Math 267 Quiz 5 – Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. Suppose we want to solve the nonhomogeneous second order differential equation

$$y'' - 8y' - 20y = 24\sin 2x \qquad (*)$$

by the method of undetermined coefficients.

   (a) If we guess that the particular solution of the nonhomogeneous equation has the form $y_p = A\sin 2x + B\cos 2x$, use the method of undetermined coefficients to find y_p.

$$y_p' = 2A\cos 2x - 2B\sin 2x$$

$$y_p'' = -4A\sin 2x - 4B\cos 2x$$

$$24\sin 2x = y_p'' - 8y_p' - 20y_p$$

$$= -4A\sin 2x - 4B\cos 2x - 16A\cos 2x + 16B\sin 2x$$
$$- 20A\sin 2x - 20B\cos 2x$$

$$= (-24A + 16B)\sin 2x + (-16A - 24B)\cos 2x$$

when $x = 0$: $0 = -16A - 24B \implies B = \dfrac{-2}{3}A$

when $x = \dfrac{\pi}{4}$: $24 = -24A + 16B \implies 24 = \dfrac{-104}{3}A$

$$\implies A = -\dfrac{9}{13}, \quad B = \dfrac{6}{13}$$

$$y_p = -\dfrac{9}{13}\sin 2x + \dfrac{6}{13}\cos 2x$$

(b) Write down the general solution to the *homogeneous* differential equation corresponding to (*).

$$y'' - 8y - 20y = 0$$

$$\Rightarrow \quad 0 = r^2 - 8r - 20$$

$$= (r - 10)(r + 2)$$

$$\Rightarrow \quad r = 10, -2$$

$$y = C_1 e^{10x} + C_2 e^{-2x}$$

(c) Write down the general solution to the nonhomogeneous differential equation (*).

$$y = C_1 e^{10x} + C_2 e^{-2x}$$

$$\pm \frac{9}{13} \sin 2x + \frac{6}{13} \cos 2x$$