LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MNM TEAM

Focused Tutorial on Quantum Computing

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MNM-Team, Ludwig-Maximilians-Universität München

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MNM-Team


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Outlook of today's tutorial

- First part: Introduction to mathematics of QC

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  - QuBit states

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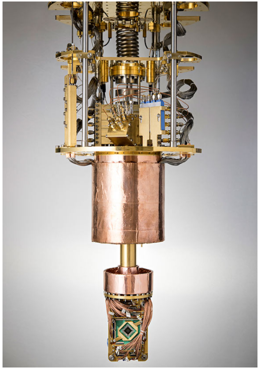
  - Variational quantum circuits
  - Binary classification

Exercises

- Course Material: https://github.com/mnm-team/qc-focused-tutorial-23/
- Sheet 1: Pen and Paper
- Sheet 2: Jupyter notebooks

What is Quantum Computing?

- Exploiting quantum mechanical effects for computations



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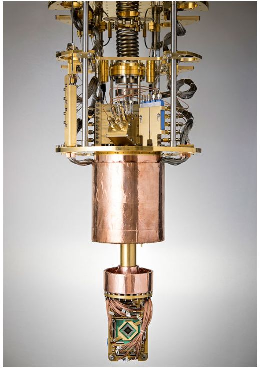

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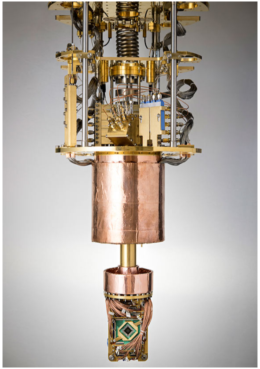

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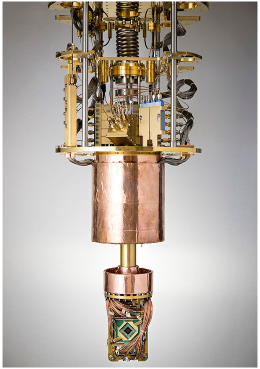
- Optimal hardware yet to be determined



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Single QuBit States

- QuBit state: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ or a *superposition* of $|0\rangle$ and $|1\rangle$

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- Represented by a two-dimensional *normalized* vector:

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha |0\rangle + \beta |1\rangle \qquad \alpha, \beta \in \mathbb{C}$$

where

$$|\alpha|^2 + |\beta|^2 = 1$$

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$\rightarrow$ We have to measure multiple times to approximate the original quantum state

Dirac notation (bra-ket notation)

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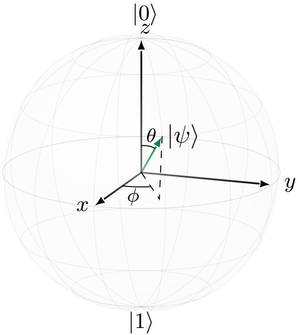
- “bra” is complex conjugate transpose of ket, i.e. a row vector: $\langle\psi| = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix}$
- “bra-ket” is an inner product yielding a scalar:
  $\langle\psi, \psi'\rangle = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \cdot \begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = \alpha^*\alpha' + \beta^*\beta' \in \mathbb{C}$

Bloch Sphere

We can interpret a single QuBit vector as a point on a three dimensional plane:



$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|1\rangle$$

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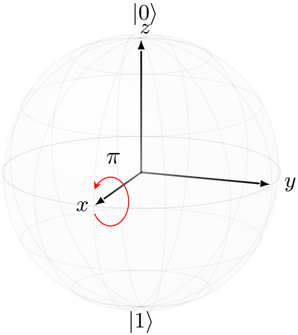
- We can represent operations on QuBits in the quantum circuit model:

$$|\psi\rangle \;\text{—}\boxed{U}\text{—}\; |\psi'\rangle$$

Single QuBit Gates I

Pauli matrices:

- $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 



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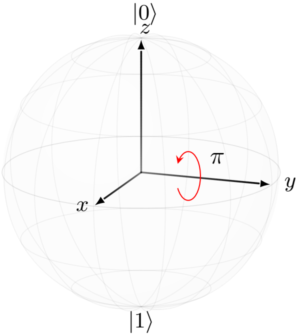

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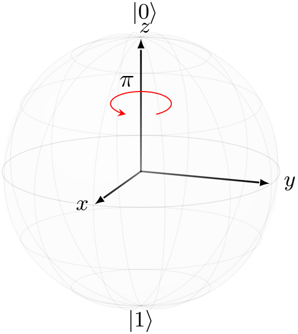
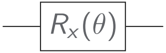
- $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad -\boxed{Z}-$

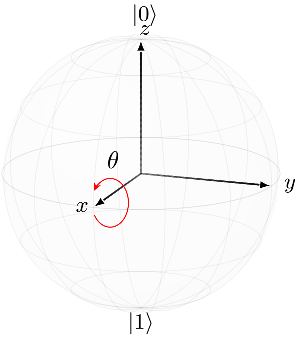


Arbitrary Rotations

$R_?(\theta)$-Gate: Rotation by θ around the X/Y/Z axis (?: X, Y, or Z)

- $R_x(\theta) = \begin{pmatrix} \cos(\frac{\theta}{2}) & -i\sin(\frac{\theta}{2}) \\ -i\sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{pmatrix}$





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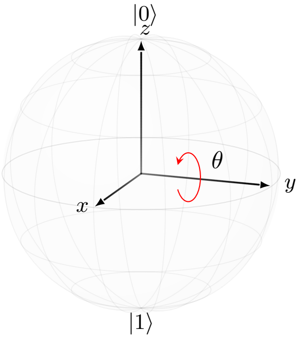

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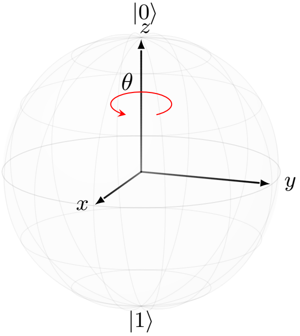
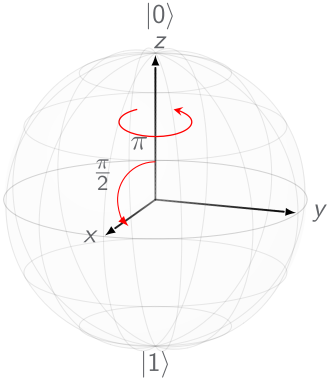
- $R_z(\theta) = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$ —$\boxed{R_z(\theta)}$—



Single QuBit Gates II

- Hadamard-Gate (H-Gate)

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



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  - $H\,|0\rangle = \frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle$
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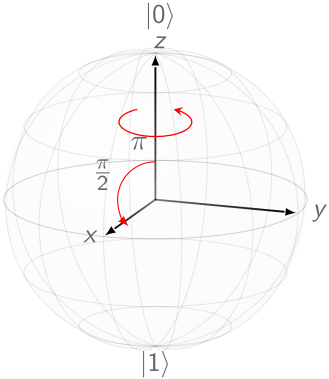

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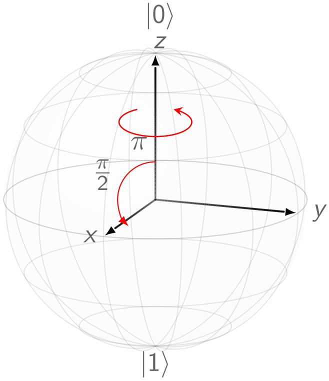

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- Probability of measuring $|0\rangle$ for state $H|0\rangle$ or $H|1\rangle = \frac{1}{2}$

Quantum Registers

- Multiple QuBits can be combined to a *quantum register* by calculating their tensorproduct:

$$\begin{pmatrix} \alpha_{n-1} \\ \beta_{n-1} \end{pmatrix} \otimes \begin{pmatrix} \alpha_{n-2} \\ \beta_{n-2} \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} \alpha_0 \\ \beta_0 \end{pmatrix}$$

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- Quantum operations on quantum registers are represented by the tensorproduct of the corresponding matrices (dimension: $2^n \times 2^n$)

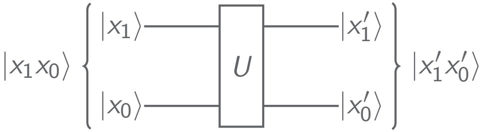
Two QuBit Register

$$|\psi\rangle = a_{00}\,|00\rangle + a_{01}\,|01\rangle + a_{10}\,|10\rangle + a_{11}\,|11\rangle$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Multi-QuBit Gates

We can also apply gates on multiple QuBits:



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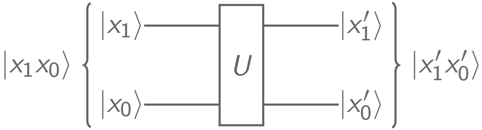


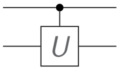
For instance, we can combine the existing single QuBit gates using the tensor product:

$$X \otimes X =$$

Controlled QuBit Operations

- An operation on a QuBit may depend on the state of another QuBit $\rightarrow$ Any single-QuBit gate can have a "controlled version"

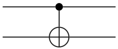
$$CU = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & u_{00} & u_{10} \\ 0 & 0 & u_{01} & u_{11} \end{pmatrix}$$



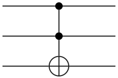
Controlled QuBit Operations

- An operation on a QuBit may depend on the state of another QuBit $\rightarrow$ Any single-QuBit gate can have a “controlled version”
- Most common: CX/CNOT and CCX/CCNOT/Toffoli Gate

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



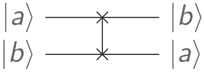
$$CCX = \begin{pmatrix} & & & 0 & 0 \\ & \mathbb{I}_6 & & \vdots & \vdots \\ & & & 0 & 0 \\ 0 & \cdots & 0 & 0 & 1 \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}$$



The SWAP Operation

- The SWAP operation “swaps” the states of two qubits:

$$\text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



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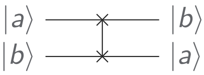
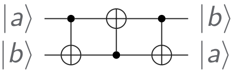
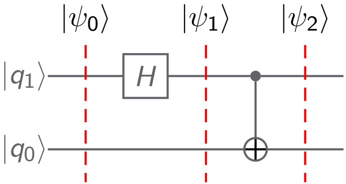


- It can be implemented with three CNOT gates:



Entanglement I

By applying a controlled gate (e.g., CNOT) on QuBits in superposition we can create entanglement:



e.g., $|q_1 q_0\rangle = |00\rangle$:

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$\Rightarrow$ We either measure both QuBits in $|0\rangle$ **or** in $|1\rangle$.

Entanglement II

Definition

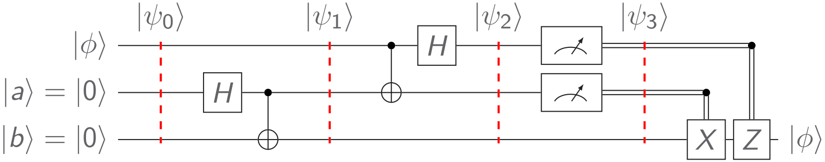
QuBits are *entangled* with each other iff the QuBit state cannot be represented as a tensor product of single QuBit states.

$$\text{e.g., } \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

No-Cloning Theorem

- Entanglement is **not** equivalent to cloning a QuBit state
- It is not possible to clone or copy an arbitrary QuBit state
- With entanglement, it is possible to teleport an arbitrary state from one QuBit to another

Teleportation



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- The expectation value is calculated with respect to the eigenvalues of the observable, not the measurement bases

Pauli-Z Matrix as Observable

- Z matrix: $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
- The expectation value can be calculated with $\langle\psi| Z |\psi\rangle$
- Examples:
  1. $\langle 0| Z |0\rangle =$
  2. $\langle 1| Z |1\rangle =$
  3. $\frac{1}{\sqrt{2}}(\langle 0| + \langle 1|) \cdot Z \cdot \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) =$

Summary Classical vs. Quantum Computing

	Classical Computing	Quantum Computing
Basic States	Either 0 or 1	Superposition of 0 and 1
Operations	Logic (boolean) gates	Unitary reversible gates
Copying States	Yes	No
Readout	No information loss	Superposition is destroyed; information loss

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- Near-term goal: design quantum circuits/algorithms that...

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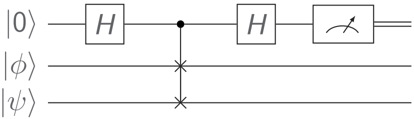
- Near-term quantum algorithms: Variational Quantum Algorithms (VQAs)

Swap Test I

Means to measure the distance d of two quantum states $|\phi\rangle$ and $|\psi\rangle$

$d = |\langle\psi|\phi\rangle|^2$ (angle^2 between $|\psi\rangle$ and $|\phi\rangle$)



Swap Test II

- At the end of the circuit the probability of measuring $|0\rangle$ is the following

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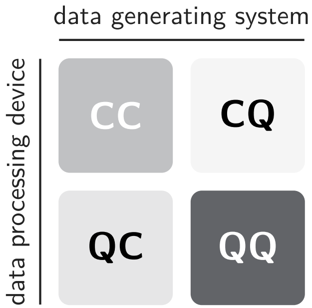
- With multiple repetitions of the SWAP test, the distance d can be approximated with any precision

Overview



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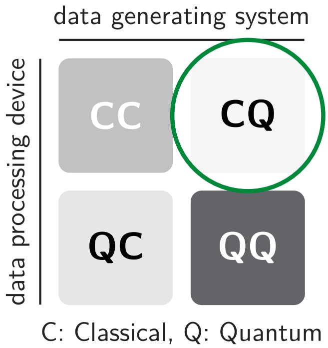
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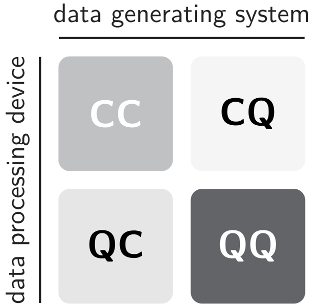

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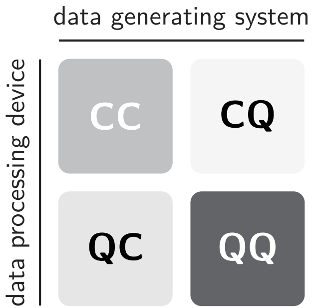

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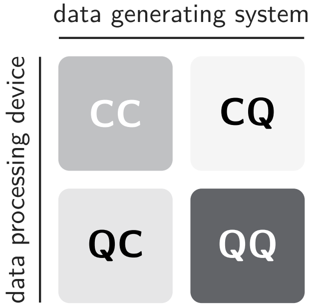

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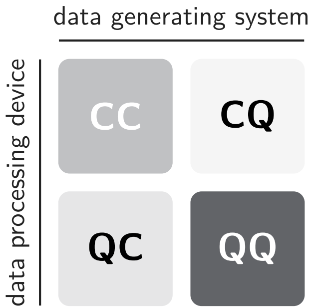

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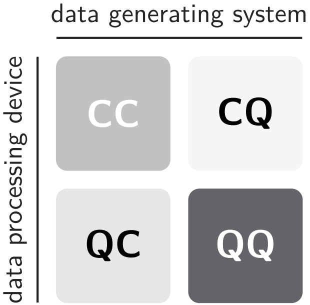

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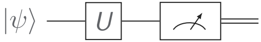
  - "Standard" quantum circuit: 

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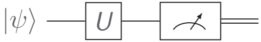
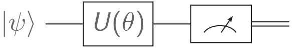
  - "Standard" quantum circuit: 

  - Variational quantum circuit: 

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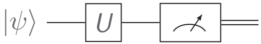
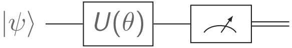
  - "Standard" quantum circuit: 

  - Variational quantum circuit: 

→ Goal: optimize the parameter value(s) for the application

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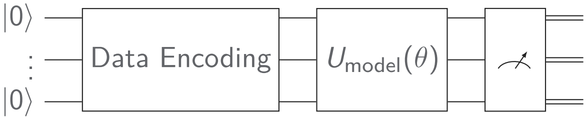
Quantum Circuit



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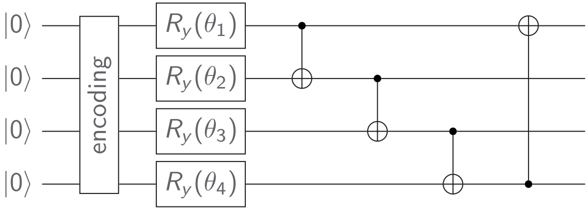

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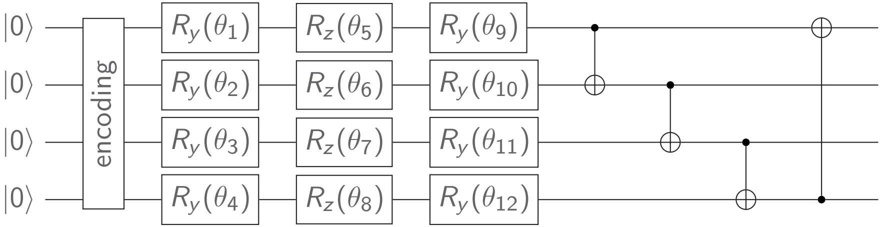

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- E.g., for binary classification we could take the **sign function** as classification function.

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- Quantum projects in Munich: Munich Quantum Valley, MuniQC Atoms/SC

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