Link Influence Entropy (LInE)

Abhishek Chakraborty

Indian Institute of Space Science and Technology
Department of Space, Government of India
Thiruvananthapuram 695547, India

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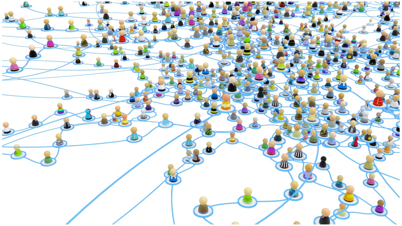
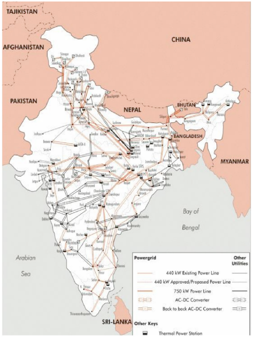
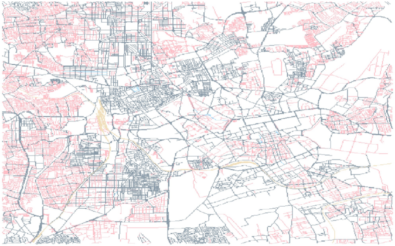
Examples of a few real-world networks



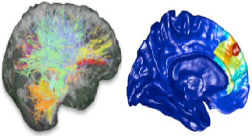
Social Network



Power Grid Network



Bangalore Road Network



Biological Network

Complex networks in a nutshell

- "Network" – Any complex interlocking system (ref. to transportation of river, canals, and railways [1839 AD])

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- Composed by many interacting parts where a nodal behavior affects the whole network

Complex networks in a nutshell

- "Network" – Any complex interlocking system (ref. to transportation of river, canals, and railways [1839 AD])
- Complex network is a graph with non-trivial topology
- Entities are nodes and their interactions mimic as links
- Composed by many interacting parts where a nodal behavior affects the whole network
- Few real-world complex network examples:
  - **Communication** (PSTN, computer, wireless)
  - **Transportation** (Road, railways, air)
  - **Technological** (Internet, WWW, power grid)
  - **Social** (Groups, friendships, organizations, diseases spreading)
  - **Online social** (Facebook, Twitter, LinkedIn, citation)
  - **Biological** (Protein interactions, food web, neural, metabolic)
  - **Many more. . .**

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Our proposed metric, **Link Influence Entropy (LInE)**, efficiently incorporates link influence to identify nodal influence of a network

Major contributions[1]

1. Efficiently identified influence of a node by incorporating link influence with **Link Influence Entropy (LInE)** metric

2. Measured influence stability of nodes in a dynamic network

3. Extensively studied LInE performance on various synthetic complex networks and a few real-world networks
   - **Synthetic networks:** Regular grid, small-world, Erdős-Rényi (ER), scale-free, and spatial wireless networks
   - **Real-world networks:** Zachary's karate club networks and Wireless Mesh Network (WMN) testbed

4. Classified complex networks based on LInE values
   - Regular network > Small-world network > ER network > Scale-free network > Spatial wireless network

[1] Priti Singh, Abhishek Chakraborty, and BS Manoj. "Link Influence Entropy". In: *Physica A: Statistical Mechanics and its Applications* 465 (2017), pp. 701–713.

Outline of the talk

Link Influence Entropy (LInE)

LInE metric

- Addition or removal of certain link impacts the Average Path Length (APL), defined by $\mathbf{APL} = \mathbf{2} \times \sum_{\mathbf{i} \neq \mathbf{j}} \mathbf{d}(\mathbf{i},\ \mathbf{j})/[\mathbf{N} \times (\mathbf{N} - \mathbf{1})]$

- LInE behavior primarily influences by link location along with network APL

- LInE of a network can be quantified by

$$H = -\sum_{i \neq j} p_{ij} \log p_{ij}, \quad p_{ij} = \frac{|APL - APL_{\overline{ij}}|}{\sum_{i \neq j} |APL - APL_{\overline{ij}}|}$$

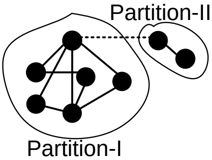
$APL_{\overline{ij}}$ — APL after removal of a link between nodes i and j

- Influence of a node, say node i, can be evaluated as

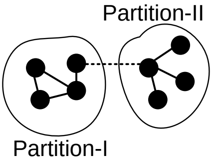
$$p_i = \frac{\sum_j p_{ij}}{2}, \quad 0 \leqslant p_i \leqslant 1$$

Handling partitioned network with LInE

- Removal of certain link, known as bridge link, from a network may cause network partitions
- Two situations arise when a bridge link (dashed line in the figures) is removed



(a) $APL_{\overline{ij}} = APL_{\overline{ij(I)}}$, as Partition-I contains more than 80% of nodes.



(b) $APL_{\overline{ij}} = \frac{APL_{\overline{ij(I)}} + APL_{\overline{ij(II)}}}{2}$, as either of the two partitions have size more than 20% of total number of nodes.

Handling dynamic networks with LInE

- LInE value for dynamic networks has to be evaluated for a particular time instant k

$$H^k = -\sum_{i \neq j} p_{ij}^k \log p_{ij}^k, \quad p_{ij}^k = \frac{|APL^k - APL_{\overline{ij}}^k|}{\sum_{i \neq j} |APL^k - APL_{\overline{ij}}^k|}$$

APL^k and $APL_{\overline{ij}}^k$ are the corresponding values at k^{th} instant

- Influence of node i at k^{th} instant can be evaluated as

$$p_i^k = \frac{\sum_j p_{ij}^k}{2}, \quad 0 \leqslant p_i \leqslant 1$$

Identifying influence stability of dynamic networks

- Change of a nodal influence with time gives an estimate of influence stability of that node
- A node is more stable if all links properties attached to the node remains unchanged with time
- Influence stability of node i, i.e. δ_i, can be calculated as

$$\delta_i = \frac{1}{\max(\Delta p_i^k / \Delta k)}$$

Δp_i^k — Change of p_i^k w.r.t. the mean value
Δk — Difference in network topology between k^{th} and $(k-1)^{th}$ time instants

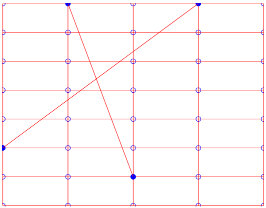
- Steep change in δ_i represents highly dynamic w.r.t. the network

Performance evaluation of LInE

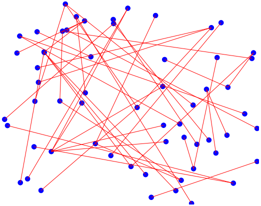
Experimental setting

- LInE performance is evaluated using five network models
  - Regular grid network
  - Small-world (SW) network
  - Erdős-Rényi (ER) network
  - Scale-free (SF) network
  - Spatial wireless network
- LInE is also tested on a real-world network and a dynamic Wireless Mesh Network (WMN) testbed
- LInE performance is compared with following metrics
  - Degree based entropy
  - Betweenness Centrality (BC) based entropy

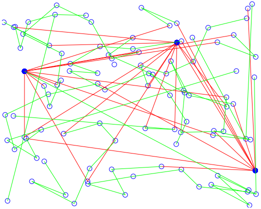
Examples of various network topologies



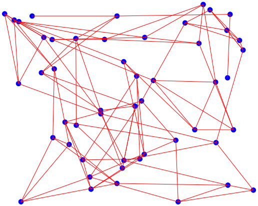
(a) An example small-world network.



(b) An example ER random network.



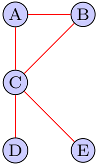
(c) An example scale-free network where *Red lines* are connected to hub-nodes.



(d) An example spatial wireless network.

Betweenness Centrality (BC)

- Measures the extent of one node that lies in shortest paths between every pair of nodes in the network
- Measures the *importance* of the node in making the long distance communications in the network



- BC of i^{th} node in an N node network is measured as:

$$BC(v_i) = \sum_{v_i \neq v_j \neq v_k} \frac{g_{jk}(v_i)}{g_{jk}}$$

g_{jk} is the total number of shortest paths from j to k, and $g_{jk}(v_i)$ is the number of paths that pass through v_i

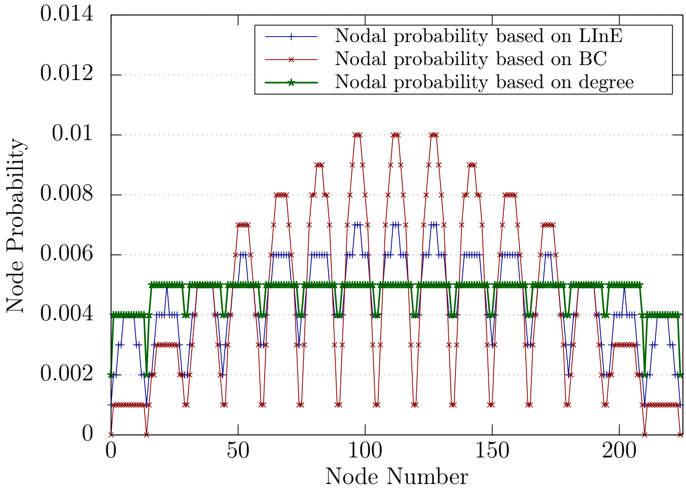
- Normalized BC w.r.t. star topology network:

$$BC^{'}(v_i) = \frac{BC(v_i)}{[(N-1)(N-2)/2]}$$

Node	$BC^{'}$
A	0
B	0
C	5/6
D	0
E	0

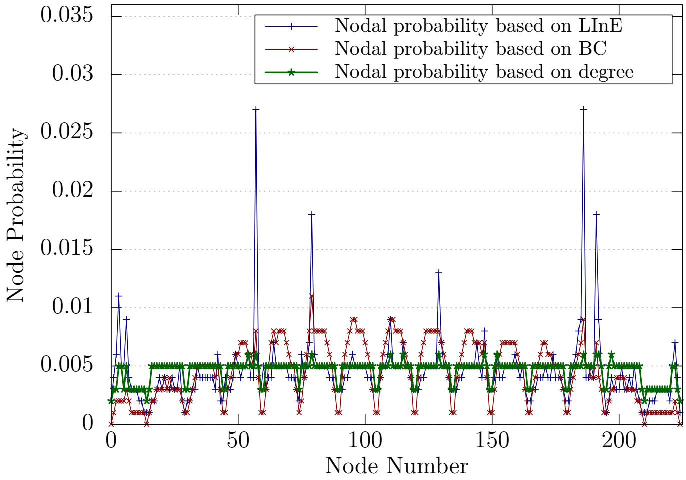
LInE behavior in complex networks

LInE behavior in regular grid network



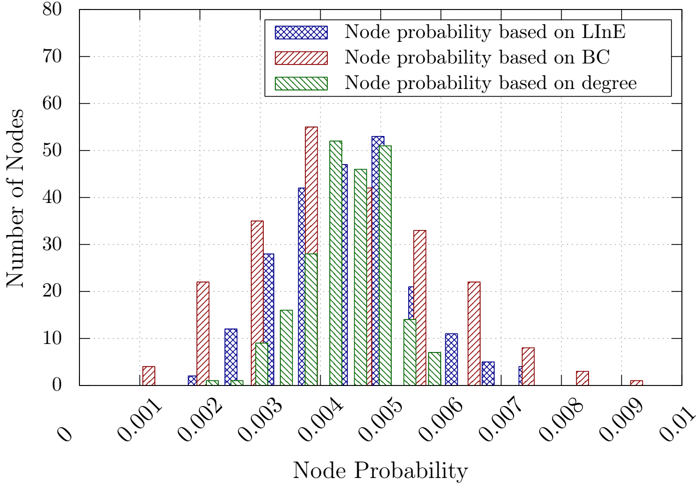
Variation of node probability based on LInE, BC, and degree entropy in a 15 $\times$ 15 grid network. Here, 'X'-axis represents node number, where node 1 is located at the bottom-left and node 225 is located at the top-right position in the grid.

LInE behavior in SW network



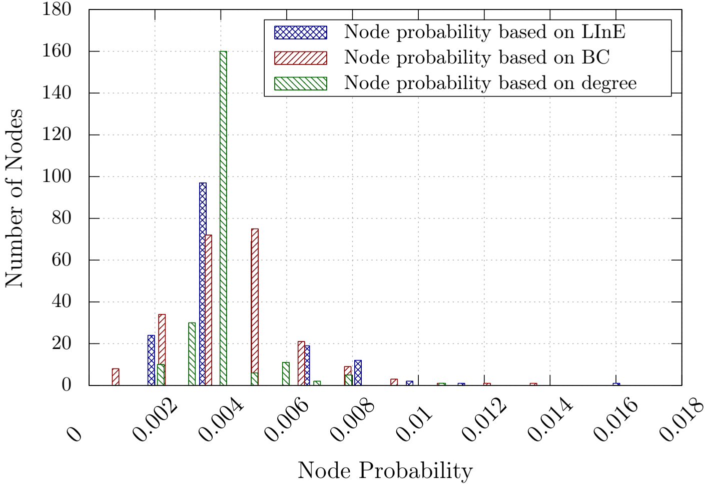
Variation in nodal probability based on LInE, BC, and degree in a small-world network where the number of LLs is approximately 4% of the nodes. Here, 'X'-axis represents node number, where location of the nodes is similar as described in last slide.

LInE behavior in ER network



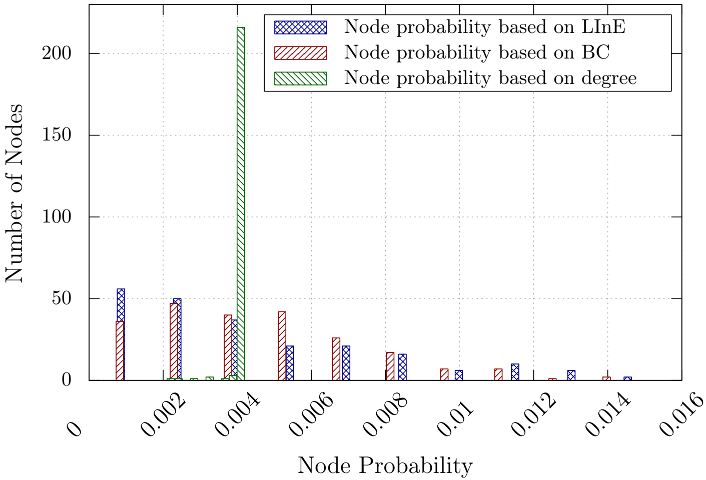
Nodal probability distribution in an ER network. The ER network with randomly distributed nodes is modeled with $p = 0.017$ and maximum number of links that can be made by each node is equal to 4.

LInE behavior in SF network



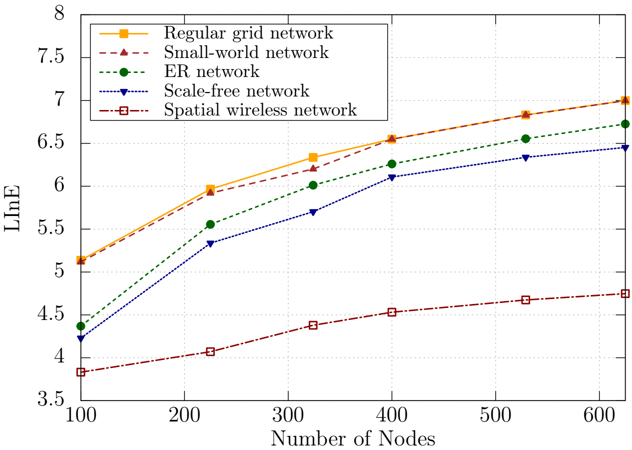
Nodal probability distribution in a scale-free network. The scale-free network is modeled such that the probability of links created by each node follows a power-law distribution.

LInE behavior in spatial wireless network



Nodal probability distribution in a spatial wireless network. Here, node probability of degree based entropy is centered around 0.005 (see *Green histogram* at 0.004).

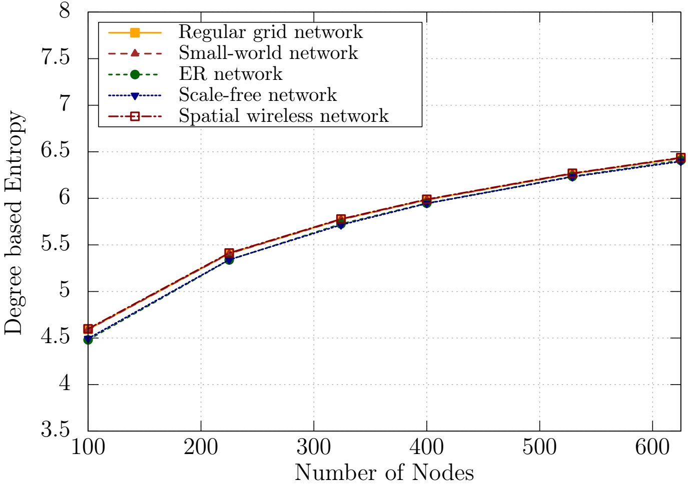
Identifying types of complex networks I



LInE vs number of nodes for various nodes.

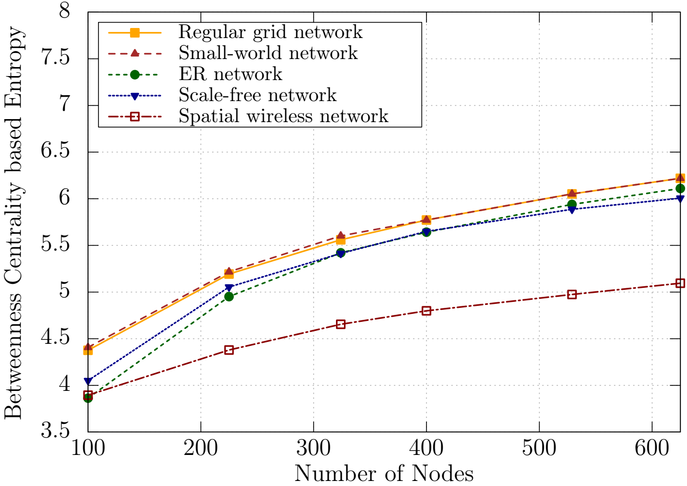
regular network > small-world network > ER network > scale-free network > spatial wireless network

Identifying types of complex networks II



Degree based entropy vs number of nodes for various nodes.

Identifying types of complex networks III

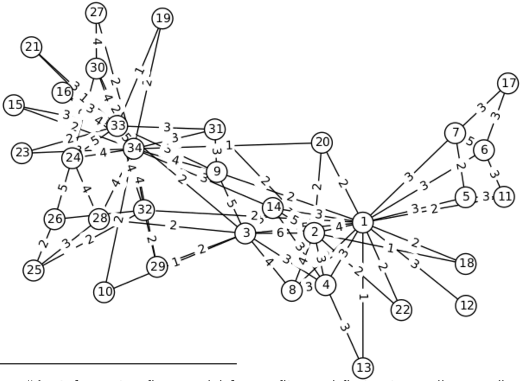


BC based entropy vs number of nodes for various nodes.

LInE behavior in real-world networks

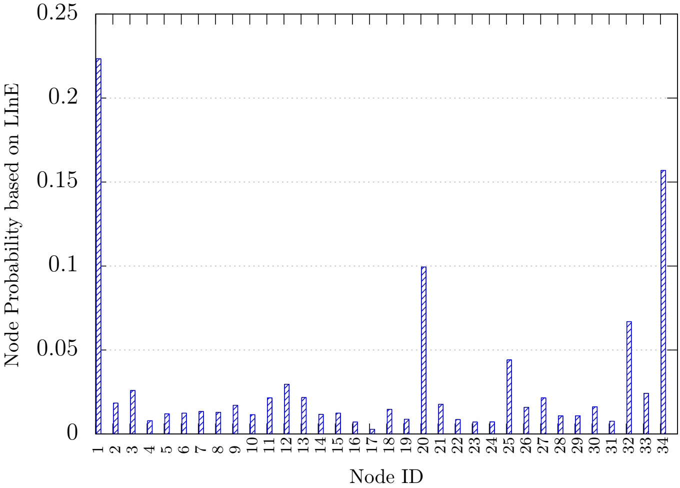
LInE behavior in a real-world network

- Zachary's karate club network[2] is an example of a social network of 34 members at a US university karate club
- Due to some conflict between the club administrator (node 1) and the karate instructor (node 34), club members divided into two groups



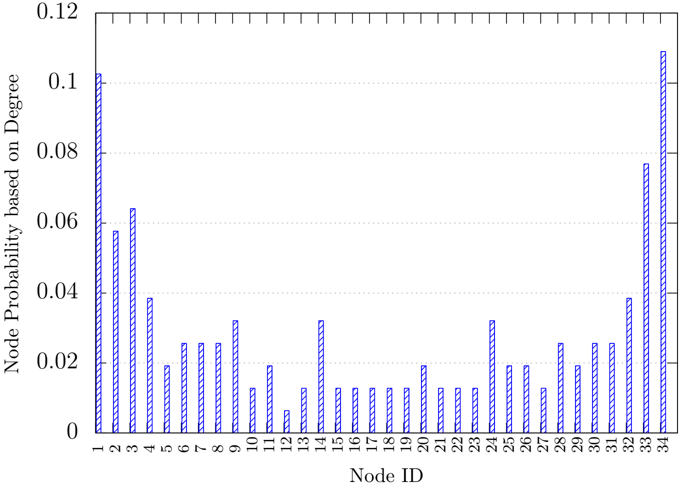
[2] Wayne W Zachary. "An information flow model for conflict and fission in small groups". In: *Journal of anthropological research* (1977), pp. 452–473.

Identifying influential nodes in karate club I



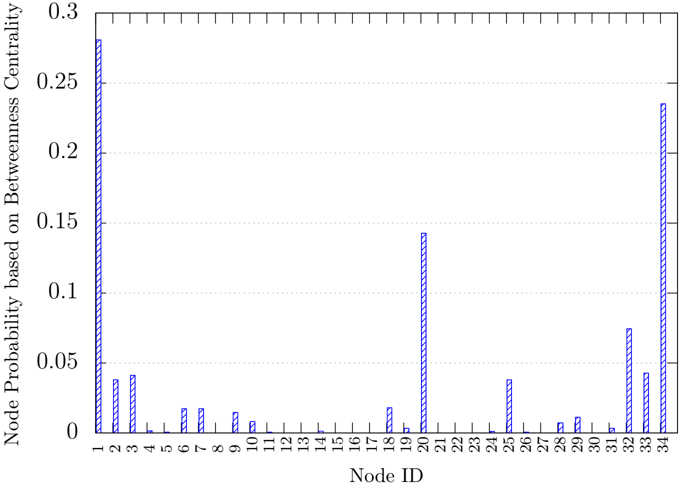
Probability of nodal influence of Zachary's karate club network based on **LInE**.

Identifying influential nodes in karate club II



Probability of nodal influence of Zachary's karate club network based on **degree**.

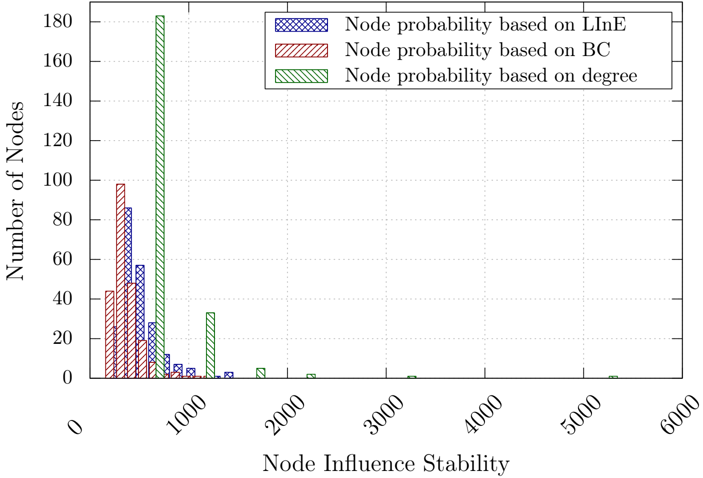
Identifying influential nodes in karate club III



Probability of nodal influence of Zachary's karate club network based on **BC**.

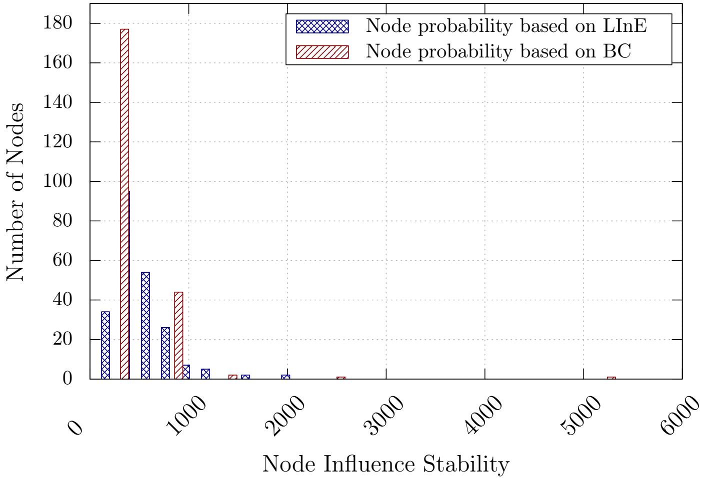
LInE behavior in dynamic networks

Observations on influence stability distribution I



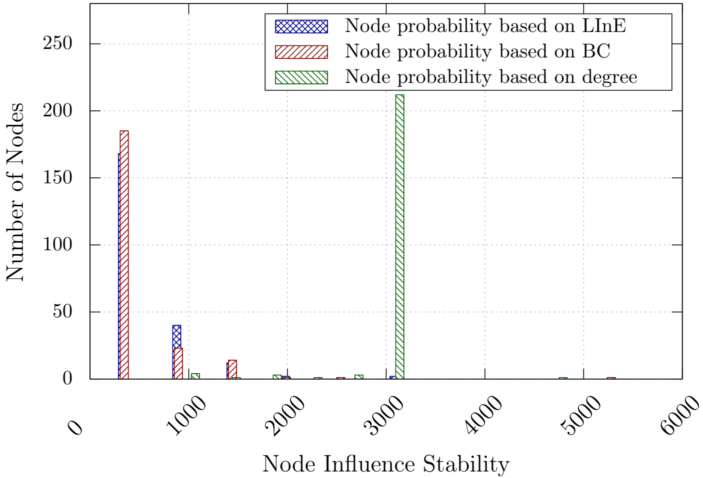
Node influence stability distribution in an **Erdős-Rényi (ER) network**.

Observations on influence stability distribution II



Node influence stability distribution in a **scale-free network**.

Observations on influence stability distribution III

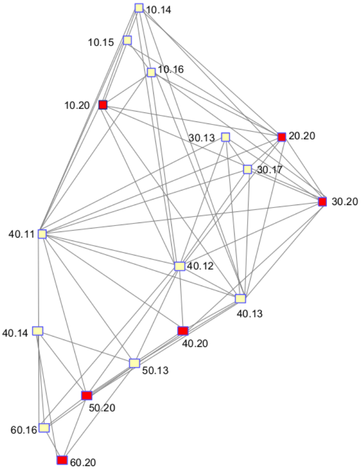


Node influence stability distribution in a **spatial wireless network**.

LInE behavior in WMN testbed

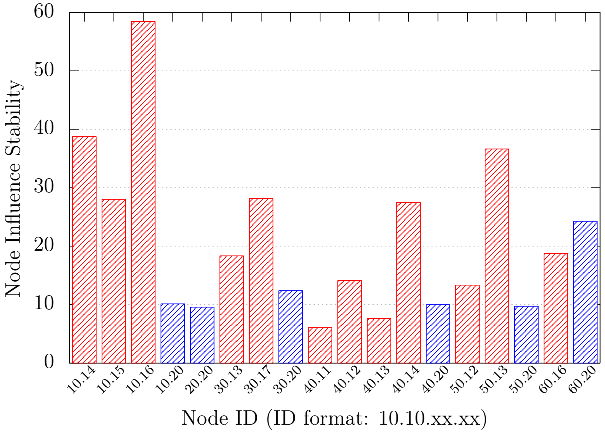
LInE behavior in WMN testbed

- A real-world Wireless Mesh Network (WMN) testbed comprised of
  - Six fixed mesh routers (*Magenta*)
  - A few mobile nodes (*Green-yellow*)
- The figure shows a WMN topology at a particular time instant
- All nodes in the testbed have IP address pattern for the least significant 16 bits as XX.XX
  - Mesh router with XX.20



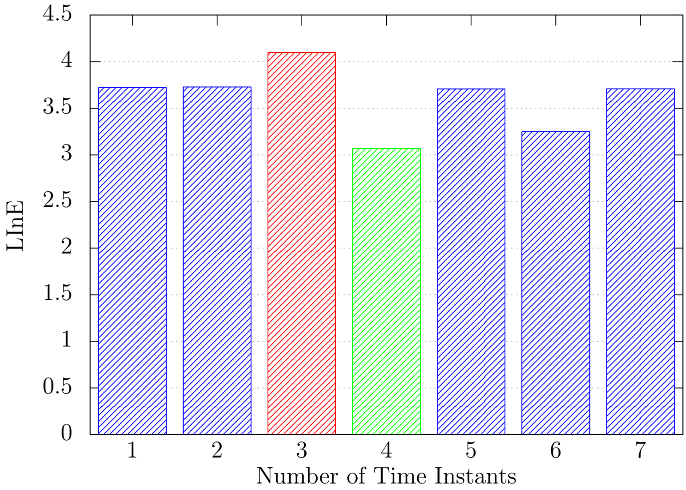
WMN topology at the 1^{st} time instant.

Experimental observations on our WMN testbed I



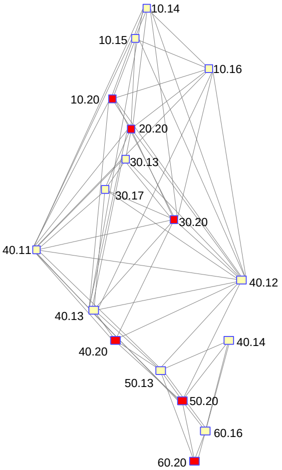
The influence stability of the mesh routers are plotted in *Blue* colored histograms, and that for the mobile nodes are shown in *Red* colored histograms. Higher value corresponds to a more stable node.

Experimental observations on our WMN testbed II

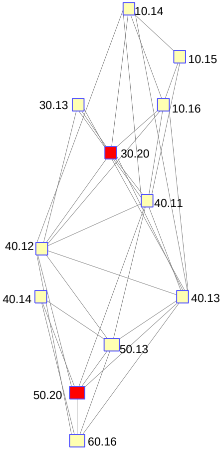


LInE variation at 7 different time instants. The highest (3^{rd} instant) and the lowest (4^{th} instant) LInE value instants are shown in *Red* and *Green* colored histograms.

Experimental observations on our WMN testbed III



The WMN network topology at the 3^{rd} instant (corresponding to the highest LInE value).



The WMN network topology at the 4^{th} instant (corresponding to the lowest LInE value).

Observations and conclusions

- Proposed a new metric, **Link Influence Entropy (LInE)**, which
  - Incorporates influence of individual link to measure nodal importance
  - Also measure influence stability of nodes in dynamic networks
- It has been observed that
  - Degree based entropy may not always differentiate nodal influence
  - Betweenness centrality (BC) based network entropy cannot incorporate link influence and unable to measure influence stability
- LInE incorporates partitioned network scenarios while identifying nodal influence
- It is found that
  - LInE is highest in regular grid networks and lowest in spatial wireless networks

Questions?

abhishek2003slg@ieee.org

Thank You.

abhishek2003slg@ieee.org