Section Notes : 2 2-7-2017

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- Questions?
  - Homework or otherwise

- Outline

Derivation of imaging system (perhaps a little generous to call it a derivation

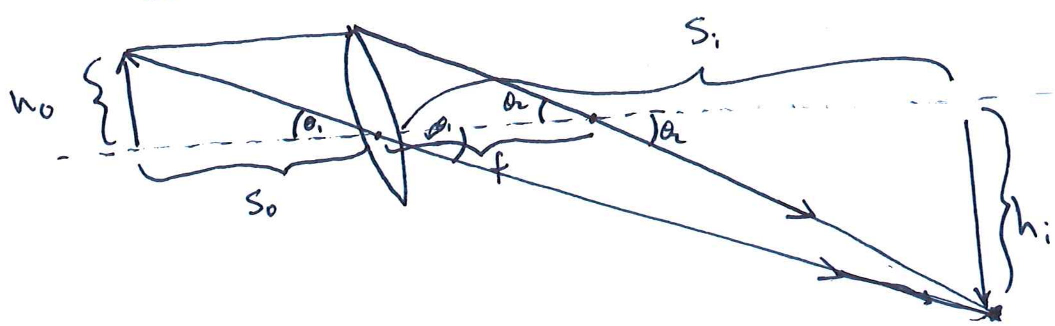
- Problems
  - ~~eyeballs~~ eyeballs
  - Glasses
  - Adding close lenses
  - Microscopy telescope (4F)
  - ~~Telephoto~~ Zoom lens

- Imaging Equation

We will derive this for an object outside of focal length of the converging lens, but the result is true in general.

3 rules of lenses

- i) rays parallel to the axis must ~~be focused~~ go through the focus on the other side
- ii) rays that go through the center don't change k-vector (θ) on either side
- iii) rays through focus will be parallel on the other side (admittedly i) & iii) are redundant.



We have 2 similar triangles here!

Cont. Imaging equation.

~~Triangle~~ Triangle 1

θ_1) ~~so~~ $\tan\theta_1 = \frac{h_o}{s_o} = \frac{h_i}{s_i} \Rightarrow$

$$\boxed{\frac{h_i}{h_o} = \frac{s_i}{s_o} = M} \;!$$

The magnification, M, by definition is $\frac{h_i}{h_o}$!

Consider triangle 2

θ_2) $\tan\theta_2 = \frac{h_o}{f} = \frac{h_i}{s_i - f} \Rightarrow \frac{h_i}{h_o} = \frac{s_i - f}{f}$, which from θ_1)

$\hookrightarrow = \frac{s_i}{s_o}$!

$$\frac{s_i - f}{f} = \frac{s_i}{s_o} \Rightarrow \underbrace{s_o s_i - f s_o = s_i f}$$

perhaps more familiar

$$\boxed{\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}}$$

(sometimes also written as

$$\boxed{(s_i - f)(s_o - f) = f^2}$$

You can get this readily from completing the square/factoring.

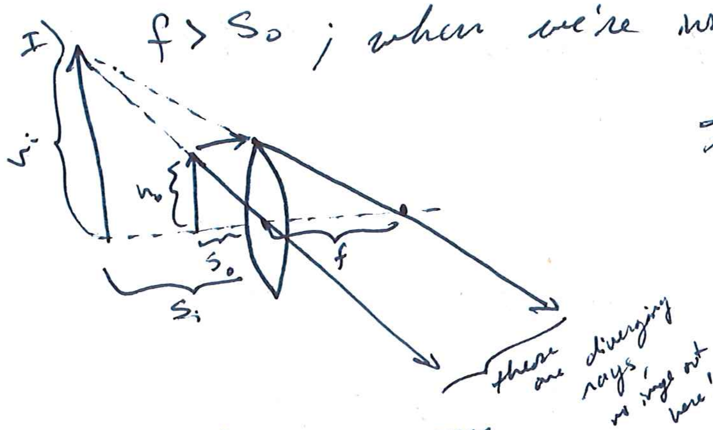
This figure forms a <u>real image</u>:

all rays from object at ~~hopso~~ (s_o, h_o) go to (s_i, h_i).

2 important cases

I) $f > S_o$; when we're inside the focus.

This gives us a virtual image!



$$S_o S_i - S_o f = S_i f$$

$$S_i (S_o - f) = S_o f$$

$$S_i = \frac{S_o f}{S_o - f} \quad \text{if } f > S_o > 0$$

$$S_i < 0$$

$$\& \; \frac{f}{S_o - f} > 1$$

$$\Rightarrow |S_i| > S_o$$

$$\& \; h_i > h_o$$

Note there are some minus signs to be careful with when using the ~~lens~~ imaging eqn from before.

- Wait, so if it's a virtual ~~system~~ image?, how can I see it??

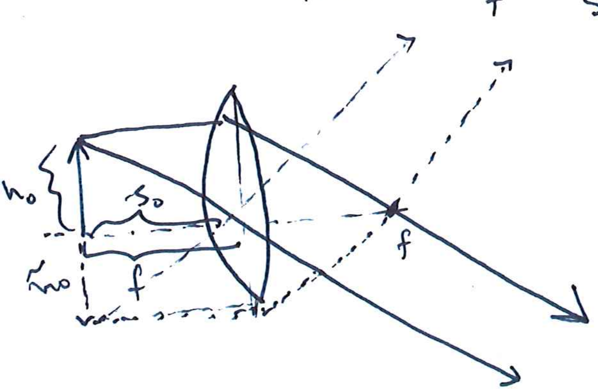
  Answer, your eye is a second optical system!

  The diverging rays then mapped to your retina will "see" the virtual image since that is where the ~~points~~ rays go back to.

II) Preview, when $S_o = f$

from $\frac{1}{f} - \frac{1}{S_o} = \frac{1}{S_i}$ we should see the problem of $S_o \to f$

It implies that $S_i \to \infty$



these two rays are parallel

position is mapped to angle!

ⓐ This case of $s_o = f$ is super very incredibly important for Fourier Optics and an important technique called Fourier Filtering.

So, to be pedantic let's see how this in a bit more generally.

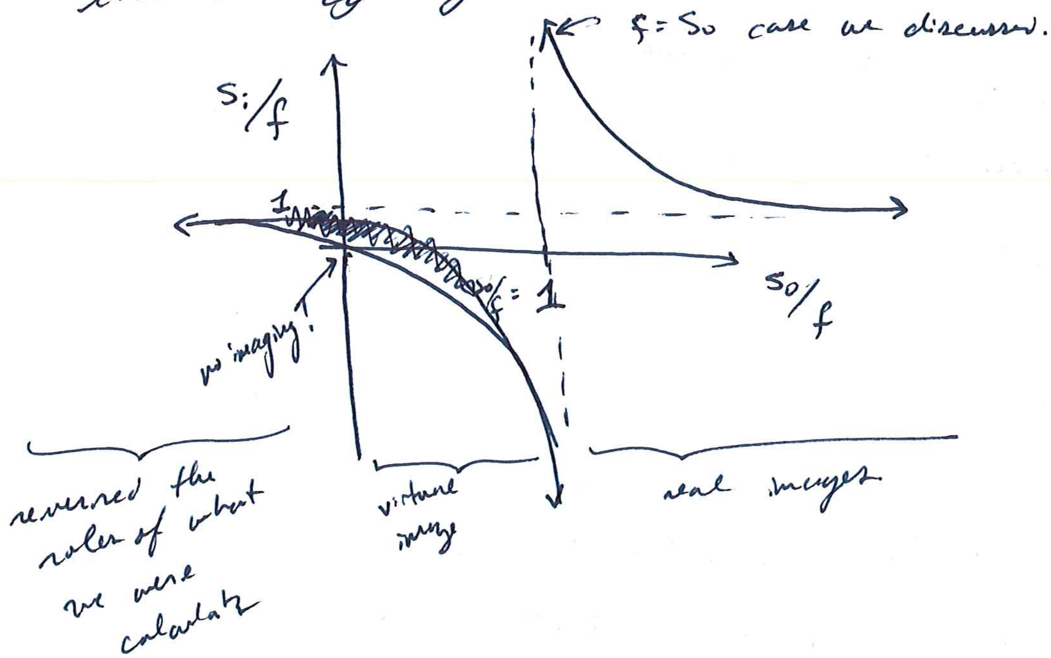
$$\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o} \Rightarrow \underbrace{s_i = \frac{s_o f}{s_o - f}}_{\text{Let's find everything relative to } f} = \frac{s_o f}{f\left(\frac{s_o}{f}-1\right)} = \frac{f\,(s_o/f)}{\left(\frac{s_o}{f}-1\right)}$$

$$\Rightarrow \frac{s_i}{f} = \frac{s_o/f}{(s_o/f - 1)} \qquad \Rightarrow y = \frac{s_i}{f}$$

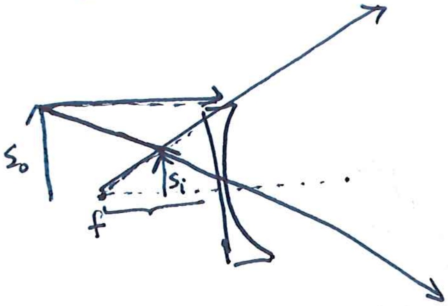
$$x = s_o/f$$

So what does this look like in general?

$$y = \frac{x}{x-1}$$



III) $(f \rightarrow -f)$ Diverging lens



$$s_i = \frac{s_o f}{s_o - f}$$

if $f \rightarrow -f$

$$\Rightarrow \frac{-s_o|f|}{s_o + |f|}$$

$$\Rightarrow \frac{-s_o}{\left(\frac{s_o}{|f|} + 1\right)} = s_i$$

& $s_o = |f| \Rightarrow \frac{s_o}{2} = s_i$

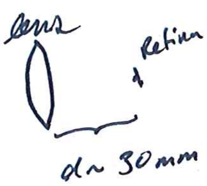
for all $s_o > 0$

we get $|s_i| < |s_o|$

Applications:

a) eyeballs:

Healthy eyeballs are composed of a lens and a retina (for our purposes). The lens is special because it can change focal length.



Healthy eyes can focus from far away up to ~120mm. So what are the range of focal lengths?

far away? ; $s_o \Rightarrow \infty$

$$\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$$

$s_i \sim f$

so $\boxed{f = 30\text{ mm}}$

close up? $s_o \sim 120$ mm

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{120}$$

$\boxed{f = 24\text{ mm}}$

So the typical question is for far/near sighted people, what is wrong with their eyes?

near sighted – (myopic)

~~[illegible]~~ eye too large! or f can't be large enough

If you're near sighted, what sort of lens do you need diverging or converging?

(6)

Consider eyeballs now w/ far-sighted?

(far-sighted) (hyperopia)

eye is too short! or f is too large.

also not focused!

Do you need Converging or diverging lenses?

Fun (maybe) question (I)!

who has read Lord of the Flies?

Was "piggy" famously myopic or hyperopic?

So ~~which~~ which lenses would he have had in his lenses?

Diverging! So the whole story w/ using these glasses to start a fire is B.S. and the story would've been completely different!

Fun (maybe) question (II)!

Are our eyes useful under water?

$f \sim \frac{1}{\Delta n}$ → air, perfect. → water, super far sighted!

$$P = \frac{1}{f} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

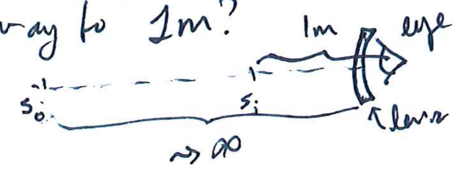
$\Delta n = n_2 - n_1$

for air, $\Delta n \sim 0.4$!

for water, $\Delta n \sim 0.05$!

Glasses (Multi-lens systems)

Let's say I'm near-sighted (I am actually) and I can only see from 1 m or closer, how do I bring objects far away to 1m?



$s_i = -1$

$s_o \to \infty$

$$\frac{1}{f} = \frac{1}{-1} + \cancel{\frac{1}{\infty}}^{\,0}$$

$$\boxed{f = -1\text{m}}$$

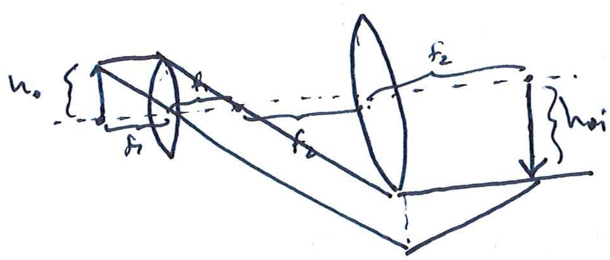
(7) Eyes & pinholes

As a separate note, for those of you who are near-sighted do you naturally squint when you can't see something? Consider an aperture in front of the eye that is very small.

effectively corrects (well kind of) this problem.

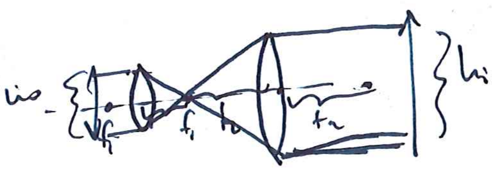
But, of course, the image gets way dimmer since you block out most of the light.

(4f) Imaging (multi-lenses)



$$\boxed{M = -\frac{f_2}{f_1}}$$

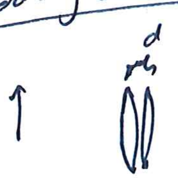
another way to see this is ⇒



In practice, this is super useful for many reasons!

Note too that this implies that as I move an object around in the plane being imaged, the offset inverted on the other side is moved by a factor of M!

Adding close lenses



we assume $d \ll f, o, i$

lens 1: $\frac{1}{s_0} + \frac{1}{s_1} = \frac{1}{f_1} = P_1$ ← this is the "power" of the lens which is measured in "diopters"

lens 2: $\frac{1}{-s_1} + \frac{1}{s_2} = \frac{1}{f_2}$

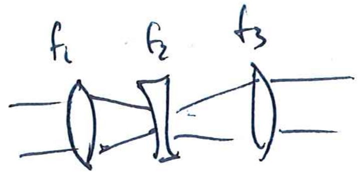
$$\frac{1}{s_0} - \frac{1}{f_1} + \frac{1}{s_2} = \frac{1}{f_2}$$

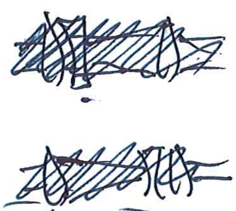
$$\frac{1}{s_0} + \frac{1}{s_2} = \frac{1}{f_2} + \frac{1}{f_1} = P_1 + P_2$$

so the power of nearby lenses add.

(8) However, especially for those of you interested in fotography, you sometimes want to ~~move~~ change your zoom (adjust M). And especially not by changing f_1 & f_2!

So now we need 3 lenses





~~For a simple example:~~
Assume $f_1 = f_3 = \frac{1}{2}$ m

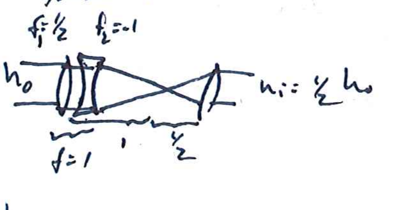
$f_2 = -1$ m

from our close lens addition we know

$P_{eff} = P_1 + P_2 = \frac{1}{f_{eff}}$

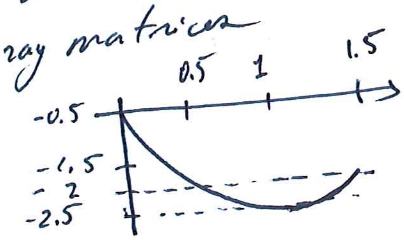
$(2 - 1) = \frac{1}{1} = f_{eff} = 1$

take two extremes



If we use mathematica and ray matrices

$\Rightarrow$



Another way to get an idea of how to get f_{eff} from two lenses & distance we could solve the transfer matrices for

where $d << f_1, f_2$

$$\Rightarrow \frac{1}{f_{eff}} = \frac{-1}{f_1} + \frac{-1}{f_2} + \frac{d}{f_2 f_1}$$

Gullstrand's Equation

$$P = P_1 + P_2 - d\frac{P_1 P_2}{n}$$

use P_2 w/ $-f$

$$\underbrace{P = P_1 - |P_2| + P_1|P_2|d}$$!

this gives an adjustable focal length! Use a third lens to fix aberrations / image to the right place.