Name: KEY

Section 6

Math 267 Quiz 1 – Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. (a) Show that for any constants A and B, the function $y = A\sin(4t) + B\cos(4t)$ solves the differential equation:

$$y'' + 16y = 0$$

$$y' = 4A\cos 4t - 4B\sin 4t$$

$$y'' = -16A\sin 4t - 16B\cos 4t$$

$$y'' + 16y = (-16A\sin 4t - 16B\cos 4t) + 16(A\sin 4t + B\cos 4t)$$

$$= (-16+16)A\sin 4t + (-16+16)B\cos 4t$$

$$= 0$$

(b) What is the order of the differential equation in part (a)? second

2. (a) Find the form of the general solution to the differential equation $\frac{d^2y}{dx^2} = xe^{2x}$

$$\frac{dy}{dx} = \int xe^{x}\,dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$$

$$u = x \quad du = dx \qquad = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C_1$$

$$dv = e^{2x}\,dx \quad v = \frac{1}{2}e^{2x}$$

$$y = \int \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C_1\,dx = \frac{1}{2}\int xe^{2x}\,dx - \frac{1}{8}e^{2x} + C_1 x$$

$$= \frac{1}{2}\left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right) - \frac{1}{8}e^{2x} + C_1 x + C_2$$

$$= \frac{1}{4}xe^{2x} - \frac{1}{4}e^{2x} + C_1 x + C_2$$

(b) What is the particular solution that solves the differential equation in part (a) subject to the initial conditions $y(0) = 5$ and $y'(0) = 0$.

$$0 = y'(0) = 0 - \frac{1}{4} + C_1 \Rightarrow C_1 = \frac{1}{4}$$

$$5 = y(0) = 0 - \frac{1}{4} + 0 + C_2 \Rightarrow C_2 = \frac{21}{4}$$

$$y = \frac{1}{4}xe^{2x} - \frac{1}{4}e^{2x} + \frac{1}{4}x + \frac{21}{4}$$