Mathieu Noreau
Physics 512

Assignment 3

September 30, 2022
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For a), we obtain the plot:

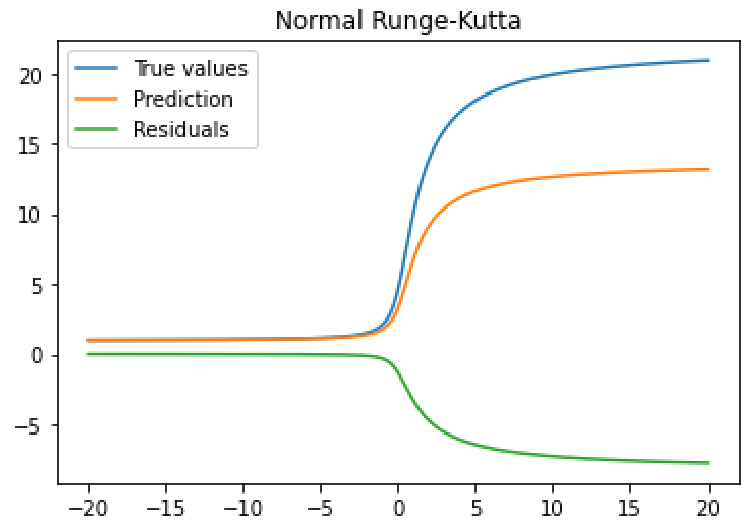

Figure 1: Solution of the initial-value-problem for standard RK4

For b) We take an argument from Numerical Recipes, which shows that to suppress the leading-term errors of the two methods, using big-O notation and some function ϕ independent on h, which yield:

$$y(x+2h) = y_1 + (2h)^5\phi + O(h^6) + \dots$$
$$y(x+2h) = y_2 + 2(h)^5\phi + O(h^6) + \dots$$
$$\implies \Delta = y_2 - y_1 = (y(x+2h) - (2h)^5\phi - O(h^6) - \dots)$$
$$-(y(x+2h) - 2(h)^5\phi - O(h^6) - \dots)$$
$$\implies \Delta = y(x+2h) - y(x+2h) - 32h^5\phi + 2h^5\phi - O(h^6) - \dots$$
$$\implies \Delta = -15 \cdot 2h^5\phi - O(h^6) - \dots$$

Thus we can improve our guess using y_2 by adding $\frac{\Delta}{15}$ as an error correction term. We should note that this correction comes at the cost of running the Runge-Kutta 3 times, thus tripling the amount of function evaluations we have, from 4 in a standard 4th-order Runge-Kutta to 12 Instead of using 200 steps, we will only use 66 steps in order to not have more function evaluations. This yields:

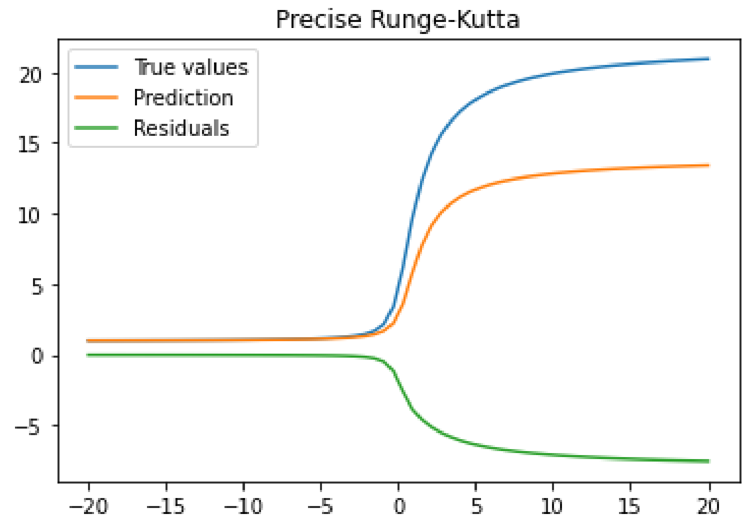

Figure 2: Solution of the initial-value-problem for more accurate RK4

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Our ratio of lead over uranium is:

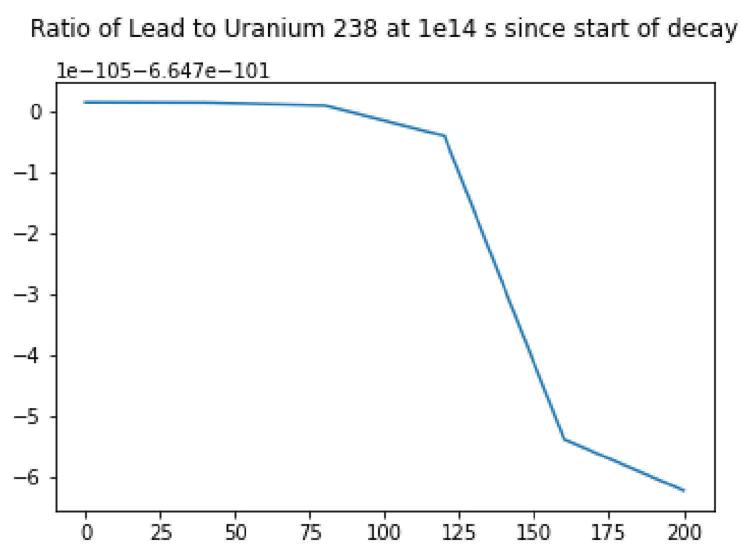

Figure 3: Lead over Uranium 238

In this figure, we see that the ratio of lead to uranium is decreasing, which shouldn't happen even though we use the Randau method to integrate.

If instead we want a ratio of Thorium 230 to Uranium 234, we can obtain:

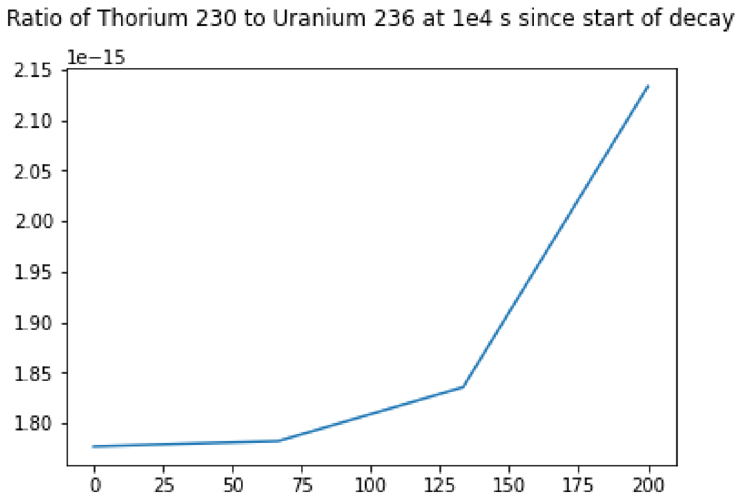

Figure 4: Thorium over Uranium

Which is simply consistent with our knowledge, as Uranium 234 desintegrates at a quicker pace than Thorium 230

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We have the equation:

$$\frac{z - z_0}{a} = (x - x_0)^2 + (y - y_0)^2$$
$$x' = (x - x_0)^2, y' = (y - y_0)^2$$
$$z = a \cdot (x' + y' + z_0)$$

Assuming a centered paraboloid, $x' = x^2$, $x' = y^2$ I can't really figure it out in time