KEY

Name: _____

Math 267 Quiz 3 – Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. Find the general solution to the linear differential equation below.

$$\frac{dy}{dt} + \frac{2y}{t} = 15e^{-t^3}$$

$$e^{\int P(t)\,dt} = e^{\int \frac{2}{t}\,dt} = e^{2\ln t} = t^2$$

$$t^2 \frac{dy}{dt} + 2ty = 15t^2 e^{-t^3}$$

$$\Rightarrow \left[t^2 y \right]' = 15t^2 e^{-t^3}$$

$$t^2 y = \int 15t^2 e^{-t^3}\,dt$$

$$= \int -5e^u\,du$$

$$u = -t^3$$
$$du = -3t^2\,dt$$
$$-du = 3t^2\,dt$$

$$= -5e^u + C = -5e^{-t^3} + C$$

$$\Rightarrow y = -\frac{5}{t^2}e^{-t^3} + \frac{C}{t^2}$$

2. Find the general solution to the homogeneous differential equation below.

$$x^5 y' - x^4 y + 4x^2 y^3 = 0$$

$$y = vx \qquad y' = v + xv'$$

$$x^5 (v + xv') - x^5 v + 4x^5 v^3 = 0$$

$$\Rightarrow \qquad x^6 v' + 4x^5 v^3 = 0$$

$$\Rightarrow \qquad -xv' = 4v^3$$

$$\Rightarrow \qquad \int \frac{-1}{4v^3} \, dv = \int \frac{1}{x} \, dx$$

$$\Rightarrow \qquad \frac{1}{8v^2} = \ln |x| + C$$

$$\Rightarrow \qquad v^2 = \frac{1}{8 \ln |x| + C}$$

$$\Rightarrow \qquad v = \pm \frac{1}{\sqrt{8 \ln |x| + C}}$$

$$\Rightarrow \qquad y = \pm \frac{x}{\sqrt{8 \ln |x| + C}}$$