Name: KEY Section 7

Math 267 Quiz 8 – Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. Solve the following system of linear differential equations by using eigenvalues and eigenvectors.

$$\frac{dy}{dt} = y - v$$

$$\frac{dv}{dt} = 2y + 4v$$

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$$

$$0 = \det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix}$$

$$= (1-\lambda)(4-\lambda) + 2$$

$$= \lambda^2 - 5\lambda + 6 = (\lambda - 3)(\lambda - 2)$$

$$\Rightarrow \lambda = 3, 2$$

$\lambda = 3$:
$$\begin{pmatrix} -2 & -1 \\ 2 & 1 \end{pmatrix} \xrightarrow{R_2 + R_1 \to R_2} \begin{pmatrix} -2 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\xrightarrow{-\frac{1}{2}R_1 \to R_1} \begin{pmatrix} 1 & 1/2 \\ 0 & 0 \end{pmatrix} \Rightarrow v_1 + \frac{1}{2} v_2 = 0$$

$$\Rightarrow \quad v_1 = -\frac{1}{2} v_2 \quad \overset{v_2=1}{\Rightarrow} \quad \begin{pmatrix} -1/2 \\ 1 \end{pmatrix} \text{ is an eigenvector}$$

$$\lambda = 2: \quad \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix} \xrightarrow{R_2 + 2R_1 \to R_2} \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix} \xrightarrow{-R_1 \to R_1}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow v_1 + v_2 = 0 \quad \Rightarrow \quad v_1 = -v_2$$

$$\underset{v_2=1}{\Rightarrow} \quad \begin{pmatrix} -1 \\ 1 \end{pmatrix} \text{ is an eigenvector}$$

$$\begin{pmatrix} y \\ v \end{pmatrix} = c_1 e^{3t} \begin{pmatrix} -1/2 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{2} c_1 e^{3t} - c_1 e^{2t} \\ c_1 e^{3t} + c_2 e^{2t} \end{pmatrix}$$