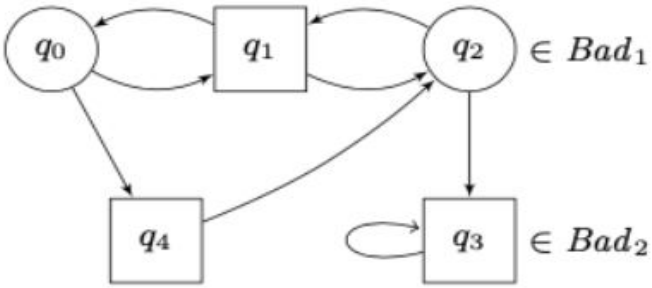
Why Admissible strategies?

- Scenarios where Player 0 has no winning strategy
- What if Player 1 helps?
- Play rationally and not in an adversarial manner
- Both meet their objectives



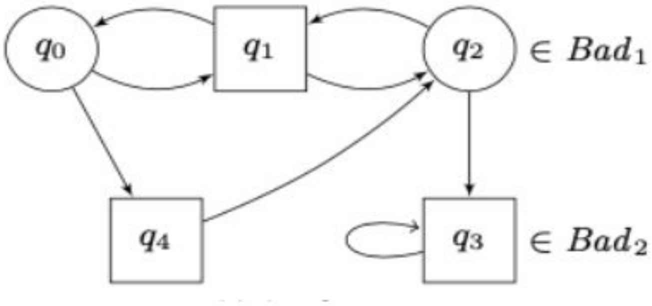
Dominance of strategies

- **Intuition** → Consider q_0. If we go to q_4, we are bound to lose. How? q_4 will send it to q_2.
- What if we go to q_1? We **may** win in that case if Player 1 helps. As seen above, we look for a **win-win situation.**
- **Dominance →** We can say $q_0 \rightarrow q_1$ dominates $q_0 \rightarrow q_4$.

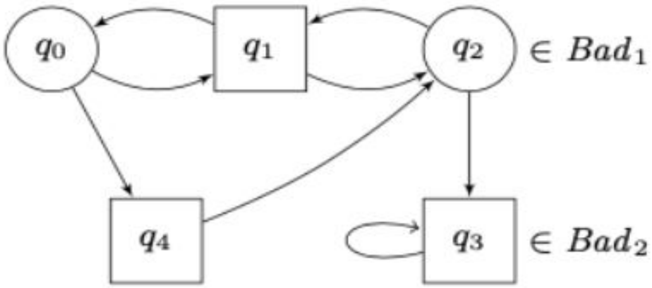


Dominance of strategies

- Notion of **value.**
- **Use of subroutine** to find winning strategies.
- **Implemented algorithm** from scratch to iteratively find dominated strategies and remove them.
- And the strategies left are **admissible** in nature.



Code Demonstration to find admissible strategies



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RESULTS AND FUTURE WORK

RESULTS

01

SAFETY GAMES

Implemented algorithm to find winning strategies

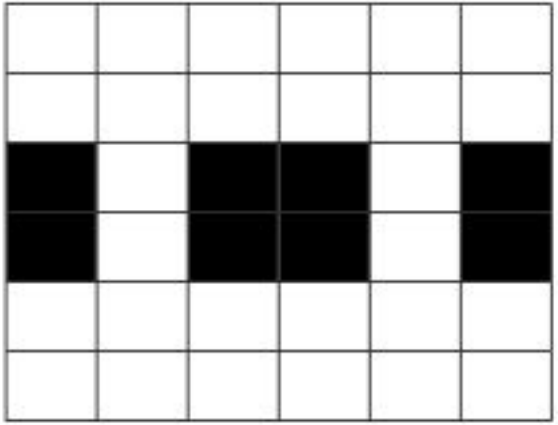
02

ADMISSIBLE STRATEGIES

Implemented algorithm to find admissible strategies

Problem in Robot Motion Planning

- Given an N x N **grid** with **2 robots** with some obstacles, we have an uncontrolled robot which moves in an unconstrained way and a controlled robot which can sense the other robot in a **1 unit distance** only (corners included).
- **Objective** → Avoid collision between the two and let them finish their tasks.



WORK TILL NOW AND FUTURE WORK

PART 1

Get **acquainted** with the domain

PART 2

Algorithm to find strategies in **safety games**

PHASE 3

Algorithm to find **Admissible** strategies

PHASE 4

Describe **Robot Motion** Planning in **LTL**

PHASE 5

Convert **LTL** to a safety game (complex)

PHASE 6

Merge everything to try to solve the robot motion problem.

ACKNOWLEDGEMENT

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We would like to express our gratitude to our mentor Prof. Purandar Bhaduri for giving us the opportunity to explore this new field of Infinite Games.

THANK YOU

We are open to questions, if any.