MECH 5130 Theory of Finite Element Method
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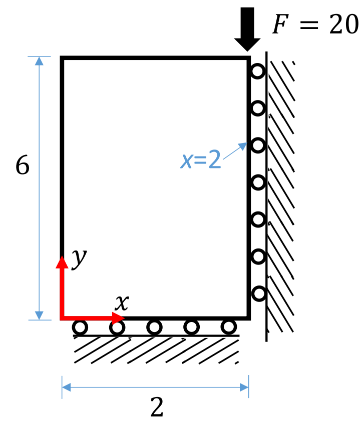
Project: Application of Finite Elements

For the following assignment, please work only one of these problems. Assemble your results in the form of slides for an 8 minute presentation. Show all the details of what you did along with investigating big lessons learned. Submit code you used in Blackboard.

(1) **Stress singularities**: For the model shown below (E=70,000, ν=0.33, plane stress, $t_z = 0.3$) do the following:



a. Create at least four different models in your own numerical program (Matlab for most) using 4-noded elements, where the nominal element size of each consecutive model is halved (for example, nominal element size = 1, 0.5, 0.25, 0.125). Plot the nodes and elements for each model.

b. Plot the σ_{yy} field for each nominal element size.

c. Plot σ_{yy} as a function of y at x=2 for each nominal element size in one plot.

d. Repeat a-c using 8-noded quadratic elements.

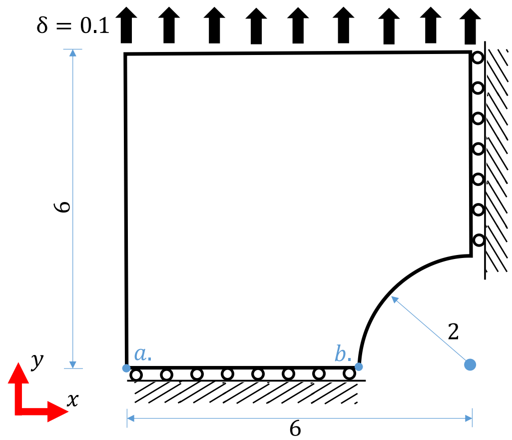
e. Time each model and plot the number of elements vs cpu time for both quadratic and linear elements.

f. Comment about what happens to the stress at y=6 with decreasing nominal element size. Where does the effect of the point load seem to dissipate?

g. What general lessons can you draw from this example? Under what circumstances would this model be sufficient? Under what circumstances would it be insufficient?

h. Come up with a strategy to introduce load in a way that does not introduce a stress singularity and quantitatively show this.

(2) **Mesh Convergence**: For the model shown below (E=70,000, ν=0.33, plane stress, $t_z = 0.3$) do the following:



a. Create at least 6 different models in Abaqus (using the python scripting interface) with 4-noded bilinear elements, where the nominal element size of each consecutive model is halved (for example, nominal element size = 2, 1, 0.5, 0.25, 0.125, 0.0625). Show the mesh for each model.

b. Plot σ_{yy} at locations a. and b. as a function of nominal element size for all the models all in one plot.

c. Propose a metric to determine at what nominal element size the stress is "converged". Give equations and justify your method. Plot your metric vs nominal element size for both locations.

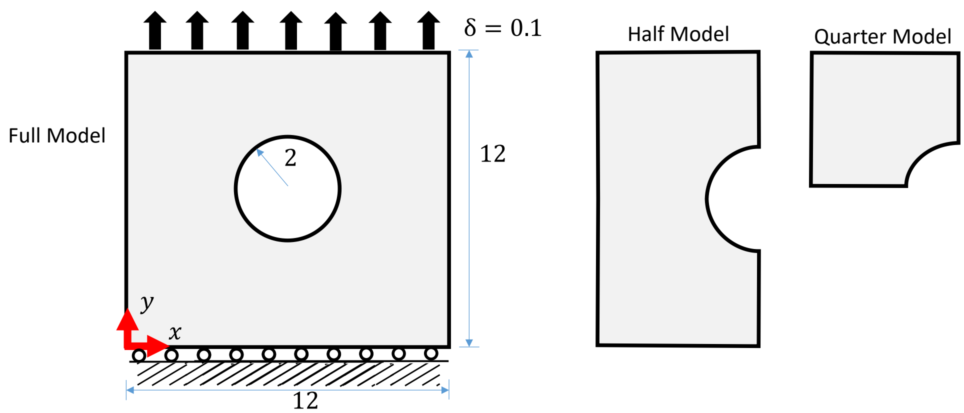
d. Discuss the mesh/element convergence of both locations and what this could mean generally. Give one other example to illustrate your point.

e. Repeat a. using 8-noded quadrilateral elements and compare the plots from c. between the element types. Comment about h-method and p-method modelling. Cite your references that you use to formulate your answer.

f. Time each model and plot the number of elements vs cpu time for both quadratic and linear

g. What is the least number of elements you can use to get a converged solution? Show your mesh and results

(3) **Symmetry**: For the model shown below do the following:



a. In your own finite element code, create a quarter, half, and full model of the plate with a hole in the middle using symmetry and appropriately scaling boundary conditions. Use the largest converged mesh for isotropic material properties and prescribed displacement loading and show convergence. Plot the nodes and elements of the mesh for each model and indicate the boundary conditions for each model.

b. Plot σ_{yy} in each model using an isotropic, plane stress model where *E*=70,000, ν=0.33, and $t_z = 0.3$. Use the same size and color scale for all three models.

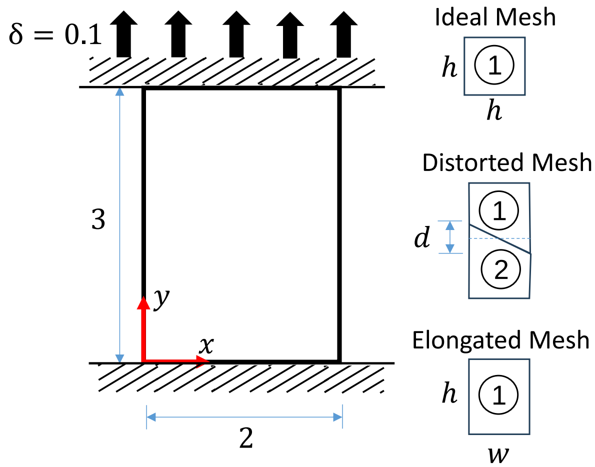
c. Plot σ_{yy} in each model using an isotropic, plane stress model where *E*=70,000, ν=0.33, and $t_z = 0.3$ and an applied body force of $\mathbf{f}^b = [0, 20y]$. Do not apply the displacement to the top surface. Use the same size and color scale for all three models.

d. Plot σ_{yy} in each model using an isotropic, plane stress model where *E*=70,000, ν=0.33, and $t_z = 0.3$ and an applied body force of $\mathbf{f}^b = [10,10]$. Do not apply the displacement to the top surface, and fully clamp the bottom surface for the full model. Use the same size and color scale for all three models.

e. Plot σ_{yy} in each model using a transversely isotropic plane stress model where E_1=70,000, E_2=20,000=E_3, G_{12}=18,000, ν_{12}=0.28, and $t_z = 0.3$. The 1-axis of the material is oriented 30° counter-clockwise from the x-axis. Fully clamp the bottom surface of the full model. Use the same size and color scale for all three models.

f. Discuss what the requirements are for using symmetry in models and how your model showed this.

(4) **Distorted Elements**: For the model shown below do the following:



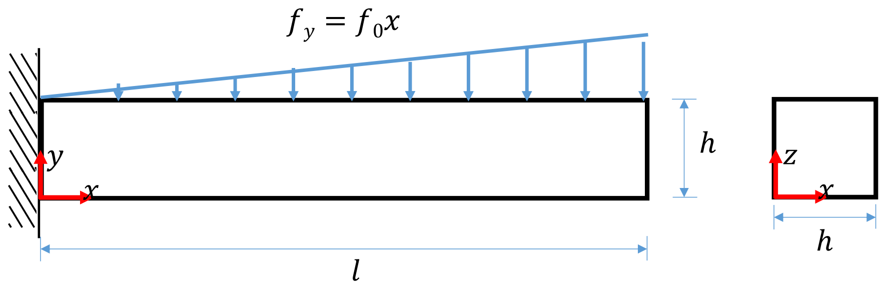
a. Create an ideal mesh for plane stress model where E=70,000, ν=0.33, and $t_z = 0.3$., identify a metric for showing elemental convergence, and find the maximum elemental size for convergence for 1) fully integrated 4-noded linear elements, 2) fully integrated 8-noded quadratic elements, 3) reduced integration 4-noded and 4) reduced integration 8-noded elements. Compare convergence of all 4 in a plot.

b. Distort the mesh of each element by d and plot the elemental convergence of each type of element for 5 different values of d (for d=0, 0.25h, 0.5h, 0.75h, 1.0h)

c. Add integration points to the linear/quadratic elements and compare their convergence to the convergence from part b.

d. Elongate the elements from a. to be h/w = 1, 2, 3, 5 and show the convergence of each element type from a.

(5) **Bending**: For the model shown below do the following:



a. For a chosen material and loads that keep deformation in the small strain/rotations regime, show elemental convergence (using tip displacement) for at least four h/l ratios (1 to 0.01) for fully integrated 4-noded linear elements, 2) fully integrated 8-noded quadratic elements, 3) reduced integration 4-noded and 4) reduced integration 8-noded elements. Compare convergence of all 4 in a plot with at least 4 data points.

b. Compare the converged result with an appropriate analytical model.

c. Use literature to find advice about modelling bending and hourglassing / shear locking and report about how your study reflects this advice

(6) **Characterization**: When choosing a test for material characterization, it is important to try to isolate the aspect of the material that you are trying to measure. For example, we used dogbone specimens for tensile tests so that we could have a gauge section with only tensile stresses that are free of gripping stress concentrations to measure the Young's modulus and Poisson's ratio.

Measuring the shear modulus of a material can be difficult and there is no single perfect test. The best test would be one in which the shear stresses are isolated and the shear stress / strain can be cleanly found to get the shear modulus. You are tasked with deciding on the shear test for a client. There are two types of tests that you will compare: the notched shear specimen test and the shear frame test, shown below:

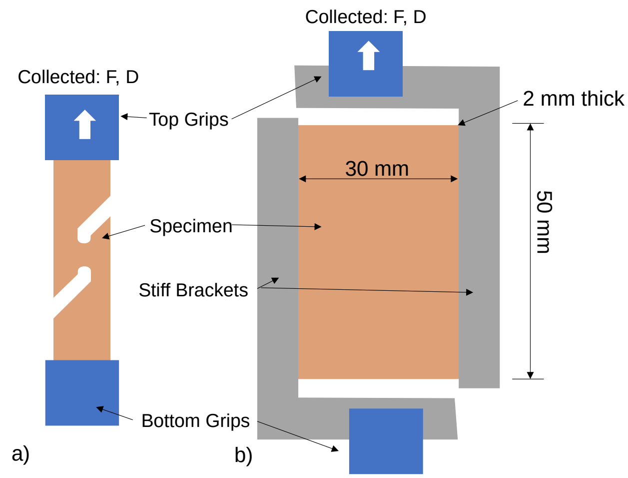


Figure 1 Two shear tests to select from: a) notched shear specimen, and b) shear frame.

The notched shear specimen has notches cut into a strip, which isolates a small gauge section in the middle which should be under shear loading. The shear frame test has a much larger specimen mounted on stiff L-brackets which are pulled to apply load on the sides of the specimen. For both tests, the force in the grips is measured and divided by the cross-sectional area over which the shear load is applied to get the shear stress. The strain can be obtained two ways. First, the crosshead displacement can be used to compute the shear strain, assuming that all of the displacement is in the gauge section of the notched shear specimen or assuming that the displacement is evenly distributed in the shear frame specimen. This is the most convenient method. Second, digital image correlation (DIC) can be used to get the shear strain in the gauge section, which requires more work.

a. Run both tests virtually with your Matlab code. Find and show a converged mesh (based on some criteria you define).

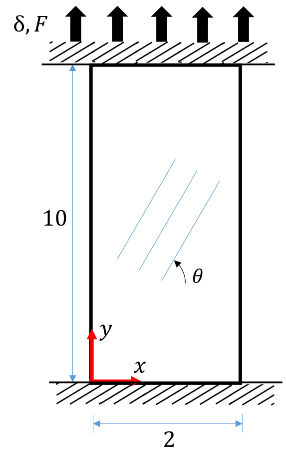
b. For both models, calculate the as-measured value of the shear modulus based on:

   a. The applied force to get the shear stress, and the measured displacement to get the shear strain

   b. The applied force to get the shear stress, and the value of shear strain that you would get in the gauge section (as if you were using DIC, averaging the value of shear strain over an area).

c. Knowing the shear modulus of the material you are using in Matlab, calculate the % error that you would get in the lab for both tests and both methods of obtaining strain. You will use this along with any illustrations you need to make solid arguments for recommending one test over another and whether it is worth it to measure strain with DIC or not.

(7) **Transversely isotropic materials**:



a. You have a rectangular specimen made up of a transversely anisotropic material such as a carbon fiber reinforced plastic where the fiber direction is much stiffer than the transverse direction. You can cut extremely thin specimens of the geometry shown above oriented in any direction, or in other words θ can be any value from 0 to 90°. What is the minimum number of experimental tensile tests that you would need and which angles would you use to fully characterize the in-plane properties of the material?

b. If each experiment had a measuring tolerance of +- 1% for the stiffness, what is your unknown tolerance for the various material properties?

c. If you tested a specimen at angle θ until failure and you believe that the fiber direction tensile stress drives failure, how would you find the failure stress of the material based on the failure load of the tensile test?