Practice Mid-Term Examination Questions

Physics 175

March 21/23, 2017

Name: ______________________________

Instructions

You will have 90 minutes to work on the exam. The exam is a total of 100 points. Turning in your formula sheet is worth X points. Budget your time wisely, using the point values as a guide. Use empty pages for computation.

An equation sheet containing all required equations (and then some!) can be found in the back.

You are not allowed to use any material (books, lecture notes) other than *double-sided sheet with your handwritten notes*, which you're required to hand in together with your exam. The use of a calculator is permitted, but no other electronic device (cell phone, laptops, etc.) are allowed during the exam.

Extra paper is available should you require it. Please write your name on *every* sheet that you attach to your exam.

Problem	**Score**
1	___
2	___
3	___
4	___
5	___
TOTAL	___

GOOD LUCK!

1 Lenses

(a) Lens Equations

What is the lens equation?

(b) Imaging: Conceptual

What is the focal length of a lens?

Answer this in words or using the lens equation

(c) Imaging

Where is an object imaged to if it is placed $z_o = 2f$ in front of the lens at $z = 0$?

2 Fourier Transforms

(a) Important Examples

What is the fourier transform of f(x)?

$$f(x) = \delta(x - x_o) + \delta(x + x_o)$$

What is the fourier transform of f(x)?

$$f(x) = \sin k_x x_o$$

(b) 4F Systems

What mathematical function do lenses perform?

(c) 4F imaging

Consider a 4F imaging system where a gaussian beam, with flat phase front at $z = -f_1$ is imaged by a lens with focal length f_1 to a plane $z = f_1$ where it impinges on an ideal grating with spacing d.

The electric field at $z = -f_1$ is given below:

$$E(z = -f_1) = G_{0,\sigma}(x)$$

The mask function $M(x')$, ideal grating in this case, is given functionally below:

$$M(x) = \text{III}_d(x) = \sum_{n=-\infty}^{\infty} \delta(x - nd)$$

Refer to equation sheet at end of test if confused about these functions

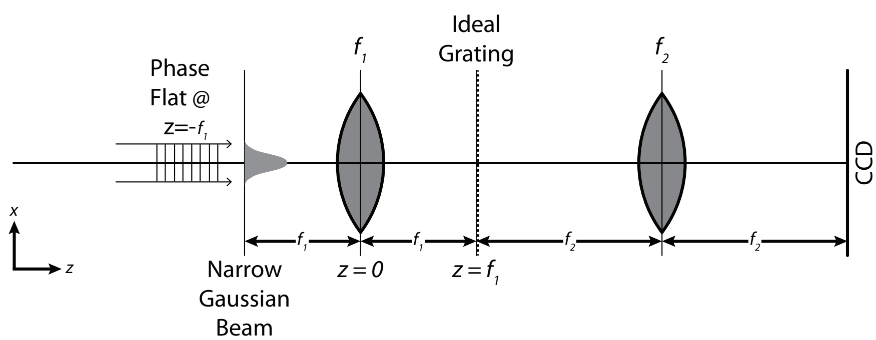

Figure 1: 4F imaging system. Problem **2a**

1. Write on the figure where the "Fourier Plane" is for this imaging system.
2. Determine what the electric field is (as a function of k_x) just before hitting the ideal grating/mask at $z = f_1 - \epsilon$, ($\epsilon \ll f_1$).
3. Rewrite the electric field as a function of transverse coordinate x' at $z = f_1 - \epsilon$, ($\epsilon \ll f_1$).
4. Determine the electric field just after illuminating the ideal grating, $M(x')$, as a function of transverse coordinate x' at $z = f_1 + \epsilon$, ($\epsilon \ll f_1$).
5. Determine the electric field now at the CCD chip ($z = f_1 + 2f_2$). (For simplicity of the solution, assume that $2\pi/d \gg \sigma$.

Hint: Remember that when changing from k-space to real dimensions we use the substitution $k_x \rightarrow \frac{2\pi}{\lambda f} x'$.

Hint: The scaling theorem and convolution theorem may be *very* useful in this problem.

3 Filters by eye

(a) Filters

For the following image, guess which resultant image is a function of the input image (top left) and the fourier filter (top right)

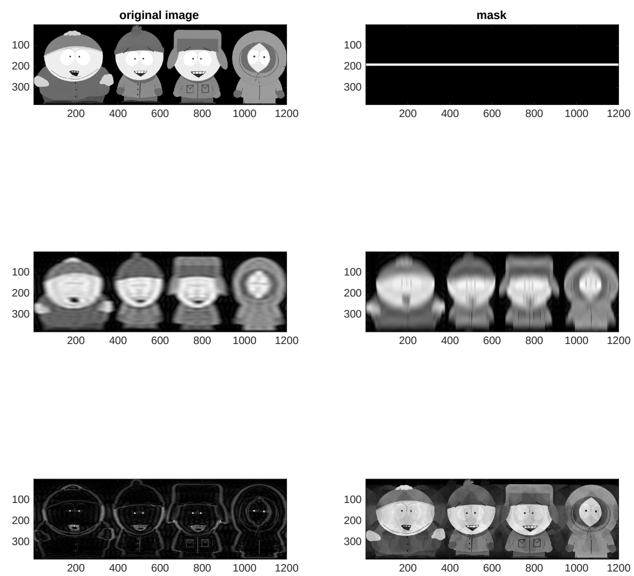

Figure 2: 4F imaging system with filtering. Top Left is the original image. The Filter in the fourier plane is the top right image.

For the following image, guess which resultant image is a function of the input image (top left) and the fourier filter (top right)

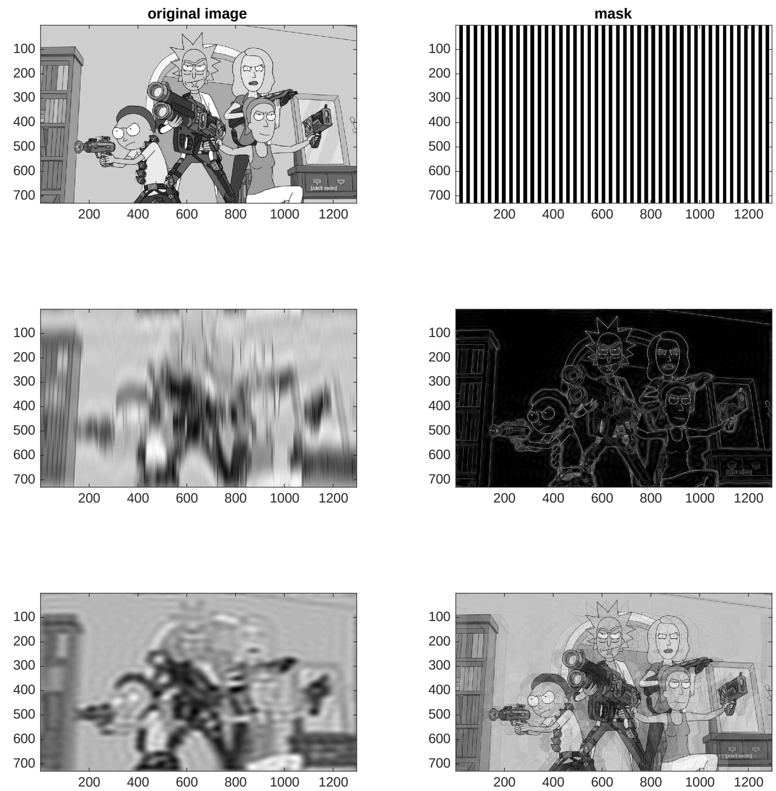

Figure 3: 4F imaging system with filtering. Top Left is the original image. The Filter in the fourier plane is the top right image.

For the following image, guess which resultant image is a function of the input image (top left) and the fourier filter (top right)

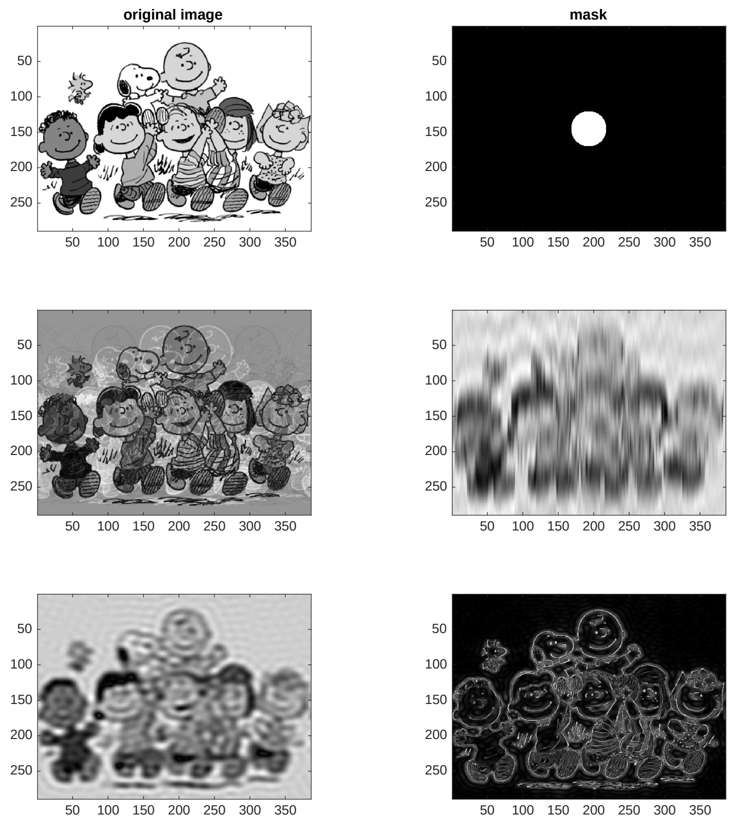

Figure 4: 4F imaging system with filtering. Top Left is the original image. The Filter in the fourier plane is the top right image.

For the following image, guess which resultant image is a function of the input image (top left) and the fourier filter (top right)

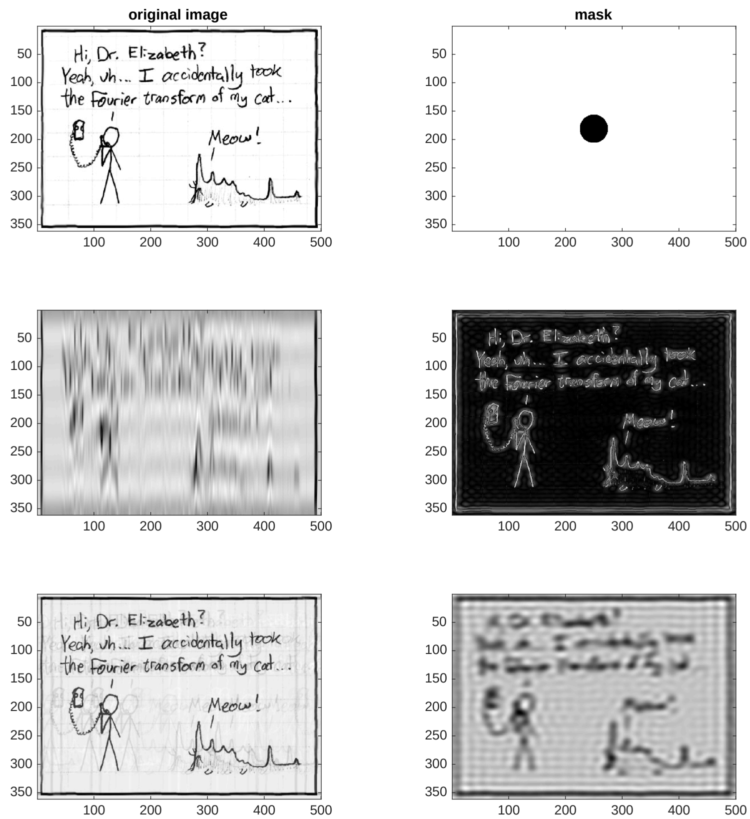

Figure 5: 4F imaging system with filtering. Top Left is the original image. The Filter in the fourier plane is the top right image.

4 Gaussian Modes

(a) Qualitative Question 1

What equation does the "Gaussian Beam" satisfy?

Paraxial Helmholtz Equation

(b) Qualitative Question 2

What is the "rayleigh range" defined as?

(c) Less Qualitative Question 3

If I want a waist of w_o, and I place a lens of f at a location where the radius of curvature is infinite, and the beam has a waist of w':

1. Where will the minimum waist be located?
2. What f should the lens be?

5 Cavities

Given a cavity with two mirrors of equal magnitude of curvature (the sign you can adjust based on placement) separated by $6z_o$, what must the radius of curvatures be?

1. Consider the case where the mirrors are placed symmetrically on either side of the minimum waist.
2. Consider the case where the mirrors are placed both on one side of the minimum waist.

6 Polarization

(a) General Question 1

Using the Jones Vector notation, how do we find if another polarization is orthogonal?

(b) General Question

What does a $\lambda/2$ wave plate do? What are the “fast" and “slow" axes in the form of its matrix representation?

7 True/False (15p)

Circle **T** or **F**, as appropriate to the statement.

1. A diverging lens makes an object appear farther than it actually is. **T / F**
2. The focal length of a glass focusing lens underwater is typically shorter than in air. **T / F**
3. If a laser is collimated on earth and directed into space, the beam will eventually begin to diverge (neglect atmospheric effects). **T / F**
4. When a plano-convex lens is used to focus parallel rays, the curved side should face the parallel rays, to suppress aberrations. **T / F**
5. Two lossless mirrors can be arranged in a way such that 100% of light of two distinct frequencies is being transmitted. **T / F**
6. Two lossless mirrors can be arranged in a way such that 100% of white light is being transmitted. **T / F**
7. Two lossless mirrors can be arranged in a way such that 100% of light of a particular frequency is reflected. **T/F**
8. The Free Spectral Range (FSR) is purely determined by the cavity length.
9. The Finesse of the cavity is determined by both the cavity length and the reflectivity.
10. Metals make good lossless ideal mirrors.

YOU ARE DONE!

Equations

Special Functions

$$\Pi_H(x) = \begin{cases} 1 & : |x| < 1/2 \\ 0 & : |x| \geq 1/2 \end{cases}$$

$$Ш_L(x) = \sum_{n=-\infty}^{\infty} \delta(x - nL)$$

$$G_{\mu,\sigma}(x) = \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

δ Function Identities

$\int_{-\infty}^{\infty} dx f(x)\delta(x-\alpha) = f(\alpha)$	Definition
$\int_{-\infty}^{\infty} dx e^{\pm ikx} = 2\pi\delta(k)$	Decomposition

Fourier Transforms

$f(x)$	$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx f(x) e^{-ikx}$
$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk g(k) e^{ikx}$	$g(k)$
$f(x)g(x)$	$f(k) * g(k)$
$f(x) * g(x)$	$f(k)g(k)$
$f(x-\alpha)$	$f(k-2\pi\alpha)$
$f(x\alpha)$	$\frac{f(k/\alpha)}{\alpha}$
$\delta(x-\alpha)$	$\frac{1}{\sqrt{2\pi}}\exp -ik\alpha$
$\exp(i\alpha x))$	$\sqrt{2\pi}\delta(k-\alpha)$
$\Pi_H(x/L)$	$\frac{L}{\sqrt{2\pi}}\frac{\sin(kL/2)}{kL/2}$
$Ш_L(x)$	$\frac{\sqrt{2\pi}}{L}Ш_{2\pi/L}(k)$
$G_{\mu,\sigma}(x)$	$\sigma e^{-ik\mu} G_{0,1/\sigma}(k)$

Maxwell's Equations

$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	Electric Gauss' Law
$\nabla \cdot \mathbf{B} = 0$	Magnetic Gauss' Law
$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$	Faraday's Law
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0\varepsilon_0 \frac{\partial}{\partial t}\mathbf{E}$	Ampere's Law

EM Boundary Conditions

$\mathbf{D} = \epsilon\mathbf{E} + \mathbf{P}$	Electric Displacement
$\mathbf{B} = \mu\mathbf{H}$	Magnetic Field
$\mathbf{D}_{n1} = \mathbf{D}_{n2}$	Normal Electric
$\mathbf{E}_{t1} = \mathbf{E}_{t2}$	Transverse Electric
$\mathbf{B}_{n1} = \mathbf{B}_{n2}$	Normal Magnetic
$\mathbf{H}_{t1} = \mathbf{H}_{t2}$	Transverse Magnetic

Wave Optics

$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$	Speed of Light
$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$	Wave Equation
$\mathbf{E}(\mathbf{x}, t) = E_0\mathbf{J}_0 \exp i(\omega t - \mathbf{k}\cdot\mathbf{x} + \phi)$	Plane Wave
$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$	Poynting Vector
$\langle S\rangle = \frac{1}{2\mu_0 c}\lvert\mathbf{E}_0\rvert^2$	Plane Wave Intensity (t avg)
$\nabla^2 A + \lvert\mathbf{k}\rvert^2 \frac{\partial A}{\partial z} = 0$	Helmholtz
$\nabla_T^2 A - 2i\lvert\mathbf{k}\rvert \frac{\partial A}{\partial z} = 0$	Parax. Helmholtz

Ray Optics

$P = \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$	Lensmaker's Equation
$\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$	Imaging Equation
$n_1 \sin\theta_1 = n_2 \sin\theta_2$	Snell's Law
$\tan\theta_B = \frac{n_2}{n_1}$	Brewster Angle
$r = \frac{n_1 - n_2}{n_1 + n_2}$	Normal Fresnel

Gaussian Beams

$z_0 = \frac{\pi w_0^2}{\lambda}$	Rayleigh Range
$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}$	Waist
$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$	Curvature
$\frac{1}{f} = \frac{1}{R} + \frac{1}{R'}$	Lens

Cavities

$\mathrm{FSR} = \frac{c}{2d}$	Free Spectral Range
$\mathscr{F} = \pi \frac{\sqrt{\lvert r_1\rvert\lvert r_2\rvert}}{1-\lvert r_1\rvert\lvert r_2\rvert}$	Finesse
$\nu_{\mathrm{FWHM}} = \frac{\mathrm{FSR}}{\mathscr{F}}$	Full Width Half Max

Polarization, Jones Calculus

$\mathbf{J} = \frac{1}{A^2+B^2}\begin{pmatrix} A \\ B \exp i\phi \end{pmatrix}$	Jones Vector
$\mathbf{J}_1 \cdot \overline{\mathbf{J}_2}$	$\mathbb{C}$ dot product
$\mathbf{J} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Horiz. Polarization
$\mathbf{J} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$	Vert. Polarization
$\mathbf{J} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$	Right Handed
$\mathbf{J} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$	Left Handed
$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	Polarizer $\hat{\mathbf{x}}$
$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$	$\lambda/4$ Plate (fast axis $\hat{\mathbf{x}}$)
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\lambda/2$ Plate (fast axis $\hat{\mathbf{x}}$)
$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	Rotation Matrix

Quantum Optics

f $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\psi(t) = \hat{\mathscr{H}}\psi(t)$	Schrodinger

Trigonometry

$\sin^2\theta + \cos^2\theta = 1$	Pythagorean
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\beta\sin\alpha$	Sine sum
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	Cosine sum
$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$	Double Angle Sine
$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$	Double Angle Cosine