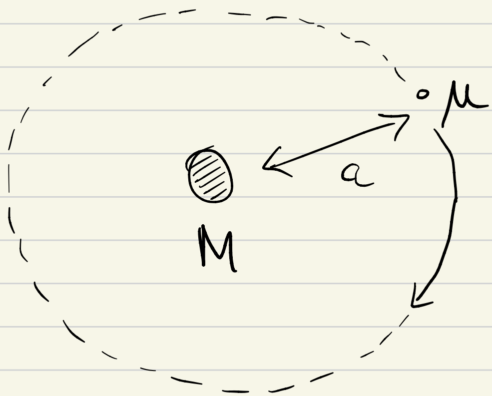
07/11/2024

GW Signals From A Binary

⟹ Consider a binary with components m_1, m_2.

$$M = m_1 + m_2$$
$$\left.\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}\right\} \quad \mu = \frac{m_1 m_2}{M}$$

"reduced mass"



KEPLER III

$$\mu \omega^2 a = \frac{\mu M}{a^2}$$

$$\underbrace{\omega}_{=2\pi f_{orb}} = \sqrt{\frac{M}{a^3}}$$

$$h^{ij} = \frac{2}{r}\frac{d^2}{dt^2}\left[I^{ij}(t-r)\right]$$

⟹ Reduced mass in orbit around total mass

Binary in (xy) plane
$$\left\{\begin{aligned} x^i(t) &= \left(a\cos\Phi(t),\ a\sin\Phi(t),\ 0\right) \\ \frac{d\Phi}{dt} &= \omega \end{aligned}\right.$$

$$I^{ij} = \begin{bmatrix} \frac{\mu a^2}{2}\left(1+\cos 2\Phi(t)\right) & \frac{\mu a^2}{2} \sin 2\Phi(t) & 0 \\ \frac{\mu a^2}{2} \sin 2\Phi(t) & \frac{\mu a^2}{2}\left(1-\cos 2\Phi(t)\right) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\ddot{I}^{ij} = \begin{bmatrix} -2\mu a^2\omega^2 \cos 2\Phi(t) & -2\mu a^2\omega^2 \sin 2\Phi(t) & 0 \\ -2\mu a^2\omega^2 \sin 2\Phi(t) & 2\mu a^2\omega^2 \cos 2\Phi(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$h^{ij} = -\frac{4\mu a^2\omega^2}{r} \underbrace{\begin{bmatrix} \cos 2\Phi & \sin 2\Phi & 0 \\ \sin 2\Phi & -\cos 2\Phi & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\begin{pmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{pmatrix}}$$

CHIRP MASS $\Longrightarrow$ $$\mathcal{M} = \mu^{3/5} M^{2/5}$$

$$= \left[\frac{q}{(1+q)^2}\right]^{3/5} M$$

$(q = M_2/M_1)$

$$h_+ = \underline{-4\mu a^2\omega^2} \times \cos 2\Phi(t)$$

$$= \frac{-4\mathcal{M}^{5/3}\omega^{2/3}}{r} \times \cos 2\Phi(t)$$

$$h_\times = \underline{-4\mu a^2\omega^2} \times \sin 2\Phi(t)$$

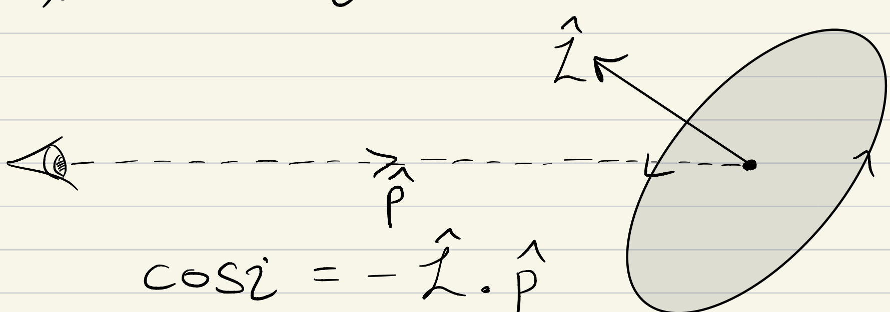
$$= \frac{-4\mathcal{M}^{5/3}\omega^{2/3}}{r} \times \sin 2\Phi(t)$$

$$\Rightarrow \boxed{\begin{aligned} \omega_{gw} &= 2\omega \\ f_{gw} &= 2 f_{orb} \end{aligned}}$$

... for binary <u>NOT</u> in (xy) plane $\Rightarrow$

$$h_+(t) = -2\mathcal{M}^{5/3}\omega(t)^{2/3}(1+\cos^2 i)\cos 2\Phi(t)$$

$$h_\times(t) = -4\mathcal{M}^{5/3}\omega(t)^{2/3}\cos i \sin 2\Phi(t)$$



$$\cos i = -\hat{L}\cdot\hat{p}$$

FACE-ON $\Rightarrow i = 0$; EDGE-ON $\Rightarrow i = \pi/2$

$\Longrightarrow$ How do we get $\omega(t)$ and $\Phi(t)$?

$$\frac{d\omega}{dt} = \frac{96}{5} \mathcal{M}^{5/3} \omega^{11/3}$$

$$\omega(t) = \omega_0 \left[1 - \frac{256}{5} \mathcal{M}^{5/3} \omega_0^{8/3} (t - t_0) \right]^{-3/8}$$

$$\frac{d\Phi}{dt} = \omega$$

$$\Phi(t) = \Phi_0 + \frac{1}{32 \mathcal{M}^{5/3}} \left[\omega_0^{-5/3} - \omega(t)^{-5/3} \right]$$

ASIDE:

$$\frac{d\omega}{dt} = \frac{96}{5} \mathcal{M}^{5/3} \omega^{11/3} \frac{1 + \frac{73}{24} e^2 + \frac{37}{96} e^4}{(1 - e^2)^{7/2}}$$

$$\frac{de}{dt} = \frac{-304}{15} \mathcal{M}^{5/3} \omega^{8/3} e \, \frac{1 + \frac{121}{304} e^2}{(1 - e^2)^{5/2}}$$

$\Longrightarrow$ What about observables?

$$z(t, \hat{\Omega}) = \frac{1}{2} \frac{p^i p^j}{(1 + \hat{\Omega} \cdot \hat{p})} \Delta h_{ij}$$

$$= \sum_{A = +, \times} \underbrace{F^A(\hat{\Omega}, \psi)}_{\frac{1}{2} \frac{p^i p^j}{(1 + \hat{\Omega} \cdot \hat{p})} e^A_{ij}(\hat{\Omega}, \psi)} \Delta h_A(t)$$

"ψ" is GW polarization angle $\Longleftarrow$

$$s(t,\hat{\Omega}) = \int_0^t z(t',\hat{\Omega})\,dt'$$

$$= \sum_{A=+,\times} F^A(\hat{\Omega},\psi)\int_0^t \Delta h_A(t')\,dt'$$

$$= \sum_{A=+,\times} F^A(\hat{\Omega},\psi)\Big[\underbrace{s_A(t)}_{\text{EARTH}} - \underbrace{s_A(t_p)}_{\text{PULSAR}}\Big]$$

$$s_+(t) = \frac{\mathcal{M}^{5/3}}{D_L\,\omega(t)^{1/3}}\left[-(1+\cos^2 i)\sin 2\Phi(t)\right]$$

$$s_\times(t) = \frac{\mathcal{M}^{5/3}}{D_L\,\omega(t)^{1/3}}\left[2\cos i\cos 2\Phi(t)\right]$$

$\int_0^t h_{+,\times}(t')\,dt'$ involves $\int_0^t \sin 2\Phi(t')\,dt' \approx \frac{-1}{\omega}\cos 2\Phi(t)$

and cosine equivalent for weakly evolving sources.

$\Longrightarrow$ i.e. $d\omega/dt \sim 0$ over $\sim$ decades.

Earth term : # parameters = 8

$$\Longrightarrow \{\underbrace{\theta,\phi}_{\text{sky location}},\underbrace{i,\psi}_{\text{orientation}},\Phi_0,\mathcal{M},D_L,\omega_0\}$$

degenerate in Earth term

Pulsar term : # additional parameters $= N_p$

$\Rightarrow \{ L_p \ \forall \text{ pulsars} \}$

$$t_p = t - L_p(1 + \hat{\Omega}.\hat{p}).$$

... if $L \sim$ kpc $\Rightarrow \underbrace{\Delta t \sim 10^3 \text{ years.}}_{\text{lots of frequency + phase evolution.}}$

$$\omega_p = \omega_0 \left[1 + \frac{256}{5} \mathcal{M}^{5/3} \omega_0^{8/3} L_p (1 + \hat{\Omega}.\hat{p}) \right]^{-3/8}$$

i.e. $\omega_p < \omega_0$

$\Rightarrow$ what is $\Delta\omega = \omega - \omega_p$?

$\Delta\omega \sim \frac{d\omega}{dt} \Delta t$ --- $\Delta t \sim 1000$ years $\sim 10^{10}$ s

$\mathcal{M} = 10^9 M_\odot$, $\omega = 10^{-8}$ Hz $\longmapsto \Delta\omega \sim$

$\mathcal{M} = 10^{10} M_\odot$, $\omega = 10^{-7}$ Hz $\longmapsto \Delta\omega \sim$

$$\Phi(t_p) = \Phi_0 + \underbrace{\Phi_p}_{= \frac{1}{32\mathcal{M}^{5/3}} \left[\omega_0^{-5/3} - \omega(t_{p,0})^{-5/3} \right]} + \frac{1}{32\mathcal{M}^{5/3}} \left[\omega(t_{p,0})^{-5/3} - \omega(t_p)^{-5/3} \right]$$

⇒ In principle, all we need to compute the PULSAR TERM is the stuff that goes into the EARTH TERM, + L_p.

⇒ In practice, σ_{L_p} can be tens % and therefore $\gg \lambda_{gw}$.

↳ easier to search for separate pulsar distance and pulsar-term phase parameters.

∴ CW (continuous wave) PTA search has

intrinsic RN	$\xrightarrow{\text{at least}}$	$2N_p$
chromatic terms	$\xrightarrow{\text{at least}}$	$2N_p$ (0 for DMX)
GW background	$\xrightarrow{\text{at least}}$	2
CW parameters	$\xrightarrow{\text{at least}}$	$8+2N_p$

$\geq 6N_p + 10$ parameters

IPTA DR3 ... $N_p \sim 120 \Rightarrow \geq 730$.