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Quantum Mechanics with Spontaneous Localization and the Quantum Theory of Measurement (*).

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Summary. — Quantum mechanics with spontaneous localization (QMSL) is a recently proposed stochastic modification of the N-body Schrödinger equation consistent both with microphysics and macrophysics. QMSL is applied here to the measurement problem. It is shown that the replacement of standard quantum mechanics by QMSL has the only effect of producing an actual reduction of the wave function.

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1. – Introduction.

The quantum theory of measurement, in spite of the many efforts which have been devoted to clarify the conceptual problems that it has raised, has

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not yet found a completely satisfactory formulation. We do not want to discuss here extensively this matter; we shall limit ourselves to recall some basic difficulties which emerge already in the most simplified description of the measurement process—von Neumann's ideal measurement scheme. As is well known, these difficulties cannot be overcome by simply taking into account the complexity of the apparatus or its nonideal functioning. For a lucid discussion of these points we refer the reader to the analysis presented by D'ESPAGNAT [1].

Let A be the apparatus devised to measure the observable l of a system S. The ideal measurement scheme assumes [2] the existence of a unitary operator U representing the time evolution during the S-A interaction such that

$$U|\psi_n\rangle|A_0\rangle = |\psi_n\rangle|A_n\rangle\,, \tag{1.1}$$

where $|\psi_n\rangle$ is the system eigenstate belonging to the eigenvalue l_n of l, $|A_0\rangle$ is a suitable initial state of A and $|A_n\rangle$ are eigenstates of an observable of A whose further detection yields the desired information about the result of the measurement. When the apparatus is triggered by the system in the state $|\psi\rangle = \sum_n c_n|\psi_n\rangle$, the linear nature of quantum evolution implies that

$$U|\psi\rangle|A_0\rangle = \sum_n c_n|\psi_n\rangle|A_n\rangle\,. \tag{1.2}$$

Equation (1.2) has the desired feature of exhibiting a complete correlation between the eigenstates $|\psi_n\rangle$ of the measured observable and the final apparatus states $|A_n\rangle$. This same equation, however, gives rise to the basic problem of the quantum theory of measurement, when it is confronted with the quantum-mechanical postulate of the wave packet reduction, claiming that, after the measurement, one is left with a statistical mixture of the states $|\psi_n\rangle|A_n\rangle$ with weights $|c_n|^2$. The problem arises from the fact that, if one accepts that any bounded self-adjoint operator of the S+A system can be measured, the physical predictions about the outcomes of later experiments, which can be derived from the pure state (1.2) and from the described statistical mixture, are different.

It has to be noted that eq. (1.2) does not describe the completion of the measurement process. In fact, to get the desired information about the meas-

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(1) B. D'ESPAGNAT: *Conceptual Foundations of Quantum Mechanics* (Benjamin, Reading, Mass., 1976).

(2) The measurement scheme summarized by eqs. (1.1) and (1.2) cannot be strictly correct when there are additive conserved quantities and the operator l does not commute with them. However, the unavoidable malfunctioning of the apparatus which occurs in such cases can be controlled to any desired accuracy by making the apparatus sufficiently massive. For a detailed analysis of this point see, *e.g.*, G. C. GHIRARDI, A. RIMINI and T. WEBER: *J. Math. Phys.* (*N. Y.*), **24**, 2454 (1983).

ured system, one has to find out the state of the measuring apparatus. This requires the use of a second apparatus B devised to measure A, and so on. In this way one ends up with the von Neumann chain

$$U\Big(\sum_n c_n|\psi_n\rangle\Big)|A_0\rangle|B_0\rangle|C_0\rangle \ldots = \sum_n c_n|\psi_n\rangle|A_n\rangle|B_n\rangle|C_n\rangle \ldots , \tag{1.3}$$

where U describes now all subsequent interactions with the various measuring apparatuses. The final state in eq. (1.3) is again a pure state, not a statistical mixture, so that, to reconcile this fact with the wave packet reduction, one has to break the chain, *i.e.* to assume that some element of it has the property of inducing the transition to the statistical mixture. Due to the complete correlation of the indices, it is completely irrelevant at what stage the chain is broken. However, the transition to the statistical mixture cannot be described as a quantum process governed by the linear evolution equations of quantum mechanics.

Various solutions for this puzzling situation have been considered, corresponding to very different attitudes. Some authors plainly accept two principles of evolution for physical systems, some invoke the consciousness of the observer as responsible for the transition to the statistical mixture, some claim that, taking into account the irreversible character of the amplification process always present in a quantum measurement, all problems are solved. At any rate, a remark by JAUCH [3] is of prominent importance to make the theoretical description of a quantum measurement compatible with textbook quantum mechanics. This remark is that the states (1.2) or (1.3) cannot be distinguished from the corresponding statistical mixtures if we admit that the class of observables for a macro-object (like the apparatus pointer) is restricted to an Abelian set. On the basis of his result JAUCH claimed that there is no problem in the theory of quantum measurement.

Without entering into the details of this debate, we say that we are not completely satisfied with Jauch's point of view because of two reasons. The first reason is that it is based in an essential way on the *a priori* identification of a class of objects for which the quantum rules are not fully valid. The second reason is that it clearly points towards a nonobjectivistic interpretation of quantum mechanics.

If one assumes that macroscopic objects are particular quantum systems, then the possible occurrence of linear superpositions of macroscopically distinguishable states gives rise to problems similar to the ones discussed above, even when the behaviour of a macroscopic object by itself and not its particular role as a measuring instrument is considered. Recently, considerable progress

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[3] J. M. JAUCH: *Helv. Phys. Acta*, **37**, 293 (1964).

towards a satisfactory description of macroscopic bodies has been made in the framework of quantum mechanics with repeated measurements ([4,5]). Successively, some of us have shown ([6,7]) how one can obtain a unified description of microscopic and macroscopic objects by considering a modification of quantum mechanics to which we shall refer as quantum mechanics with spontaneous localization (QMSL). The basic assumption of QMSL is that all microconstituents of any physical system are subjected to processes, corresponding formally to approximate position measurements, occurring at random times with appropriate mean frequencies. In ref. ([6,7]) we have shown that, with very high accuracy (for suitable choices of the parameters of the spontaneous localizations), QMSL leaves unaltered the quantum predictions for microscopic particles and for the internal dynamics of macro-objects, while it leads to the classical behaviour for the external motion of macro-objects, including the prohibition of superpositions of distant states.

Taking into consideration the basic physical features of QMSL, one can guess that it should also yield a consistent objectivistic solution of the measurement problem. Suggestions in this sense have already been given in ref. ([7]) and a special discussion has been presented very recently by BELL ([8]). The present paper is devoted to an investigation of the consequences of QMSL for the measurement process.

In sect. **2** we sketch and discuss a simplified model for the measurement process of a type already considered in the literature. Section **3** gives a brief description of QMSL. Section **4** investigates the consequences of a consistent application of QMSL to the model presented in sect. **2**. It is shown that a satisfactory solution of the problem of quantum measurement emerges naturally from the formalism. Section **5** is devoted to show that the conclusions obtained in sect. **4** for the particular model considered there hold in general.

([4]) A. BARCHIELLI, L. LANZ and G. M. PROSPERI: *Nuovo Cimento B*, **72**, 79 (1982); *Found. Phys.*, **13**, 779 (1983); in *Proceedings of the International Symposium on the Foundations of Quantum Mechanics*, edited by S. KAMEFUCHI, H. EZAWA, Y. MURAYAMA, M. NAMIKI, S. NOMURA, Y. OHNUKI and T. YAJIMA (The Physical Society of Japan, Tokyo, 1983), p. 165.

([5]) A. RIMINI: in *Proceedings of the Meeting of Theoretical Physics, Amalfi, 1983*, edited by A. GIOVANNINI, F. MANCINI, M. MARINARO and A. RIMINI (ESI, Naples, 1984), p. 275.

([6]) G. C. GHIRARDI, A. RIMINI and T. WEBER: in *Quantum Probability and Applications* II, edited by L. ACCARDI and W. VON WALDENFELS, *Lecture Notes in Mathematics*, Vol. **1136** (Springer-Verlag, Berlin, 1985), p. 223; in *Fundamental Aspects of Quantum Theory*, edited by V. GORINI and A. FRIGERIO (Plenum Press, New York, N. Y., and London, 1986), p. 57.

([7]) G. C. GHIRARDI, A. RIMINI and T. WEBER: *Phys. Rev. D*, **34**, 470 (1986).

([8]) J. S. BELL: in *Schrödinger—Centenary Celebration of a Polymath*, edited by C. W. KILMISTER (Cambridge University Press, Cambridge, 1987), p. 41.

2. – A model for the measurement process.

In this section we describe a model for the interaction between the measured system and the measuring apparatus, corresponding to a reasonable schematization of an ideal measurement process. Models analogous to the one presented here have already been discussed by various authors ([1,9-11]).

In many practical cases the triggering of the apparatus induced by its interaction with the measured system results, after some elementary processes have been properly amplified, in the fact that a macroscopic part of the apparatus—the pointer—takes a well-defined position. The possible final positions of the pointer correspond to the possible different eigenvalues of the measured quantity. If one is not particularly interested in describing the amplification process, it is an acceptable schematization to consider the measuring apparatus as consisting of its pointer alone. Our apparatus A will then be a single macroscopic object described through its one-dimensional centre-of-mass position Q only. We also assume that the observable l of the system S to be measured has purely discrete nondegenerate spectrum with eigenvalue equation

$$l|\psi_n\rangle = l_n|\psi_n\rangle \,. \tag{2.1}$$

We indicate by

$$|\psi\rangle = \sum_n c_n|\psi_n\rangle \tag{2.2}$$

the state of the system S at the initial time $t = 0$.

We assume for the S+A system the Hamiltonian

$$H = H_{\mathrm{A}} + H_{\mathrm{int}}\,, \tag{2.3}$$

where $H_{\mathrm{A}} = P^2/2M$ is the free Hamiltonian for the apparatus and

$$H_{\mathrm{int}} = \mathrm{d}\beta/\mathrm{d}t\, f(l)\, P \tag{2.4}$$

describes the S-A interaction. We have disregarded for simplicity the inessential free motion of S. In eq. (2.4), β is a dimensionless function of t having

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([9]) J. VON NEUMANN: *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, N. J., 1955).

([10]) J. S. BELL and M. NAUENBERG: in *Preludes in Theoretical Physics: in Honor of V. F. Weisskopf*, edited by A. DE SHALIT, H. FESHBACH and L. VAN HOVE (North Holland, Amsterdam, 1966).

([11]) A. PERES and W. K. WOOTTERS: *Phys. Rev. D*, **32**, 1968 (1985).

the shape indicated in fig. 1, so that $\mathrm{d}\beta/\mathrm{d}t$ describes the switching on and off of the interaction during the interval (t_0, t_1). The operator f is a function of the observable l having the dimensions of a length and such that all distances $|f(l_m) - f(l_n)|$ are macroscopic. At the initial time, the apparatus is in a well-localized state $|A_0\rangle$ with mean values of position and momentum equal to zero

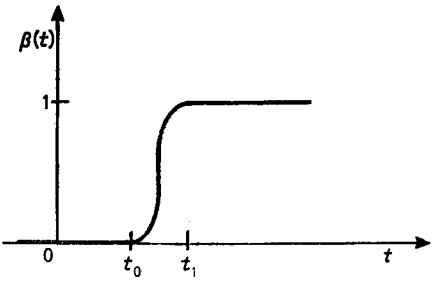

Fig. 1. – Shape of the function $\beta(t)$.

and with spread ΔQ such that

$$\Delta Q \ll |f(l_m) - f(l_n)|\,, \qquad \forall m \neq n. \tag{2.5}$$

The Schrödinger evolution of the initial state of the S+A system yields the state

$$|\Phi(t)\rangle = \exp\left[-iH_{\mathrm{A}}t/\hbar\right] \exp\left[-i\beta(t)f(l)P/\hbar\right] |\psi\rangle|A_0\rangle = \tag{2.6}$$

$$= \sum_n c_n \exp\left[-iH_{\mathrm{A}}t/\hbar\right] \exp\left[-i\beta(t)f(l_n)P/\hbar\right] |\psi_n\rangle|A_0\rangle = \sum_n c_n|\psi_n\rangle|A_n(t)\rangle\,,$$

where

$$|A_n(t)\rangle = \exp\left[-i\beta(t)f(l_n)P/\hbar\right] \exp\left[-iH_{\mathrm{A}}t/\hbar\right] |A_0\rangle\,. \tag{2.7}$$

The state $|A_n(t)\rangle$ has $\langle Q\rangle = \beta(t)f(l_n)$, $\langle P\rangle = 0$, and its position spread is very little increased with respect to the initial value ΔQ, since A is a macroscopic object. For $t > t_1$, $\langle Q\rangle = f(l_n)$, so that, due to condition (2.5), the states $|A_n(t)\rangle$ are practically mutually orthogonal and, considered also the macroscopic character of the distances $|f(l_m) - f(l_n)|$, they can be identified with the states $|A_n\rangle$ of eq. (1.2).

The final state $|\Phi(t)\rangle$, for $t > t_1$, is a linear superposition of states for which the apparatus is localized in macroscopically far apart regions. This fact gives rise to the puzzling situation described in sect. **1**. One has then to resort to the wave packet reduction which transforms $|\Phi(t)\rangle$ into the corresponding statistical mixture. The existence of specific phase relations among the states appearing in (2.6) is reflected in the fact that the corresponding statistica

operator

$$\varrho = \sum_{n,m} c_n c_m^* |\psi_n\rangle\langle\psi_m| \otimes |A_n\rangle\langle A_m| \tag{2.8}$$

has nonvanishing off-diagonal matrix elements. Due to the n-Q correlation, the nondiagonal features with respect to the system index n and to the apparatus co-ordinate Q are linked. Precisely, the matrix element $\langle Q'|\langle\psi_m|\cdot\varrho|\psi_n\rangle|Q''\rangle$ is nonzero only when Q' and Q'' are in a neighbourhood ΔQ of $f(l_m)$ and $f(l_n)$, respectively. These properties are to be confronted with those of the statistical operator

$$\tilde{\varrho} = \sum_n |c_n|^2 |\psi_n\rangle\langle\psi_n| \otimes |A_n\rangle\langle A_n| , \tag{2.9}$$

consequent to the assumption of the wave packet reduction. The operator $\tilde{\varrho}$ is diagonal with respect to n and, due to the n-Q correlation, diagonal in Q within a distance ΔQ.

3. – Quantum mechanics with spontaneous localization.

We give here a brief account of QMSL, since we want to discuss the measurement process from the point of view of such a modification of standard quantum mechanics. As discussed in ref. ([6,7]), we assume that any microscopic constituent of any physical system be subjected, at random times according to a Poisson process, to a localization process described, in the language of the statistical operator, by the map $\varrho \to T_q[\varrho]$, where $T_q[\cdot]$ is the operation

$$T_q[\cdot] = \sqrt{\alpha/\pi}\int_R \mathrm{d}x \exp\left[-\alpha(q-x)^2/2\right]\cdot\exp\left[-\alpha(q-x)^2/2\right] . \tag{3.1}$$

We assume for simplicity that each microscopic constituent has only one degree of freedom corresponding to motion in one dimension; q is the position operator.

For a single microscopic particle, if λ is the mean frequency of the process described by (3.1), the above assumption amounts to postulating that the statistical operator obeys a modified Schrödinger equation

$$\mathrm{d}\varrho/\mathrm{d}t = -(i/\hbar)[H, \varrho] + \lambda(T_q[\varrho] - \varrho) , \tag{3.2}$$

containing a non-Hamiltonian term. The main feature of eq. (3.2) is ([7]) that, if $\varrho = |\psi\rangle\langle\psi|$ with $|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$, the states $|\psi_1\rangle$ and $|\psi_2\rangle$ being localized in two regions of extension smaller than $1/\sqrt{\alpha}$ and separated by a distance larger than $1/\sqrt{\alpha}$, the state ϱ is transformed, in times of the order of $1/\lambda$, into a statistical mixture of $|\psi_1\rangle$ and $|\psi_2\rangle$.

For a composite system having N constituents, the assumption of the occurrence of spontaneous-localization processes for all constituents gives, for the N-body statistical operator, the evolution equation

$$\mathrm{d}\varrho/\mathrm{d}t = -(i/\hbar)[H, \varrho] + \sum_{i=1}^{N} \lambda_i (T_{q_i}[\varrho] - \varrho)\,, \tag{3.3}$$

where $T_{q_i}[\cdot]$ is the operation (3.1) with the position operator q_i in place of q, and λ_i is the mean localization frequency for the i-th constituent.

We now consider the implications of eq. (3.3). Let Q indicate the centre-of-mass co-ordinate and r be a shorthand for all relative co-ordinates. The centre-of-mass dynamics can be deduced from eq. (3.3) by considering the reduced statistical operator ϱ_Q obtained by taking the partial trace $\mathrm{tr}^{(r)}$ of ϱ on the internal degrees of freedom. The operation $T_{q_i}[\varrho]$ has the noticeable property (7)

$$\mathrm{tr}^{(r)}\{T_{q_i}[\varrho]\} = T_Q[\mathrm{tr}^{(r)}\varrho] \equiv T_Q[\varrho_Q]\,. \tag{3.4}$$

In virtue of eq. (3.4), when H splits into the sum of the centre-of-mass and relative Hamiltonians H_Q and H_r, we get immediately from eq. (3.3)

$$\mathrm{d}\varrho_Q/\mathrm{d}t = -(i/\hbar)[H_Q, \varrho_Q] + \Big(\sum_{i=1}^{N} \lambda_i\Big)\{T_Q[\varrho_Q] - \varrho_Q\}\,. \tag{3.5}$$

Therefore, the centre of mass of any composite system obeys the same equation (3.2) as a single microscopic particle, but with frequency $\lambda = \sum_i \lambda_i$ for the occurrence of the spontaneous localization. Stronger conclusions can be drawn in special but significant cases. For example, if the considered body is such that the internal-motion Hamiltonian gives rise to a sharp (with respect to $1/\sqrt{\alpha}$) localization of the internal co-ordinates, as happens (for a suitable α) for an insulating solid, one can prove (7) that, to a very good (depending on the value of α) approximation, one has

$$T_{q_i}[\varrho] = T_Q[\varrho]\,. \tag{3.6}$$

The physical meaning of eq. (3.6) is that a localization of a single constituent of an almost rigid system is equivalent to a localization of the centre of mass. Use of eq. (3.6) into eq. (3.3) shows that, under the above approximation, if the initial statistical operator has the form $\varrho = \varrho_Q \otimes \varrho_r$, it remains of the same form and the statistical operators ϱ_Q and ϱ_r obey eq. (3.5) and the equation

$$\mathrm{d}\varrho_r/\mathrm{d}t = -(i/\hbar)[H_r, \varrho_r]\,, \tag{3.7}$$

respectively. We can then conclude that in the considered case the internal and the centre-of-mass motions decouple, the internal motion of the solid remaining unaffected by the localization process and the centre-of-mass motion being affected by such a process with a frequency equal to the sum of the frequencies for all single constituents.

Due to the above amplification of the spontaneous-localization frequency, we have the possibility of choosing the parameters λ_i and α in such a way to satisfy two different requirements: on the one side that, for a system with a relatively small number of constituents, its dynamics coincides for any practical purpose with the standard quantum mechanics; on the other that, for an almost rigid system containing a macroscopic number of constituents (say $N \approx 10^{23}$), the dynamical changes which occur (only) for its centre-of-mass motion forbid the occurrence of superpositions of far apart states and induce ([7]) a behaviour coinciding in practice with the classical one. As an example, suppose, as discussed in ref. ([7]), that the parameters of the spontaneous localization have values

$$\lambda_i \approx 10^{-16}\ \mathrm{s}^{-1}, \qquad 1/\sqrt{\alpha} \approx 10^{-5}\ \mathrm{cm}. \tag{3.8}$$

According to this choice, a single constituent undergoes a spontaneous localization once every $10^{8 \div 9}$ y; moreover, when a system with few constituents has linear dimensions much smaller than $1/\sqrt{\alpha}$, the localization itself is, for the internal dynamics, almost ineffective. Therefore, the standard quantum mechanics remains valid for microscopic systems. On the other hand, the amplification of the frequency for a macroscopic body ($\lambda \approx 10^7\ \mathrm{s}^{-1}$ for $N \approx 10^{23}$) has conceptually important consequences, as sketched above and discussed in detail in ref. ([7]).

4. – Wave packet reduction. The model.

In this section we consider the implications of QMSL for the simplified model of a measurement process described in sect. **2**. Since the system S to be measured is microscopic, because of the extremely small value of the microscopic λ frequencies, we can consistently disregard possible spontaneous localizations for it during the measurement process. Furthermore, in the spirit of the schematization of sect. **2**, consisting in describing the whole apparatus A through the centre of mass of its pointer only, we envisage the pointer as an almost rigid macroscopic object, so that, as shown in sect. **3**, the spontaneous localizations on its microconstituents can be entirely accounted for by the last term in eq. (3.5), while its internal motion remains decoupled and unaffected by them. On the whole, the modifications to be introduced in the dynamical equation of the model when QMSL is used in place of standard quantum me-

chanics amount simply to the non-Hamiltonian term corresponding to the spontaneous localization of the pointer. We have then

$$(4.1)\qquad \mathrm{d}\varrho/\mathrm{d}t = -(i/\hbar)[P^2/2M,\ \varrho] - (i/\hbar)(\mathrm{d}\beta/\mathrm{d}t)[f(l)P,\ \varrho] + \lambda(T_Q[\varrho] - \varrho)\,,$$

ϱ being the statistical operator for the S+A system. The frequency λ, according to the discussion in sect. **3**, is large and the localization length $1/\sqrt{\alpha}$ can be assumed to be small with respect to all macroscopic distances $|f(l_m) - f(l_n)|$.

It is almost evident that the new term in eq. (4.1) leads to a final statistical operator of the type (2.9) when the mean time between two successive localizations of the pointer is large enough to allow for the completion without disturbance of the dynamical evolution described in sect. **2**, *i.e.* when $\lambda^{-1} > t_1 - t_0$. However, eq. (4.1) can be explicitly solved quite in general, showing that the same conclusion is valid for any λ. A procedure analogous to that used in ref. (7) for the free particle shows that the solution of eq. (4.1) is given by

$$(4.2)\qquad \langle Q'|\langle\psi_m|\varrho(t)|\psi_n\rangle|Q''\rangle = 1/(2\pi\hbar)\int_R \mathrm{d}k \int_R \mathrm{d}y \cdot$$

$$\cdot \exp\,[-iky/\hbar]\, F_{mn}(k,\, Q' - Q'',\, t)\langle Q' + y|\langle\psi_m|\varrho_0(t)|\psi_n\rangle|Q'' + y\rangle\,,$$

where

$$(4.3)\qquad F_{mn}(k,\, Q' - Q'',\, t) = \exp\,[-\lambda t]\cdot$$

$$\cdot \exp\left[\lambda\int_0^t \mathrm{d}\tau \exp\left[-(\alpha/4)[k\tau/M - (Q' - Q'') + (f(l_m) - f(l_n))\,\gamma_t(\tau)]^2\right]\right].$$

In eq. (4.2) $\varrho_0(t)$ indicates the solution of eq. (4.1) for $\lambda = 0$ (no spontaneous localizations) and in eq. (4.3) $\gamma_t(\tau)$ stays for $\beta(t) - \beta(t - \tau)$. It is easily checked that $\varrho(0) = \varrho_0(0)$. From eq. (2.6) one has

$$(4.4)\qquad \langle Q'|\langle\psi_m|\varrho_0(t)|\psi_n\rangle|Q''\rangle = c_m c_n^* \langle Q'|A_m(t)\rangle\langle A_n(t)|Q''\rangle\,,$$

the states $|A_n(t)\rangle$ being given by eq. (2.7). We also note that, for $m = n$, the function F_{mn} reduces to the function

$$(4.5)\qquad F(k,\, Q' - Q'',\, t) = \exp\,[-\lambda t]\exp\left[\lambda\int_0^t \mathrm{d}\tau \exp\left[-(\alpha/4)[k\tau/M - (Q' - Q'')]^2\right]\right]$$

introduced and studied in ref. (7).

We are interested in $\varrho(t)$ when the S-A interaction is over, *i.e.* for $t > t_1$. For such values of t the function $\gamma_t(\tau)$ has the shape indicated in fig. 2, so that,

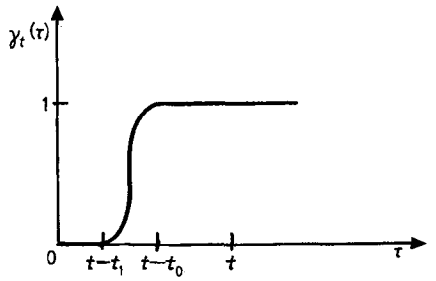

Fig. 2. – Shape of the function $\gamma_t(\tau)$ for $t > t_1$.

splitting the integration interval in eq. (4.3), one finds

$$F_{mn}(k, Q'-Q'', t) = F(k, Q'-Q'', t-t_1) \exp[-\lambda t_1] \cdot \cdot \exp\left[\lambda \int_{t-t_1}^{t} d\tau \exp\left[-(\alpha/4)[k\tau/M - (Q'-Q'') + (f(l_m) - f(l_n))\gamma_t(\tau)]^2\right]\right]. \tag{4.6}$$

The last factor is smaller than $\exp[\lambda t_1]$, so that

$$F_{mn}(k, Q'-Q'', t) < F(k, Q'-Q'', t-t_1). \tag{4.7}$$

The n-Q correlation present in the ϱ_0 factor according to eqs. (4.4) and (2.7) suppresses all contributions in eq. (4.2) unless for

$$|Q'-f(l_m)| < \Delta Q, \quad |Q''-f(l_n)| < \Delta Q. \tag{4.8}$$

For $n \neq m$, due to condition (2.5), this implies that $|Q'-Q''|$ is of the same order as $|f(l_m)-f(l_n)|$ and in turn that

$$|Q'-Q''| \gg 1/\sqrt{\alpha}. \tag{4.9}$$

For such values of $Q'-Q''$, it has been shown in ref. [7] that the function F has the property

$$F(k, Q'-Q'', t) < \exp[-\delta\lambda t], \tag{4.10}$$

δ being a factor (designated by β there) of the order of 1. Therefore, for $m \neq n$,

$$F_{mn}(k, Q'-Q'', t) < \exp[-\delta\lambda(t-t_1)], \tag{4.11}$$

showing that, after the completion of the S-A interaction, the matrix elements

of ϱ off-diagonal with respect to the system indices are exponentially damped with a « mean life » of the order of $1/\lambda$.

For $m = n$, the matrix elements of $\varrho(t)$ can be written

$$\langle Q'|\langle\psi_n|\varrho(t)|\psi_n\rangle|Q''\rangle = |c_n|^2 1/(2\pi\hbar)\int_R \mathrm{d}k \int_R \mathrm{d}y \cdot \cdot \exp\left[-iky/\hbar\right] F(k, Q'-Q'', t)\langle Q'|A_n(t)\rangle\langle A_n(t)|Q''\rangle \,. \tag{4.12}$$

According to eq. (2.7) and to ref. (7), this is, apart from the weight factor $|c_n|^2$, the same statistical matrix one obtains in QMSL for a free particle which is at the initial time in a pure state having $\langle Q\rangle = f(l_n)$, $\langle P\rangle = 0$ and position spread equal to ΔQ. It was shown in ref. (7) that, for a macroscopic particle and for a wide range of choice of the parameters, the only effect of spontaneous localization is to bring the length within which coherence manifests itself from the initial value ΔQ to a very small value (this process is, of course, carried out through a transition to a mixture), without, nevertheless, perturbing motion in an essential way for extremely long times.

Collecting the above results we conclude that the final statistical operator is diagonal in the system indices as wanted and, as far as the apparatus variables are concerned, for a given value n of the system index, it is in a state (possibly a mixture, depending on the initial width ΔQ) correlated in the right way to n. No embarassing long-distance coherence remains. We have, therefore, shown that a consistent use of QMSL for the considered model of measurement leads to a solution of the quantum measurement problem. The reduction of the wave packet is a natural and direct consequence of the new unified principle of evolution embodied in eq. (3.3).

5. – Wave packet reduction. The general case.

In the previous section we have applied consistently the formalism of QMSL to a particular, idealized model of the measurement process. Here we want to discuss the same matter without making reference to a specific model, keeping only those general features of the process which must be there in order that one can speak about a measurement. Our discussion is of a qualitative character, of course, but it is, we think, quite convincing.

Any measurement process must be characterized by the fact that, if the apparatus A is triggered by an eigenstate $|\psi_n\rangle$ of the measured observable l of the system S, when the interaction is over the system S+A is in a state $|\Phi_n\rangle$ exhibiting a strict correlation between the initial state $|\psi_n\rangle$ of S and the final state $\varrho_n^{\mathrm{A}} = \mathrm{tr}^{(\mathrm{S})}|\Phi_n\rangle\langle\Phi_n|$ of A. In the ideal case $|\Phi_n\rangle = |\psi_n\rangle|A_n\rangle$ with mutually orthogonal $|A_n\rangle$'s. The process leading from the initial to the final state may be very complicated and in general it will involve an interaction

of S with some microscopic part of A and an amplification process leading to the final situation described by the state $|\Phi_n\rangle$. We do not analyse this process and focus our attention on the final apparatus states. In all cases, the different states ϱ_n^A must correspond to macroscopically distinguishable physical situations of the apparatus. This statement requires to make precise what is meant by such an expression. We assume that two macroscopically distinguishable states ϱ_n^A and ϱ_m^A are characterized by the fact that a macroscopic number N of microconstituents of A are confined in different spatial regions separated by a macroscopic distance. This is a specific assumption which, however, seems to be correct in all practical cases. Typically, this happens when the apparatus has a macroscopic pointer showing the result of the measurement. Also when consideration is given to measuring apparatuses of a very different type, one can arrive at the same conclusion. For example, when the measurement is performed by means of a spark chamber or a Geiger counter, the ionization process and the setting up of the electric discharge which makes the counter fire correspond to the fact that in the two states (counter activated or not) a macroscopic number of electrons and ions are confined in different spatial regions. Similarly, if a particle is detected by a photographic emulsion and induces the dissociation of some BrAg molecules, when the photographic process is completed, there is an amplification effect leading to the formation of many Ag atoms and successively all remaining Br and BrAg atoms and molecules are removed from the emulsion, so that again in the two cases (blackened or unblackened plate) a macroscopic number of constituents are confined in different spatial regions.

Let us now come back to QMSL. Its basic assumption is that every microconstituent of a system is subjected, with its own frequency λ_i, to a spontaneous-localization process expressed, in the language of the statistical operator, by the operation

$$T_{q_i}[\varrho] = \sqrt{\alpha/\pi}\int_R \mathrm{d}x \exp\left[-\alpha(q_i - x)^2/2\right] \varrho \exp\left[-\alpha(q_i - x)^2/2\right], \tag{5.1}$$

where q_i is the position operator of the i-th constituent and ϱ is the statistical operator for the system under consideration. In our case, for an arbitrary initial state

$$|\psi\rangle = \sum_n c_n |\psi_n\rangle \tag{5.2}$$

of S, when the measurement process is completed, the operator ϱ has the form

$$\varrho = |\Phi\rangle\langle\Phi| = \sum_{n,m} c_n c_m^* |\Phi_n\rangle\langle\Phi_m| . \tag{5.3}$$

A comment about eq. (5.1) is appropriate here. The effect of a spontaneous

localization, when expressed in the language of the statistical operator, is somewhat ambiguous because the statistical ensemble corresponding to a given statistical operator is not unique, in general. For example, the statistical operator $T_{q_i}[\varrho]$ of eq. (5.1) can be written as

$$T_{q_i}[\varrho] = \sum_m R_m \varrho R_m , \tag{5.4}$$

where

$$R_m = [(\alpha/2)^m/m!]^{\frac{1}{2}} q_i^m \exp[-\alpha q_i^2/4] . \tag{5.5}$$

Expression (5.1) has a simple interpretation as a decomposition of the statistical ensemble into subensembles in which the particle i is localized, while eq. (5.4) corresponds to a quite different decomposition. Within the conceptual framework of QMSL one has to stick to the interpretation of $T_q[\varrho]$ as a localization, *i.e.* to assume, following eq. (5.1), that the process corresponds to the jumping of the system from the state $|\Phi\rangle$ to the states $\exp[-\alpha(q_i - x)^2/2]|\Phi\rangle$ with an appropriate probability distribution dependent on $|\Phi\rangle$ (in such a way that, if the particle i is already localized in the state $|\Phi\rangle$, nothing happens).

We now ask what is the effect of the spontaneous-localization process on the final state $|\Phi\rangle$ of the system S+A. The S part of the system contains few components, so that the probability that anyone of them undergoes a localization is, assuming a very small value for the λ_i's, very small for a long time interval. On the other hand, the constituents of A can be divided into two parts. The first part contains the particles which occupy the same position in all terms in $|\Phi\rangle$. The probability that any of these particles undergoes a localization is large, but they do not actually perform any jump because they are in the same position in all terms in $|\Phi\rangle$. The second part contains those particles which occupy different, macroscopically distant positions in the various terms in $|\Phi\rangle$ corresponding to the various values of the system index n. The number N of such constituents is still very large, so that the probability that any of these undergoes a localization is large even in a small time interval. A single localization of one of these particles is sufficient to cause the jump of the whole system S+A from $|\Phi\rangle$ to one of its terms $|\Phi_n\rangle$. Therefore, the jump—the reduction—actually takes place in a time of the order of $(N\lambda_i)^{-1}$.

The above argument has been developed as if the transition from the initial state $\sum_n c_n|\psi_n\rangle|A_0\rangle$ to the state $\sum_n c_n|\Phi_n\rangle$ would take place in the absence of localizations and only subsequently the localization process would become active. The actual situation will be much more complicated. However, the model example of the previous section and the results of ref. (7), showing that the localization process is very soft in the sense that it is practically ineffective except for the suppression of the unwanted superpositions, give us full confidence in the general validity of our description of quantum measurement.

The choice of the frequency parameters mentioned in sect. **3** ($\lambda_1 \approx 10^{-16}\ \mathrm{s}^{-1}$) gives, for $N \approx 10^{23}$, corresponding to a fraction of a gram, a time for the reduction process of the order of 10^{-7} s—a short time. However, lighter systems still have to be considered as macroscopic. In particular, pointers in the generalized sense used in this section are macroscopic systems often much lighter than one gram. To have short reduction times also in these cases requires increasing the frequency parameters. According to the results of ref. (7), there is a wide margin for that.

One can say that QMSL is (or it will be, when fully developed) a unifying theory in a twofold sense. First, as shown in our previous work, the same principles describe both the classical behaviour of macroscopic bodies and the quantum behaviour of microscopic systems. Second, as shown here, the uniqueness of principles for all kinds of systems allows us to describe the measurement process in a completely consistent way, thereby unifying the Schrödinger equation and the reduction postulate.

In our opinion, finally, there is no serious reason in the framework of QMSL not to interpret the wave function as a real property of a physical system.

● RIASSUNTO

La meccanica quantistica con localizzazione spontanea (QMSL) è una modificazione stocastica recentemente proposta dell'equazione di Schrödinger a N corpi in accordo sia con la microfisica che con la macrofisica. In questo lavoro la QMSL è applicata al problema della misurazione. Si mostra che la sostituzione della meccanica quantistica usuale con la QMSL ha il solo effetto di dar luogo a un'effettiva riduzione della funzione d'onda.

Квантовая механика со спонтанной локализацией и квантовая теория измерения.

Резюме (*). — Квантовая механика со спонтанной локализацией представляет недавно предложенную стохастическую модификацию N-частичного уравнения Шредингера, согласующуюся и с микрофизикой и с макрофизикой. В этой работе квантовая механика со спонтанной локализацией применяется к проблеме измерения. Показывается, что замена стандартной квантовой механики на квантовую механику со спонтанной локализацией приводит единственно к реальному упрощению волновой функции.

(*) *Переведено редакцией.*