Re: Your paper ("Measuring Multipartite ent...")

M.N.R.*

(Dated: April 29, 2016)

Hello Philipp,

Thank you for providing such a detailed email! I've started looking at some of the observables you discussed in your email. So firstly, I'm sorry for the length of this email, but I wanted to attach some plots so that I could ask some follow up questions about these observables and if I understand the inequality in the methods section of your paper correctly.

It's definitely true that the Ising model that we map to is the one you wrote in your email. The real Hamiltonian we have to simulate is a Bose-Hubbard Hamiltonian with an appropriate tilt term. This allows us to roughly scan the orange region in the phase diagram in FIG. 1. Where the parameter we are scanning is $g = (E - U)/U$ where E is the applied tilt we vary slowly in time and U is the static on-site interaction. In all of these plots, we adiabatically scan forward and back, and terminate the experiment at different times during the adiabatic ramp.

Experimentally we can actually create finite size sytems that we initalize with a fixed atom number and read out the on-site occupation number at the end of the experiment. With this we can determine if we are faithfully producing states that exist within this spin subspace of the entire Bose-Hubbard Hilbert space (and if so the population in each of these states). This allows us to compute the types of observables and order parameters from your email.

So it seems that the odd and even cases actually produce quite different results. When starting with an even-number of sites and passing through this paramagnetic to antiferromagnetic resonance. There is a single ground state into which the atoms transfer. While for the odd-numbers of sites, there are $(N+1)/2$ states, where there is a domain boundary left over in the system (assuming that the hard wall boundary conditions for the BH model map to pinned-spins on the bonds at the edge of the system). To try and clarify what I mean I added a cartoon of these states in FIG. 2. Also, separately, N is the number of sites initialized with an atom, not the number of spins.

To try and prove the purity of our system throughout this transition we both evolved the Hamiltonian forward from the paramagnetic side to the anti-ferromagnetic side and then reverse the Hamiltonian from the anti-ferromagnetic side to the paramagnetic side to find our overlap with the initial Mott Insulator state. Plots of the population found for the significant states in the 4 and 5 site cases are shown in FIG. 3,4. Note that for technical reasons we did not apply the entanglement entropy measurements here (happy to discuss this).

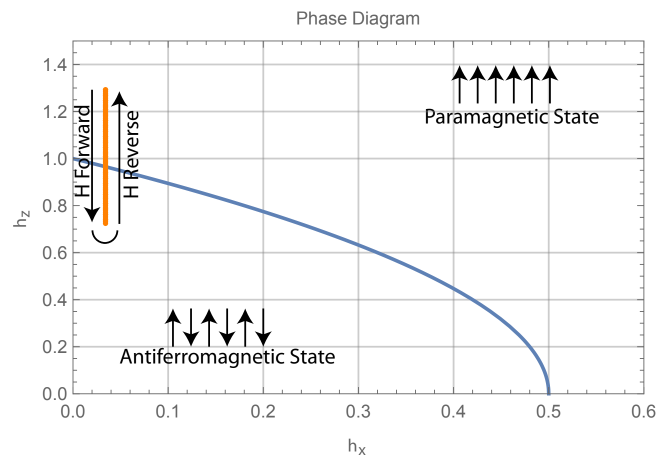

FIG. 1. Cartoon of the phase diagram and where we can measure the Ising model.

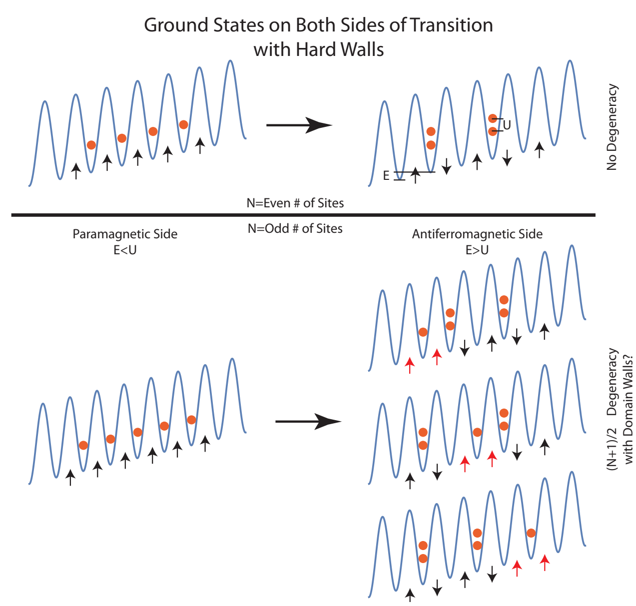

FIG. 2. Cartoon showing the hopping process for a finite size system going through the PM to AFM transition. The AFM states are on the right hand side of the figure and the PM states are on the left hand side.

So now in both the even and odd cases I've plotted both the theory (using the trotter expansion) and data (after being mapped to the spin basis) for the magnetization, Neel order parameter, and staggered magnetization. In subplot 1 5,6. I've plotted the magnetization $\langle \sum_i s_i^z \rangle$, and it's variance $\langle (\sum_i s_i^z)^2 \rangle - \langle \sum_i s_i^z \rangle^2 \times 10$.

In subplot 2, I've plotted the Neel Order Parameter,

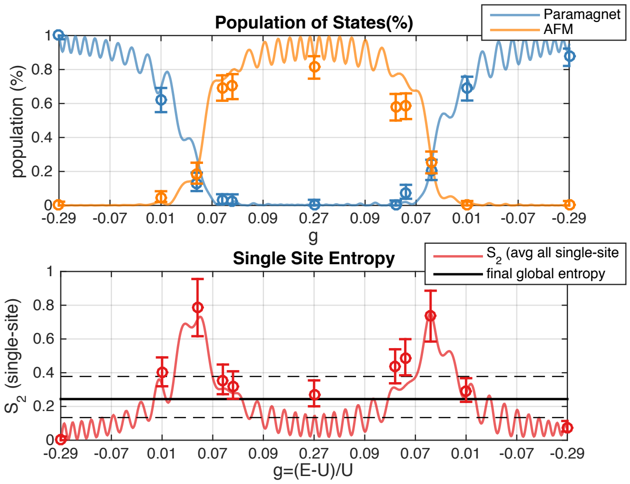

FIG. 3. Plot showing the dynamics of the 4-site system by plotting the population in pertinent states across the PM to AFM transition. All values are plotted vs. the tuning parameter "g" where $g = (E - U)/U$. The averaged single-site entropy is also plotted in the second subplot.

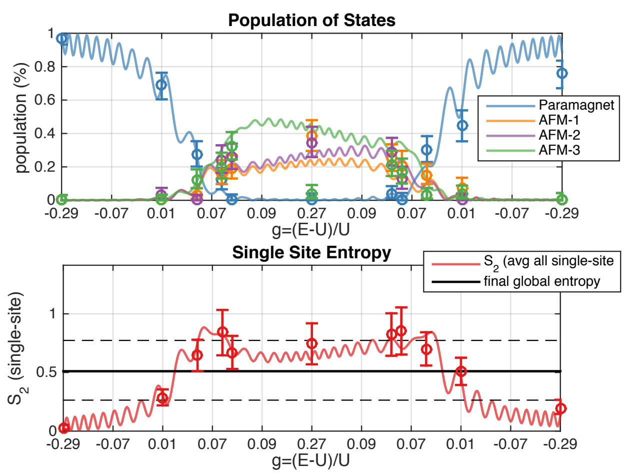

FIG. 4. Plot showing the dynamics of the 5-site system by plotting the population in pertinent states across the PM to AFM transition. All values are plotted vs. the tuning parameter "g" where $g = (E - U)/U$. The averaged single-site entropy is also plotted in the second subplot.

$\langle\left(\sum_i(-1)^i s_i^z\right)^2\rangle$ and it's variance x10.

In subplot3, I've plotted the staggered magnetization $\langle\sum_i(-1)^i s_i^z\rangle$ and it's variance x10.

I should also clarify here that I've defined $S_i^z = (1/N_{spins})\sigma_i/2$ This is related to one of my questions. As discussed in the previous email, it seems the staggered magnetization is the appropriate linear observable to detect this sort of entanglement near this phase transition (which seems true for the even-site case). But to clarify the inequality at in the methods of your paper (mostly due to my funny operator definition), if I properly adjust the FQ I measure for the factors of N_{spins} (or N_{spins}^2 I think in this case) then my FQ is then $FQ = 4N_{spins}^2\left[\langle\left(\sum_i(-1)^i S_i^z\right)^2\rangle - \langle\sum_i(-1)^i S_i^z\rangle^2\right]$?

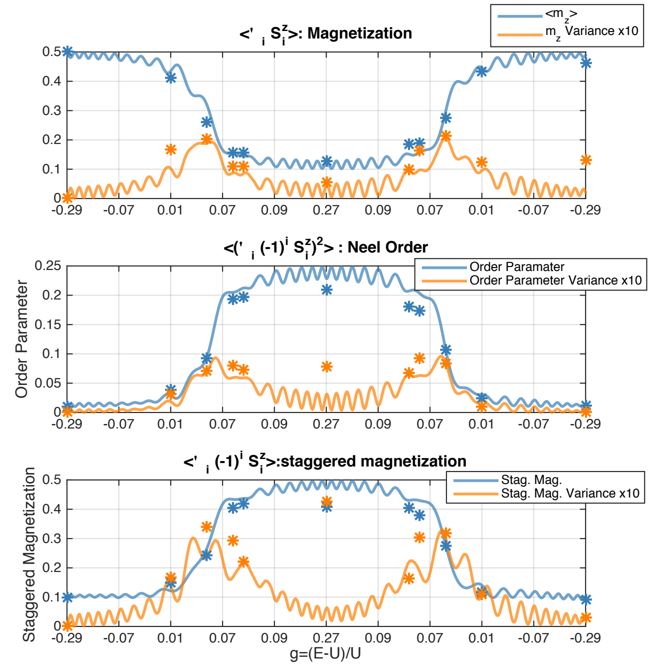

FIG. 5. Plots showing observables and their variances (x10) for the 4-site system: the Magnetization, Néel Order Parameter, and staggered Magnetization. All values are plotted vs. the tuning parameter "g" where $g = (E - U)/U$.(For some reason the Σ came out as $'$ in the titles of all the subplots)

In other words: this $4N_{spins}^2$ multiplied by the S_i^z, as I defined it, is what I use to compare to the inequality $F_q > \lfloor\frac{N}{m}\rfloor m^2 + \left(N - \lfloor\frac{N}{m}\rfloor m\right)^2$ where N_{spins} is the number of spins in my chain and m is the degree of m+1-partite entanglement?

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Part of my next question is related to the degeneracy in the odd-site case. They should in theory exist with relatively equal probability (not exactly what is measured, but for this question's sake we'll pretend they are at equal probability) which gives them actually a very different expectation value for the staggered magnetization due to the domain wall flipping the order of the density wave after it. So in order of top-to-bottom of the cartoon FIG.**??**, these states should give values of (+2, 0, -2) /N, which means that the expectation value of the anti-ferromagnetic degenerate state ends up being zero. Therefore, for both the paramagnetic side and the anti-ferromagnetic side the staggered magnetization operator has zero expectation value, though one side exhibits non-zero variance. Moreover, the fluctuations existf at both the phase transition and in the antiferromagnetic phase, which makes the curves for the staggered

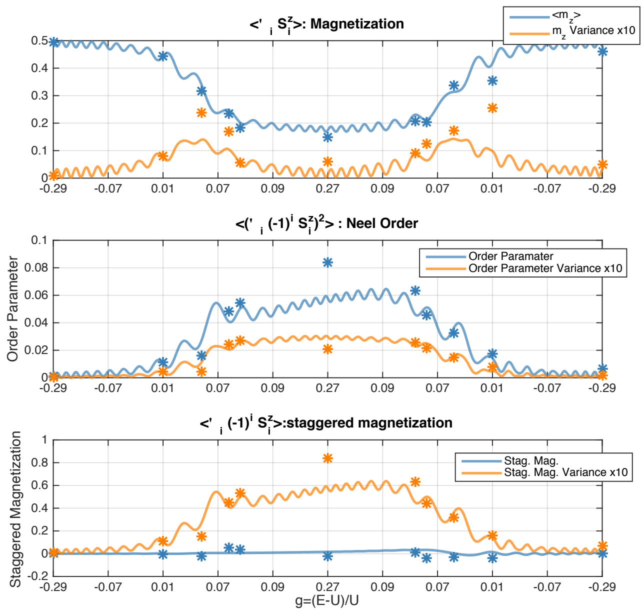

FIG. 6. Plots showing observables and their variances (x10) for the 5-site system: the Magnetization, Néel Order Parameter, and staggered Magnetization.All values are plotted vs. the tuning parameter "g" where $g = (E-U)/U$.(For some reason the Σ came out as $'$ in the titles of all the subplots)

magnetization varianc ein subplot 3 of FIG. 5,6 persist in this region. This is probably good because it should also imply there is some real spatial entanglement in this phase, but wouldn't have the same scaling behavior near the transition as for the case of an even length chain. Does this seem correct or reasonable? Does this also imply that perhaps there is a more suitable observable that should still have scaling behavior enar the transition in both (even and odd) cases?

So lastly I have a question about finding what the right observable might be in the Bose-Hubbard picture for this system. Is there an obvious corresponding observable for this picture that is similar to $\langle \sum_i (-1)^i S_i^z \rangle$ witness used in the spin case? My guess would be something like $\langle \sum_i (-1)^i (1-n_i) \rangle$ such that the staggered $|2020\rangle$ state might map to something like ± 4, yielding large fluctuations in the afm phase and at the transition. Is using absolute values for this oeprator OK in terms of later applying the entanglement inequality? I only ask since it would then seem something like $\langle | \sum_i (-1)^i (1-n_i) | \rangle$ would also have zero variance in the afm/staggered phase, but peak around the transition.

Again thank you very much for the help you've already given and pardon for the length of this email. I just wanted to be clear about what we're doing and the effects we're seeing.

Best,

Matthew

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