MECH 5130 Theory of Finite Element Method
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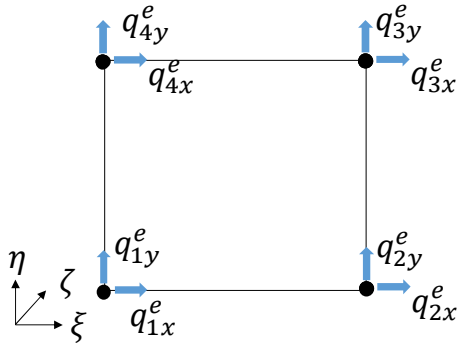
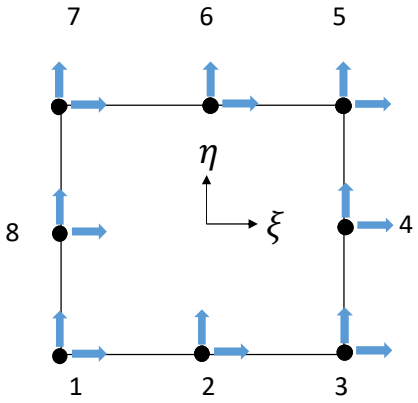
Homework 2: Isoparametric Shape Functions

4-noded 2D element (linear shape functions)



$$\hat{\mathbf{x}} = \begin{bmatrix} (0,0) \\ (12,-1) \\ (15,8) \\ (-1,10) \end{bmatrix} \begin{bmatrix} (\hat{x}_1, \hat{y}_1) \\ (\hat{x}_2, \hat{y}_2) \\ (\hat{x}_3, \hat{y}_3) \\ (\hat{x}_4, \hat{y}_4) \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ -0.1 \\ 0.1 \\ 0.1 \\ -0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} q^e_{1x} \\ q^e_{1y} \\ q^e_{2x} \\ q^e_{2y} \\ q^e_{3x} \\ q^e_{3y} \\ q^e_{4x} \\ q^e_{4y} \end{bmatrix}$$

8-noded 2D element (quadratic shape functions)

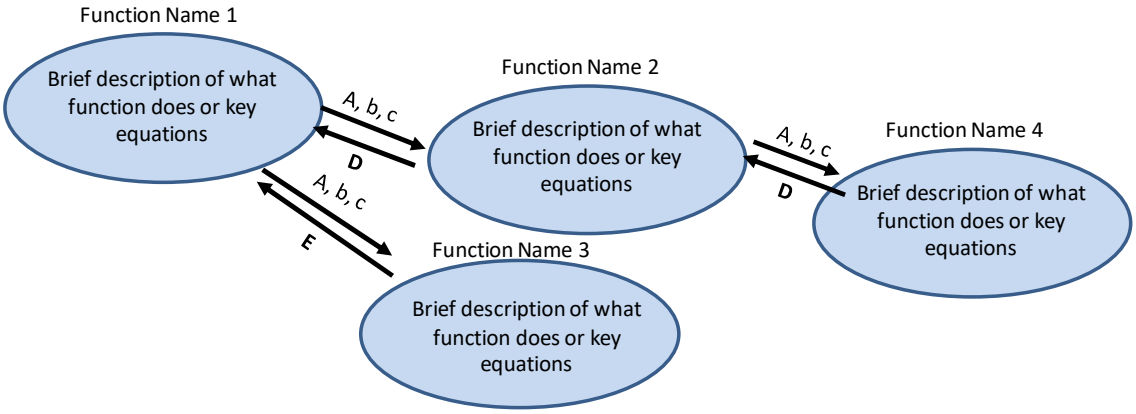


$$\hat{\mathbf{x}} = \begin{bmatrix} (0,0) \\ (6,0.5) \\ (12,-1) \\ (11,5) \\ (15,8) \\ (6,11) \\ (-1,10) \\ (1,5) \end{bmatrix} \begin{bmatrix} (\hat{x}_1, \hat{y}_1) \\ (\hat{x}_2, \hat{y}_2) \\ (\hat{x}_3, \hat{y}_3) \\ (\hat{x}_4, \hat{y}_4) \\ (\hat{x}_5, \hat{y}_5) \\ (\hat{x}_6, \hat{y}_6) \\ (\hat{x}_7, \hat{y}_7) \\ (\hat{x}_8, \hat{y}_8) \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ -0.1 \\ 0.2 \\ -0.3 \\ 0.2 \\ -0.3 \\ 0.2 \\ -0.3 \\ 0.1 \\ -0.1 \\ 0.0 \\ 0.1 \\ 0.0 \\ 0.0 \end{bmatrix}$$

Complete (1), (2) and (3) for pre-homework submission. For each of the elements shown above, do the following:

(1) Derive the shape functions, N_i in terms of some point in the natural coordinate system, (ξ, η).

(2) Create a "bubble diagram" of the functions needed for (4). Make sure to find the most abstract tasks and create one bubble chart that works for all of the functions. Show the interdependency of the functions, using arrows connecting functions as shown below, where an incoming arrow should be labelled with the inputs for the function and an outgoing arrow labeled with the outputs. Indicate in RED the functions which will need to be customized for each element.



(3) Write pseudo-code for each of the functions in the bubble chart. Get to enough detail that you have worked out all the indices and dimensions of any arrays needed. Look up key functions needed and show that the arrays you give to the functions are in the right form.

(4) In Matlab, python, or your preferred language, write a function or object for each shape function, and write a function or object for the displacement and position at a point, $\boldsymbol{\xi}$, based on nodal displacements and nodal coordinates. Plot the following:

a. Plot the value of the shape functions within the element in $\boldsymbol{\xi}$ - space. Plot each shape function in contour plot, as a function of ξ and η. Discuss how these plots can be used for verification that your code and formulation is correct.

b. Plot the value of the shape functions within the element in **x**- space. Plot each shape function in contour plot, as a function of x and y. Discuss how these plots can be used for verification that your code and formulation is correct.

c. Plot the displacement field within the element in $\boldsymbol{\xi}$ - space. Plot each displacement component in a contour plot, as a function of ξ and η.

d. Plot the displacement field within the element in **x**- space. Plot each displacement component in a contour plot, as a function of x and y.

(5) In Abaqus, create a 1-element model for each of the **linear** elements and apply the nodal displacements you used for (2d). Compare the results by generating the same plot from (2d) with the same color scale.