Assignment 6

1

We will multiply our fourier transform with $e^(-2\pi xni/N$ to shift our function by n. Shifting our gaussian and comparing with the numpy roll() function, we have:

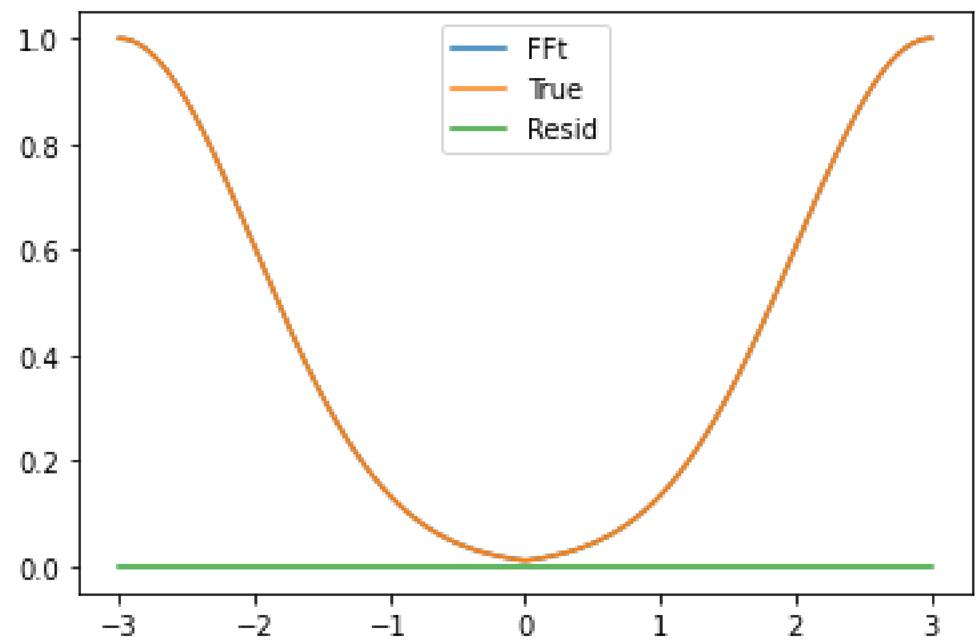

Figure 1: Shifting by correlation function, numpy function and residuals

2

2.1

We have $f \star g = \int f(x)\overline{g(x+y)}$

$$
\begin{aligned}
f \star g = \int \left[\int F(k) e^{2\pi x k i} dk \cdot \int \overline{G(k')} e^{-2\pi k'(x+y)i} dk' \right] dx \\
= \iiint F(k) \overline{G(k')} e^{-2\pi x(k-k')i} e^{-2\pi k' y} dx dk dk' \\
= \iint F(k) \overline{G(k')} e^{-2\pi k' y i} \delta(v - v') dk' dk \\
\int F(k) \overline{G(k)} e^{-2\pi k y i} dk' dk = ift(F(k)\overline{G(k)})
\end{aligned}
$$

2.2

Looking at correlation functions of gaussians that are shifted by some proportion of the initial gaussian we see that the shift is inverted as numpy roll() shifts items to the right and these functions are shifted to the left

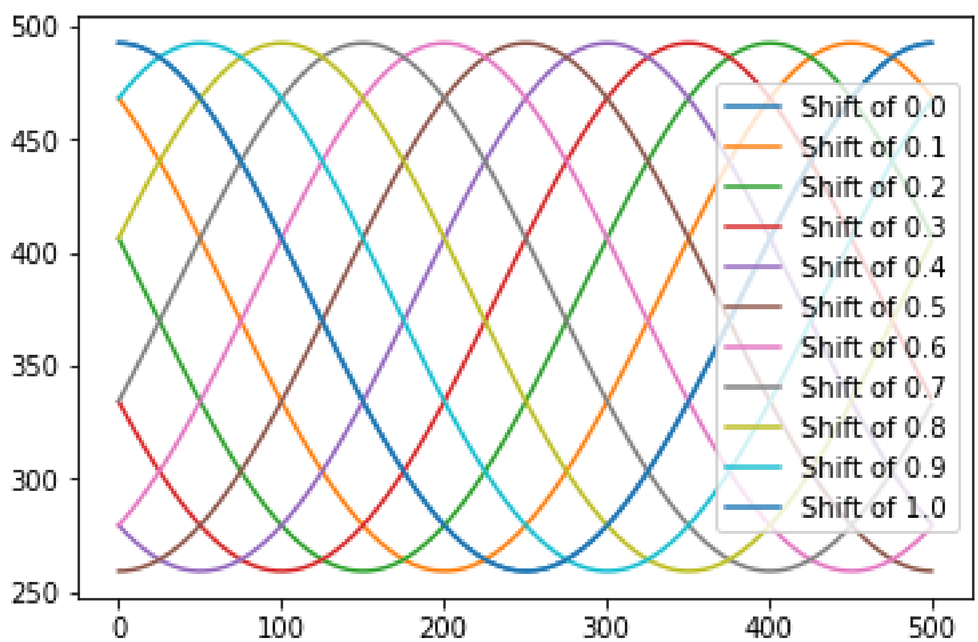

Figure 2: Multiple correlation functions

3

We will pad the arrays up to the nearest power of two and test it for the functions $\arctan x$ and $sinx$

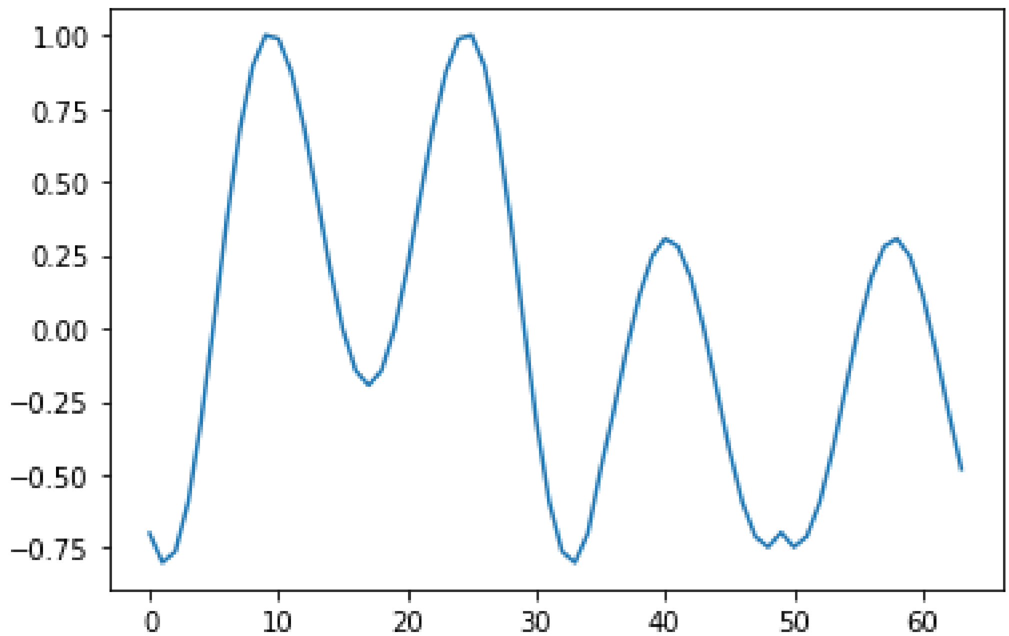

Figure 3: Convolution of padded functions

We can see that we avoid completely noisy arrays but also have some vermuch finer functions from the original:

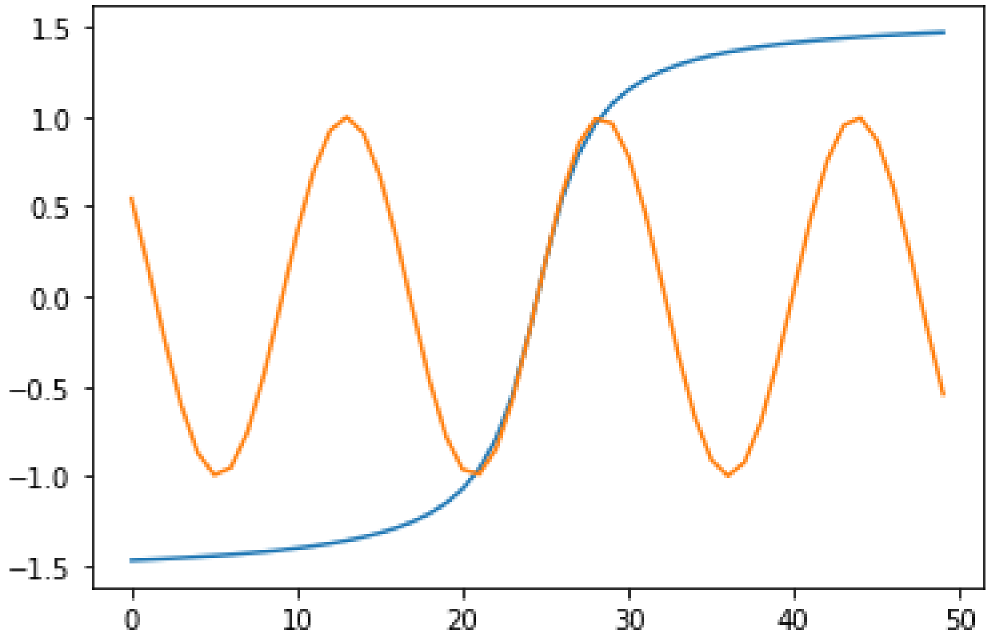

Figure 4: Initial sine and arctangent

4

4.1

We have

$$\sum_{x=0}^{N-1} e^{\frac{-2\pi ikx}{N}} = \sum_{x=0}^{N-1} \alpha^x = \frac{1-\alpha^N}{1-\alpha} = \frac{1-e^{-2\pi ikN/N}}{1-e^{\frac{-2\pi ik}{N}}} = \frac{1-e^{-2\pi ik}}{1-e^{\frac{-2\pi ik}{N}}}$$

4.2

When k approaches 0, our sum approaches:

$$\sum_{0}^{N-1} 1 = N$$

We define k as $0 < k < N$. Then

$$\mathbf{Re}\{e^{-2\pi ik}\} = \cos{(2\pi k)} = 1, k \in \mathbf{N}$$

And the imaginary part is equal to 0 We get a similar result for the case $k = aN + c, a \in \mathbf{N}, c < N$.

$$\mathbf{RE}\{e^{-2\pi i(aN+c)}\} = \mathbf{RE}\{e^{-2\pi iaN} \cdot e^{-2\pi ic}\} = \cos(2\pi aN) \cdot \cos(2\pi c) = 1 \cdot 1 \tag{1}$$

and 0 for the imaginary part, again So in both cases, the numerator will be null.

4.3

Let us use $k' = \frac{1}{4}$ as an example, so :

$$\sum_{x=0}^{N-1} e^{\frac{-\pi ix}{2N}} \cdot e^{-2\pi ikx/N} = \frac{e^{-2\pi ik} \cdot e^{2i\pi k+i}}{1 - e^{-(i\pi(4k+1)/(2N))}} \tag{2}$$

Which we can plot as :

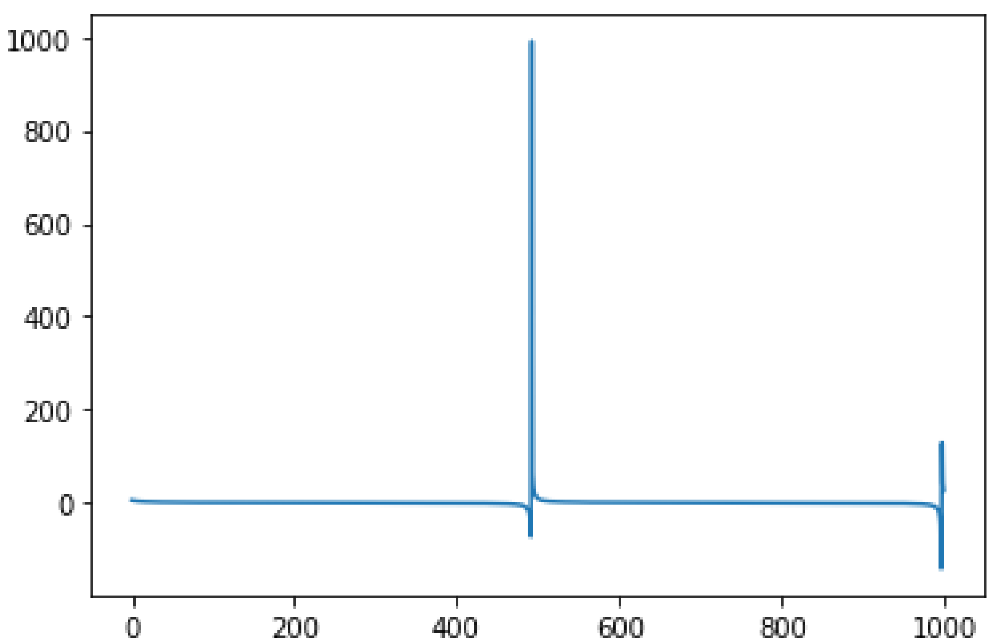

Figure 5: Spectral leak of the supposed delta function

As expected, our result is not exactly a delta function. Applying the suggested windowing function on the numpy-calculated Fourier transform yields:

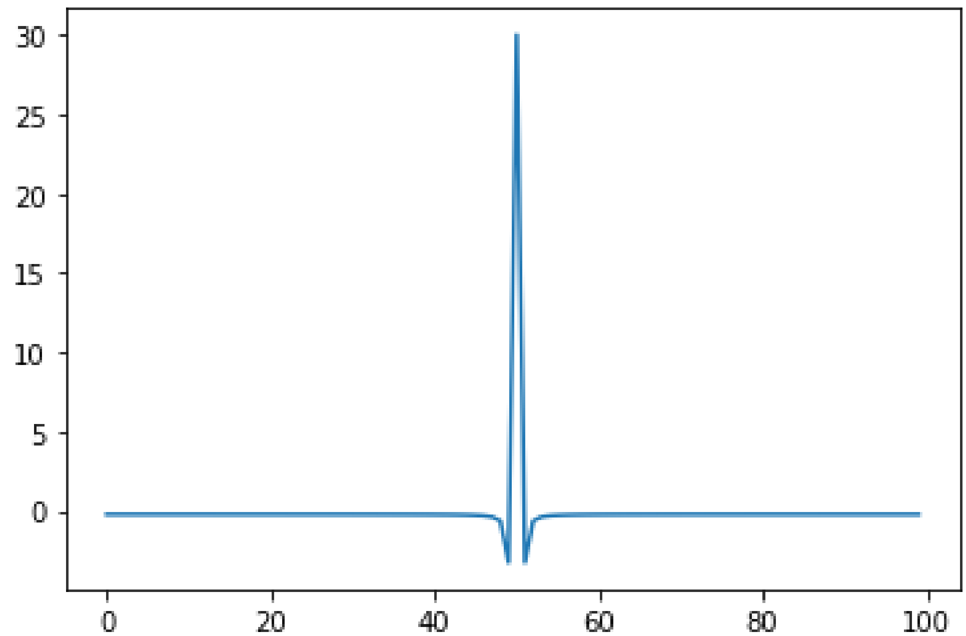

Figure 6: Windowed Fourier transform

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Did not have time to make progress on LIGO data due to wisdom teeth removal.