INFO 6205 - Fall 2020

Program Structures & Algorithms

Assignment 1

1. Task:

The task on hand was to conclude a relationship between the number of steps and the distance traveled in a two-dimensional random walk scenario, where the minimum step size is 1.

2. Output

To produce the following output, I ran the program for the Steps range of {1 to 1200} repeating each experiment 500 times and averaging the values for every individual value of steps. This would run the program for 600,000 times and would yield us enough data to analyze the relationship between the mean distance and steps. The data created let me create the following graph.

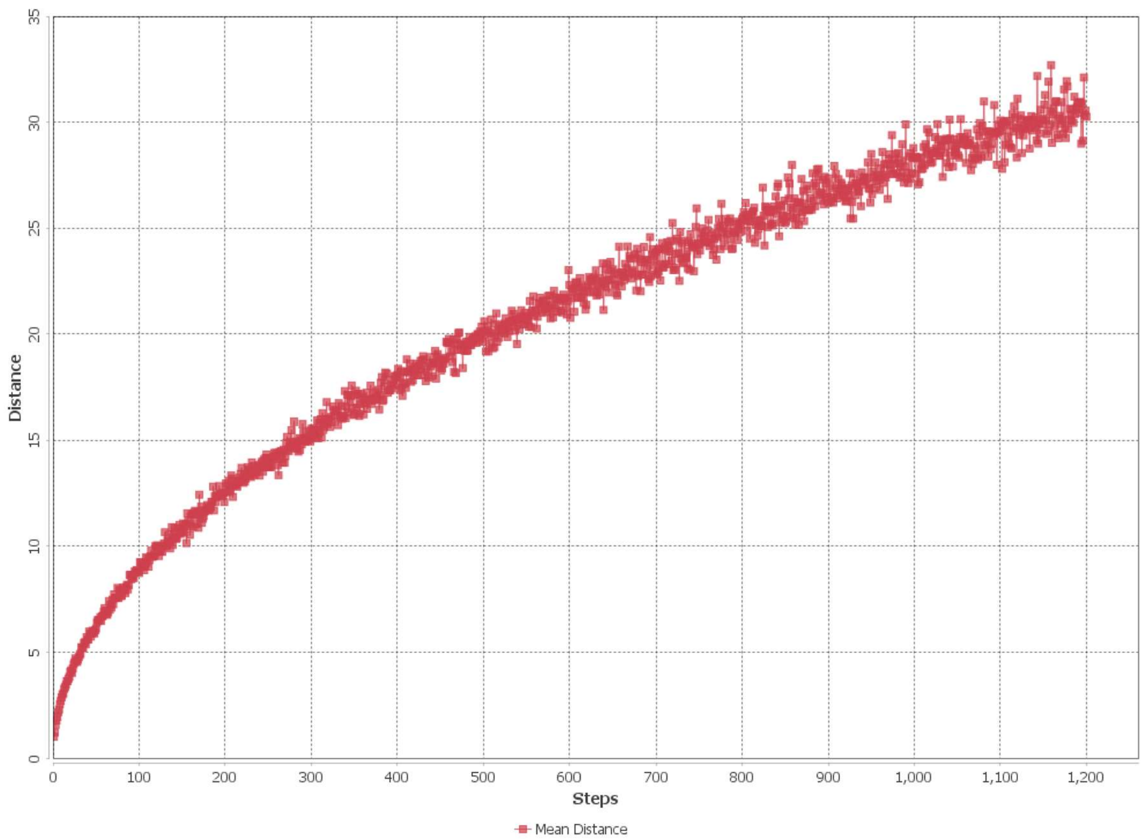


Figure 1: Plot of Distance against Steps

Console Output:

```
1 steps, distance: 1.0 over 500 experiments
2 steps, distance: 1.249248916810283 over 500 experiments
3 steps, distance: 1.579233695494924 over 500 experiments
4 steps, distance: 1.6951238460464433 over 500 experiments
.
.
.
.
1198 steps, distance: 30.137620143071896 over 500 experiments
1199 steps, distance: 31.91350205951836 over 500 experiments
1200 steps, distance: 30.23428232565557 over 500 experiments
Process finished with exit code 0
```

3. Relationship conclusion

The relationship we can conclude is the following:

$$Average\ Euclidean\ Distance\ \propto \sqrt{Number}\ of\ Steps$$

or

$$Average\ Euclidean\ Distance = k * \sqrt{Number}\ of\ Steps$$

$$Where\ k = 0.8865675444083764\ or\ \sqrt{\pi}/2$$

4. Evidence to support the relationship

From Figure. 1, and the mentioned Console Output from Section 2, I noticed that the Euclidean distance is proportional to the square root of the Steps the drunkard walked. I plotted the graph of the square-root of Steps against Euclidean distance to explore the relationship further.

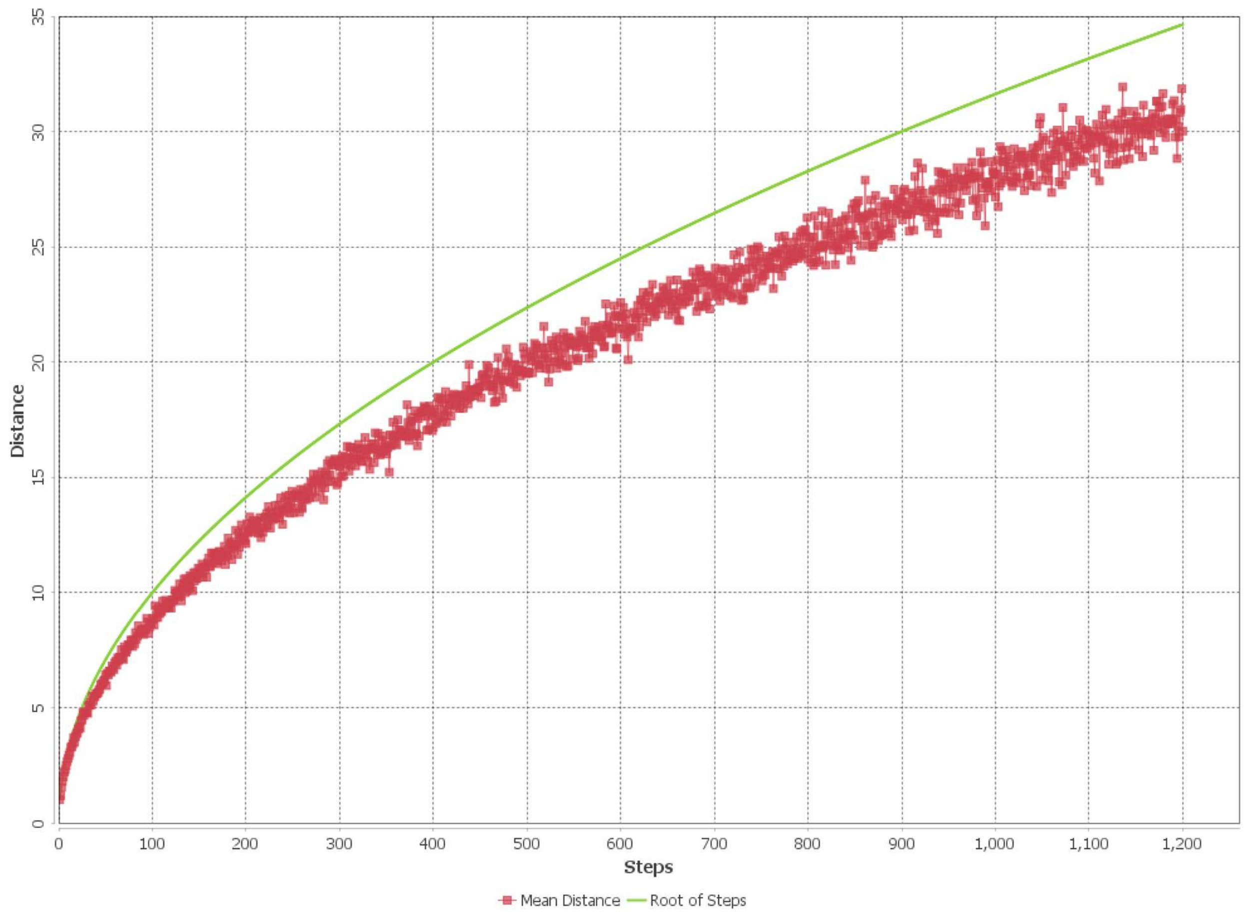


Figure 2: Graph of Euclidean Distance when compared to sq-root of Steps.

From the graph above we can conclude that the Euclidean Distance of random steps is proportional to the square root of the number of steps, but not exactly equal. Hence, we can state

$$Average\ Euclidean\ Distance = k * \sqrt{Number\ of\ Steps}$$

Now to calculate the value of the coefficient k, I devised an experiment. For the range of steps {1 to 10,000} and for each step experiment repeated for 100,000 times, I calculated the Euclidean Distance and Square Root of the number of Steps. For each step, the ratio between the Euclidean Distance and the Square Root was stored in a list. The list was then averaged over in the end. The average yielded us the value 0.8865675444083764 as previously mentioned in section 3.

$$k = 0.886567544408474$$

Using this value as the coefficient, if we plot a graph with steps = {1, 1200} and repeating the experiment 500 times, we get the following.

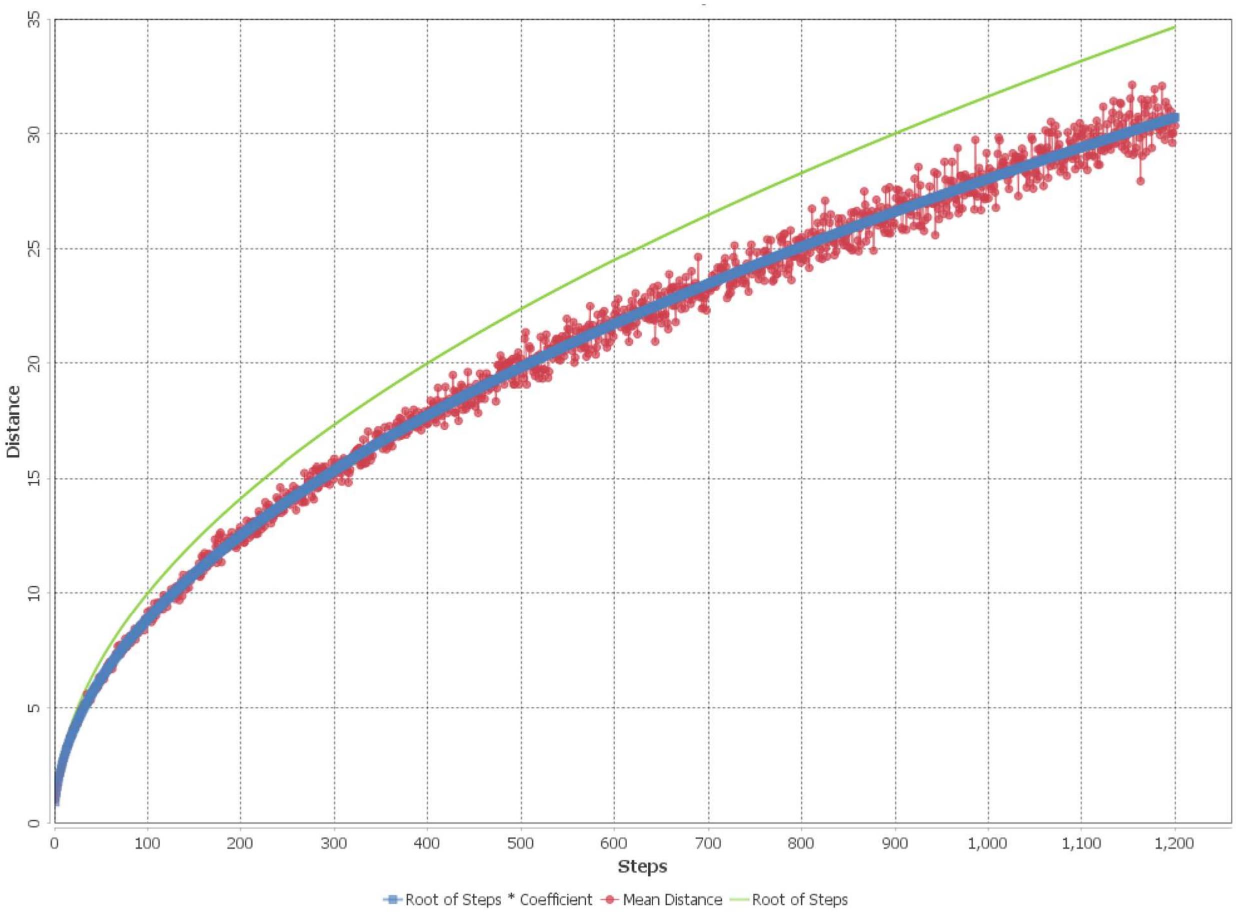


Figure 3: Graph with sq root of Steps multiplied with coefficient.

As we can see from the above graph that clearly, that multiplying the coefficient yields a perfectly overlapping graph.

Hence, we can state

$$Average\ Euclidean\ Distance = k * \sqrt{Number}\ of\ Steps$$

$$Where\ k = 0.8865675444083764$$

From further reading on the internet, I came across the exact mathematical formula of calculating the approximate average Euclidean distance in case of a 2-dimensional random walk scenario [1][2],

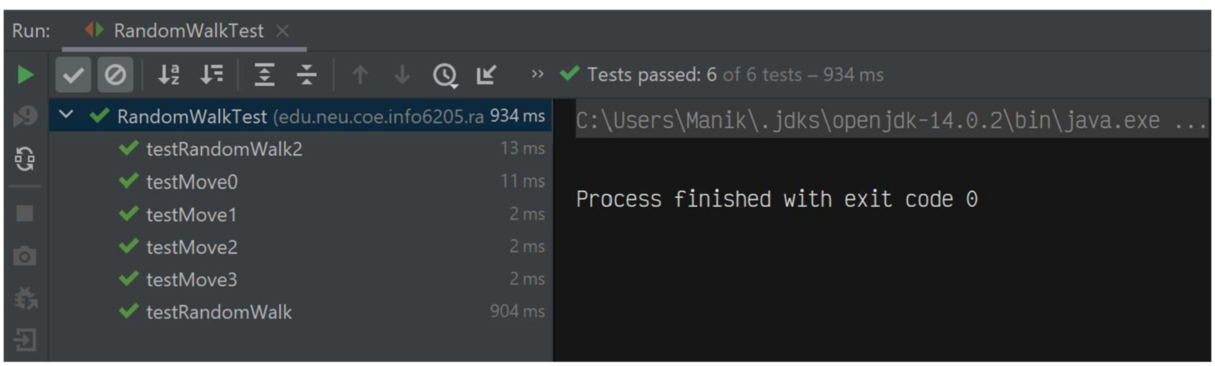
$$Average\ Euclidean\ Distance = \sqrt{\pi}/2 * \sqrt{Number\ of\ Steps}$$

From the above equation, the value of $\sqrt{\pi}/2$ can be calculated to 0.886226925452758..... which we can see to be very similar to the value of the coefficient we calculated using the RandomWalk.java program.

Some example values of actual mean Euclidean distances, and distances calculated using the above formula.

Steps	**Mean Euclidean Distances**	**K*Root(steps)**
40	5.4732152963444705	5.607145479825893
100	8.977560357150278	8.865675444083763
250	13.964345851717646	14.01786369956473
700	24.481542551907122	24.456372429657765
1150	30.426321385045977	30.130255931074245

5. Screenshot of Unit test passing



6. References

[1] - Hey! You Can Find Pi With a Random Walk. Here's How. (2017-3-14). Rhett Allain. https://www.wired.com/2017/03/hey-can-find-pi-random-walk-heres/

[2] - Henry (https://math.stackexchange.com/users/6460/henry). Expected Value of Random Walk. (2012-01-28). https://math.stackexchange.com/q/103170