Quantum Ising Phase Transition

Rubidium Makas*
Department of Physics, Harvard University, Cambridge, Massachusetts, 02138, USA
(Dated: June 6, 2016)

Transitions between strongly correlated phases of matter harbor a great deal of intersting characteristics near the phase transition point. Using a quantum gas microscope[1] we are able to study quantum effects that depend strongly on very low-entropy states. We study the quantum phase transitions of an Ising-like model by mapping the single-site atom occupation in a tilted Mott insulator to an ising spin system[**?**], [2]. The recent development of tools to perform state-readout have enabled a more in depth investigation of this transition in affording access to the correlation functions, entanglement entropy, and scaling of both these quantities with respect to the finite system size.

- Things we have:
- Entanglement Entropy and Ising Phase Transition
- State Readout
- Magnetization and Variance
- Correlation Lengths item Order Parameter item Degeneracy, Scaling, Domain Wall Excitation
- Eigenstate Population in the two extremes
- Things we Don't have:
- Diagonal Entropies, Purities, and B.S.
- Thermalization if we sit at the gap
- open boundaries across the transition
- Figures we probably want:
- Reminder Cartoon of what system looks like and fixed chain length – show mapping
- How we read out the state? Could probably be in the supplementary to show mapping more thoroughly
- Fisher Information, Magnetization, Order Parameter, reversing the hamiltonian and entropy for 4-6?
- Check for domain formation/density in 4-8 and plot
- correlation lengths and any other scaling arguements possible.

INTRODUCTION

Phase transitions and their dependence or effect on correlation lengths, entanglement entropy, and system size scaling behavior have been a point of theoretical and experimental interest for (insert time period). Partially due to their universality class, understanding and studying these systems provides deeper insight into the underlying or ubiquitous behaviors of many systems and remains both challenging and elusive for scaling in many systems. Recent technological and theoretical developments have enabled more in depth studies and understanding possible.

Here we use the highly controlled environment of ultracold atoms in optical lattices as a paradigmatic quantum simulator for studying state read-out, correlated phases of matter, and scaling behaviors of the system. By initializaing these experiments with very low entropy Mott insulating states we are able to create very pure finite one-dimensional systems to study the fluctuations, correlations, and entanglement within the system near the Ising-like phase transition and the ground states of the two phases.

Ising Transition As demonstrated in a previous experiment, the Ising model transition from a paramagnetic (PM) phase to an antiferromagnetic (AFM) phase when chaning the strength of an effectively applied h_z [2].

This also makes the system particularly apt for mapping to the Ising spin hamiltonian where the hamitlonian is described by a spin interaction term and an alignment with external applied magnetic fields. In our system the Bose-Hubbard model is mapped onto this spin hamiltonian by initializaing the system into a finite size unity filling Mott Insulator in a tilted lattice. This phase describes the paramagnetic state within the Ising hamiltonian. In this model the spins are initialized "up" and the dipole excitation (de-excitation) operator is given by $d_i^\dagger = a_{i+1}^\dagger a_i^\dagger$. In particular, an $n = 1$ Mott insulator in a tilted lattice can effectively be described by the interaction-assisted hopping of an atom onto its neighbor when the tilt between neighboring sites is equal to the on-site interaction of atoms. The relative energy scales of the tilt E, interaction U, and tunneling J, set what path across this transition is accessible in this experiment 1.

EXPERIMENTAL IMPLEMENTATION

We study continuous one-dimensional paramagnetic to antiferromagnetic transitions with tilted Mott Insulators

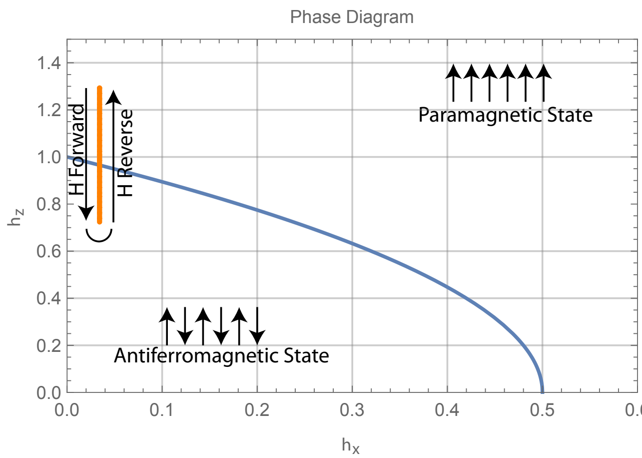

FIG. 1. The accessible range of the phase diagram explored by this mapping.

.

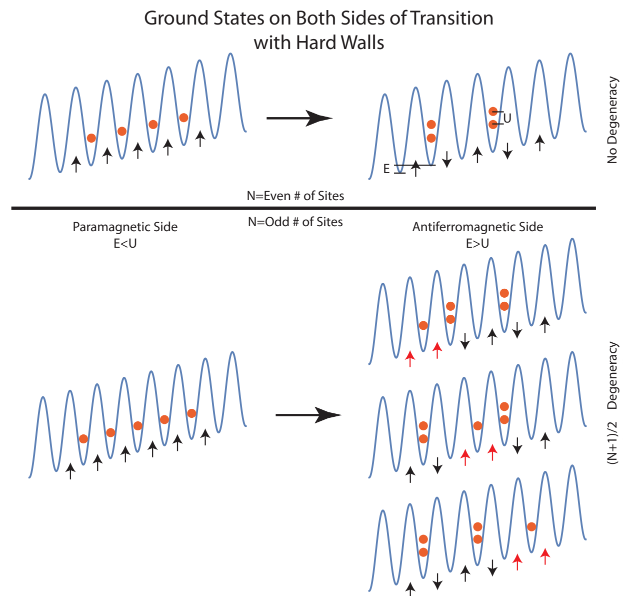

FIG. 2. Differences between the even and odd cases of fixed boundary conditions.

.

made up of bosonic ^{87}Rb atoms in a quantum gas microscope setup. The Hamiltonian for the system is given by the Bose-Hubbard Hamiltonian 1

$$H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) + \sum_i E\, i\, n_i \quad (1)$$

where $a_i^\dagger(a_i)$ is the bosonic creation(annihlation) operator at site i and $n_i = a_i^\dagger a_i$ is the number operator. The nearest-neighbor tunneling J and the on-site interaction U are tunable via the depth of the optical lattice in the directions of the one-dimension Ising chain (the y-direction), and the external gradient E is set by a magnetic field gradient. In this experiment we adiabatically transfer a unity filling Mott Insulating state into a staggered density wave in simulation of a paramagnetic(PM) state into an anti-ferromagnetic(AFM) state by changing this magnetic field gradient linearly in time. The initial state for the PM to AFM transition is obtained from a low-entropy two-dimensional Mott insulator using a single-site addressing scheme (see Methods). We prepare one or two rows of atoms along the y-direction of a deep optical lattice ($V_x = V_y = 45\ E_r$), as shown in Figure **??**. Then the PM to AFM transition is then performed by keeping the tubes along the x-direction decoupled and tilting the system in the $y$direction at a reduced depth of $V_y \sim 16E_r$. By freezing the dynamics at variable points during the linear gradient ramp and expanding the one-dimensional tubes we can measure the on-site occupation in each system which effectively allows us to measure the spin system along the z-direction (see Methods). The read out of the entire manybody-wave function in the on-site occupation (Fock state) basis provides access to single-site entropy of the system and the many-body state population within the Fock basis. Within the spin subspace this allows for measurement of the correlation lengths of the system in the two phases (and near the transition), system size scaling behavior, and measurement of the order parameter.

As proposed by [**?**]This hopping is what generates the spin-flips in our ising model which is then algebraically described by the dipole creation operator $d_i^\dagger = a_{i+1}^\dagger a_i$.

QUANTUM PHASE TRANSITION AND RESULTS

Quantum Phase Transition Perhaps just show phase diagram and give the mapping and $\lambda = \frac{E-U}{J}$

It is important and instructive to also notice that the critical λ_c does not occur at the naive transition of $E = U, \lambda = 0$ but due to the quantum fluctuations actually occurs at $\lambda_c \approx 1.85$ /citeSubir2002. This theoretically predicted value agrees very well both with the exact diagonalization for this system and the measured peak in the single-site entropy (and maybe Fisher Information).

Fisher Information, Entropy, and Magnetization Discuss how the single-site occupation for the Mott insulator gives an entropy measure of $S_2 = -\log(\mathrm{Tr}_i\, \rho_{ii})$. There is also the $f_Q = 4\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$

$$O = \left\langle \left(\frac{1}{N} \sum_j (-1)^j S_z^j \right)^2 \right\rangle \quad (2)$$

$$\langle m_z \rangle = \left\langle \frac{1}{N} \sum_j S_z^j \right\rangle \quad (3)$$

$$\langle \sigma m_z \rangle = \left\langle \left(\frac{1}{N} \sum_j S_z^j \right)^2 \right\rangle - \left\langle \frac{1}{N} \sum_j S_z^j \right\rangle^2 \tag{4}$$

$$\langle s m_z \rangle = \left\langle \frac{1}{N} \sum_j (-1)^j S_z^j \right\rangle \tag{5}$$

$$\langle \sigma s m_z \rangle = \left\langle \left(\frac{1}{N} \sum_j (-1)^j S_z^j \right)^2 \right\rangle - \left\langle \frac{1}{N} \sum_j (-1)^j S_z^j \right\rangle^2 \tag{6}$$

Even vs. Odd Cases By fixing the system size we have imposed boundary conditions on the type of transitions that the model is allowed to make. With an interesting aspect to this being that the parity of the total system size is stronlgy coupled to the states within the AFM regime. The imposed boundary conditions for the tilted Mott insulator are based on the non-existence of atoms to the left or right of the chain. This means that during the AFM transition no atom can hop onto the right most site or hop out of the left most site. The analogous condition in the spin model would be that the spin chain made up of $\mathrm{N_{sites}} + 1$ spins has the left- and right-most spins pinned along the initial direction of the paramagnetic phase. This means that for a Mott insulator with an even number of sites in the chain then the system is able to relax into a completely AFM ordered state with no degeneracies. However, for the odd-site case, the pinned outter spins require that a single domain wall be formed within the chain. In the $\lambda \to \infty$ limit, the location of this wall is translationally invariant throughout the chain and so an (N+1)/2 degeneracy is formed in the ground state to nearly perfect AFM orderd state, where N is the number of sites/atoms in the system.

This gives us the ability to use superselection as the way to read out the single-site entropy. This also allows us to post-select on the system such that we can be more certain our system didn't have a lot of heating and mostly remained within the correct model (quote some number for our 'loss' going from tilted Mott Insulator to Quantum Ising Model). This also allowed us to reverse the Hamiltonian dynamics such that if we are adiabatic we will simply map back to our original "ground" state. This proves the "purity" of our system. Can also say something about verifying a quantum super-position in the degenerate case considering that a statistical mixture of these states doesn't come back to the Mott.

Unfixed Size: This allows us get a handle of what the correlation length really is. Measuring within the finite systems ans post-selecting throws out a lot of data. Admittedly this is the best thing to do, but for very long systems you might argue that this should still provide an

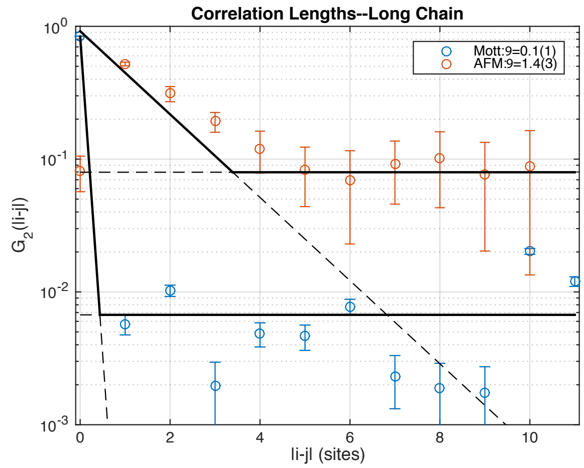

FIG. 3. who knows.

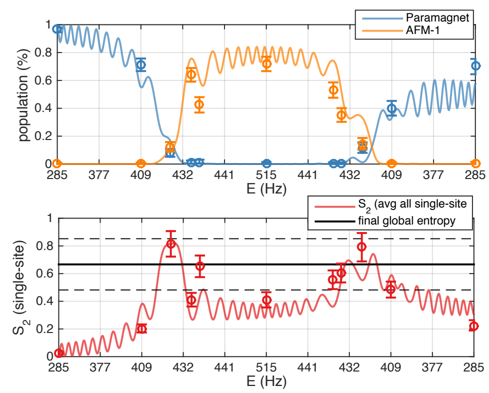

FIG. 4. Realizing a system with frustration.

.

answer of some sort. Could still try to find "domains" that form in the system. But so far I don't see anything in this but it could be that in the way that I'm checking it isn't the write thing and could check this more carefully.

Scaling and FQI near QPT Hopefully talk about how ξ changes near the transition and how how it scales in system size? Unclear.

CONCLUSION

The quantum gas microscope architecture provides a very clean way to initialize and measure quantum systems. The procedure perscribed in this paper involves the initialization of a very pure state Mott insulator in a tilted lattice. One special, or at least convenient aspect, is that this tilt breaks the translational symmetry of the system and thus conveniently makes the eigenbasis of the Bose-Hubbard system configurational basis of the lattice sites. As long as E,$U \gg J$ then the Hamiltonian is

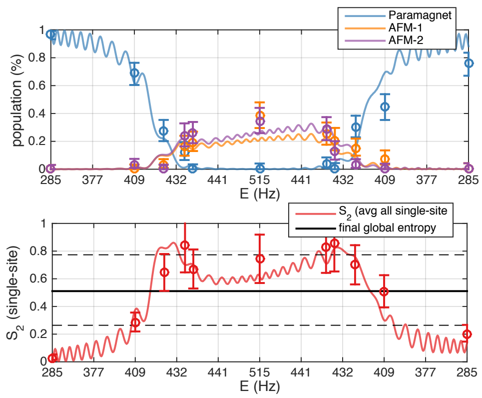

FIG. 5. Realizing a system with frustration.

.

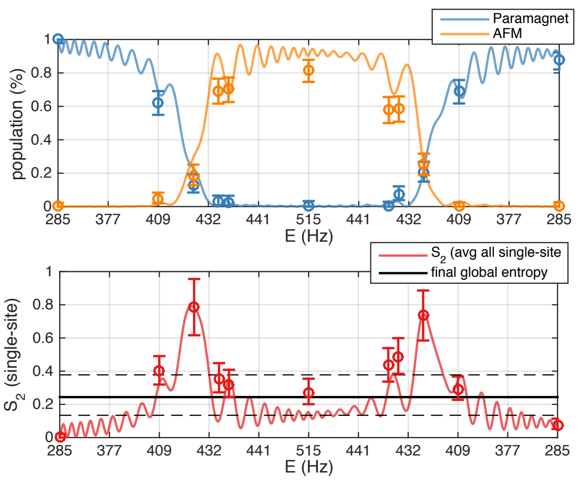

FIG. 6. Realizing a system without frustration.

almost entirely diagonal which would enable interesting studies in reading out popuations within the eigenbasis of a many-body system where intermediate dynamics can be adiabatically mapped to their eigenstate populations. This opens up the doors to proposals to measure quantities such as diagonal entropies and their relations to thermodynamic quantities or ... something.

METHODS

Sample preparation. Using a quantum gas microscope[1] with single-site resolution readout and potential projection [**?**] we are able to realize low-entropy, finite-size quantum systems with high fidelity. A quantum degenerate cloud of ultracold ^{87}Rb is confined to a single layer of a two-dimensional optical lattice with lattice constant $d = 680$ nm and a recoil energy of $E_r = 2\pi \times \frac{h}{8m_{Rb}d^2} \approx 2\pi \times 1240$ Hz, where h is Planck's constant and m_{Rb} is the atomic mass of ^{87}Rb. The starting point for our experiments is a two-dimensional Mott insulator in the $|F, m_F\rangle = |1, -1\rangle$ hyperfine state. To prepare the initial state for the Ising transition, we use a spatial light modulator (SLM), capable of generating arbitrary beam profiles, to "cut" two rows of atoms from the atomic cloud of finite and fixed length: In a deep lattice of depth $V_x = V_y = 45\ E_r$, we superimpose a blue-detuned ($\lambda = 760$ nm) beam, with a Hermite-Gauss profile along the (short) x-direction and a flattop potential along the (long) y-direction. Subsequently, we switch off the lattice in the x-direction and turn on a large anti-confining beam. Only atoms in the rows coinciding with the nodes of the Hermite-Gauss beam are retained, while all other atoms are expelled from the system. We thus obtain one or two isolated rows of atoms with a typical single-site loading fidelity of 98% and length varying from $4 \rightarrow 8$ sites. After adiabatically turning off the addressing beam and bringing the x-lattice back to its original depth of 45 E_r, we tilt the lattice to a gradient of $\approx 284Hz/site$. At this point, we lower the depth of the lattice $\approx 16E_r$ such that the bare tunneling J$\approx 9(1)Hz$. We then begin the adiabatic ramp of this finite system at unity filling to the staggered density wave state by ramping from a the inital gradient of $284Hz \rightarrow 514Hz$. This is also correspondingly a ramp of our parameter $\lambda = -13.2 \rightarrow 12.3$. The lattice along the $y$direction remains at $45Er$, such that the tunneling outside of the 1-dimensional tubes is supressed on the experimental time scales and the measurements take place in two decoupled tubes such that they may be separated and the on-site occupation number read out in each experiment. While our on-site readout procedure is only sensitiveto the parity of a site occupation, the expansion of these one-dimensional tubes into large orthogonal tubes alows for the spatial separation of the atoms such that we are then able to resolve the on-site occupation number.

Data analysis. Our in-situ read-out procedure is only sensitive to the parity of a site occupation. In order to resolve the on-site occupation number, we expand each one-dimensional system into orthogonal one-dimension tubes such that each site is expanded by uniformly delocalizing the atom across aproximately ~ 50 lattice sites. The vast majority of states containing more than ¿2 atoms per site is very small and therefore the likelihood of miss-measuring the on-site occupation due to parity projection in the delocalized tubes is very small ($\leq 2\%$). In addition, since we load fixed chain lengths for some of the systems, we can also post-select on retaining the total atom number corresponding to unity filling in the system. This increases the fidelity of experimental outcomes that contribute to the spin model and also excludes systems that are heated out of the system or bad-state initialization. This also provides the added benefit then that the exact density matrix for a single-site is simply the particle number population on that site. This is due to the supersymmetry of the entire state having a fixed particle number and thus preventing any coherences existing between different fock states of ρ_A and ρ_B.

$$f_q = \left[\left\lfloor\frac{N}{m}\right\rfloor m^2 + \left(N - \left\lfloor\frac{N}{m}\right\rfloor m\right)^2\right] \frac{\mathcal{O}_{max} - \mathcal{O}_{min}}{N} \quad (7)$$

Simulation and lattice parameters. All theory plots are results of a direct numerical solution of the Schrödinger Equation with Hamiltonian (1) in the Fock space of two particles on 25 lattice sites. Lattice depths are calibrated using modulation spectroscopy to higher bands, with a typical error of $XX\%$. We obtain the parameters J and U of the Hamiltonian from a numerical band structure calculation, where a measurement of the interaction using photon-assisted tunneling in a tilted lattice serves as a calibration for U

Numerics and Theory All simulation was performed with exact diagonalization of the Hamiltonian at all times during the dynamics by using the Suzuki-Trotter expansion. The discretization of the time steps was found to not change for >100,000 steps for the entire evolution of the system. It is imporant to mention that all theoretical calculations are done for the full Bose-Hubbard Hamiltonian so the theory curves are compted for the entire BH Hilbert space and then for comparisons to the spin observables the dipole mapping procedure mentioned above is used to find the expected population/contribution to these states from the tilted bose hubbard system. While addmittedly the dynamics of the system are largely dominated by only the states within the subspace of the dipole states, since the Hilbert space of the Bose Hubbard system expands factorially compared to the spin hamiltonian hilbert space, higher order virtual processes could become more of a problem at larger system sizes.

ξ Relation to Weighted Sum The correlation for a Ising-like system is described as an exponential decay as long as the sites are of the normalized two-point correlator are not too close 8.

$$g_2(|i-j|) = |\langle S_i^z S_j^z\rangle - \langle S_i^z\rangle\langle S_j^z\rangle| \sim e^{-|i-j|/\xi} \quad (8)$$

Unfortunately fitting small finite systems can be very noisy due to residual oscillations in the system. One very natural way to find the extend of the correlations on length in the system is to consider a weighted sum of the correlations by their distance 9.

$$X = \frac{\sum_{d=0}^{N} d g_2(d)}{\sum_{d}^{N} g_2(d)} \quad (9)$$

If we assume the form of 8 to describe $g_2(d)$, then in the infinite and continuous case, the sums become integrals and $X \to \xi$. However, these discrete sums over finite chain lengths are also easily solvable simply being geometric sums. In which case $X(N,\xi)$ is dependent upon the number of the sum is truncated too (chain length) and the correlation length of the system **??**.

$$X = \frac{\sum_{n=0}^{N} n r^n}{\sum_{n=0}^{N} r^n} \quad (10)$$

where $r = e^{-1/\xi}$.

$$X = \frac{r}{1-r} - \frac{(N+1)r^{N+1}}{1-r^{N+1}} \quad (11)$$

OTHER MATH

$$X = \frac{\sum_{n=0}^{N} n r^n}{\sum_{n=0}^{N} r^n} \quad (12)$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad (13)$$

$$\sum_{n=0}^{N} r^n = \frac{1-r^{N+1}}{1-r} \quad (14)$$

$$\sum_{n=0}^{N} n r^n = r\frac{\partial}{\partial r}\sum_{n=0}^{N} r^n \quad (15)$$

$$r\frac{\partial}{\partial r}\sum_{n=0}^{N} r^n = r\frac{\partial}{\partial r}\frac{1-r^{N+1}}{1-r} = \frac{-(N+1)r^{N+1}}{1-r} + \frac{r(1-r^{N+1})}{(1-r)^2} \quad (16)$$

$$\frac{r\frac{\partial}{\partial r}\sum_{n=0}^{N} r^n}{\sum_{n=0}^{N} r^n} = \frac{r}{1-r} - \frac{(N+1)r^{N+1}}{1-r^{N+1}} \quad (17)$$

WHERE FIGURE ARE KEPT

- Correlatin Length long chain: /02Feb16/02/afm4
- 4 thru 8 site theory: /02Feb16/08/theory
- 4 site data: /02Feb16/08/
- 5 site data: /02Feb16/08/
- 6 site data: /02Feb16/09/
- 7 site data: /02Feb16/11/

- 8 site data: /02Feb16/12/
- Adiabaticity: /01Jan16/28/
- Adiabaticity: /02Feb16/04/

* greiner@physics.harvard.edu

[1] W. S. Bakr, J. I. Gillen, A. Peng, S. Fölling, and M. Greiner, Nature **462**, 74 (2009).
[2] J. Simon, W. S. Bakr, R. Ma, M. E. Tai, P. M. Preiss, and M. Greiner, Nature **472**, 307 (2011).

	Theory AFM	Theory PM
4 Sites	0.4%	0.4%
5 Sites	1%	2.7%
6 Sites	1%	1.5%
7 Sites	1.3%	3%
8 Sites	1.5%	2.25%

TABLE I. This is a table quantifying the percentage of states accessed throughout the dynamics in both the code and the data that fall within the spin subspace described in the text .

	BH Hilbert Space	Ising Hilbert Space
4 Sites	35	5
5 Sites	126	8
6 Sites	462	13
7 Sites	1716	21
8 Sites	6435	34

TABLE II. Listing of the hilbert space sizes in comparision for both the full Bose Hubbard Model and the Ising subspace mapped to as a part of the transition investigated in the text.