Bulk Heating Effects as Tests for Collapse Models

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We discuss limits on the noise strength parameter in wave function collapse models implied by bulk heating effects, and examine the role of the noise power spectrum in comparing experiments of different types. We suggest possible new bulk heating experiments that can be performed subject to limits placed by natural heating from radioactivity and cosmic rays. The proposed experiments exploit the vanishing of thermal transport in the low temperature limit.

There is increasing interest in testing wave function collapse models [1], and in particular the continuous spontaneous localization (CSL) model, by searching for effects associated with the small noise which drives wave function collapse when nonlinearly coupled in the Schrödinger equation. The original proposals for the noise coupling strength were so small that devising suitable experiments was problematic, but the situation has changed with the suggestion [2] that latent image formation, such as deposition of a developable track in an emulsion or in an etched track detector, already constitutes a measurement embodying wave function collapse. A recent cantilever experiment of Vinante et al. [3] has set bounds consistent with the enhanced parameters suggested in [2], and reports a possible noise signal. Thus, it is timely to consider further experiments [4] which could detect or rule out a noise coupling with the strength suggested by [3].

For a body comprised of a group of particles of total mass M, the secular center-of-mass energy gain for white noise with mass-proportional coupling is given by the standard formula [5]

$$\frac{dE}{dt} = \frac{3}{4}\lambda\frac{\hbar^2}{r_C^2}\frac{M}{m_N^2} \quad , \tag{1}$$

with m_N the nucleon mass and λ the coupling parameter for white noise with noise correlation length r_C and no frequency spectrum cutoff. Dividing by M, Eq. (1) can be rewritten as a formula for the energy gain rate per unit mass,

$$\frac{dE}{dt\,dM} = \frac{3}{4}\lambda\frac{\hbar^2}{r_C^2}\frac{1}{m_N^2} \quad . \tag{2}$$

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For the noise coupling parameter $\lambda = 10^{-7.7}\mathrm{s}^{-1}$ suggested in [3], and the conventional value $r_C = 10^{-5}$cm, Eq. (2) corresponds to

$$\frac{dE}{dt\,dM} \simeq 40\frac{\mathrm{MeV}}{\mathrm{g\ s}} \simeq 0.64 \times 10^{-8}\frac{\mathrm{W}}{\mathrm{kg}} \quad . \tag{3}$$

Since heating rates of 100pW/kg $= 10^{-10}$W/kg are attained in low temperature experiments [6], with the limit accounted for by modeling energy deposition from radioactive decays and penetrating muons [7], the residual heating from unknown sources is limited to roughly 10^{-11}W/kg. This means that the effective λ for bulk heating is at most $3.1 \times 10^{-11}\mathrm{s}^{-1}$, ruling out a collapse model white noise interpretation of the excess noise reported in [3].

A similar bound on the bulk heating rate is given by the earth energy balance. Table 6.3 of de Pater and Lissauer [8] gives the luminosity to mass ratio for solar system objects, with a value 6.4×10^{-12}W/kg for earth. Estimates of primordial and radiogenic sources of earth heat roughly account for this, but have uncertainties that could allow $\sim 3 \times 10^{-12}$W/kg to come from unknown heating sources. This places a limit of $\sim 10^{-11}\mathrm{s}^{-1}$ on the effective λ for bulk heating. There is a caveat here, because as noted in [2], when the effects of dissipation are included, as in the model of Bassi, Ippoliti, and Vacchini [9], the rate of heat production can vanish at large times where a limiting temperature is reached. For example, with the parameters of [9], a limiting temperature of 0.1 K is reached on a time scale of billions of years, giving a current noise-induced Earth heat production rate smaller than given by Eq. (2), permitting a larger effective λ.

The effective CSL model λ for bulk heating experiments can be strongly reduced if the noise is non-white, with a power spectrum cutoff. This is already suggested by experimental limits on spontaneous gamma ray emission from germanium [10], which shows that the noise strength suggested in [2] is ruled out unless the noise power spectrum cuts off at an angular frequency below $\sim 15\,\mathrm{keV}/\hbar \sim 2\times 10^{19}\mathrm{s}^{-1}$. For bulk heating of solids the noise couples through longitudinal acoustic phonon excitation, and with $r_c \sim 10^{-5}$cm, heating takes place only if the noise has frequency components of at least $\omega_L(|\vec{q}| = r_c^{-1}) \sim v_s|\vec{q}| \sim 0.4 \times 10^{11}\mathrm{s}^{-1}$, with $\omega_L(\vec{q})$ the longitudinal phonon frequency at wave number $\vec{q}$, and v_s the speed of sound (which for this estimate we have taken as 4000 m/s characteristic of copper at low temperature). Quantitative calculation [11] shows that for non-white noise with power spectrum $\lambda(\omega)$ heating a solid by phonon excitation, Eq. (2) is replaced by

$$\frac{dE}{dt\,dM} = \frac{3}{4}\lambda_{\mathrm{eff}}\frac{\hbar^2}{r_C^2}\frac{1}{m_N^2} \quad , \tag{4}$$

with $\lambda_{\rm eff}$ given by

$$\lambda_{\rm eff} = \frac{2}{3\pi^{3/2}} \int d^3 w e^{-\vec{w}^2} \vec{w}^2 \lambda(\omega_L(\vec{w}/r_c)) \quad . \tag{5}$$

When $\lambda(\omega)$ is a constant independent of ω, Eq. (5) reduces to $\lambda_{\rm eff} = \lambda$, and Eq. (2) is recovered, but when there is a frequency cutoff below the phonon excitation frequency, the effective noise coupling is strongly reduced. Thus, if the noise reported in [3] at the very low cantilever frequency of $8174{\rm s}^{-1}$ were due to CSL, it would be further evidence for non-white CSL model noise.

Assuming now a maximum value $\lambda_{\rm eff} \sim 10^{-11}{\rm s}^{-1}$ consistent with low temperature experiments and non-dissipative earth heating, let us explore possible alternative experiments for detecting or further bounding $\lambda_{\rm eff}$. *Feasibility of such experiments assumes (a) that the background heating rate from cosmic ray muons and radioactive decays can be reduced much below* $3 \times 10^{-12}{\rm W/kg}$ *by shielding, underground operation, and careful choice of materials, and (b) all other known sources of heat leaks in ultralow temperature experiments, such as vibrations, relaxation from two-level systems, and hydrogen ortho-para conversion, have been suppressed.* To present estimates we multiply $3 \times 10^{-12}{\rm W/kg}$ by the solid density ρ, so that the maximum allowed value of $\lambda_{\rm eff}$ corresponds to a volume heating rate of

$$H = 3 \times 10^{-15} \rho {\rm W/cm}^3 \quad , \tag{6}$$

with ρ the density in units ${\rm g/cm}^3$. We consider two geometries for which the heat transport problem is effectively one dimensional and easily solved.

1. For a sphere of radius R cm and density ρ, the total heating rate is $4\pi R^3 H/3$, and so in equilibrium the rate of escape of heat from unit area of the surface will be

$$\dot{Q}_{\rm sphere} = RH/3 \tag{7}$$

in units ${\rm W/cm}^2$. This must balance the rate of transport of heat per unit area from the surface of the sphere, at temperature T_1, to the surrounding cryostat surfaces, at temperature T_2. This is given [6] in units ${\rm W/cm}^2$ by the formula

$$\dot{Q}_{\rm transport} = 5.67 \times 10^{-12} \epsilon (T_1^4 - T_2^4)[K]^4 + 0.02 a P[{\rm mbar}](T_1 - T_2)[K] \quad , \tag{8}$$

with the first term the Stefan-Boltzmann equation for radiative heat transfer, and the second term coming from gas particle conduction in the cryostat. Here $a \leq 1$ is an "accommodation coefficient" for gas particles on the cryostat walls, which can be as small as 0.02 for a clean metal surface in contact with helium gas, and $\epsilon \leq 1$ is the emissivity for radiative transfer. At equilibrium, the surface temperature of the sphere T_1 is determined by equating $\dot{Q}_{\rm sphere}$ to $\dot{Q}_{\rm transport}$. Taking as an example the density of lead $\rho = 11.4\,{\rm g/cm^3}$, and a sphere of radius $R = 50\,{\rm cm}$ (which would fit in the CUORE underground experiment cryostat [12]) , we have

$$\dot{Q}_{\rm sphere} = 5.7 \times 10^{-13} \quad , \tag{9}$$

while taking $a = 0.02$ and $P = 10^{-6}$mbar gives

$$\dot{Q}_{\rm transport} = 5.67 \times 10^{-24} \epsilon (T_1^4 - T_2^4)[{\rm mK}]^4 + 4 \times 10^{-13} (T_1 - T_2)[{\rm mK}] \quad , \tag{10}$$

both in units W/cm^2. Evidently, for millikelvin T_1 and T_2 radiative heat transfer is completely negligible and Eqs. (9) and (10) are of similar size. The heating rate H can therefore be estimated by measuring the surface equilibrium temperature as a function of the gas pressure.

Similar reasoning gives the temperature distribution inside a sphere of material with thermal conductivity $k(T)$. At a given distance r from the center of the sphere, the energy transport rate through the spherical surface of radius r is equal to

$$E_{\rm out} = -4\pi r^2 k(T) \frac{dT}{dr} \quad , \tag{11}$$

which at equilibrium must balance the heating rate of the volume within radius r,

$$E_{\rm in} = \frac{4\pi}{3} r^3 H \quad , \tag{12}$$

giving the differential equation

$$-k(T) \frac{dT}{dr} = \frac{1}{3} r H \quad . \tag{13}$$

Integrating from the center of the sphere at radius 0 to radius R, with respective temperatures T_c and T_1, this gives

$$-\int_{T_c}^{T_1} k(u) du = \frac{R^2 H}{6} \quad . \tag{14}$$

For $k(u) = \hat{k}_0 u^\beta$, this becomes

$$-\frac{\hat{k}_0}{1+\beta} (T_1^{1+\beta} - T_c^{1+\beta}) = \frac{R^2 H}{6} \quad , \tag{15}$$

which gives

$$(T_c^{1+\beta} - T_1^{1+\beta})^{1/(1+\beta)} = \left[\frac{1+\beta}{\hat{k}_0 K^{1+\beta}} \frac{R^2 H}{6}\right]^{1/(1+\beta)} [\mathrm{K}] \quad . \tag{16}$$

To give a numerical estimate, for the good thermal insulator Torlon 4203 [13], with density 1.42 g cm^{-3} and with $k(T) = 6.13 \times 10^{-3} (T/\mathrm{K})^{2.18} \mathrm{W}/(\mathrm{m\ K})$, so that $\beta = 2.18$ and $\hat{k}_0 \mathrm{K}^{3.18} = 6.13 \times 10^{-3} \mathrm{W/m}$, the right hand side of Eq. (16) for $R = 50\,\mathrm{cm}$ is 6.1 mK.

We have used Torlon as a convenient example for estimates, but probably it would not be the most suitable material for these experiments. As discussed by Pobell [6], amorphous materials are usually rich in two-level systems leading to slow relaxation processes with time-dependent heat release. Moreover, polymers and plastics like Torlon may easily absorb impurities which can also give unreliable thermal properties. For realistic experiments, it would be better to use crystalline insulators (such as sapphire or silicon) or superconducting metals. In both cases the thermal conductivity is much higher than that of Torlon at $T > 100\mathrm{mK}$, but drops as T^3 and approaches the conductivity of plastic materials like Torlon in the mK range.

2. Another geometry with easily solvable heat transfer would use a long cylinder (or parallelepiped, or more generally a rod of uniform cross section) of length L with the "near" end fastened to a heat sink that provides a large enough heat transport rate so that the heat transport from all other surfaces given by Eq. (8) can be ignored. Then the heat transport problem is one dimensional, and the analog of Eq. (13) is the differential equation

$$k(T)\frac{dT}{dz} = (L - z)H \quad . \tag{17}$$

Again taking $k(u) = \hat{k}_0 u^\beta$ and integrating, the analog of Eq. (16) relating the "far" to the "near" end temperatures at $z = L$ and $z = 0$ respectively is

$$(T_{\mathrm{far}}^{1+\beta} - T_{\mathrm{near}}^{1+\beta})^{1/(1+\beta)} = \left[\frac{1+\beta}{\hat{k}_0 K^{1+\beta}} \frac{L^2 H}{2}\right]^{1/(1+\beta)} [\mathrm{K}] \quad . \tag{18}$$

For a rod with the parameters quoted above, and $L = 50\,\mathrm{cm}$, the right hand side of Eq. (18) is 8.6 mK.

In a variant of the rod geometry, one can attach to the "far" end of the rod an object of larger size, which acts as a "CSL noise absorber". The CSL heat released in the absorber can be much larger than that in the rod. The temperature at the "far" end of the rod will then be determined by matching the heat flow per unit area within the rod with the heat flow per unit area $\dot{Q}_{\rm ABS}$ entering the rod from the absorber, so that Eq. (17) becomes

$$k(T)\frac{dT}{dz} = (L-z)H + \dot{Q}_{\rm ABS} \quad , \tag{19}$$

which on integration gives

$$(T_{\rm far}^{1+\beta} - T_{\rm near}^{1+\beta})^{1/(1+\beta)} = \left[\frac{1+\beta}{\hat{k}_0 K^{1+\beta}}\left(\frac{L^2 H}{2} + \dot{Q}_{\rm ABS} L\right)\right]^{1/(1+\beta)} [{\rm K}] \quad . \tag{20}$$

This provides the freedom of independently tuning the rod thermal conductivity (by making the cross section arbitrarily small) and the heat input from CSL (by choosing the material and size of the CSL absorber). From an experimental point of view this gives a very flexible design. Of course, one must make sure that the heat transport mechanisms of Eq. (8) from the surfaces of both the rod and the absorber are kept negligible.

To conclude, bounds on the effective noise coupling $\lambda_{\rm eff}$ for bulk heating of solids show that in designing experiments to test the enhanced rate [2], [3] that makes latent image formation a measurement, it will be important to take the power spectrum of the noise into account. Because thermal transport rates vanish at zero temperature, millikelvin and submillikelvin experiments to further improve the bounds on $\lambda_{\rm eff}$ may be feasible. However, underground operation is probably necessary in order to evade the limiting heating rate from cosmic rays.

We wish to thank Angelo Bassi for helpful comments.

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