Mathieu Noreau
Physics 512

Assignment 7

November 21, 2022
Jonathan Sievers

1

We can observe that there are indeed planes in the c-generated random numbers

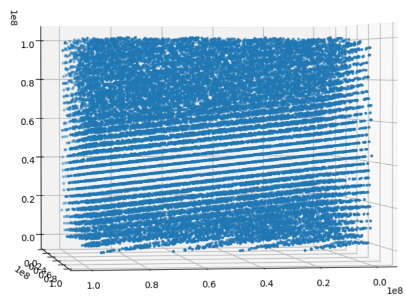

Figure 1: Planes in trios of random points in native c

[H] However, those points are not present in the python random base library, although the documentation that the generator should not be used for any cryptographic purpose.

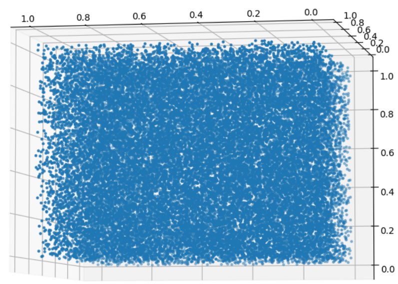

Figure 2: Trios of random points in the python default randomness library

And indeed, although we can faintly make out patterns in this graph, they are definitely not planes

2

We start with the distribution $-\frac{1}{\sqrt{2\pi}}e^{-0.5x^2}$ and will generate only positive number in it. And we want to generate e^{-x} We have an acceptance probability distribution of

$$\begin{aligned}
p(x) &= c \cdot \frac{\frac{1}{\sqrt{2\pi}}e^{-0.5x^2}}{e^{-x}} \\
&= \frac{c}{\sqrt{2\pi}}e^{-0.5x^2+x} \\
p_{max}(x) = p(1) &= \frac{c}{\sqrt{2\pi}}e^{\frac{1}{2}} \leq 1 \\
\implies c \leq \sqrt{\frac{2\pi}{e}} &\implies c_{max} = \sqrt{\frac{2\pi}{e}} p(x) = \frac{1}{\sqrt{e}}e^{-0.5x^2+x}
\end{aligned}$$

Once we have this, we just need to generate random gaussian-distributed positive numbers and to reject them accordingly There seems to be a bug in my implementation as it returns

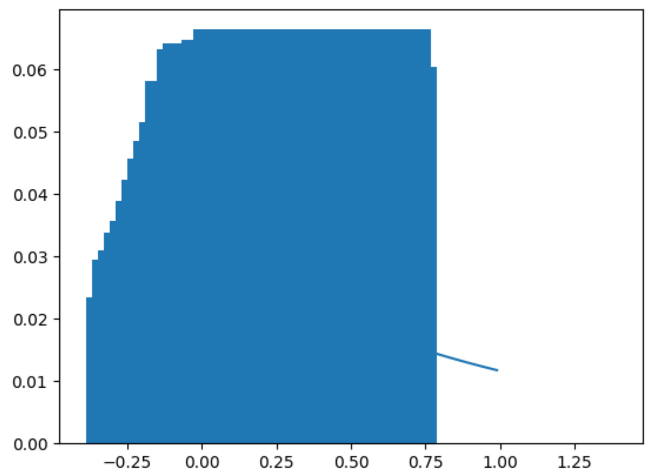

Figure 3: Bounding function and random distribution that it should contain

3

We start off with our $0 < u < \sqrt{p(v/u)} \implies 0 < u < \sqrt{e^{-v/u}} \implies 0 < v < -u \cdot \ln u^2$ Getting $v = g(u) = u \cdot \ln u^2$, which has zeros at $u = 1$ We also have $\frac{dv}{du} = -\ln u^2 - 2$ which has a zero as $u = e^{-1}$. We can then generate u from $[0, 1]$ and v from $[0, e^{-1}]$, with condition $u \leq \sqrt{e^{-v/u}}$. Then return v/u

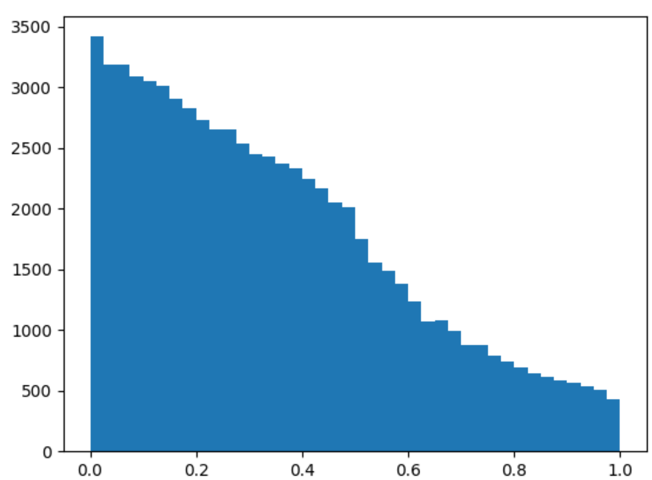

Figure 4: Ratio of uniform that create an exponential function

We can see that we have a distribution that deviates from the expected exponential at the middle.

I got Covid this week, in addition to my teeth extraction last week, great time. Sorry for the late submission.