Autonomous Mobile Robotics: Dynamic Control, Global Path Planning, and Trajectory Tracking Across Three Robot Platforms

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Outline

Introduction

Autonomous mobile robots require reliable path planning and accurate trajectory tracking for safe navigation. This project addresses three core problems: **(i)** generating feasible, collision-free paths, **(ii)** designing controllers for smooth and stable trajectory tracking across different robot models, and **(iii)** tracking a recorded trajectory with a mecanum robot.

The work is divided into three independent tasks:

1. **Task A – Skid-Steer Robot: Modeling & Control**
   - Kinematic–dynamic modeling of a 3-DoF skid-steer platform.
   - Nonlinear two-loop controller (position + dynamic loops).
   - Goal: stable convergence to a desired trajectory.
2. **Task B – Unicycle Robot: Planning & Tracking**
   - A* grid-based planner with obstacle avoidance.
   - Smoothed path followed using Pure Pursuit.
   - Goal: smooth, low-error tracking.
3. **Task C – Mecanum Robot: Trajectory Tracking**
   - Tracking of recorded XY trajectory.
   - Goal: Robot follows the recorded trajectory.

Skid Steer Robot - Objective:

- A four wheel skid steering mobile robot is a differential-drive platform with planar motion (x, y, θ).
- **Objective:** Explore two key aspects of robotic motion
  1. Designing and implementation of PD controller.
  2. Perform trajectory tracking objective.
- These two components are implemented and tested independently in Matlab simulation.

Skid Steer Robot - Kinematic Model

The robot pose and body velocities are related as:

$$\dot{\boldsymbol{\eta}} = J(\psi)\boldsymbol{\zeta}, \quad \boldsymbol{\eta} = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}, \quad \boldsymbol{\zeta} = \begin{bmatrix} u \\ v \\ r \end{bmatrix}$$

$$J(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Desired trajectory:

$$\boldsymbol{\eta}_d(t) = \begin{bmatrix} x_d(t) \\ y_d(t) \\ \psi_d(t) \end{bmatrix},$$

Skid Steer Robot - Error Definition

Position error in inertial frame:

$$\boldsymbol{e}_p = \boldsymbol{\eta}_d - \boldsymbol{\eta}$$

Convert to body frame to decouple orientation:

$$\boldsymbol{e}_b = J^T(\psi)\boldsymbol{e}_p$$

Differentiating:

$$\dot{\boldsymbol{e}}_b = \dot{J}^T(\psi)\boldsymbol{e}_p + J^T(\psi)\dot{\boldsymbol{e}}_p$$

Since $\dot{\boldsymbol{e}}_p = \dot{\boldsymbol{\eta}}_d - \dot{\boldsymbol{\eta}} = \dot{\boldsymbol{\eta}}_d - J(\psi)\boldsymbol{\zeta}$,

$$\boxed{\dot{\boldsymbol{e}}_b = J^T(\psi)\dot{\boldsymbol{\eta}}_d - S(r)\boldsymbol{e}_b - \boldsymbol{\zeta}}$$

where

$$S(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

represents the coupling between rotation rate r and position error.

Skid Steer Robot - Nonlinear Error Dynamics

From the previous slide:

$$\dot{\boldsymbol{e}}_b = J^T(\psi)\dot{\boldsymbol{\eta}}_d - S(r)\boldsymbol{e}_b - \boldsymbol{\zeta}$$

We define a desired body velocity that includes both feedforward and feedback terms:

$$\boxed{\boldsymbol{\zeta}_{des} = J^T(\psi)\dot{\boldsymbol{\eta}}_d + K_p\boldsymbol{e}_b}$$

The actual velocity tracking error:

$$\tilde{\boldsymbol{\zeta}} = \boldsymbol{\zeta}_{des} - \boldsymbol{\zeta}$$

Substitute into $\dot{\boldsymbol{e}}_b$:

$$\boxed{\dot{\boldsymbol{e}}_b = -S(r)\boldsymbol{e}_b - K_p\boldsymbol{e}_b + \tilde{\boldsymbol{\zeta}}}$$

Thus, the outer loop ensures position convergence provided $\tilde{\boldsymbol{\zeta}} \to 0$.

Skid Steer Robot - Dynamic Model

The robot's body dynamics:

$$\boldsymbol{D}(\zeta)\dot{\zeta} + \boldsymbol{n}_v(\zeta) = \boldsymbol{\tau}$$

where

$$\boldsymbol{D} = \begin{bmatrix} m & 0 & -my_{bc} \\ 0 & m & mx_{bc} \\ -my_{bc} & mx_{bc} & I_z + m(x_{bc}^2 + y_{bc}^2) \end{bmatrix}$$

and

$$\boldsymbol{n}_v = \begin{bmatrix} -mr(v + x_{bc}r) \\ mr(u - y_{bc}r) \\ mr(x_{bc}u + y_{bc}v) \end{bmatrix}$$

$\boldsymbol{\tau}$ = generalized body forces (wheel torques)

Skid Steer Robot - Inner Loop Control Law

To drive $\zeta \to \zeta_{des}$, choose

$$\boxed{\boldsymbol{\tau} = \boldsymbol{D}(\zeta)\left[\dot{\zeta}_{des} + K_d(\zeta_{des} - \zeta)\right] + \boldsymbol{n}_v(\zeta)}$$

Then

$$\boldsymbol{D}(\zeta)\dot{\tilde{\zeta}} + K_d\boldsymbol{D}(\zeta)\tilde{\zeta} = 0$$

and $\tilde{\zeta} \to 0$ exponentially.
Substituting into the outer loop:

$$\dot{\boldsymbol{e}}_b = -S(r)\boldsymbol{e}_b - K_p\boldsymbol{e}_b$$

This guarantees asymptotic convergence of $\boldsymbol{e}_b(t) \to 0$.

Using Lyapunov theory we can easily prove this

$$\boxed{\zeta(t) \to \zeta_{des}(t) \quad \Longrightarrow \quad \eta(t) \to \eta_d(t)}$$

Skid Steer Robot - Circular Trajectory Example

Let the desired trajectory be a circle of radius $R = 2$ and angular rate $\omega_d = 0.2$:

$$\begin{aligned} x_d(t) &= R\cos(\omega_d t), \\ y_d(t) &= R\sin(\omega_d t), \\ \psi_d(t) &= \omega_d t + \psi_0 \end{aligned}$$

Then

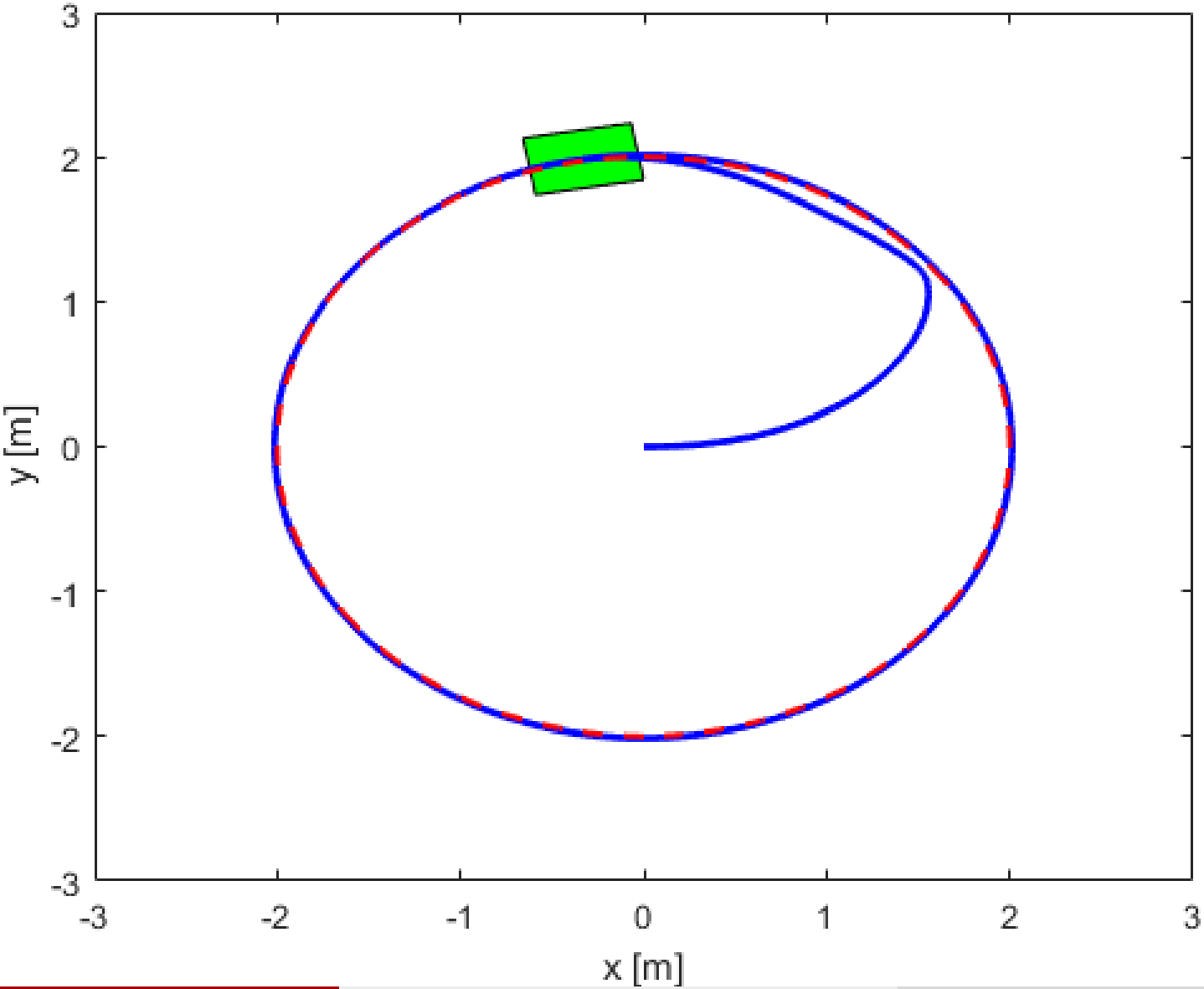
$$\dot{\boldsymbol{\eta}}_d = \begin{bmatrix} -R\omega_d \sin(\omega_d t) \\ R\omega_d \cos(\omega_d t) \\ \omega_d \end{bmatrix}, \quad \ddot{\boldsymbol{\eta}}_d = \begin{bmatrix} -R\omega_d^2 \cos(\omega_d t) \\ -R\omega_d^2 \sin(\omega_d t) \\ 0 \end{bmatrix}$$

Skid Steer Robot - Summary of Control Law

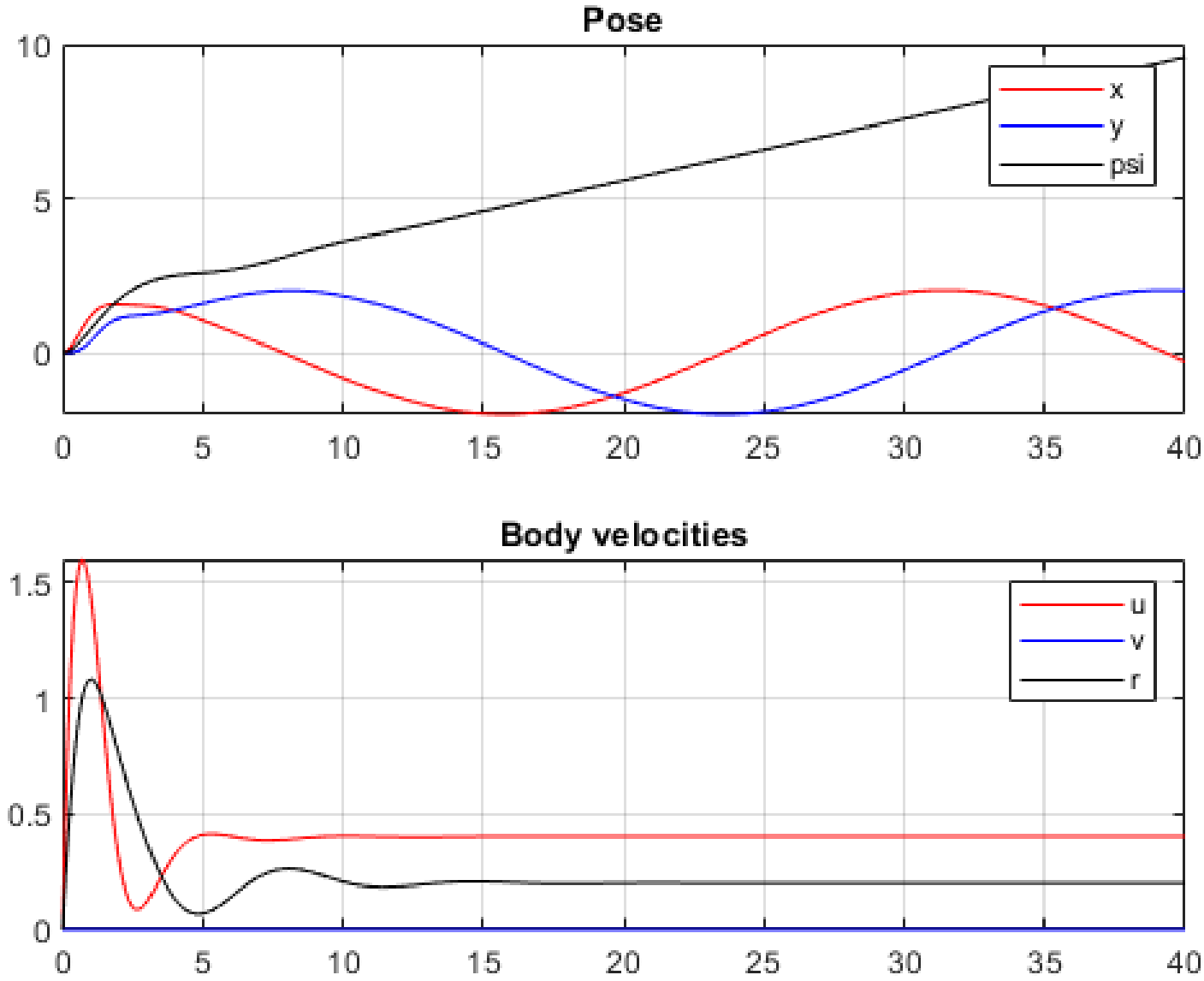
This nonlinear two-loop controller:

- Tracks arbitrary smooth trajectories (including circular motion)
- Compensates for full robot dynamics
- Ensures $\boldsymbol{e}_b, \tilde{\boldsymbol{\zeta}} \to 0$ via Lyapunov stability

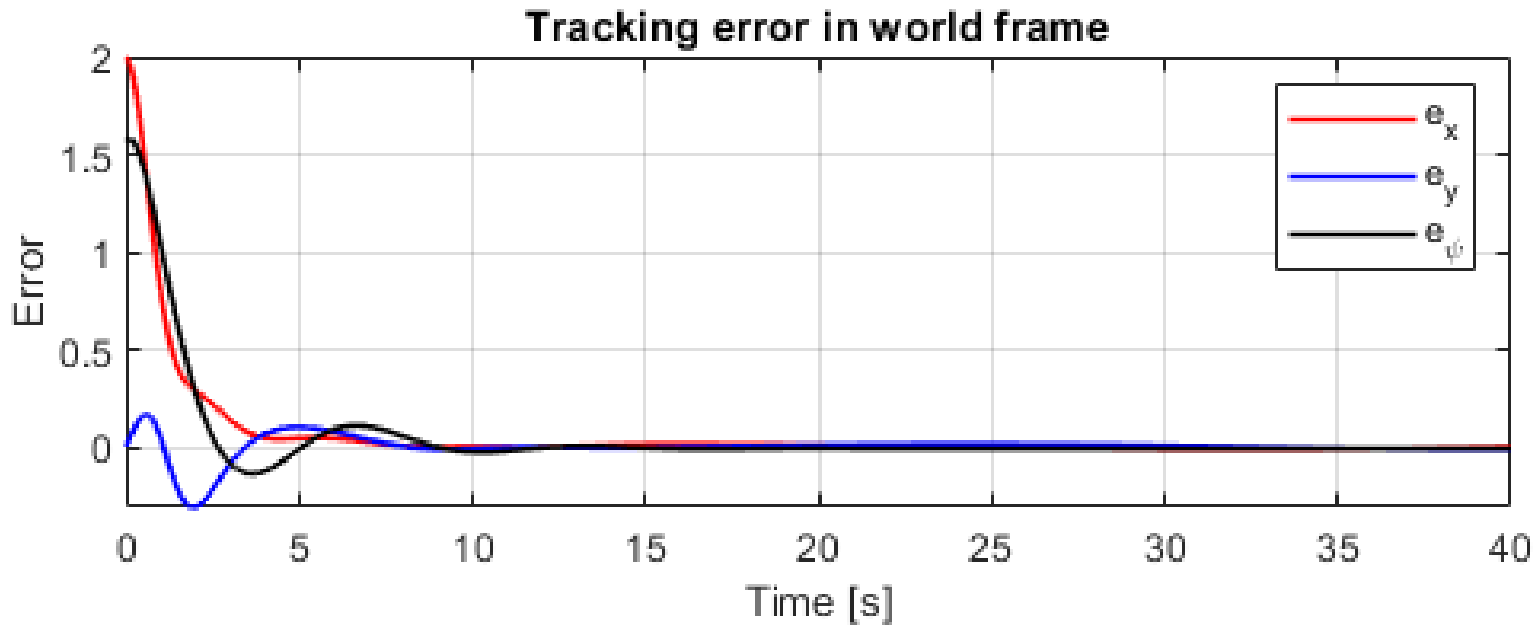
Skid Steer Robot - Robot Pose

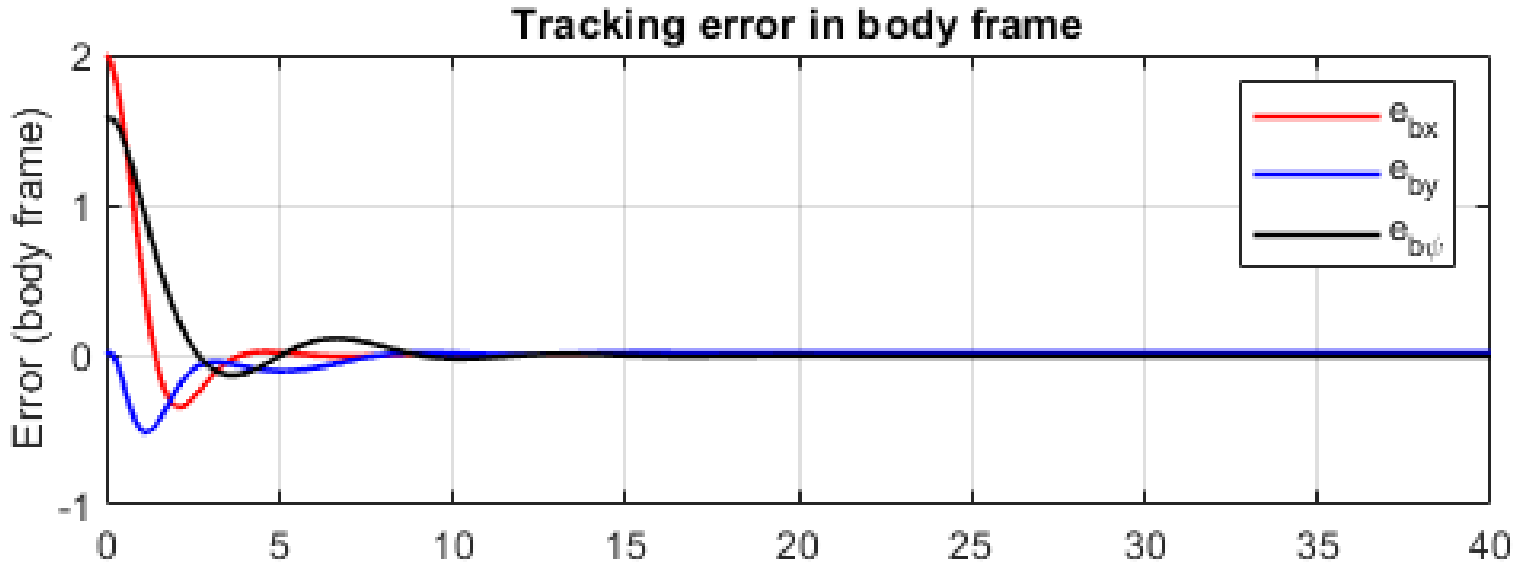


Skid Steer Robot - Robot Pose

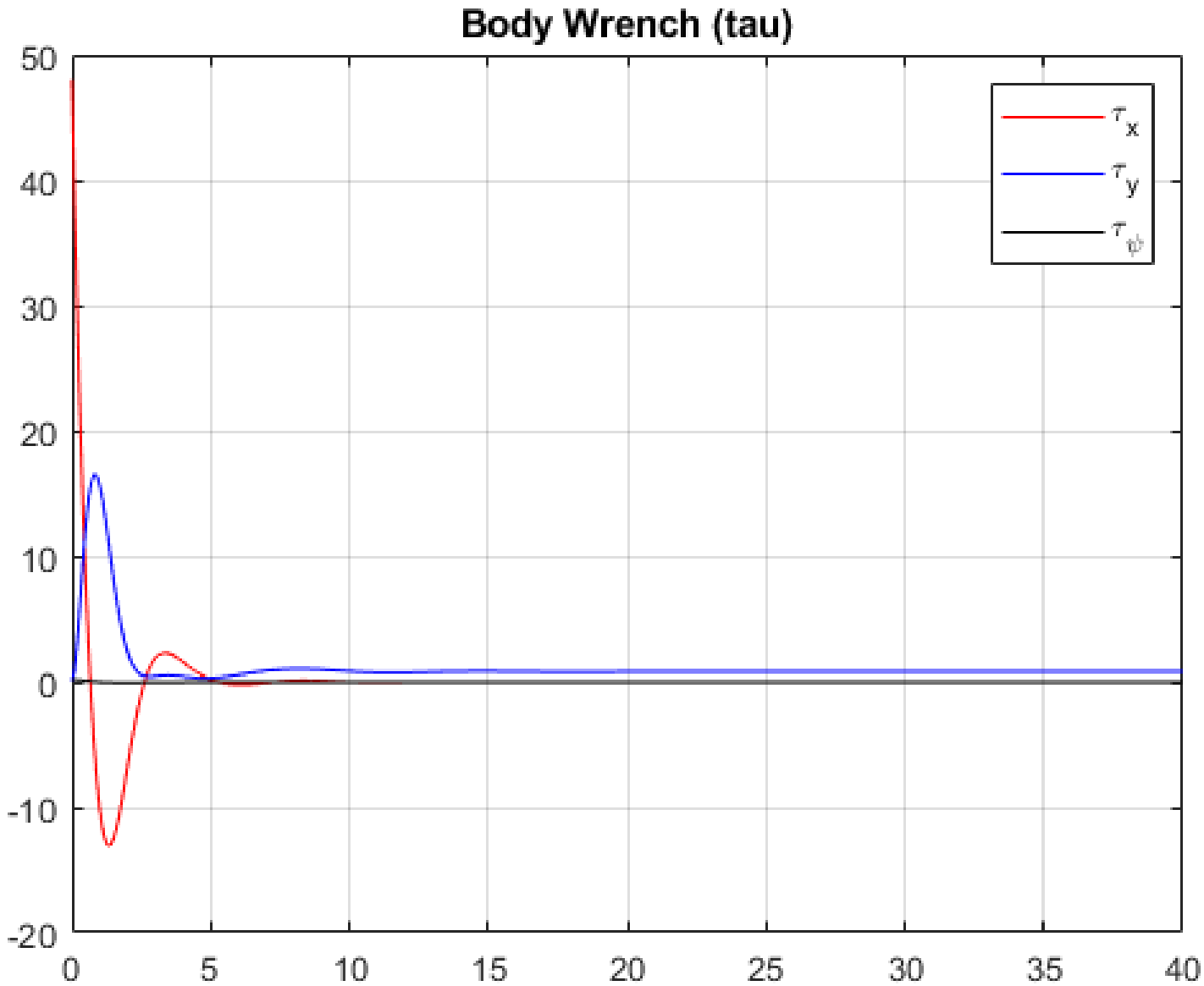


Skid Steer Robot - Tracking Errors





Skid Steer Robot - Control Inputs



Skid Steer Robot - Wheel Inputs

Unicycle Robot - Introduction & Motivation

A unicycle mobile robot is a differential-drive platform with planar motion (x, y, θ).
Objective: Path planning using the **A* algorithm**.

Unicycle Robot - Degrees of Freedom & Kinematic Model

The robot configuration and control inputs are:

$$\boldsymbol{\eta} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \qquad \boldsymbol{u} = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

The kinematic equations of motion are:

$$\dot{x} = v\cos\theta, \quad \dot{y} = v\sin\theta, \quad \dot{\theta} = \omega$$

This simplified kinematic model assumes pure rolling contact and forms the basis for trajectory tracking simulations.

Part I: Path Planning using A* Algorithm

Objective: Generate a collision-free path between a start and goal position on a 2D occupancy grid.

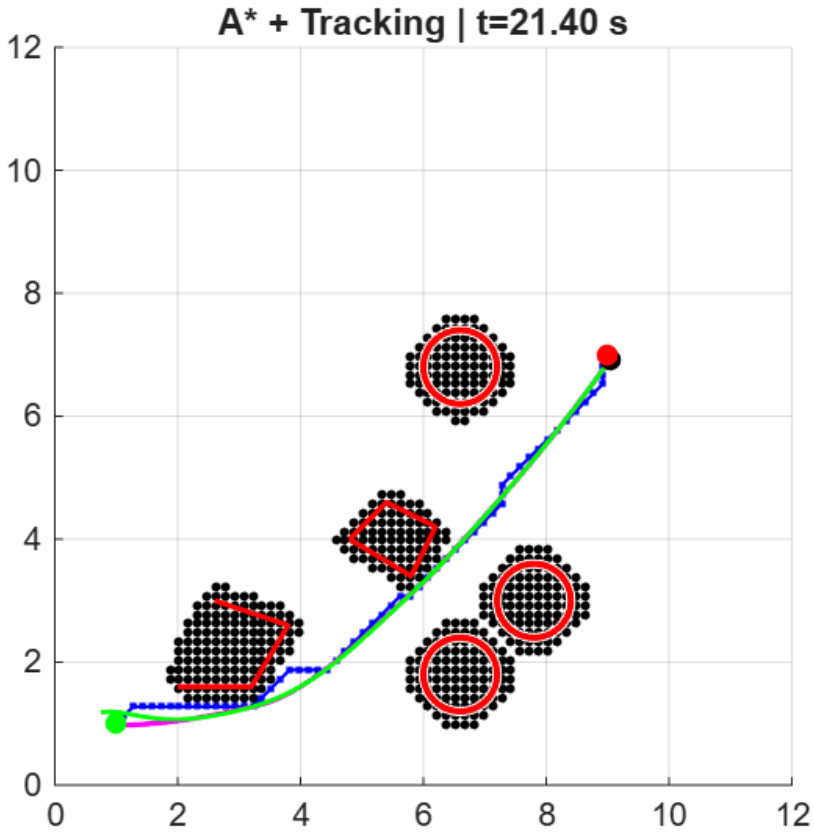
Algorithm:

- The environment is represented as a grid map with obstacles.
- Each node is evaluated using the cost function:

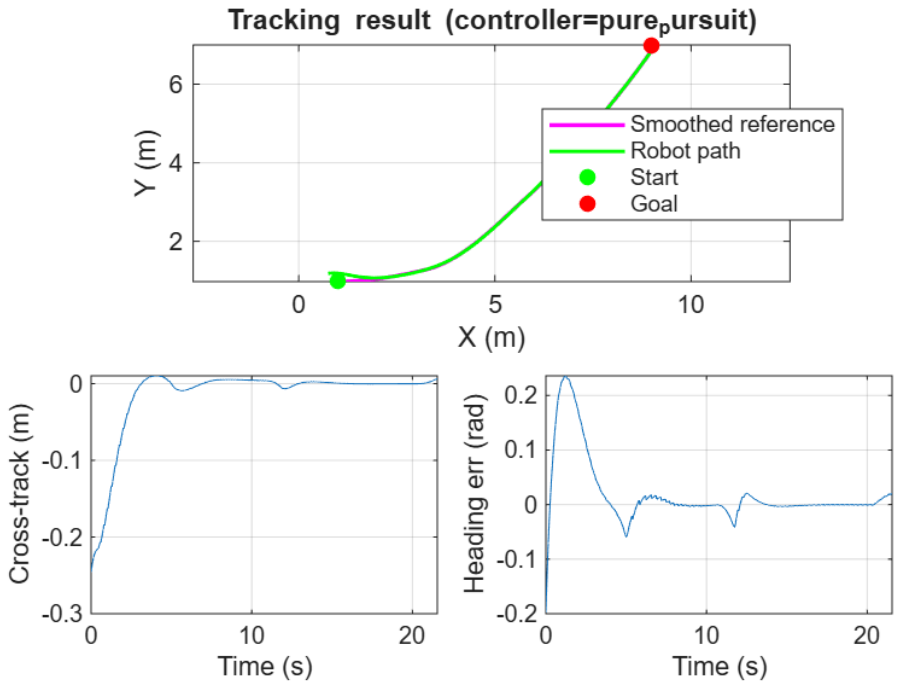
$$f(n) = g(n) + h(n)$$

  where $g(n)$ is the cost from the start, and $h(n)$ is the heuristic (Euclidean distance to goal).
- The node with minimum $f(n)$ is expanded until the goal is reached.

A* Environment and Path Metrics



Environment Map and Paths



Path metrics

Part I: A* Simulation Results & Observations

- The algorithm successfully planned a feasible path around obstacles.
- The number of nodes expanded and computation time were recorded for performance analysis.
- The generated path is discrete and grid-based; further smoothing would be required for physical implementation.
- **Limitations:**
  - Sensitive to map resolution and heuristic scaling.
  - Path not dynamically feasible for a real unicycle robot without smoothing.

Unicycle Robot - Comparison & Discussion

- The A* planner provides a discrete obstacle-free route, suitable for global navigation.
- The Pure Pursuit controller enables smooth local trajectory following when a reference path is known.
- These two components together represent the core of a full autonomous navigation system, even though implemented separately here.
- Future work: integrate both modules by smoothing A* output for tracking and including dynamic constraints.

Unicycle Robot - Implementation Requirements & Conclusion

- **Hardware Requirements:**
  - Sensors: wheel encoders, IMU, GPS or camera for localization.
  - Actuators: two DC motors with differential drive.
  - Controller: Raspberry Pi / Arduino / STM32 for onboard control.
- **Applications:**
  - Warehouse navigation, indoor delivery, and academic robotics.

Problem: Trajectory Tracking with Mecanum Robot

- **Goal**: Make a Mecanum-wheeled robot follow a recorded real-world trajectory.
- **Challenge**:
  - Recorded data is noisy, discontinuous
  - High-frequency acceleration $\rightarrow$ control instability
  - Dynamics mismatch $\rightarrow$ drift
- **Why Mecanum?** Omnidirectional motion, but kinematically complex.

Wheel Kinematics

Wheel angles:

$$\rho = (\theta_1, \theta_2, \theta_3, \theta_4)$$

Relationship between wheel speeds and body velocity:

$$\dot{q}^B = J\dot{\rho},$$

$$J = \frac{r}{4}\begin{bmatrix} -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ \dfrac{1}{l_1+l_2} & -\dfrac{1}{l_1+l_2} & \dfrac{1}{l_1+l_2} & -\dfrac{1}{l_1+l_2} \end{bmatrix}.$$

Wheel speeds from body velocity:

$$\dot{\rho} = J^\dagger \dot{q}^B,$$

$$J^\dagger = \begin{bmatrix} -1 & 1 & l_1+l_2 \\ -1 & 1 & -(l_1+l_2) \\ -1 & -1 & l_1+l_2 \\ 1 & 1 & l_1+l_2 \end{bmatrix}. \tag{5}$$

Dynamic Model

Four-wheel mecanum robot dynamics:

$$M\ddot{\rho} + D\dot{\rho} = \tau + \tau_d. \tag{6}$$

Mass matrix:

$$M = \begin{bmatrix} \Gamma & -m_1 & m_2 - m_1 & m_1 \\ -m_1 & \Gamma & m_2 - m_1 & m_1 \\ m_1 & m_2 - m_1 & \Gamma & -m_1 \\ m_2 - m_1 & m_1 & -m_1 & \Gamma \end{bmatrix},$$

where:

$$m_1 = \frac{I_w}{r^2(l_1 + l_2)}, \quad m_2 = \frac{mr^2}{8}, \quad D = \text{diag}(D_1, D_2, D_3, D_4),$$

$$\Gamma = m_1 + m_2 + I_m.$$

Why It Works: Key Fixes

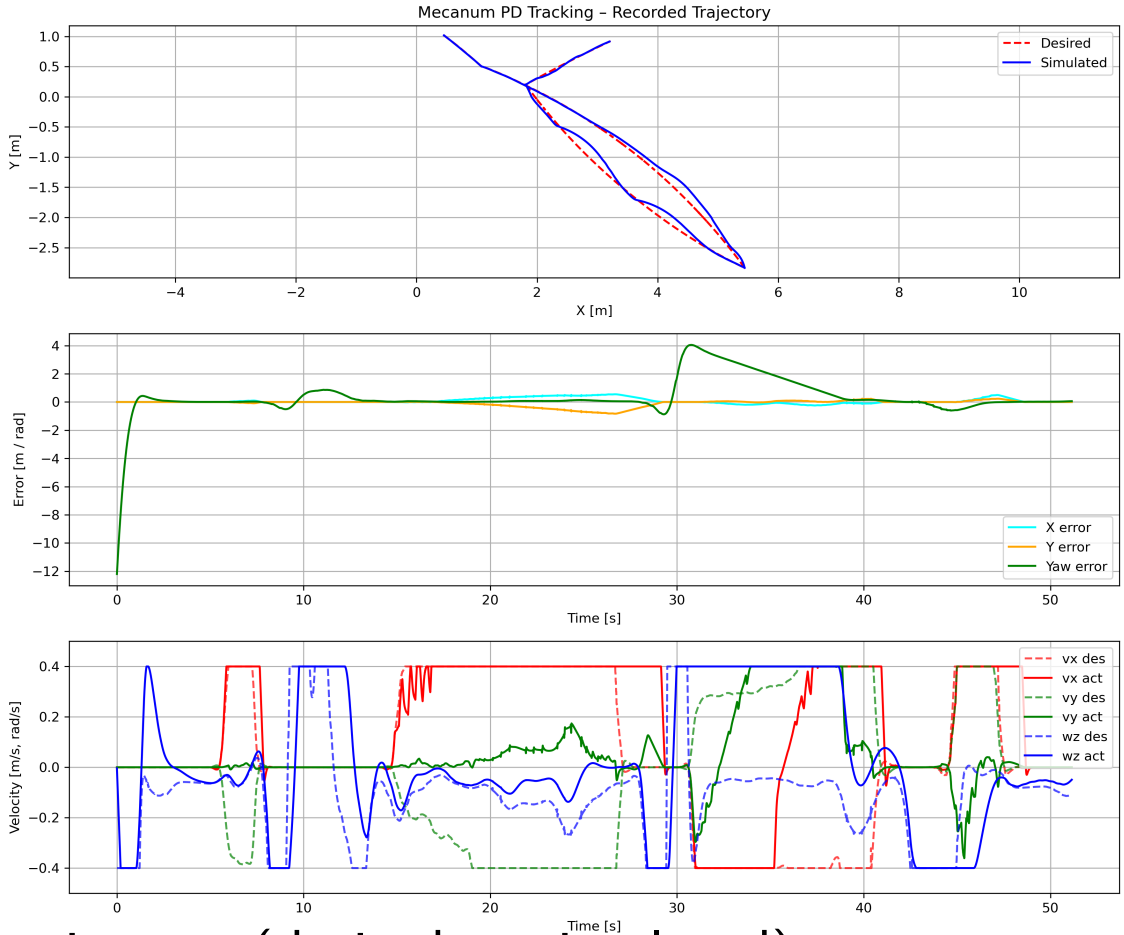
- **Smooth Interpolation**:

$$\text{Butterworth} \rightarrow \text{Cubic Spline}$$

- **Physical Damping**: No artificial decay ($\nu \not= 0.97$)
- **Velocity Clipping**: Respect robot limits

Result: RMS error < 5 cm, stable tracking on real data.

Simulation Results



- **Top**: XY trajectory (desired vs simulated)
- **Middle**: Tracking errors
- **Bottom**: Velocity tracking

Robust tracking despite noisy recorded input.

Overall Conclusion

- The project addressed complementary navigation modules for three robots:
  - **Skid–steer:** dynamic model + nonlinear two–loop control.
  - **Unicycle:** A* global planner + Pure Pursuit tracking.
  - **Mecanum:** reference trajectory and velocity tracking.
- **Key results:**
  - Skid–steer controller ensured stable tracking of the full kinematic–dynamic model.
  - A* generated collision–free paths; Pure Pursuit tracked paths with low error.
  - Mecanum robot accurately followed XY trajectories and maintained acceptable tracking errors despite noise and dynamics mismatch.
- **Takeaway:** The work illustrates the separation between global planning and local motion control which are the core pillars of autonomous navigation.
- **Future work:** integrate planner + controller, enforce dynamic feasibility, and perform hardware validation.

THANK YOU