

Name: _____

KEY

Section 6

Math 267 Quiz 2 – Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. Find the general solution to the differential equation below using separation of variables.

$$\frac{dy}{dx} = \frac{y(1+10y)(1-10y)}{2}$$

$$\int \frac{2}{y(1+10y)(1-10y)} dy = \int 1 dx$$

$$\frac{2}{y(1+10y)(1-10y)} = \frac{A}{y} + \frac{B}{1+10y} + \frac{C}{1-10y} \Rightarrow 2 = A(1-100y^2) + By(1-10y) + Cy(1+10y)$$

when $y=0$: $A=2$

when $y = \frac{1}{10}$: $\frac{2}{10}C = 2 \Rightarrow C = 10$

when $y = -\frac{1}{10}$: $-\frac{2}{10}B = 2 \Rightarrow B = -10$

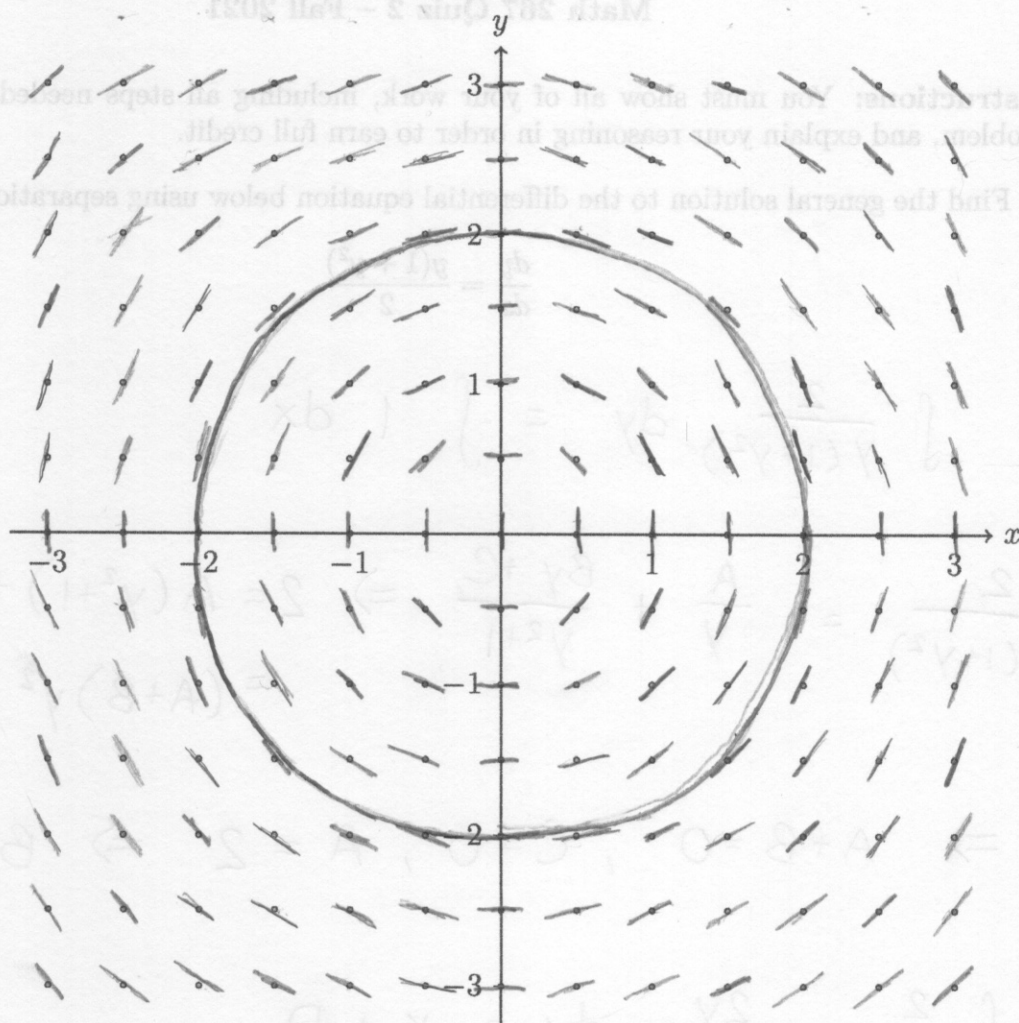
$$\int \left(\frac{2}{y} - \frac{10}{1+10y} + \frac{10}{1-10y} \right) dy = x + D$$

$$2 \ln|y| - \ln|1+10y| - \ln|1-10y| = x + D$$

$$\Rightarrow \frac{y^2}{1-100y^2} = De^x \Rightarrow De^{-x} = \frac{1}{y^2} - 100$$

$$\Rightarrow y = \frac{1}{\sqrt{100 + De^{-x}}}$$

2. (a) Sketch the slope field of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$ on the axes below.



- (b) On your slope field in part (a), use the directions of the slopes to sketch an example of curve that solves the differential equation. Describe the shape of the solution curves.

The solution curves are circles centered around the origin.