Using Extreme Value Theory to analyze market risk of Apple stock

ACTSC445 Assignment 3 Question 1

Apple (NASDAQ:APPL)

Apple is a U.S based technology company that manufactures and offers a variety of consumer electronic products ranging from personal computers, smartphones, as well as services such as Apple Store, Apple Music. Apple operates in 175 countries and regions with over 511 retail stores globally (as of May 14, 2021). Apple is valued at over $2.08 trillion (as of March 15, 2021).

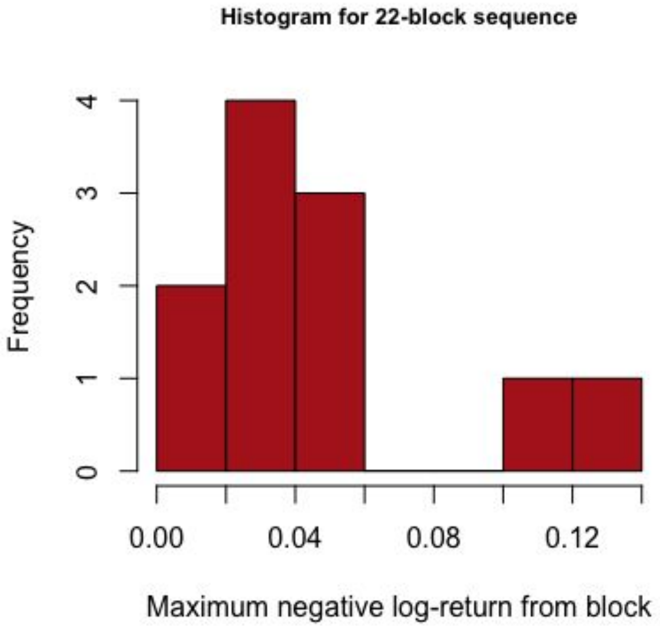
For our data, we will be using the apple stock negative daily log-returns from July 1, 2019 to June 30, 2020

Block maximum based method

We first generate a histogram for the

22-block sequence

$\{M_(22, 1), M_(22, 2), \ldots, M_(22, 11)\}$



Block maximum based method

We then fit the sequence $\{M_(22,1), M_(22,2), ..., M_(22,11)\}$ with the GED in which we get estimates:

Loc: 0.02583, Scale: 0.01402, Shape: 0.69567

and we use these estimates for the GED distribution function

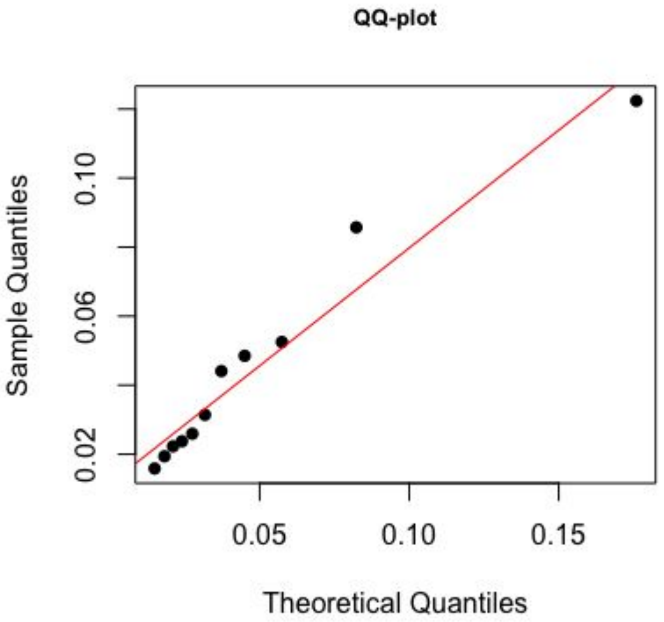
$$H_{\xi,\mu,\sigma}(x) = \exp\left[-\left(1+\xi\frac{x-\mu}{\theta}\right)^{-1/\xi}\right]$$

with a density

$$h_{\xi,\mu,\sigma}(x) = \frac{1}{\theta}\left(1+\xi\frac{x-\mu}{\theta}\right)^{-(1+1/\xi)} \exp\left[-\left(1+\xi\frac{x-\mu}{\theta}\right)^{-1/\xi}\right]$$

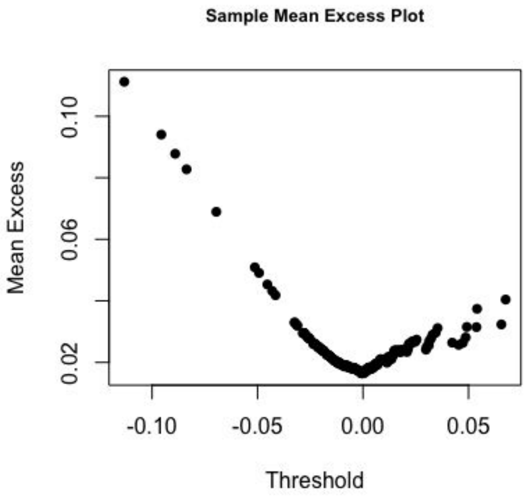
Block maximum based method

Now we conduct an analysis for goodness of fitting using the QQ-plot. We see that the points lie close on the regression line which highly suggests that the fitting is good enough.



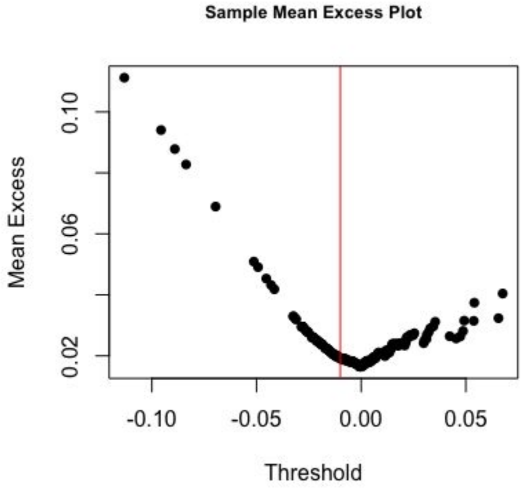
Threshold Exceedance Based Method

Consider the sample mean excess plot of the negative log-returns



Threshold Exceedance Based Method

We choose mu = -0.01 since it looks that the points start to experience oscillation when threshold > -0.01



Threshold Exceedance Based Method

Using mu = -0.01, we fit the GPD to the exceedances of the negative log-returns

We get estimates:

Shape = -0.5349712, Scale = 0.089668

We use these estimates for the GPD function

$$G_{\xi,\theta}(x) = 1 - \left(1 + \xi \frac{x}{\theta}\right)^{-1/\xi}$$

Threshold Exceedance Based Method

Using the estimates and the developed GPD, we calculate 99% VaR(X) and 99% CVaR(X) where X is the negative log-return R.V and F is the GPD function. The expressions are calculated as:

$$\mathrm{VaR}_\alpha(X) = u + \frac{\theta}{\xi}\left[\left(\frac{1-\alpha}{\overline{F}(u)}\right)^{-\xi} - 1\right]$$

$$\mathrm{CVaR}_\alpha(X) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_s(X)\,ds = \frac{\mathrm{VaR}_\alpha}{1-\xi} + \frac{\theta - \xi u}{1-\xi}$$

Threshold Exceedance Based Method

We get:

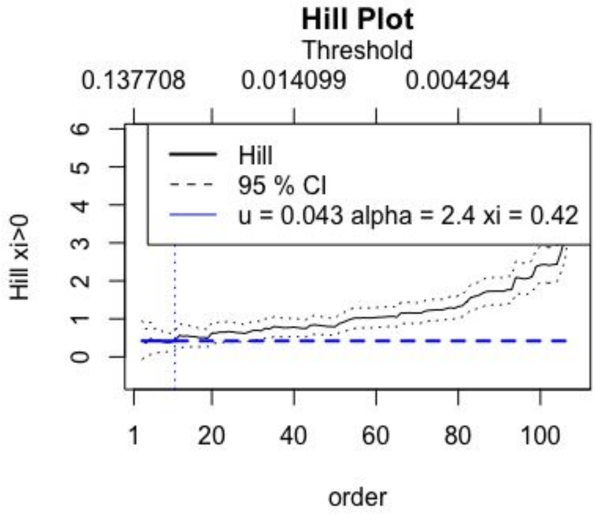
99% $VaR(X) = 0.1398125$

99% $CVaR(X) = 0.1460163$

Hill's method

We first produce a Hill plot and find

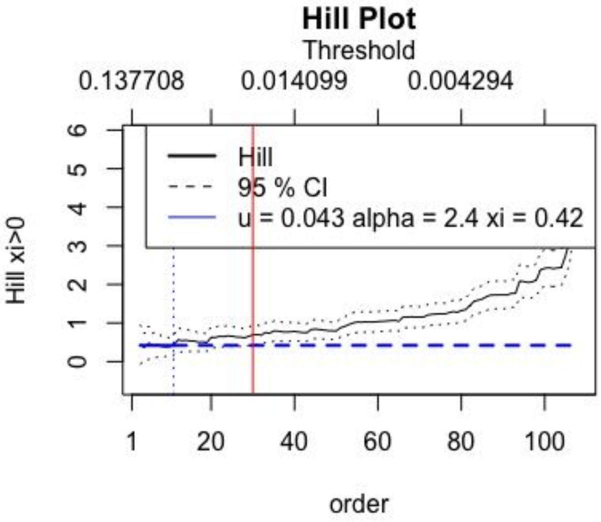
a reasonable value for k



Hill's method

It looks that when k = 25, the hill plot looks to be

flat around k so we use it to calculate the Hill's estimator



Hill’s method

We calculate the Hill’s estimator as:

$$\widehat{\alpha}_{k,n}^{(\mathrm{H})} = \left(\frac{1}{k} \sum_{j=1}^{k} \ln X_{j,n} - \ln X_{k,n} \right)^{-1}$$

where k = 25, n = 251, and $X_{j,n}$ is the j-th largest data point in the negative log-return dataset

Based on the calculation, we get Hill estimate = 1.580521

Hill’s method

Now we calculate 99% VaR(X) and 99% CVaR(X) using the Hill’s estimate. We first find expressions for 99% VaR(X) and 99% CVaR(X)

P1 d) Based on the Hill's Estimator Based tail estimates, the estimated survival function for X is:

$$\hat{\bar{F}}(x) = \frac{k}{n}\left(\frac{x}{X_{k,n}}\right)^{-\hat{\alpha}_{k,n}^{(H)}} \qquad x > X_{k,n}$$

we will use this to find the expression for $VaR_{0.99}(X)$

Let $VaR_{0.99}(X) = v$

$$\hat{\bar{F}}(v) = 1 - 0.99$$

so we get:

$$\frac{k}{n}\left(\frac{v}{X_{k,n}}\right)^{-\hat{\alpha}_{k,n}^{(H)}} = 1-0.99$$

$$\left(\frac{v}{X_{k,n}}\right) = \left(\frac{n}{k}(1-0.99)\right)^{\frac{-1}{\hat{\alpha}_{k,n}^{(H)}}}$$

$$v = X_{k,n}\left(\frac{n}{k}(1-0.99)\right)^{\frac{-1}{\hat{\alpha}_{k,n}^{(H)}}}$$

Hence:

$$VaR_{0.99}(X) = X_{k,n}\left(\frac{n}{k}(0.01)\right)^{\frac{-1}{\hat{\alpha}_{k,n}^{(H)}}}$$

P1 d) By property 1 of CVaR

$$CVaR_{0.99}(X) = VaR_{0.99}(x) + \frac{1}{1-0.99}\int_{VaR_{0.99}(x)}^{\infty} \bar{F}(x)\,dx$$

- let $v = VaR_{0.99}(x)$
- from the previous calculation, $\hat{\alpha}_{k,n}^{(H)} = 1.580521$
  where $k = 25$

we will use these info. to calculate $CVaR_{0.99}(x)$

$$CVaR_{0.99}(x) = v + \frac{1}{1-0.99}\int_v^{\infty} \frac{k}{n}\left(\frac{x}{X_{k,n}}\right)^{-\hat{\alpha}_{k,n}^{(H)}} dx$$

$$= v + \frac{1}{0.01}\left(\frac{k}{n}\right) X_{k,n}^{\hat{\alpha}_{k,n}(H)} \int_v^{\infty} x^{-\hat{\alpha}_{k,n}(H)}\,dx$$

$$= v + \frac{1}{0.01}\left(\frac{k}{n}\right) X_{k,n}^{\hat{\alpha}_{k,n}(H)} \left[\left. \frac{x^{-\hat{\alpha}_{k,n}(H)+1}}{-\hat{\alpha}_{k,n}(H)+1} \right|_v^{\infty} \right]$$

$$= v + \frac{X_{k,n}^{\hat{\alpha}_{k,n}(H)} k}{0.01n}\left[0 - \frac{v^{-\hat{\alpha}_{k,n}(H)+1}}{-\hat{\alpha}_{k,n}(H)+1} \right]$$

since $\lim_{x\to\infty} \frac{x^{-\hat{\alpha}_{k,n}(H)+1}}{-\hat{\alpha}_{k,n}(H)+1} = 0$

hence we get:

$$CVaR_{0.99}(x) = VaR_{0.99}(x) + \frac{X_{k,n}^{\hat{\alpha}_{k,n}(H)} k}{0.01n}\left[\frac{(VaR_{0.99}(x))^{-\hat{\alpha}_{k,n}(H)+1}}{1-\hat{\alpha}_{k,n}^{(H)}}\right]$$

where $\alpha_{k,n}^{(H)} = 1.580521$

Hill's method

Using the expressions we found, we calculate 99% VaR(X) and 99% CVaR(X) as:

99% VaR(X) = 0.0943997

99% CVaR(X) = 0.2570117

References

https://www.apple.com/ca/newsroom/2020/04/apple-services-now-available-in-more-countries-around-the-world/

https://www.statista.com/chart/24857/total-number-of-apple-stores-worldwide/

https://www.investopedia.com/news/apple-now-bigger-these-5-things/

ACTSC445 Ch05_ExtremeValueTheory_Part1

ACTSC445 Ch05_ExtremeValueTheory_Part2