Polarization & Interference.

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Section 6
Phys 175

~~Interfe~~

First, we're filming today as part of the teaching requirements for TF's in physics, so, word of caution to those who don't want to be in the video!
(P.S., this isn't public, just for internal use)

Interference:

- Mostly talk about thin films.
- Bubbles Demo!

Polarization:

- Defin's
- Jones Vectors defn
- Rotation & retardation plates.

~~Interference on thin films~~

General Interference

Ex:

$$E_{(1)} = E_1 e^{i(k_1 \cdot r - \omega_1 t + \varphi_1)}$$

$$E_{(2)} = E_2 e^{i(k_2 r - \omega_2 t + \varphi_2)}$$

$$E_{INT} = E_{(1)} + E_{(2)}$$

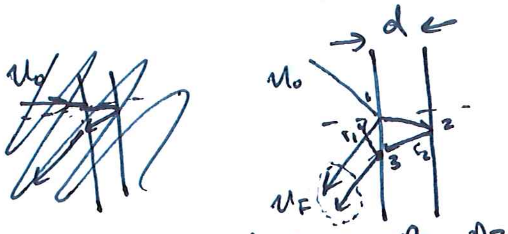
$$I = |E_{INT}|^2 = (E_{(1)}^* + E_{(2)}^*)(E_{(1)} + E_{(2)})$$

$$= |E_1|^2 + |E_2|^2 + \underline{E_1^* \cdot E_2 + E_2^* E_1}$$

$$= \ldots \quad e^{-i(k_1 \cdot r - \omega_1 t + \varphi_1)} e^{i(k_2 r - \omega_2 t + \varphi_2)} + e^{i(k_1 r - \omega_1 t + \varphi_1)} e^{-i(k_2 r - \omega_2 t + \varphi_2)}$$

$$\ldots \quad 2E_1 E_2 \cos(k_1 r - k_2 r - \omega_1 t + \omega_2 t + \varphi_1 - \varphi_2)$$

Thin film interference.



where I observe.

$$u_F = e^{ikd_1} r_1 u_0 + e^{ikd_2} r_2 u_0$$

$$u_F = u_0 \left(r_1 + r_2 e^{ik\Delta d} \right) e^{ikd_1}$$

$$\Delta d = d_2 - d_1$$

$$d_2 = (d_{12} + d_{23})\, n$$

$$d_1 = d_{14}$$

$(n_3 = n_1) < n_2$

$r_1 = -r_2$ (removing angle dependence)

$$r_{ij} \sim \frac{n_i - n_j}{n_i + n_j}$$

So, $u_F = u_0 R(\lambda, \theta) r_1 e^{ikd}$

$$R(\lambda, \theta) = (1 - e^{i\phi})$$

$$|R(\lambda,\theta)|^2 = \overset{1+1-e^{-i\phi}-e^{i\phi}}{2 - 2\cos(\phi)} \quad \xrightarrow{\text{MAX}} 4 \quad \phi = \pi, 3\pi, \ldots$$

$$\xrightarrow{\text{MIN}} 0 \quad \phi = 0, 2\pi, \ldots$$

$$\phi = \frac{2\pi}{\lambda}[d(\theta)] \underset{\theta = 0}{\approx} \frac{2\pi}{\lambda}(2d)$$

MAX $d = \lambda/4 \ldots$

MIN $d = 0,\ d = \frac{\lambda}{2}$

↑ Super thin! (easy way to measure)

Demo Time! Soap bubbles, both round & wedges.

- How thin is the top of the bubble?

Polarization. (and interference)

Remember before we had

$$\underbrace{E = E_1 + E_2}_{\text{what polarization when these?}} \Rightarrow \quad I = |E|^2 \qquad \text{(the same)}$$

What if $\vec{E}_{(1)} = E_1\,\hat{a}$ & $\vec{E}_{(2)} = E_2\,\hat{b}$

$$I = |E_1|^2 + |E_2|^2 + 2(\vec{E}_1 \cdot E_2)(\hat{a}\cdot\hat{b})\cos\theta$$

now we worry the dot product of $\hat{a}$ & $\hat{b}$

So what happens if $\hat{a} = \hat{x}$ & $\hat{b} = \hat{y}$? Then these cross terms go away!

What are different types of pol? lin, circular, ellipt. ~~How could we detect but different polarizations? ((linear for example)~~

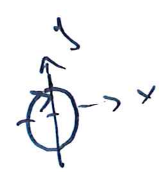
~~Polarizatio~~

How do we describe these mathematically?

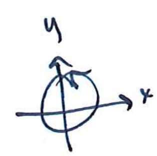
Jones Vector.

$$\vec{J} = \begin{pmatrix}\hat{E}_x \\ \hat{E}_y\end{pmatrix} \quad \text{where} \quad \vec{J}^\dagger \vec{J} = 1$$

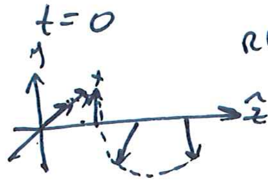
$\vec{J}_x \Rightarrow \begin{pmatrix}1\\0\end{pmatrix}$ Lin along $\hat{x}$ $\qquad \vec{J}_\circlearrowright = \begin{pmatrix}1\\-i\end{pmatrix}\frac{1}{\sqrt{2}}$ RHCP



$\vec{J}_y \Rightarrow \begin{pmatrix}0\\1\end{pmatrix}$ Lin along $\hat{y}$ $\updownarrow$ $\qquad \vec{J}_\circlearrowleft = \begin{pmatrix}1\\+i\end{pmatrix}\frac{1}{\sqrt{2}}$ LHCP



Defn's given by how my thumb points toward the source!



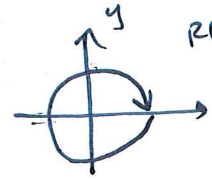
RHCP $\hat{E}_x = \cos(kz-\omega t)$

$\hat{E}_y = \sin(kz-\omega t)$

consider ~~$\hat{E}_x e^{i(kz-\omega t)}$~~ $+\hat{z}$ going wave.

~~$\hat{E}_y \sin(kz-\omega t)$~~

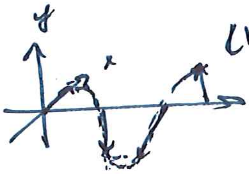
z=0



RHCP

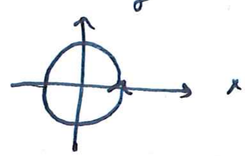
evolves w/ $e^{-i\omega t}$

the E field I see!



LHCP $\hat{E}_x = \cos(kz-\omega t)$

$\hat{E}_y = -\sin(kz-\omega t)$



Evolves in this parametric plot w ~ $e^{-i\omega t}$

Polarizers

So how can we select out a particular for ex?

Use a polarizer!

Work on linear polarization

$$\overleftrightarrow{T}_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \overleftrightarrow{T}_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Non-conserving!

$$\vec{J} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \Rightarrow \overleftrightarrow{T}_x \vec{J} = \begin{pmatrix} \alpha \\ 0 \end{pmatrix}$$

Non-unitary.

$$\overleftrightarrow{T}_y \vec{J} = \begin{pmatrix} 0 \\ \beta \end{pmatrix}$$

Also, clearly $\overleftrightarrow{T}_x \overleftrightarrow{T}_y = 0$!

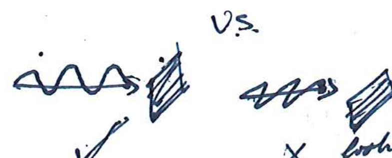
So $\overleftrightarrow{T}_x \overleftrightarrow{T}_y \vec{J} = 0$ in general.

Picture?

How can we think about these devices?

Consider, very classically, of a large λ wave impinging on a metal grill 

Which goes through?



looks like a metal barrier!

$$P_x \left(R^+(\theta)\, P_y\, R(\theta)\right) \underbrace{P_y \vec{J}}_{\text{makes polarized light}}$$

$$\vec{J}_y = \begin{pmatrix} 0 \\ E_y \end{pmatrix}$$

$$P_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad R^+(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$P_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \left(\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \right) \begin{pmatrix} 0 \\ E_y \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \left(\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & 0 \\ -\sin\theta & \cos\theta \end{pmatrix} \right) \begin{pmatrix} 0 \\ E_y \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 \\ E_y \end{pmatrix} = \begin{pmatrix} \sin^2\theta & \sin\theta\cos\theta \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ E_y \end{pmatrix}$$

$$= \begin{pmatrix} E_y \sin\theta\cos\theta \\ 0 \end{pmatrix}$$ goes to zero at $\theta = 0$ & $\theta = \pi/2$!

Then it is just rotated to x or y!

So we have polarizers and rotators, what about ~~Birefri~~ non-isotropic materials?? ~~Hole record~~

Birefringence

First, for isotropic materials, what happens to the wave in distance d?

$$S' = \begin{pmatrix} e^{ikd} & 0 \\ 0 & e^{ikd} \end{pmatrix} \Rightarrow$$ the phase just evolves for E_x & E_y separately.

Birefringent materials.

Now what about materials when

$$S'_{n_x,n_z}(d) = \begin{pmatrix} e^{ikdn_x} & 0 \\ 0 & e^{ikdn_y} \end{pmatrix}$$ where the important part

$$= \begin{pmatrix} 1 & 0 \\ 0 & e^{ikd\Delta n} \end{pmatrix} e^{ikdn_x} \qquad \Delta n = n_x - n_y$$

So physically, one axis has an effectively longer path length.

Typically, for devices this is chosen to be such that the product of $\cancel{e}\, kd\Delta n = \frac{\pi}{2} + n\pi 2$

or

$= \pi/2 + 2\pi n$

$\cancel{\ }\ \Gamma_\theta$

Ex: $\Gamma_{(\pi)} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Lin

RHCP ! → LHCP !

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

ex: $\Gamma_{(\pi)} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_x \\ -E_y \end{pmatrix}$

What is this in λ? $d\Delta n = ?$ $\frac{\lambda}{2}$

$\Gamma_{(\pi/2)}$

$= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

$\Gamma_{\pi/2} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_x \\ iE_y \end{pmatrix}$

Lin → RHCP

$\Gamma_{\pi/2} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

RHCP → lin

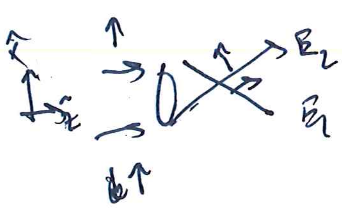
So using $R(\theta)$, Γ we can turn any polarization into any polarization! So very useful!
Note these are all unitary.

Demos

Okay now let's do some demos:

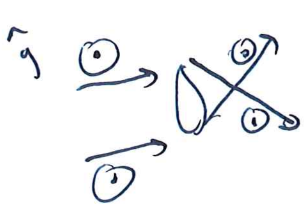
- Big Polarizers
  (hand out small ones)
- Stress induced polarization. (tilt scotch tape)
- Why are they different colors??
- Calcite crystal & image shift.

If time ask question about polarization & high NA



$$E_2 = (\hat{x} + \hat{z})\, e^{ikx + i\omega t}$$
$$E_1 = (\pm\hat{x} - \hat{z})\, e^{-ikx + i\omega t}$$

VS



$$E_1 = \hat{y}\, e^{ikx + i\omega t}$$
$$E_2 = \hat{y}\, e^{-ikx + i\omega t}$$