Section 8: Phys 175

3-28-2017

Coherence & Coherent States.

- Coherence
  - what is it?
    - spatial vs. temporal
  - Young's slits (maybe)
- Degrees of Coherence
- Coherent States
- Some coding!

Questions?

What is "coherence"?

What does "coherent" mean to you

When we say "coherence" in physics we're typically talking about the correlation between two different places in time or space.

- ~~[illegible]~~

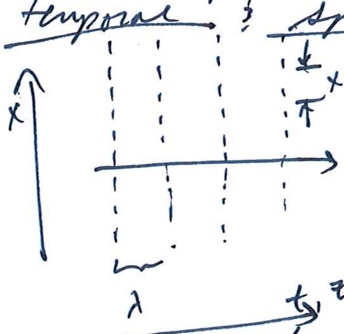
So we always must compare two things and say they're statistically correlated in their random variations with time (or space)

- funny thing about "coherent" meaning "similar" but "similar" implies two things, "coherent" can be with itself.

There are two types of this:

"temporal" & "spatial"

temporal & spatial



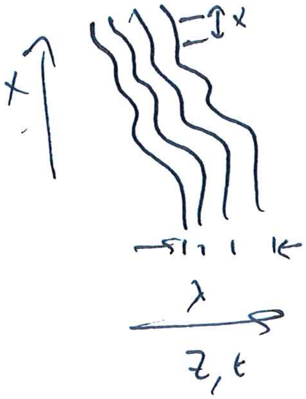
monochromatic light is temporally coherent.

τ:
ask if the E field translated by τ is correlated w/ itself.

Yes!

x:
what about across x?

what about this one?



This one is still coherent over "z,t" but not "x"

where you can see that $E(x,t) \to E(x+\Delta x, t)$

has a relation that will go to zero

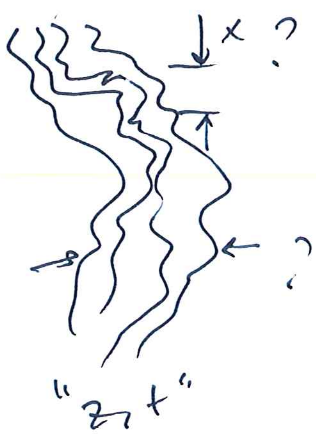
or be random after some distance

distance through the wave front

are a constant.

$$E(x,t) \to E(x, t+\Delta t)$$

what about this one.



This one is both uncorrelated

across x & t!

Okay how do we quantify this?

Degree of Coherence

$$g^{(1)}(r_1,t_1;r_2,t_2) = \frac{\langle E^*(r_1,t_1)\, E(r_2,t_2)\rangle}{\left[\langle |E(r_1,t_1)|^2\rangle \langle |E(r_2,t_2)|^2\rangle\right]^{1/2}}$$

so for spatial form

just care about the difference in $\tilde{x}$

$$g^{(1)}(\tilde{x}) = \frac{\langle E^*(x)\, E(x+\tilde{x})\rangle}{\sqrt{\langle |E^*(x)\, E(x)|^2\rangle \langle |E^*(x+\Delta x)\, E(x+\Delta x)|^2\rangle}}$$

for time

$$g^{(1)}(\tau) = \frac{\langle E^*(t)\, E(t+\tau)\rangle}{\sqrt{\langle |E^*(t)\, E(t)|^2\rangle \langle |E^*(t+\tau)\, E(t+\tau)|^2\rangle}}$$

2nd ~~ord~~ - order of Coherence

$$\cancel{g^{(n)}(r_1,t_1;r_2,t_2)} \quad g^{(2)}(r_1,t_1;r_2,t_2) = \frac{\langle E^*(r_1,t_1)\, E^*(r_2,t_2)\, E(r_1,t_1)\, E(r_2,t_2)\rangle}{\langle |E(r_1,t_1)|^2\rangle \langle |E(r_2,t_2)|^2\rangle}$$

you can do this out to some nth order of coherence.

Coherent States

This is ~~not~~ one reason why Glauber states are called coherent states.

I take $E_q \leftrightarrow a$, $E^* \leftrightarrow a^\dagger$ from our quantized fields & $\phi \langle \; \rangle$'s is the expectation value of our $|\alpha\rangle$ we choose.

The coherent states $|\alpha\rangle$ that we chose all them have $g^{(n)}(..) = 1$ for all "n"

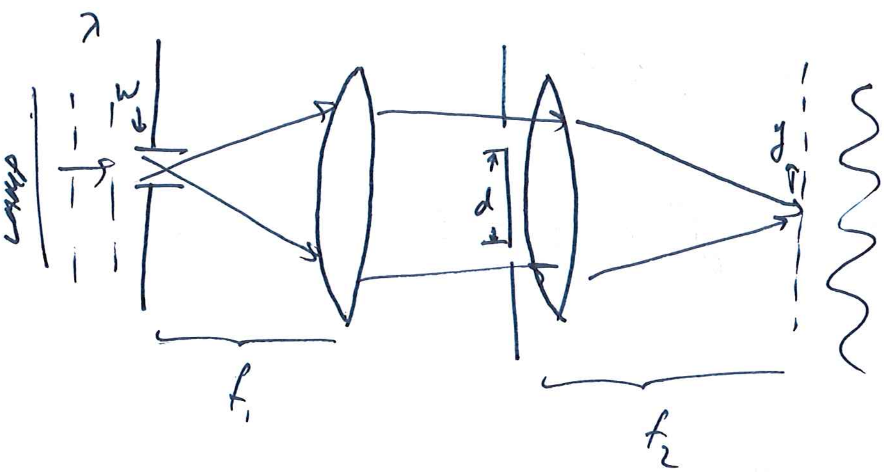
$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

where these are called Fock states or number states and describe the population of the n^{th} mode of our H.O.

(6)

An aside about coherence from the Young's slit exp. (Ref. Book by Brooker, ch. 9,10)

Consider a monochromatic flat wave front from a lamp.



$$I \sim \cos^2\left(\tfrac{1}{2} k d y \tfrac{1}{f_2}\right)$$

$$\sim 1 + \cos\left(\frac{kdy}{f_2}\right)$$

only two rays meet this condition for small w & infinitely small width of the slits separated by "d."

By making the slit wider to finite width "w." Then different pairs of beams translated to the extremes of $w \to \pm \frac{1}{2} w$ where then the beams have to travel a differential distance given by the offset.

offset of two beams

$$\Rightarrow I(w) = 1 + \cos\left(\frac{kdy}{f_2} - \frac{kdw}{2f_1}\right)$$

~~that~~

Side notes about coherent states

- Sometimes people will say coherent states are "orthogonal". Is this true?

(no)

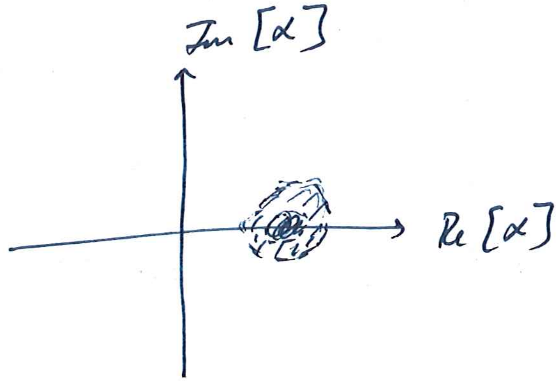
but

But, as $\alpha >> 1$, then neighboring states (ex: $\alpha = \sqrt{n}$ & $\alpha' = \sqrt{n+1}$ $\Rightarrow$ $\langle \alpha | \alpha' \rangle \to 0$) as $n \to \infty$

- Is $|\alpha\rangle$ complete? Yes, but it's over complete so you should be careful.

- Wigner Plots, how do I decompose my state into coherent states?

$$Q = |\langle \psi | \alpha \rangle|^2$$



Useful for seeing effects of how sometimes the state $|\alpha\rangle$ evolves in time.

ex: Kerr effect $H_i = \frac{U}{2} n_i (n_i - 1)$

$\longrightarrow$ can generate states where $|\psi(t')\rangle = |\alpha\rangle + |-\alpha\rangle$

Young's slit cont.

You can integrate ~~off~~ over "w"

Normalize / mean.

$$\frac{1}{w}\int_{-w/2}^{+w/2} I(w) = \frac{1}{w}\int_{-\frac{1}{2}w}^{+\frac{1}{2}w} 1\,dw + \frac{1}{w}\int_{-\frac{1}{2}w}^{+\frac{1}{2}w} \cos\left(\cancel{2\pi}\,\frac{hdy}{f_2} - \frac{hdw}{2f_1}\right) dw$$

$$= 1 + \frac{1}{w}\left[\sin\left(\frac{hdy}{f_2} - \frac{hdw}{4f_1}\right) - \sin\left(\frac{hdy}{f_2} + \frac{hdw}{4f_1}\right)\right]\frac{2f_1}{hd}$$

$$= 1 + \frac{2f_1}{hdw}\left[\cancel{\sin\left(\frac{hdy}{f_2}\right)\cos\left(\frac{hdw}{4f_1}\right)} - \cos\left(\frac{hdy}{f_2}\right)\sin\left(\frac{hdw}{4f_1}\right) - \cancel{\sin\left(\frac{hdy}{f_2}\right)\cos\left(\frac{hdw}{4f_1}\right)} - \cos\left(\frac{hdy}{f_2}\right)\sin\left(\frac{hdw}{4f_1}\right)\right]$$

$$= 1 + \underbrace{\frac{(-4f_1)}{hdw}\sin\left(\frac{hdw}{4f_1}\right)}_{\text{looks like sinc function}}\cos\left(\frac{hdy}{f_2}\right)$$

when this interference term goes to zero.

This is effectively varying the spatial coherence.

How do we calculate things?

Let's consider a system of a harmonic oscillator.

Where we have $V(x) = \frac{1}{2} m\omega^2 x^2$

w/ eigenstates defined by $|n\rangle$

w/ eigenenergies $E_n = \hbar\omega(n + 1/2)$

Then for a ~~the~~ time-independent hamiltonian.

$$\hat{H}\psi = E\psi$$

$$|n(x)\rangle = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}\, x\right)$$

$$\frac{m\omega}{\hbar} = 1$$

So as long as I know my initial state $|\psi_0\rangle$

I can always calculate the evolution of the system.

$|n(x)\rangle$ are a complete & orthogonal set of functions. so I decompose then my $|\psi_0\rangle$ in terms of $|n(x)\rangle$

$$C_n = \int \langle \psi_0(x) | n(x) \rangle \, dx$$

I then ~~write~~ know:

$$|\Psi_0(x)\rangle = \sum_{n=0}^{\infty} C_n |n(x)\rangle$$

note, that practically in a lab we are units of $\frac{E_n}{\hbar} \Rightarrow [\text{Hz}]$

where each $|n(x,t)\rangle = e^{-i\frac{E_n}{\hbar}t} |n(x,0)\rangle$

where $E_n = \hbar\omega(n+\frac{1}{2})$

So, $$|\Psi_0(x,t)\rangle = \sum_{n=0}^{\infty} C_n e^{-i\frac{E_n}{\hbar}t} |n(x,0)\rangle$$

Let's now look at some test cases.

- Code will be put online too

Side Note: In principle knowing $|\Psi_0\rangle$ can be difficult in the lab. In principle we measure things like $\langle\Psi_0|\hat{O}|\Psi_0\rangle$ so we don't deduce things about the amplitude of the Ψ but it's probability. One way to initialize a system is to do something where I am in an eigenstate of the system of H, then quickly change it to H' where I will project known $|\Psi\rangle$ to my $|\Psi\rangle$'s