MECH 5130 Theory of Finite Element Method
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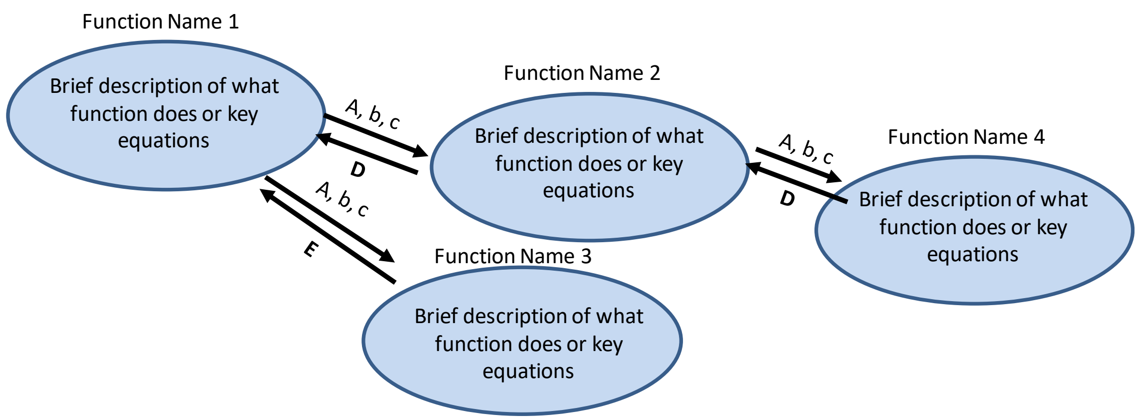
Homework 4: Gaussian Integration

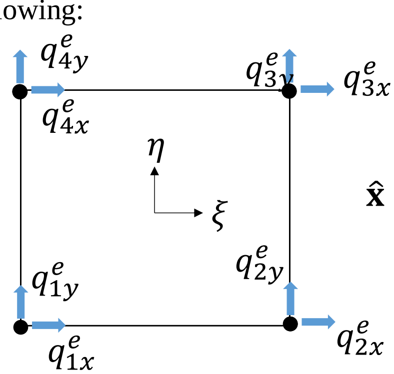
Complete (1) and (2) for pre-homework submission.

(1) Create a "bubble diagram" of the functions needed for this homework. Make sure that the starting point is clearly defined and find the most abstract tasks and create one bubble chart that works for both elements. Show the interdependency of the functions, using arrows connecting functions as shown below, where an incoming arrow should be labelled with the inputs for the function and an outgoing arrow labeled with the outputs. Indicate in RED the functions which will need to be customized for each element.



(2) Write pseudo-code for each of the new functions in the bubble chart. Get to enough detail that you have worked out all the indices and dimensions of any arrays needed. Look up key functions needed and show that the arrays you give to the functions are in the right form.

(3) For the 4-noded 2-D element shown below (E=70,000, ν=0.33, plane strain, $t_z = 1.3$), do the following:

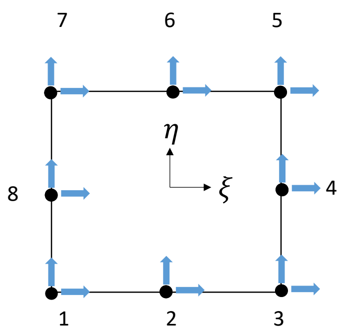


$$\hat{\mathbf{x}} = \begin{bmatrix} (0,0) \\ (12,1) \\ (15,8) \\ (-1,10) \end{bmatrix} \begin{bmatrix} (\hat{x}_1, \hat{y}_1) \\ (\hat{x}_2, \hat{y}_2) \\ (\hat{x}_3, \hat{y}_3) \\ (\hat{x}_4, \hat{y}_4) \end{bmatrix} \quad \mathbf{q} = \begin{bmatrix} 0 \\ 0 \\ 0.01 \\ -0.1 \\ 0.1 \\ 0.01 \\ -0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} q_{1x}^e \\ q_{1y}^e \\ q_{2x}^e \\ q_{2y}^e \\ q_{3x}^e \\ q_{3y}^e \\ q_{4x}^e \\ q_{4y}^e \end{bmatrix}$$

a. Plot the strain energy density, ψ ($\psi = \frac{1}{2}\varepsilon^T\sigma$), within the element in *x*/*y* coordinate system. Draw a point (or points) where you would need to evaluate the function to get an exact solution of an integral over the volume (assuming **J** is constant) with the least number of points.

b. In Matlab, python, or your preferred language, write a function to perform Gaussian Quadrature to get the element stiffness matrix. Give the element stiffness matrix for the given parameters. Plot a colormap of the matrix.

(4) For the 8-noded 2-D element shown below (*E*=70,000, *ν*=0.33, plane strain, $t_z = 1.3$), do the following:



$$\hat{\mathbf{x}} = \begin{bmatrix} (0,0) \\ (6,-0.5) \\ (12,-1) \\ (11,3) \\ (15,8) \\ (6,11) \\ (-1,10) \\ (2,5) \end{bmatrix} \begin{bmatrix} (\hat{x}_1,\hat{y}_1) \\ (\hat{x}_2,\hat{y}_2) \\ (\hat{x}_3,\hat{y}_3) \\ (\hat{x}_4,\hat{y}_4) \\ (\hat{x}_5,\hat{y}_5) \\ (\hat{x}_6,\hat{y}_6) \\ (\hat{x}_7,\hat{y}_7) \\ (\hat{x}_8,\hat{y}_8) \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} 0 \\ 0 \\ -0.1 \\ 0.1 \\ -0.2 \\ 0.3 \\ -0.2 \\ 0.3 \\ -0.2 \\ 0.3 \\ -0.1 \\ 0.1 \\ 0.0 \\ -0.1 \\ 0.0 \\ 0.0 \end{bmatrix}$$

a. Plot the strain energy density, ψ ($\psi = \frac{1}{2}\varepsilon^T\sigma$), within the element in the *x*/*y* coordinate system. Draw a point (or points) where you would need to evaluate the function to get an exact solution of an integral over the volume (assuming **J** is constant) with the least number of points.

b. In Matlab, python, or your preferred language, write a function to perform Gaussian Quadrature to get the element stiffness matrix. Give the element stiffness matrix for the given parameters. Plot a colormap of the matrix.