Astro 123
Week 10

Massive star post-main-sequence evolution is this week's topic.

Read Ostlie & Carroll Ch. 15: The Fate of Massive Stars – sections 15.1 - 15.3.

The warm-up problems this week are Q3 and Q10. Please submit your solutions by 11 pm on Monday night.

Many of the same principles we've seen in low- and intermediate-mass stellar evolution pertain to massive stars, but with some interesting additions related to very high luminosities, strong stellar winds, and especially very late-stage evolution, heavy-element nucleosynthesis, and core-collapse supernovae. (We will not discuss the gamma ray burst and cosmic ray sections at the end of the chapter but obviously feel free to read them; they are quite interesting.)

Q1: Wolf-Rayet stars are characterized by their lack of hydrogen, compactness, and very strong winds. They were first identified as a distinct class of stars (in the 19th Century) due to their broad emission lines (and lack of absorption lines). For this problem, on the "effective photospheres" of W-R stars, assume a typical W-R star has $L = 1.4 \times 10^5 L_{sun}$, $T = 50,000$K, $M = 20M_{sun}$, a wind mass-loss rate of $Mdot = -\frac{dM}{dt} = 10^{-5}\ M_{sun}\ yr^{-1}$, and a terminal wind velocity of $v_\infty = 1000$ km s^{-1}. In this problem, we'll see that the Stefan-Boltzmann law definition of a star's radius is not necessarily consistent with the $\tau = \frac{2}{3}$ definition. And if the latter is more relevant, that the radius of the star can be wavelength-dependent (!).

(a) Use the Stefan-Boltzmann based definition to compute the radius of this typical Wolf-Rayet star and then compare its surface gravity to that of the Sun. (Be ready to explain how the effective temperature of a star with a mostly-continuum spectrum could be estimated.)

(b) Derive an expression that relates the wind mass-loss rate to the velocity and density of the wind at an arbitrary radius. One way to do this is to start with the mass-continuity equation (of stellar structure) and multiply both sides by the velocity, using the definition, $v = \frac{dr}{dt}$, and the chain rule, as needed.

(c) Derive, then, a general expression for the radius of optical depth $\frac{2}{3}$ as a function of radius, mass-loss rate, opacity, and velocity. You may assume the velocity is constant[1]. You may assume you're looking along the "central ray" – pointed right at the center of the star, so that the problem is one-dimensional and all the variables are only a function of the radial coordinate, *r*.

(d) In the radio part of the spectrum, the opacity is well approximated by $\kappa = 0.15\lambda^2$ $m^2\ kg^{-1}$ where the wavelength is in centimeters (not meters... cm are traditional for the radio spectrum; think 21 cm emission of atomic hydrogen). Compute the effective radius of the star (based on the optical depth criterion) at $\lambda = 3$ cm and at $\lambda = 21$ cm. Express your answer in terms of the stellar radius, *R*, you computed in part (a).

(e) Now, think back to the fact that there are emission lines rather than absorption lines in W-R spectra. The photosphere and wind are about the same temperature, so the emission lines do not arise from the usual Kirchoff's laws physics of spectra, where hot optically thin gasses in front of a cooler background. Recalling that strong spectral lines have high opacity compared to the nearby continuum, can you explain why W-R stars have emission line spectra? You might think about the flux being proportional to intensity times the solid angle of the source (or equivalently, times the radius squared).

Q2: Ostlie & Carroll problem 15.2. Note that if you've forgotten how to include interstellar extinction (dimming due to dust scattering) in this sort of magnitude-related calculation, you can look in Ch. 3 or in Ryden & Peterson. There are three parts to this problem; please do them all. Note that for part (a) you'll first compute the absolute magnitude of eta Car and then convert that to a luminosity. You should express that luminosity in terms of L_{sun}, so it makes sense to use the Sun's properties as the comparison star in the magnitude-based calculation. While you're doing this... think about how bright eta Car would've been if it were at a distance from the Earth of 10 parsecs (which is still almost four times farther away than Sirius). Look up the apparent magnitude of the full moon and comment on the comparison. Note that the distance to eta Car is stated on the first page of the chapter (you'll need it for the next problem, too). And the extinction correction can simply be added to the right-hand side of eq. 3.6 (it doesn't actually appear to be stated in that chapter, but it is in the ISM chapter of Ryden and Peterson).

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[1] If we're looking into the wind from the outside, the velocity is approximately v_infinity for much of the wind down to a given optical depth – i.e. the wind accelerates off the surface quite rapidly and reaches the terminal velocity not far from the star's hydrostatic surface.

Q3W: Search for images of eta Car and be ready to show your favorite. Then do Carroll \& Ostlie problem 15.3.

Take a look at the evolution sequences of massive stars (both in the text and Fig. 15.3). Focus on the 25 M_{sun} star. Where in the diagram, along the evolutionary track, is the star at each of the particular phases in Conti's scheme? Think about some of the basic astrophysics we studied last week (shell burning vs.\ core burning, especially, and how you can estimate the radius from the location in the HR diagram). What is the fusion source (reactions and location – core vs. shell(s)) – at each phase? *Note that this is not a numbered question; please come prepared to answer/discuss, but nothing to write up and hand in.*

Q4: Problem 15.4.

Q5: What is the physical mechanism behind a type II supernova explosion? What are photodisintegration and electron electron capture/inverse-beta-decay? What fundamental force(s) is/are the source of the tremendous energy in the explosion? What forms does this energy take – and roughly how much in each form? What role do the neutrinos play in the explosion? What governs when the time of maximum brightness occurs (think about optical depth in an "expanding photosphere").

Q6: Problem 15.6.

Q7: Problem 15.7.

Q8: Summarize/explain what the r- and s-processes are. Find a table of nuclides online and locate ^{56}Fe. What is the step-wise process that can transform this isotope into ^{61}Ni?

Q9: Problem 15.9.

Q10W: Look at the figure in Ch. 15 showing spectra of various supernovae. Based on the widths of absorption lines, which one has the fastest expansion velocity? Use the Doppler shift equation and the half-width-at-half-maximum of the Ca II line to estimate this velocity (to one significant figure).

Q11: Problem 15.11.