Auf. 5.3

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(define (append x y)
        (cond [ (empty? x) y]
              [ (cons? x) (cons (first x) (append (rest x) y)) ] ))
```

a) (append empty l) ☰ l für alle Listen l

(append empty l)

EFUN

≡ (cond [(empty? empty) l]
[(cons? empty) (cons (first empty) (append (rest empty) l))])

PRIM, ERED, EKONG

≡ (cond [#true l]
[(cons? empty) (cons (first empty) (append (rest empty) l))])

COND-true, ERED

≡ l

b) (append l empty) ☰ l für alle Listen l.

• Induktionsanfang (Basisfall): l ≡ empty

Zu zeigen: (append empty empty) ☰ empty

(append empty empty)

EFUN

≡ (cond [(empty? empty) empty]
[(cons? empty) (cons (first empty) (append (rest empty) empty))])

PRIM, ERED, EKONG

≡ (cond [#true empty]
[(cons? empty) (cons (first empty) (append (rest empty) empty))])

COND-true, ERED

≡ empty

• Induktionsannahme: (append lst1 empty) ☰ lst1, lst2 ≡ (cons head lst1)

• Induktionschritt:

Zu zeigen: (append lst2 empty) ≡ lst2

(append lst2 empty)

EFUN, EKONG

≡ (cond [(empty? (cons head lst1)) empty]
[(cons? (cons head lst1)) (cons (first (cons head lst1)) (append (rest (cons head lst1)) empty))
]))

PRIM, ERED, EKONG

≡ (cond [#false empty]
[(cons? (cons head lst1)) (cons (first (cons head lst1)) (append (rest (cons head lst1)) empty))
]))

COND-false, ERED

≡ (cond [(cons? (cons head lst1)) (cons (first (cons head lst1)) (append (rest (cons head lst1)) empty))
]))

STRUCT-predtrue, EKONG

≡ (cond [#true (cons (first (cons head lst1)) (append (rest (cons head lst1)) empty))]))

COND-true, ERED

≡ (cons (first (cons head lst1)) (append (rest (cons head lst1)) empty))

PRIM, ERED, EKONG

≡ (cons head (append lst1 empty))

Induktionsannahme, EKONG

≡ (cons head lst1)

Induktionsannahme, PRIM

≡ lst2

Damit ist bewiesen: (append l empty) ☰ l für alle Listen l.

c) (append (cons x l1) l2) ☰ (cons x (append l1 l2)) für alle x und alle Listen l1, l2.

• Induktionsanfang (Basisfall): l1 ≡ empty

Zu zeigen: (append (cons x empty) l2) ☰ (cons x (append empty l2))

(append (cons x empty) l2)

EFUN

≡ (cond [(empty? (cons x empty)) l2]
[(cons? (cons x empty)) (cons (first (cons x empty)) (append (rest (cons x empty)) l2))])

PRIM, ERED, EKONG

≡ (cond [#false l2]
[(cons? (cons x empty)) (cons (first (cons x empty)) (append (rest (cons x empty)) l2))])

COND-false, ERED

≡ (cond [(cons? (cons x empty)) (cons (first (cons x empty)) (append (rest (cons x empty)) l2))])

STRUCT-predtrue, EKONG

≡ (cond [#true (cons (first (cons x empty)) (append (rest (cons x empty)) l2))])

COND-true, ERED

≡ (cons (first (cons x empty)) (append (rest (cons x empty)) l2))

PRIM, ERED, EKONG

≡ (cons x (append empty l2))

• Induktionsannahme: (append (cons x l0) l2) ☰ (cons x (append l0 l2)), l1 ≡ (cons z l0)

• Induktionsschritt:

Zu zeigen: (append (cons x l1) l2) ☰ (cons x (append l1 l2))

(append (cons x l1) l2)

EFUN

≡ (cond [(empty? (cons x l1)) l2]
[(cons? (cons x l1)) (cons (first (cons x l1))(append (rest (cons x l1)) l2))])

PRIM, ERED, EKONG

≡ (cond [#false l2]
[(cons? (cons x l1)) (cons (first (cons x l1))(append (rest (cons x l1)) l2))])

COND-false, ERED

≡ (cond [(cons? (cons x l1)) (cons (first (cons x l1))(append (rest (cons x l1)) l2))])

STRUCT-predtrue, EKONG

≡ (cond [#true (cons (first (cons x l1)) (append (rest (cons x l1)) l2))])

COND-true, ERED

≡ (cons (first (cons x l1)) (append (rest (cons x l1)) l2))

PRIM, ERED, EKONG

≡(cons x (append l1 l2))

Damit ist bewiesen: (append (cons x l1) l2) ☰ (cons x (append l1 l2)) für alle x und alle Listen l1, l2.

d) (append l1 (append l2 l3)) ☰ (append (append l1 l2) l3) für alle Listen l1, l2, l3

• Induktionsanfang (Basisfall): l1 ≡ empty

Zu Zeigen: (append empty (append l2 l3)) ☰ (append (append empty l2) l3)

(i) (append empty (append l2 l3))

EFUN

☰ (cond [(empty? empty) (append l2 l3)]
[(cons? empty) (cons (first empty) (append (rest empty) (append l2 l3)))])

PRIM, ERED, EKONG

☰ (cond [#true (append l2 l3)]
[(cons? empty) (cons (first empty) (append (rest empty) (append l2 l3)))])

COND-true, ERED

☰ (append l2 l3)

(ii) (append (append empty l2) l3)
Teilaufgabe (a), ERED
≡ (append l2 l3)
ETRANS
(append empty (append l2 l3)) ≡ (append (append empty l2) l3) ≡ (append l2 l3)

- Induktionsannahme: (append l0 (append l2 l3)) ≡ (append (append l0 l2) l3), l0 ≡ (cons z l1)
- Induktionsschritt:

Zu zeigen: (append l0 (append l2 l3)) ≡ (append (append l0 l2) l3)

(i) (append l0 (append l2 l3))
EFUN, EKONG
≡ (cond [(empty? (cons z l1)) (append l2 l3)]
[(cons? (cons z l1)) (cons (first (cons z l1)) (append (rest (cons z l1)) (append l2 l3)))])
PRIM, ERED, EKONG
≡ (cond [#false (append l2 l3)]
[(cons? (cons z l1)) (cons (first (cons z l1)) (append (rest (cons z l1)) (append l2 l3)))])
COND-false, ERED
≡ (cond [(cons? (cons z l1)) (cons (first (cons z l1)) (append (rest (cons z l1)) (append l2 l3)))])
STRUCT-predtrue, EKONG
≡ (cond [#true (cons (first (cons z l1)) (append (rest (cons z l1)) (append l2 l3)))])
COND-true, ERED
≡ (cons (first (cons z l1)) (append (rest (cons z l1)) (append l2 l3)))
PRIM, ERED, EKONG
≡ (cons z (append l0 (append l2 l3)))
Induktionsannahme, EKONG
≡ (cons z (append (append l0 l2) l3))

(ii) (append (append l0 l2) l3)
EFUN, EKONG
≡ (cond [(empty? (append (cons z l1) l2)) l3]
[(cons? (append (cons z l1) l2)) (cons (first (append (cons z l1) l2))
(append (rest (append (cons z l1) l2)) l3))])
PRIM, ERED, EKONG
≡ (cond [#false l3]
[(cons? (append (cons z l1) l2)) (cons (first (append (cons z l1) l2))
(append (rest (append (cons z l1) l2)) l3))])
COND-false, ERED
≡ (cond [(cons? (append (cons z l1) l2)) (cons (first (append (cons z l1) l2))
(append (rest (append (cons z l1) l2)) l3))])
STRUCT-predtrue, EKONG
≡ (cond [#true (cons (first (append (cons z l1) l2))
(append (rest (append (cons z l1) l2)) l3))])
COND-true, ERED
≡ (cons (first (append (cons z l1) l2))
(append (rest (append (cons z l1) l2)) l3))
Teilaufgabe (c), EKONG
≡ (cons (first (cons z (append l0 l2)))
(append (rest (cons z (append l0 l2))) l3))
PRIM, ERED, EKONG
≡ (cons z (append (append l0 l2) l3))
ETRANS
(append l0 (append l2 l3)) ≡ (append (append l0 l2) l3) ≡ (cons z (append (append l0 l2) l3))
Damit ist bewiesen: (append l1 (append l2 l3)) ≡ (append (append l1 l2) l3) für alle Listen l1, l2, l3.