MECH 5130 Theory of Finite Element Method
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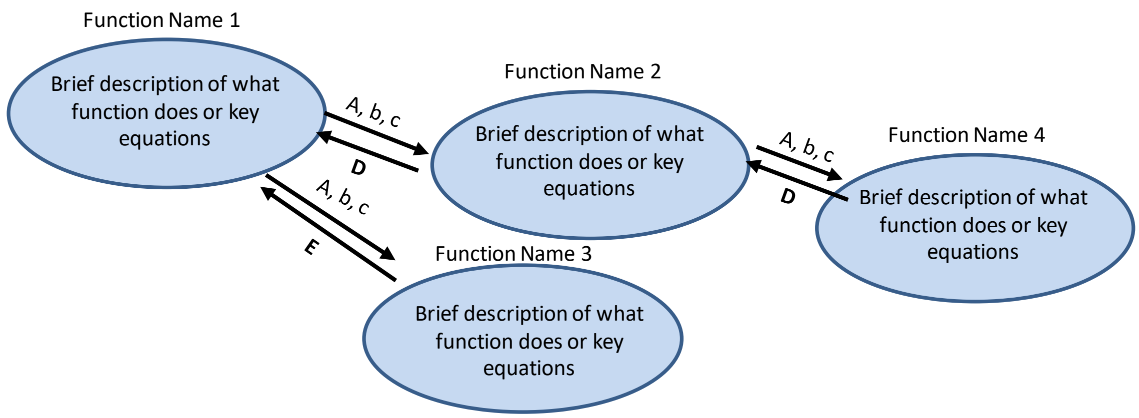
Homework 5: Load Vector

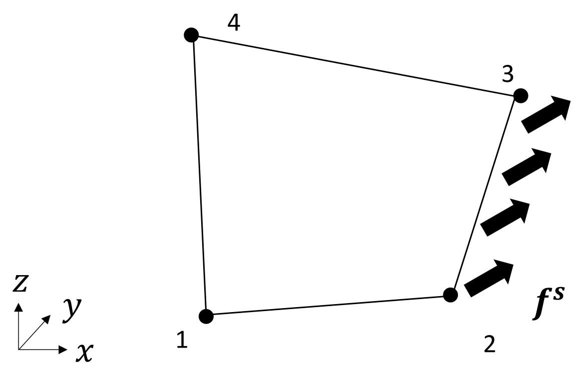
Complete (1), (2) and (3a) for pre-homework submission.

(1) Create a "bubble diagram" of the functions needed for this homework. Make sure to find the most abstract tasks and create one bubble chart that works for both elements. Show the interdependency of the functions, using arrows connecting functions as shown below, where an incoming arrow should be labelled with the inputs for the function and an outgoing arrow labeled with the outputs. Indicate in RED the functions which will need to be customized for each element.



(2) Write pseudo-code for each of the new functions in the bubble chart. Get to enough detail that you have worked out all the indices and dimensions of any arrays needed. Look up key functions needed and show that the arrays you give to the functions are in the right form.

(3) For the 4-noded 2-D element shown below (E=70,000, ν=0.33, plane stress, $t_z = 1.3$).



$$\boldsymbol{f}^s = \begin{bmatrix} 3x+4 \\ 5+y \end{bmatrix}$$

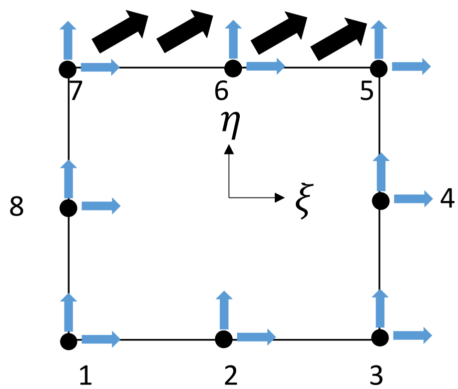
$$\hat{\mathbf{x}} = \begin{bmatrix} (0,0) \\ (12,1) \\ (15,8) \\ (-1,10) \end{bmatrix} = \begin{bmatrix} (\hat{x}_1, \hat{y}_1) \\ (\hat{x}_2, \hat{y}_2) \\ (\hat{x}_3, \hat{y}_3) \\ (\hat{x}_4, \hat{y}_4) \end{bmatrix}$$

a. Derive the force vector for $\boldsymbol{f}^s\,(x) = \begin{bmatrix} f_0 + f_1 x \\ f_2 + f_3 y \end{bmatrix}$ in terms of the nodal locations, $\hat{\mathbf{x}}$, the natural coordinate ξ and the thicknesses without performing the integral. How many integration points would you need to get an exact vector using Gaussian Quadrature?

b. Find the force vector using $f_0 = 4, f_1 = 3, f_2 = 5, f_3 = 1$, and the nodal coordinates above. (Hint: use this as a unit test)

(4) For the 8-noded 2-D element shown below (E=70,000, ν=0.33, plane strain, $t_z = 1.3$), find the force vector. State which degree of integration you used and why. State whether the integral solution is exact or not.

$$\boldsymbol{f}^s = \begin{bmatrix} 3x + 3 \\ 0 \end{bmatrix}$$



$$\hat{\mathbf{x}} = \begin{bmatrix} (0,0) \\ (6,-0.5) \\ (12,-1) \\ (11,3) \\ (15,8) \\ (6,11) \\ (-1,10) \\ (2,5) \end{bmatrix} \begin{bmatrix} (\hat{x}_1, \hat{y}_1) \\ (\hat{x}_2, \hat{y}_2) \\ (\hat{x}_3, \hat{y}_3) \\ (\hat{x}_4, \hat{y}_4) \\ (\hat{x}_5, \hat{y}_5) \\ (\hat{x}_6, \hat{y}_6) \\ (\hat{x}_7, \hat{y}_7) \\ (\hat{x}_8, \hat{y}_8) \end{bmatrix}$$