Name: KEY

Section 7

Math 267 Quiz 7 – Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. Compute the matrices below in terms of the matrices A, B, C, and $\mathbf{v}$. If the operation is not defined, write "NOT DEFINED".

$$A = \begin{pmatrix} 3 & 0 \\ 1 & -1 \\ 4 & -5 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix} \qquad C = \begin{pmatrix} -1 & \frac{7}{2} \\ 0 & 0 \\ \frac{1}{2} & 4 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

(a) $A + 2C =$ $\begin{pmatrix} 3 & 0 \\ 1 & -1 \\ 4 & 5 \end{pmatrix} + \begin{pmatrix} -2 & 7 \\ 0 & 0 \\ 1 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 7 \\ 1 & -1 \\ 4 & 13 \end{pmatrix}$

(b) $B\mathbf{v} =$ $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$

(c) $AB =$ not defined

(d) $BA =$ $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 1 & -1 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ 10 & 2 \\ 17 & 19 \end{pmatrix}$

2. Consider the system of 1st order linear differential equations:

$$\frac{dy}{dt} = 4y + 4v \qquad \frac{dv}{dt} = 6y - v$$

(a) If $\mathbf{Y}(t)$ denotes the vector-valued function $\mathbf{Y} = \begin{pmatrix} y \\ v \end{pmatrix}$, write down the coefficient matrix A such that $\frac{d\mathbf{Y}}{dt} = A\mathbf{Y}$.

$$A = \begin{pmatrix} 4 & 4 \\ 6 & -1 \end{pmatrix}$$

(b) Verify that the vector-valued functions $\mathbf{Y}_1(t) = \begin{pmatrix} 4e^{7t} \\ 3e^{7t} \end{pmatrix}$ and $\mathbf{Y}_2(t) = \begin{pmatrix} -e^{-4t} \\ 2e^{-4t} \end{pmatrix}$ both solve the given system of differential equations.

$$AY_1 = \begin{pmatrix} 16e^{7t} + 12e^{7t} \\ 24e^{7t} - 3e^{7t} \end{pmatrix} = \begin{pmatrix} 28e^{7t} \\ 21e^{7t} \end{pmatrix} = \frac{dY_1}{dt}$$

$$AY_2 = \begin{pmatrix} -4e^{-4t} + 8e^{-4t} \\ -6e^{-4t} - 2e^{-4t} \end{pmatrix} = \begin{pmatrix} 4e^{-4t} \\ -8e^{-4t} \end{pmatrix} = \frac{dY_2}{dt}$$

(c) What is the form of the general solution to the system of differential equations?

$$Y(t) = C_1 \begin{pmatrix} 4e^{7t} \\ 3e^{7t} \end{pmatrix} + C_2 \begin{pmatrix} -e^{-4t} \\ 2e^{-4t} \end{pmatrix}$$

(d) Find the specific function $\mathbf{Y}(t)$ that solves the system of differential equations subject to the initial condition $\mathbf{Y}(0) = \begin{pmatrix} 0 \\ 11 \end{pmatrix}$.

$$\begin{pmatrix} 0 \\ 11 \end{pmatrix} = Y(0) = C_1 \begin{pmatrix} 4 \\ 3 \end{pmatrix} + C_2 \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \underbrace{\begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}}_{Q} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$Q^{-1} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} \qquad \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 0 \\ 11 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$Y(t) = \begin{pmatrix} 4e^{7t} \\ 3e^{7t} \end{pmatrix} + 4\begin{pmatrix} -e^{-4t} \\ 2e^{-4t} \end{pmatrix}$$