Name: KEY Section 14

Math 267 Quiz 4 – Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. If possible, find a function $F(x, y)$ such that the equation $F(x, y) = C$ implicitly solves the following differential equation. **If it is not possible to solve the equation, explain why.**

$$e^{xy}\frac{dy}{dx} + \frac{y}{x}e^{xy} - \frac{e^{xy}}{x^2} + 4x^3 = 0$$

$$M = \frac{y}{x}e^{xy} - \frac{e^{xy}}{x^2} + 4x^3$$

$$N = e^{xy}$$

$$\frac{\partial M}{\partial y} = \frac{1}{x}e^{xy} + ye^{xy} - \frac{e^{xy}}{x} = ye^{xy} = \frac{\partial N}{\partial x} \quad \checkmark$$

$$\frac{\partial F}{\partial y} = N \Rightarrow F = \int e^{xy}\,dy = \frac{1}{x}e^{xy} + g(x)$$

$$\Rightarrow M = \frac{\partial F}{\partial x} = \frac{y}{x}e^{xy} - \frac{1}{x^2}e^{xy} + \frac{dg}{dx} \Rightarrow \frac{dg}{dx} = 4x^3$$

$$\Rightarrow g = \int 4x^3\,dx = x^4 + C$$

$$F(x,y) = \frac{1}{x}e^{xy} + x^4 + C$$

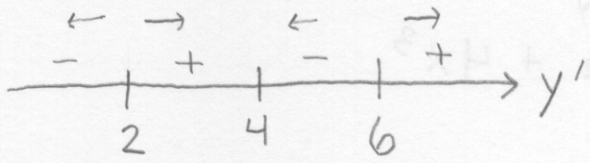
2. Consider the autonomous differential equation

$$\frac{dy}{dt} = (y-2)(y-\alpha-1)(y-2\alpha)$$

that depends on a parameter α.

(a) Find the equilibrium solutions of this differential equation when $\alpha = 3$, and identify whether each solution is stable or unstable.

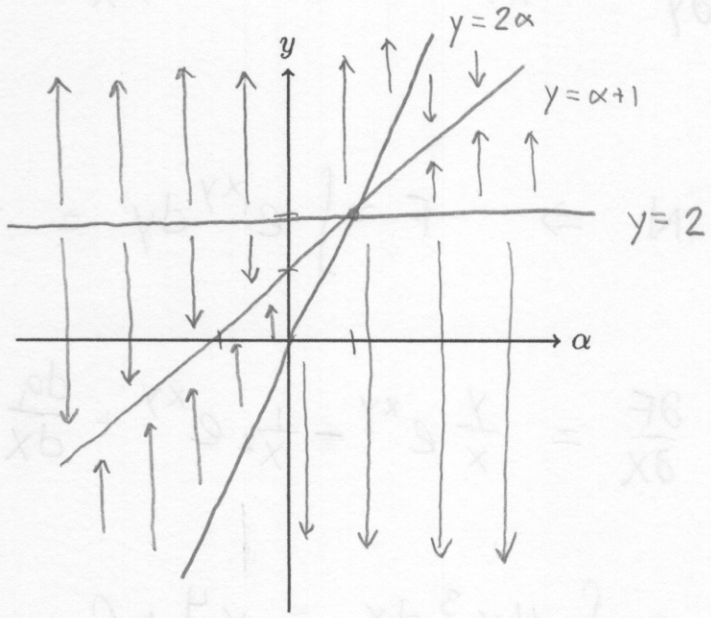
$0 = (y-2)(y-4)(y-6) \Rightarrow y = 2, 4, 6$



$y = 4$ is stable

$y = 2, 6$ are unstable

(b) Sketch the bifurcation diagram of the differential equation on the axes below.



(c) Identify all of the bifurcation points of the differential equation.

$\alpha = 1$