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Physics 512

# Assignment 1

September 17, 2022
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1. (a) We take this 4-point derivative by obtaining the derivative around our point of interest and average them:

$$\frac{\frac{f(x+2\delta)-f(x+\delta)}{\delta}+\frac{f(x-\delta)-f(x-2\delta)}{\delta}}{2}$$

These have taylor expansions:

$$f(x+\delta) \approx f(x) + f'(x)\delta + \frac{f''(x)\delta^2}{2} + ...$$
$$f(x-\delta) \approx f(x) - f'(x)\delta + \frac{f''(x)\delta^2}{2} + ...$$
$$f(x+2\delta) \approx f(x) + 2f'(x)\delta + 4\frac{f''(x)\delta^2}{2} + ...$$
$$f(x-2\delta) \approx f(x) - 2f'(x)\delta + 4\frac{f''(x)\delta^2}{2} + ...$$

Adding together $f(x+2\delta) - f(x-2\delta)$ with $-f(x+\delta) + f(x-\delta)$, we are left with $f'(x)$, which shows that the operator works

(b) To show the error, we have to take:

$$\frac{\frac{f(x+2\delta)-f(x+\delta)}{\delta}+\frac{f(x-\delta)-f(x-2\delta)}{\delta}}{2} = \frac{\frac{f(x+2\delta)(1+\epsilon g_1)-f(x+\delta)(1+\epsilon g_2)}{\delta}+\frac{f(x-\delta)(1+\epsilon g_3)-f(x-2\delta)(1+\epsilon g_4)}{\delta}}{2}$$

Then, taking the taylor expansion of the $f(\pm 2\delta)$ terms, with $g_{2\delta}$ some combination of $g_1$ and $g_4$:

$$\frac{4\delta f'(x) + 4\delta f'(x)\epsilon g_{2\delta} - f(x+\delta)(1+\epsilon g_2) + f(x-\delta)(1+\epsilon g_3}{2\delta}$$

I couldn't figure out the rest.

2. See code, not much to comment except that the code assumes it will not get arrays of order greater than 1 as inputs.

3. See code, idem as previous problem

4. Taking a polynomial with order 3, a cubic spline and a rational function of order $\frac{1}{2}$. Evaluating the interpolation methods over the range of the function, with only 4 points to interpolate with but 101 points evaluate, we get:

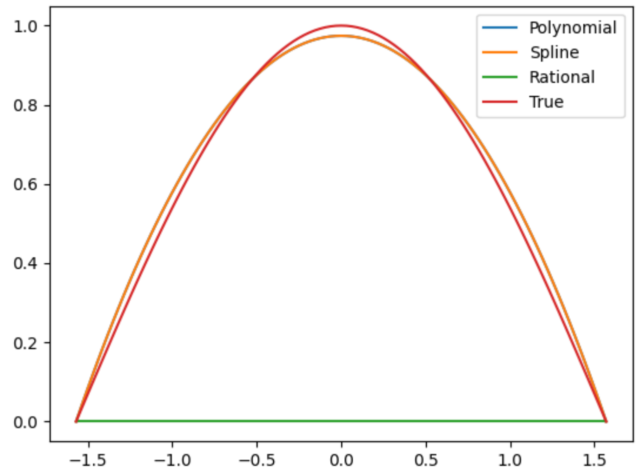


Figure 1: Different interpolation methods over a cos function

we obtain the errors:

| Function | Polyfit | Cubic Spline | Rational function |
|---|---|---|---|
| Error | $(0.02 \cdot \pm 0.01)$ | $(0.02 \pm 0.01)$ | $0.6 \pm 0.3$ |

This is somewhat as expected, as we have a good polynomial fit and spline, with a rational that functions somewhat poorly considering the ratio is very constraining

For the Lorentzian function, we up the order slightly, with now 4, for the polynomial and a rational function of order $\frac{2}{2}$, which yield:

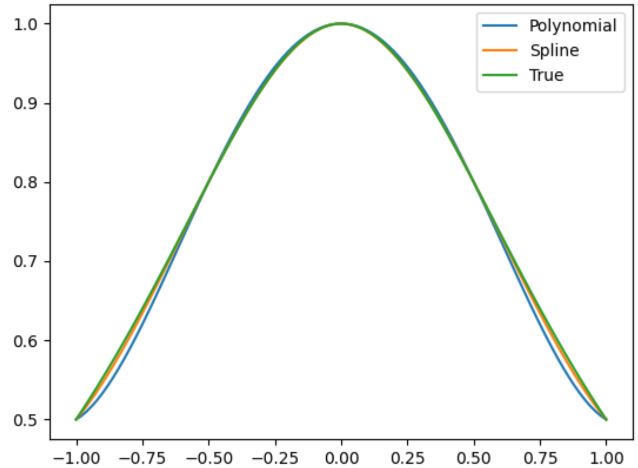


Figure 2: Different interpolation methods over a lorentzian function

Here, the rational function is not shown as it deviates too much from the original function to appear in the figure.

The errors are:

| Function | Polyfit | Cubic Spline | Rational function |
|---|---|---|---|
| Error | $(0.008 \cdot \pm 0.007)$ | $(0.002 \pm 0.002)$ | $(5 \pm 5 \cdot 10^{12}$ |

Which is reasonable except for the polynomial ratio, which yields an enormous error. Looking at a higher order for the rational function, with order of the numerator 6 and order of the denominator 7, we obtain:

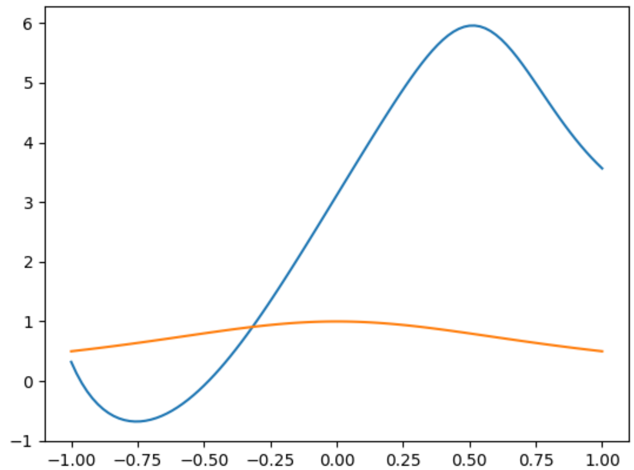


Figure 3: A lorentzian function in orange, interpolated by a rational function

This is better than the previous result, with an error of $2 \pm 2$, which is 12 orders of magnitude better than the previous orders. This can be explained, first by the sampling of more points to establish the interpolation coefficients and by the fact that as the lorentzian, this function now has a higher order at the denominator than the numerator, and thus will behave more closely to it, particularly when $x$ gets big and both function start to fall towards 0.