

Name: _____

KEY

Section 14

Math 267 Quiz 10 – Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. Find the inverse Laplace transform of the function

$$F(s) = \frac{28s + 12}{s^3 + 5s^2 - 33s + 27}$$

(HINT: $s^3 + 5s^2 - 33s + 27 = (s - 1)(s^2 + 6s - 27)$)

$$\frac{28s + 12}{s^3 + 5s^2 - 33s + 27} = \frac{28s + 12}{(s - 1)(s + 9)(s - 3)}$$

$$= \frac{A}{s - 1} + \frac{B}{s + 9} + \frac{C}{s - 3}$$

$$\Rightarrow 28s + 12 = A(s + 9)(s - 3) + B(s - 1)(s - 3) + C(s - 1)(s + 9)$$

$$\begin{aligned} \text{when } s = 1: & \quad 40 = -20A \\ & \Rightarrow A = -2 \end{aligned}$$

$$\begin{aligned} \text{when } s = 3: & \quad 96 = 24C \Rightarrow C = 4 \end{aligned}$$

$$\begin{aligned} \text{when } s = -9: & \quad -240 = 120B \Rightarrow B = -2 \end{aligned}$$

$$\mathcal{L}^{-1}\{F(s)\} = -2\mathcal{L}^{-1}\left\{\frac{1}{s - 1}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{s + 9}\right\} + 4\mathcal{L}^{-1}\left\{\frac{1}{s - 3}\right\}$$

$$= -2e^t - 2e^{-9t} + 4e^{3t}$$

2. Solve the initial value problem below by using Laplace transforms.

$$y^{(3)} + 5y'' - 33y' + 27y = 0 \quad y(0) = 0, y'(0) = 28, y''(0) = -128$$

$$0 = \mathcal{L}\{0\} = \mathcal{L}\{y^{(3)} + 5y'' - 33y' + 27y\}$$

$$= s \mathcal{L}\{y''\} + 128 + 5(s \mathcal{L}\{y'\} - 28)$$

$$- 33(sY) + 27Y$$

$$= s^2 \mathcal{L}\{y'\} - 28s + 128 + 5s^2 Y - 140$$

$$- 33sY + 27Y$$

$$= s^3 Y - 28s - 12 + 5s^2 Y - 33sY + 27Y$$

$$\Rightarrow (s^3 + 5s^2 - 33s + 27)Y = 28s + 12$$

$$\Rightarrow Y = \frac{28s + 12}{s^3 + 5s^2 - 33s + 27}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\{Y\} = -2e^t - 2e^{-9t} + 4e^{3t}$$

by Problem 1