Name: KEY   Group Number: ____________

Group Members: ________________ ________________ ________________

Math 151 Worksheet 12 – Fall 2021

Instructions: Write your name and the names of the other members of your group in the spaces provided. Everyone should work on their own worksheet, but only one worksheet from each group will be graded for the entire group, so you will have to work together and make sure you agree on your answers. Be sure to show your work and explain your reasoning.

1. Compute the following Riemann sums for the function $f(x) = \dfrac{10-5x}{x^2-2x+2}$ on $[-2, 4]$.

(a) $L_6 = f(-2)\cdot \Delta x + f(-1)\cdot \Delta x + f(0)\cdot \Delta x + f(1)\cdot \Delta x + f(2)\cdot \Delta x + f(3)\cdot \Delta x$

$\Delta x = \dfrac{4-(-2)}{6} = 1$

$= 2 + 3 + 5 + 5 + 0 - 1$

partition:

$-2 < -1 < 0 < 1 < 2 < 3 < 4$

$= 13$

(b) $M_3 = f(-1)\cdot \Delta x + f(1)\cdot \Delta x + f(3)\cdot \Delta x$

$\Delta x = \dfrac{4-(-2)}{3} = 2$

$= 2(3 + 5 - 1) = 14$

partition:

$-2 < 0 < 2 < 4$

2. Compute the following antiderivatives.

(a) $\displaystyle\int 5x^4 - \tfrac{1}{3}x^2 + 10\,dx = x^5 - \frac{1}{9}x^3 + 10x + C$

(b) $\int e^{-t}\,dt \quad = \quad -e^{-t} + C$

(c) $\int x^{7/2} - \frac{1}{5x} + \frac{2}{x^{1/3}}\,dx \quad = \quad \int x^{7/2} + \frac{1}{5}\cdot\frac{1}{x} + 2x^{-1/3}\,dx$

$$= \frac{2}{9}x^{9/2} - \frac{1}{5}\ln|x| + 3x^{2/3} + C$$

3. Compute the following derivatives.

(a) $\frac{d}{dx}\left(\int_{-5}^{x} e^{-t^2/2}\,dt\right) = \quad e^{-x^2/2}$

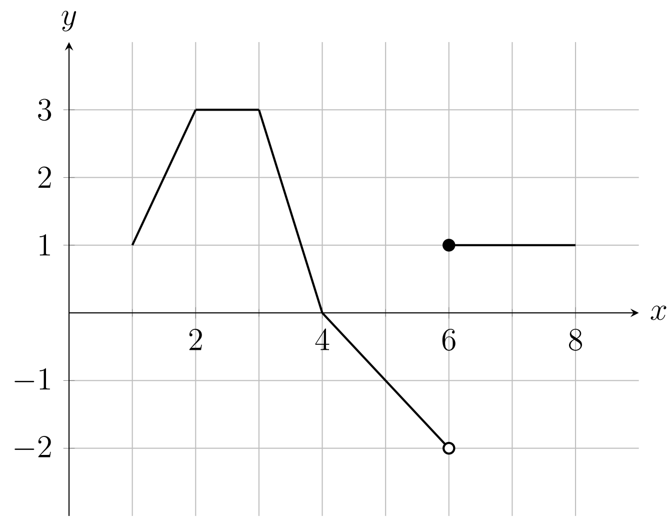
(b) $\frac{d}{dx}\left(\int_{0}^{x^2+1} (u-1)e^{u}\,du\right) = \frac{d}{dx}\left(F(x^2+1)\right)$

where $F(x) = \int_0^x (u-1)e^u\,du$ is the cumulative area function

$$= F'(x^2+1)\cdot 2x$$

$$= (x^2+1-1)e^{x^2+1}\cdot 2x = 2x^3e^{x^2+1}$$

4. Consider the graph of the function $g(x)$ defined on $[1, 8]$ shown below.



Compute the values related to the cumulative area function $G(x) = \int_1^x g(w)\, dw$, or write DNE if the value does not exist.

(a) $G(3) = \frac{1}{2}(1+3)\cdot 1 + 1\cdot 3$

$= 2 + 3 = 5$

(b) $G(7) = G(3) + \frac{1}{2}\cdot 1\cdot 3$

$-\frac{1}{2}\cdot 2\cdot 2 + 1\cdot 1$

$= 5 + \frac{3}{2} - 2 + 1 = \frac{11}{2}$

(c) $G'(5) = g(5) = -1$

(d) Where are the critical points of $G(x)$?

$x = 4, 6$

(where $G' = g = 0$ or where G' is undefined)

(e) On which intervals is $G(x)$ increasing?

$[1 \; + \; 4 \; - \; 6 \; + \; 8]$ $G' = g$

$(1, 4) \cup (6, 8)$

(f) Classify each critical point as a local max, local min, or saddle.

$x = 4$ local max

$x = 6$ local min

5. Compute the following definite integrals.

(a) $\int_{-1}^{3} 4 + 2x - x^2\, dx \quad = \left[4x + x^2 - \frac{1}{3}x^3\right]_{-1}^{3}$

$= (12 + \cancel{9} - \cancel{9}) - (-4 + 1 + \frac{1}{3}) = 15 - \frac{1}{3} = \frac{44}{3}$

(b) $\int_4^9 6\sqrt{x} + \frac{1}{\sqrt{x}}\, dx$ $= \int_4^9 6x^{1/2} + x^{-1/2}\, dx$

$$= \left[4x^{3/2} + 2x^{1/2} \right]_4^9$$

$$= (108 + 6) - (32 + 4) = 78$$

(c) $\int_1^e 5t - \frac{2}{t}\, dt$ $= \left[\frac{5}{2}t^2 - 2\ln|t| \right]_1^e$

$$= \frac{5e^2}{2} - 2\cancelto{-2}{\ln e} - \left(\frac{5}{2} - 2\cancelto{0}{\ln 1} \right)$$

$$= \frac{5e^2 - 9}{2}$$

6. Calculus students often mistakenly assume that:

$$\int_0^1 f(x)g(x)\,dx = \left(\int_0^1 f(x)\,dx\right)\left(\int_0^1 g(x)\,dx\right)$$

Find two functions $f(x)$ and $g(x)$ which show that the left and right sides of the above equation are NOT equal in general.

Consider $f(x) = x$ and $g(x) = x$.

$$\int_0^1 f(x)g(x)\,dx = \int_0^1 x^2\,dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$$ $\neq$

$$\left(\int_0^1 f(x)\,dx\right)\left(\int_0^1 g(x)\,dx\right) = \left(\int_0^1 x\,dx\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

area under curve is a triangle