1 Shifting Harmonic Trap with Gradient

1.1 Derivation

Calculating the gradient needed to shift a harmonic potential by a number of sites n.

Harmonic Trap in Hz (1)

$$E_{harm}/\hbar = \frac{1}{2\hbar} m\omega_t^2 x^2 = \frac{1}{2\hbar} m\omega_t^2 a^2 y^2 \tag{1}$$

Where y is coordinate in lattice sites, a is the lattice spacing, and ω_t is the trap frequency. A convenient way to calculate the shift is to combine the physical constants in the equation with the recoil energy in Hz(2).

$$E_{rec}/\hbar = \omega_{rec} = \frac{\hbar k^2}{2m} = \frac{\hbar\left(\frac{\pi^2}{a^2}\right)}{2m} = \frac{\hbar\pi^2}{2ma^2} \tag{2}$$

Substituting (2) into (1) leaves the harmonic potential in terms of only ω_t and ω_{rec} (3).

$$E_{harm}/\hbar = \frac{1}{2\hbar}\omega_t^2 y^2 \left(\frac{2ma^2}{\pi^2\hbar}\right)\left(\frac{\hbar\pi^2}{2}\right) = \frac{1}{4}\frac{\omega_t^2\pi^2}{\omega_{rec}} y^2 \tag{3}$$

Now consider some potential that combines both a harmonic term and a linear term to shift the position of the harmonic when completing the square (4).

$$V/\hbar = Ay^2 - By = A\left(y - \frac{B}{2A}\right)^2 + constant \tag{4}$$

Where B is defined as the gradient in kHz/site. So now if we consider a desired offset of n sites we can calculate what the gradient must be to provide this shift (5). We can ignore the constant term in this case as it just provides some overall offset to the potential and doesn't effect the dynamics of this simple system.

$$n = \frac{B}{2A} \rightarrow B = n2A = n2\left(\frac{1}{4}\frac{\omega_t^2\pi^2}{\omega_{rec}}\right) = \frac{n}{2}\frac{\omega_t^2\pi^2}{\omega_{rec}} \tag{5}$$

1.2 Quick Explicit Example

Let's consider shifting a 5 kHz trap by 150nm ($\approx a/4$). For the Rb microscope, $\omega_{rec} = 2\pi \cdot 1.24$ kHz.

$$B = \frac{(1/4)}{2} \frac{(2\pi \cdot 5)^2 \pi^2}{2\pi \cdot 1.24} = 156.282\text{kHz/site} \tag{6}$$

1.3 Kicking with Lattice

If we consider turning on a lattice that is not centered on a tight harmonic trap then this can also be used to give a kick or apply a gradient to the harmonic trap. For example if we consider doing this by $\approx a/4$ then this would be at the linear region of the lattice potential and the effect can be reasonably approximated by a linear gradient.

$$\omega_{lat} = \frac{\hbar k^2}{m} \sqrt{\frac{V_{lat}}{E_{rec}}} \tag{7}$$