

Name: _____

KEY

Section 6

Math 267 Quiz 8⁴ - Fall 2021

Instructions: You must show all of your work, including all steps needed to solve each problem, and explain your reasoning in order to earn full credit.

1. If possible, find a function $F(x, y)$ such that the equation $F(x, y) = C$ implicitly solves the following differential equation. If it is not possible to solve the equation, explain why.

$$(e^{xy} + 4y^3) \frac{dy}{dx} + \frac{y}{x} e^{xy} - \frac{e^{xy}}{x^2} = 0$$

$$M = \frac{y}{x} e^{xy} - \frac{e^{xy}}{x^2} \quad N = e^{xy} + 4y^3$$

$$\frac{\partial M}{\partial y} = \frac{1}{x} e^{xy} + y e^{xy} - \frac{e^{xy}}{x} = y e^{xy} = \frac{\partial N}{\partial x} \quad \checkmark$$

$$N = \frac{\partial F}{\partial y} \Rightarrow F = \int e^{xy} + 4y^3 dy$$

$$= \frac{1}{x} e^{xy} + y^4 + g(x)$$

$$M = \frac{\partial F}{\partial x} = -\frac{1}{x^2} e^{xy} + \frac{y}{x} e^{xy} + \frac{dg}{dx} \Rightarrow \frac{dg}{dx} = 0$$

$$\Rightarrow g = \int 0 dx = C$$

$$F(x, y) = -\frac{1}{x} e^{xy} + y^4 + C$$

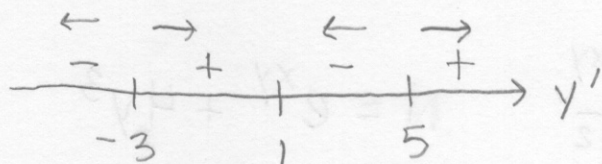
2. Consider the autonomous differential equation

$$\frac{dy}{dt} = (y-1)(y-\alpha-2)(y+\alpha)$$

that depends on a parameter α .

- (a) Find the equilibrium solutions of this differential equation when $\alpha = 3$, and identify whether each solution is stable or unstable.

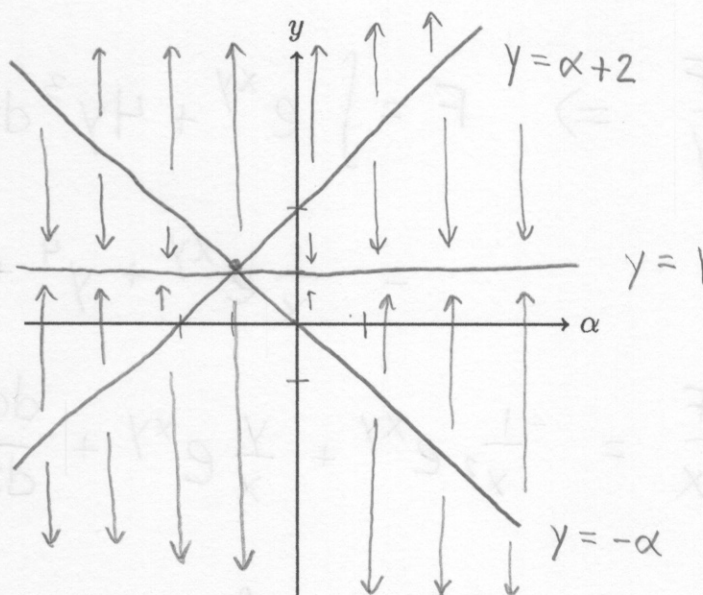
$$0 = (y-1)(y-5)(y+3) \Rightarrow y = -3, 1, 5$$



$y = 1$ is stable.

$y = -3, 5$ are unstable.

- (b) Sketch the bifurcation diagram of the differential equation on the axes below.



- (c) Identify all of the bifurcation points of the differential equation.

$$\alpha = -1$$