Backtesting the VaR of Toronto-Dominion Bank Stock

ACTSC445 Question 2

Overview of TD Bank

TD Bank is a banking company with its main headquarters in Toronto, Canada, that offers a variety of financial products and services around the world. TD Bank owns over $1.3 trillion CAD in assets with more than 15 million active customers and are ranked as one of the most leading online financial services firms. Their main services include:

- Personal and small business banking
- Wealth and asset management
- Insurance providing

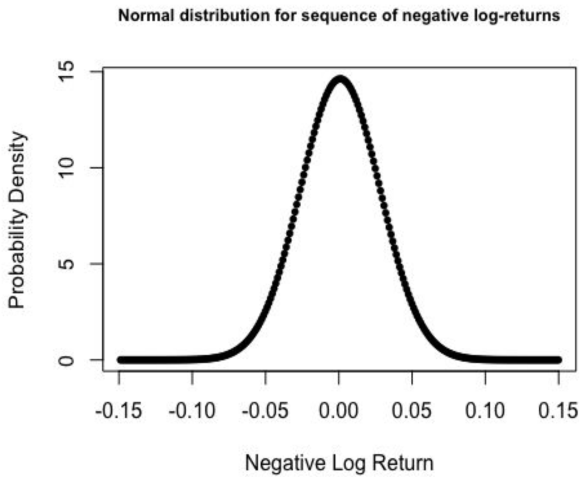
Normal distribution for sequence of negative log-returns from July 1st 2019 - June 30 2020

We first take the mean and standard deviation of all the negative log-returns then applying it to a normal distribution that is:

μ = 0.0009606819

σ = 0.02726856

Then take N(μ, σ^2)



Estimation for 95% VaR of loss variable

We fitted a normal distribution to the loss variable X that is: X ~ N(μ, σ^2)

So by definition we calculate the estimate of 95% VaR as:

$$\mathrm{VaR}_{\alpha}(X) \;=\; \mu + z_{\alpha}\sigma$$

where alpha = 0.95

Hence we get: VaR_0.95 (X) = 0.0009606819 + (1.644854) * (0.02726856)

= 0.04581347

Binomial Distribution Backtesting for VaR estimate

We first define variables:

Let R be the number of exceptions, so R = 11

Let T be the total number of observations, so T = 251

Let c be the failure rate, so c = 0.05

Binomial Distribution Backtesting for VaR estimate

We then conduct a hypothesis test. Take H0 = VaR estimate is accurate

So R follows a binomial distribution and since we have a large number of observations, we can approximate the binomial distribution by a normal distribution, that is we can take the pivotal quantity:

$$\frac{R - cT}{\sqrt{c(1-c)T}} \approx N(0,1).$$

Binomial Distribution Backtesting for VaR estimate

The pivotal quantity is calculated as:

Pivot = (11 - (0.05)(251)) / sqrt((0.05)(1-0.05)(251))

= -0.4488984

Then we calculate the p-value as:

P-value = P(|Z| <= Pivot)

= 2*(0.3267525) = 0.653505

Since p-value > 0.1, there is no evidence against H0 based on the observed data, hence we accept the VaR estimate

Likelihood Ratio Backtest for VaR estimate

Based on the observed data, we get the amount of times n_ij occurs:

n_00 = 228, n_01 = 11, n_10 = 11, n_11 = 0

So pi_0, pi_1, and pi are calculated as:

pi_0 = 0.0460251, pi_1 = 0, pi = 0.0438247

Likelihood Ratio Backtest for VaR estimate

We take the null hypothesis H0 = VaR estimate is accurate

So under the null hypothesis, the test statistic:

$$LR_{\text{ind}} = -2\ln\left(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}\right)$$

follows a χ^2 distribution of degree one.

Likelihood Ratio Backtest for VaR estimate

We calculate the test statistic as:

LR_ind = 1.013093

Note that the 95% quantile of the Chi-squared distribution with degree of freedom equal to 1, is: 3.841459

So we see that 1.013093 < 3.841459, hence we accept the VaR estimate

End

Thank you for reading!