Review Problems

Section 7:

3/21-23/2017

Lens Problems: 1

What is the lens equation?

Lens Problems: 1

$$\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$$

Lens Problems: 2

What is the focal length of a lens?

Answer this in words or using the lens equation

Lens Problems: 2

- On board

Lens Problems: 3

Where is an object imaged to if it is placed $z_o = 2f$ in front of the lens at $z = 0$?

Lens Problems: 3

$$\frac{1}{f} = \frac{1}{2f} + \frac{1}{z}$$

$$z \rightarrow 2f$$

Fourier Transforms : 1

What is the fourier transform of f(x)?

$$f(x) = \delta(x - x_o) + \delta(x + x_o)$$

Fourier Transforms : 1

$$F(x) = \mathscr{F}[f(x)] = \frac{2}{\sqrt{2\pi}} \cos k_x x_o$$

Fourier Transforms : 2

What is the fourier transform of f(x)?

$$f(x) = \sin k_x x_o$$

Fourier Transforms : 2

$$F(x) = \mathscr{F}[f(x)] = \frac{\sqrt{2\pi}}{2i}(\delta(x+x_o) - \delta(x+x_o))$$

Fourier Transforms : 3

What mathematical function do lenses perform?

Fourier Transforms : 3

- Fourier Transforms from –f to f!

Fourier Transforms : 4

Consider a 4F imaging system where a gaussian beam, with flat phase front at $z = -f_1$ is imaged by a lens with focal length f_1 to a plane $z = f_1$ where it impinges on an ideal grating with spacing d.

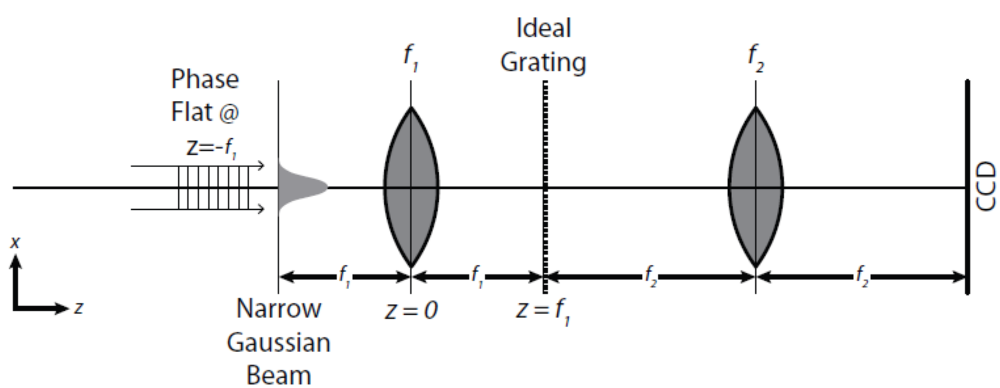
The electric field at $z = -f_1$ is given below:

$$E(z = -f_1) = G_{0,\sigma}(x)$$

The mask function $M(x')$, ideal grating in this case, is given functionally below:

$$M(x) = \text{III}_d(x) = \sum_{n=-\infty}^{\infty} \delta(x - nd)$$

Refer to equation sheet at end of test if confused about these functions



Fourier Transforms : 4

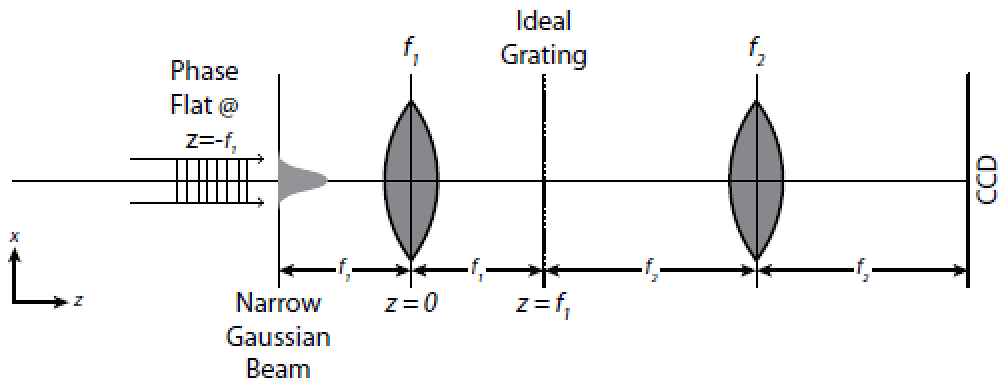
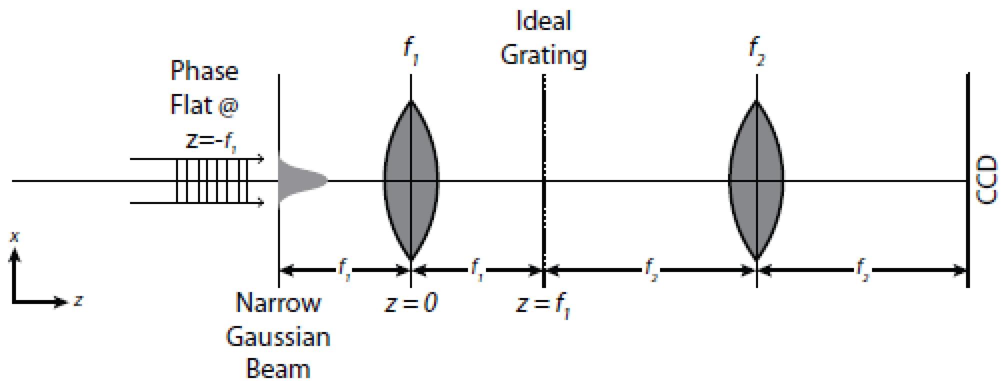


Figure 1: 4F imaging system. Problem 2a

1. Write on the figure where the “Fourier Plane" is for this imaging system.
2. Determine what the electric field is (as a function of k_x) just before hitting the ideal grating/mask at $z = f_1 - \epsilon$, ($\epsilon \ll f_1$).
3. Rewrite the electric field as a function of transverse coordinate x' at $z = f_1 - \epsilon$, ($\epsilon \ll f_1$).
4. Determine the electric field just after illuminating the ideal grating, $M(x')$, as a function of transverse coordinate x' at $z = f_1 + \epsilon$, ($\epsilon \ll f_1$).
5. Determine the electric field now at the CCD chip ($z = f_1 + 2f_2$). (For simplicity of the solution, assume that $2\pi/d \gg \sigma$.

Hint: Remember that when changing from k-space to real dimensions we use the substitution $k_x \to \frac{2\pi}{\lambda f} x'$.

Fourier Transforms : 4



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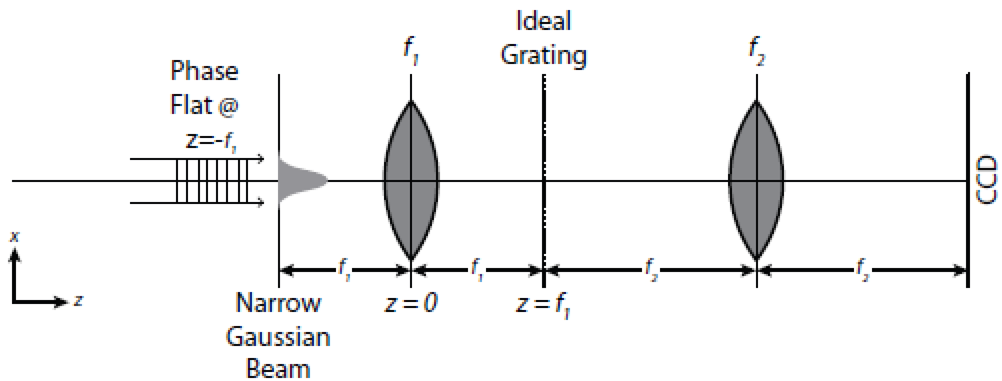
Fourier Transforms : 4

1. The fourier plane is exactly where the “Ideal Grating" is located.

Fourier Transforms : 4



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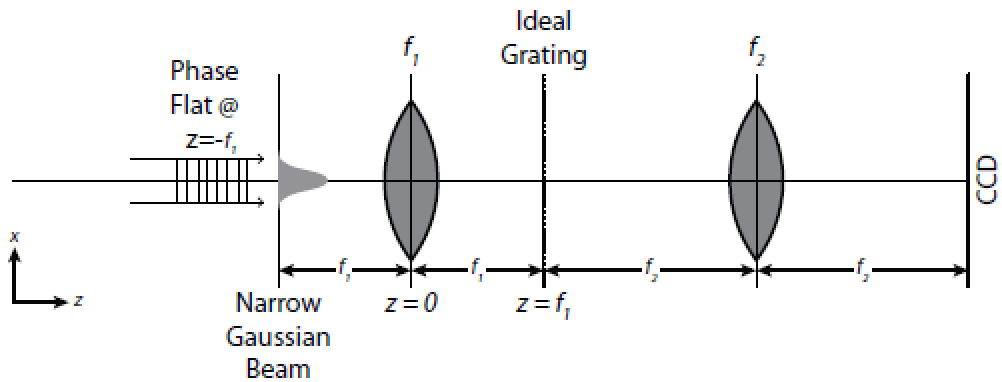
Hint: Remember that when changing from k-space to real dimensions we use the substitution $k_x \to \frac{2\pi}{\lambda f} x'$.

Fourier Transforms : 4

2. We can refer to the formula sheet at the end of the midterm where we can pull out the Gaussian fourier transform formula out. Which will give us the Gaussian just before the ideal grating.

$$\mathscr{F}\left[G_{0,\sigma}(x)\right] = G_{0,1/\sigma}(k_x)$$

Fourier Transforms : 4



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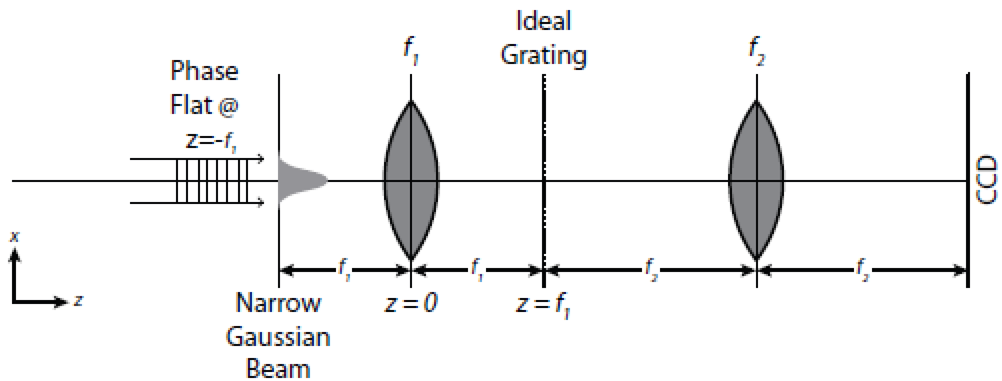
Hint: Remember that when changing from k-space to real dimensions we use the substitution $k_x \rightarrow \frac{2\pi}{\lambda f} x'$.

Fourier Transforms : 4

3. Using the hint, we can rewrite the formula above to:

$$G_{0,1/\sigma}\left(\frac{2\pi}{\lambda f_1}x'\right)$$

Fourier Transforms : 4



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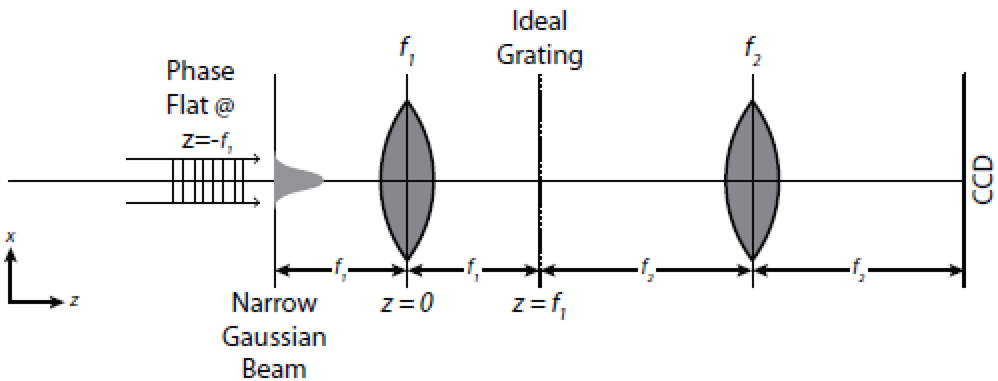
Hint: Remember that when changing from k-space to real dimensions we use the substitution $k_x \to \frac{2\pi}{\lambda f} x'$.

Fourier Transforms : 4

4. The electric field after the grating will just be the multiplication of the grating and the Gaussian beam.

$$G_{0,1/\sigma}\left(\frac{2\pi}{\lambda f_1}x'\right)\sum_{n=-\infty}^{\infty}\delta(x'-nd)$$

Fourier Transforms : 4



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Hint: Remember that when changing from k-space to real dimensions we use the substitution $k_x \to \frac{2\pi}{\lambda f} x'$.

Fourier Transforms : 4

5. Using the convolution theorem and the scaling theorem (also our knowledge about 4f systems) we can realize that at the CCD the answer is of the form

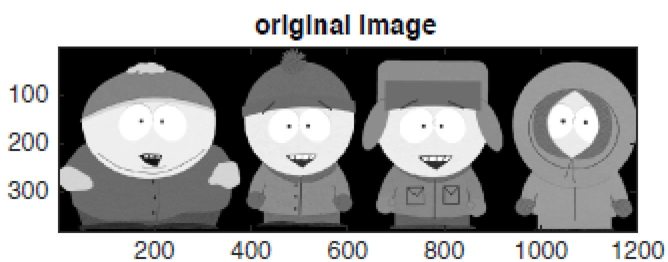
$$\mathscr{F}\left[\sum_{n=-\infty}^{\infty}\delta(x'-nd)\right]*\mathscr{F}\left[G_{0,1/\sigma}\left(\frac{2\pi}{\lambda f_1}x'\right)\right]$$

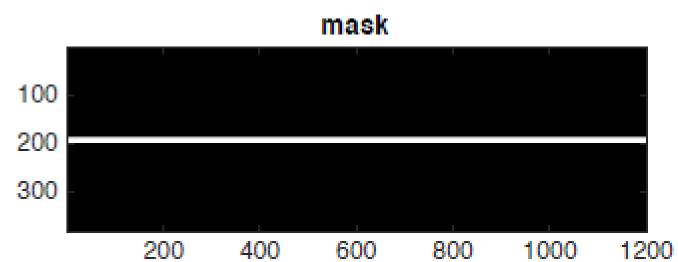
$$\left[\frac{2\pi}{d}\text{Ш}_{2\pi/d}\left(\frac{2\pi}{\lambda f_2}x'\right)\right]*\left[G_{0,\sigma}\left(\frac{f_2}{f_1}x'\right)\right]$$

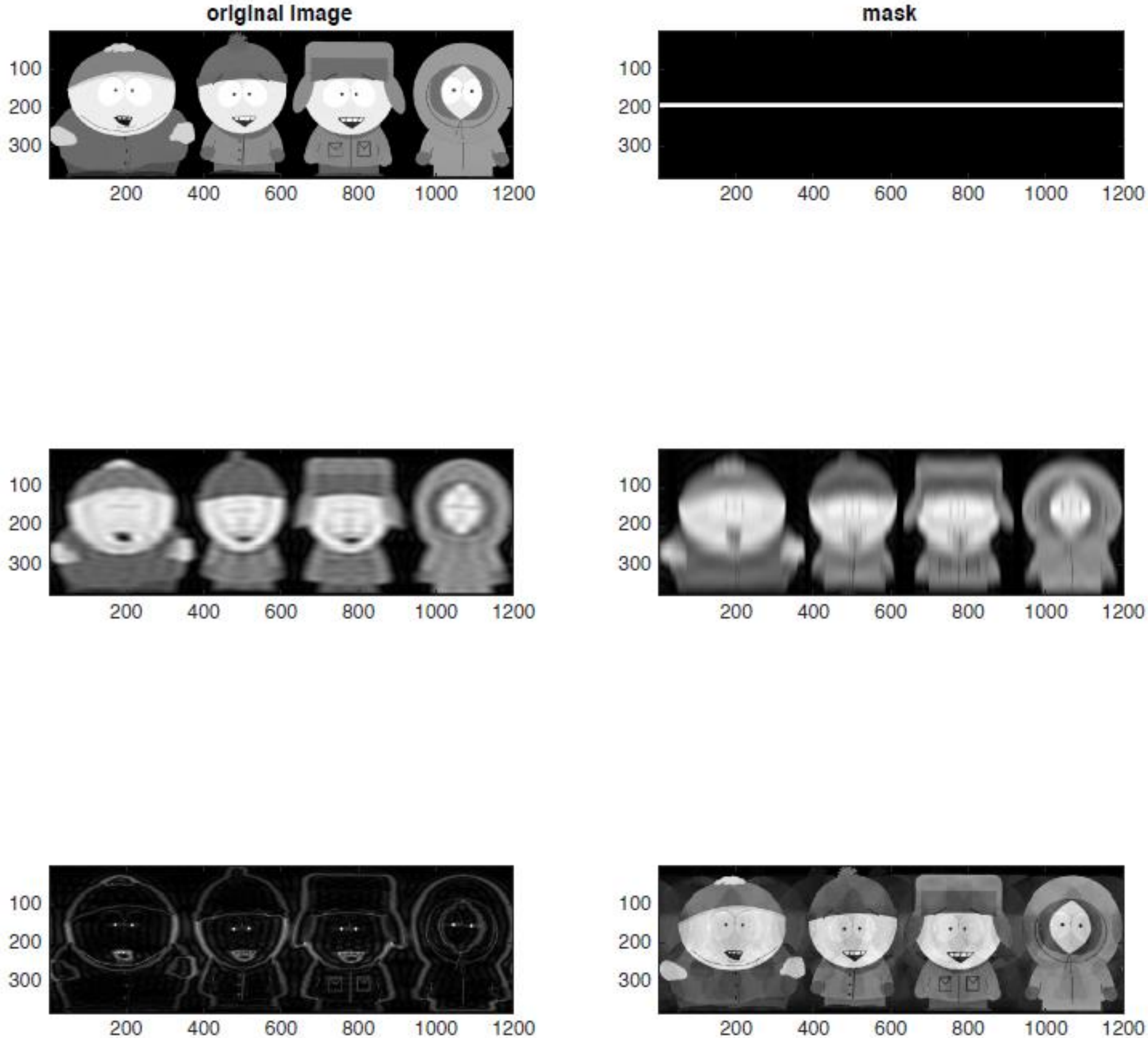
With the assumption that $\sigma \ll d$, then this answers is approximately

$$\approx \sum_{n=-\infty}^{\infty} G_{0,\sigma}\left(\frac{2\pi}{\lambda f_2}\left(x'-n\frac{2\pi}{d}\right)\frac{f_2}{f_1}\right)$$

Fourier Filters (By Eye): 1






original image
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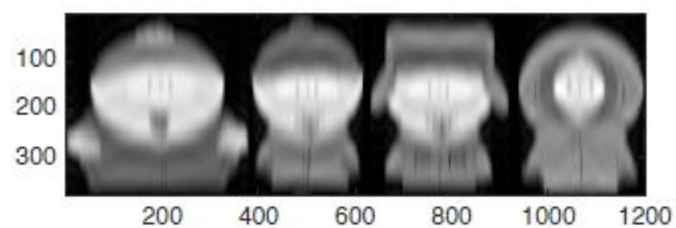
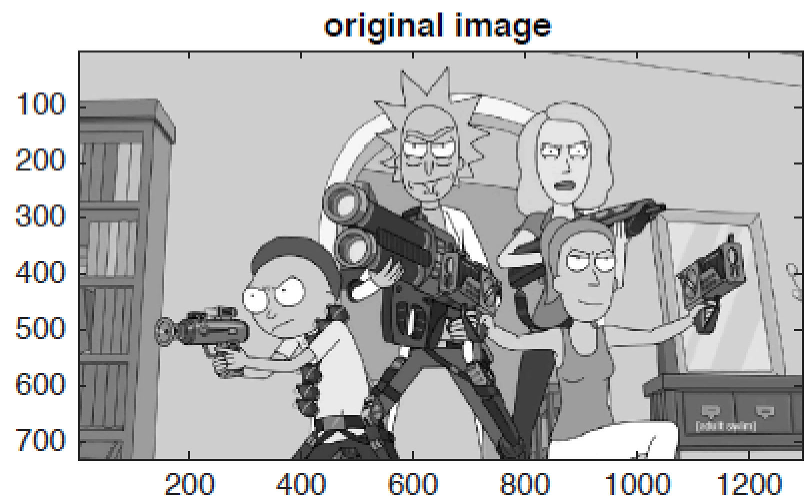
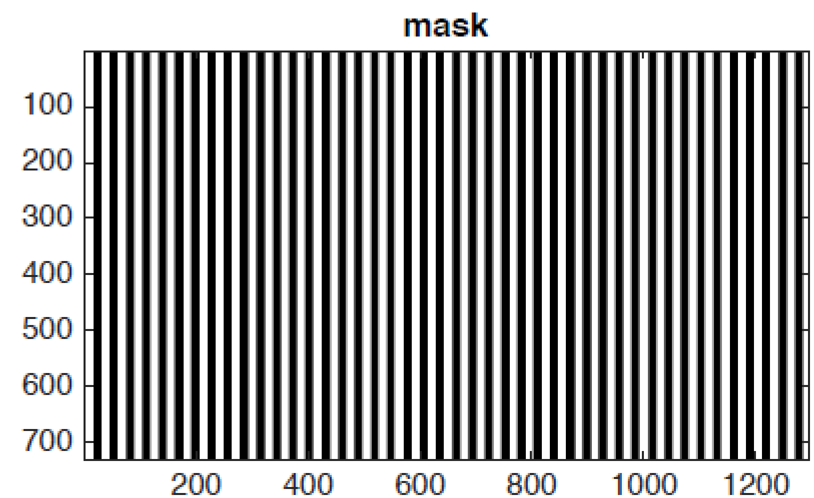
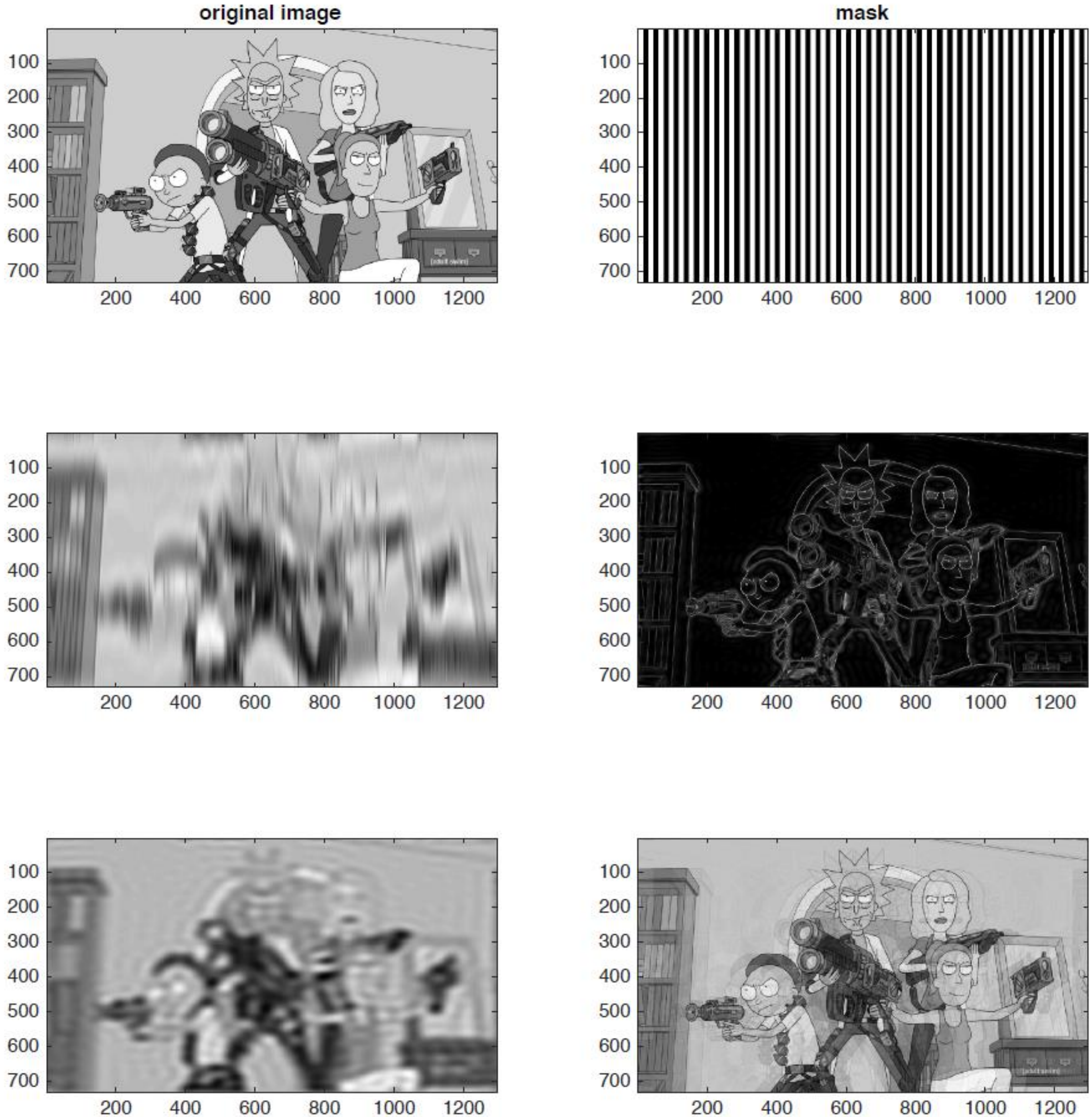
Fourier Filters (By Eye):1



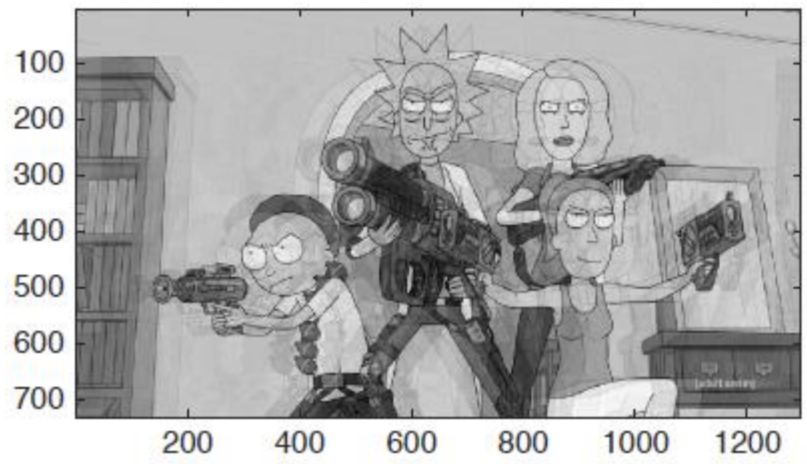
Fourier Filters (By Eye): 2



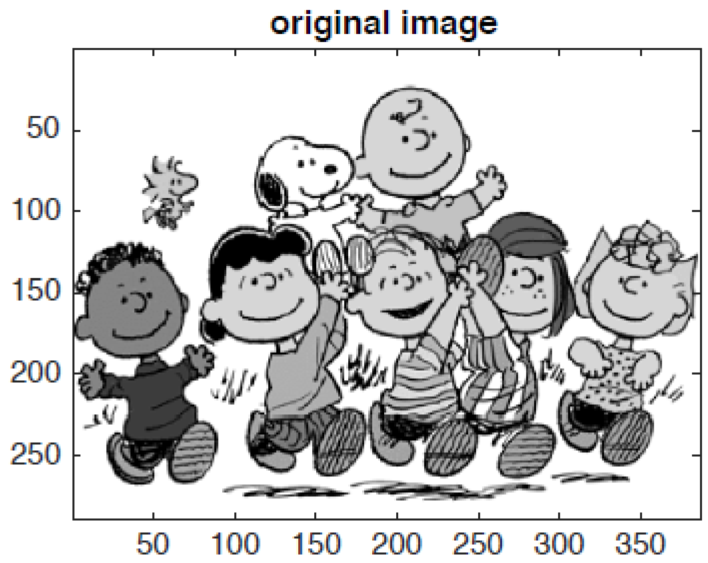


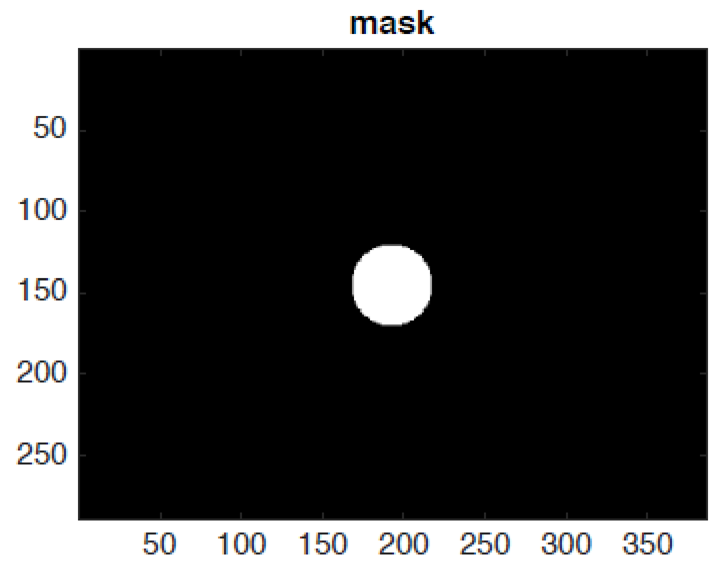

original image
mask

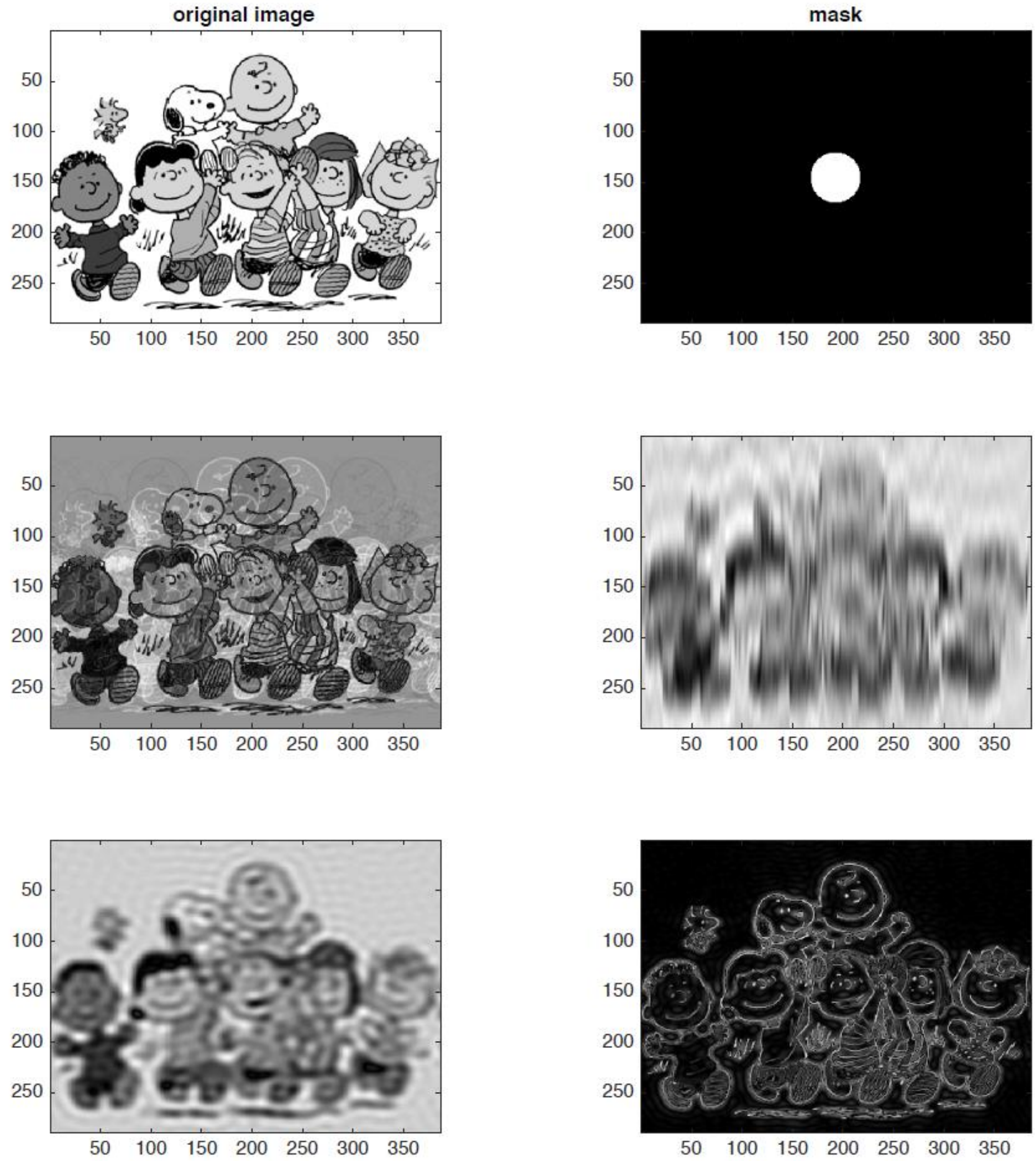
Fourier Filters (By Eye):2



Fourier Filters (By Eye): 3



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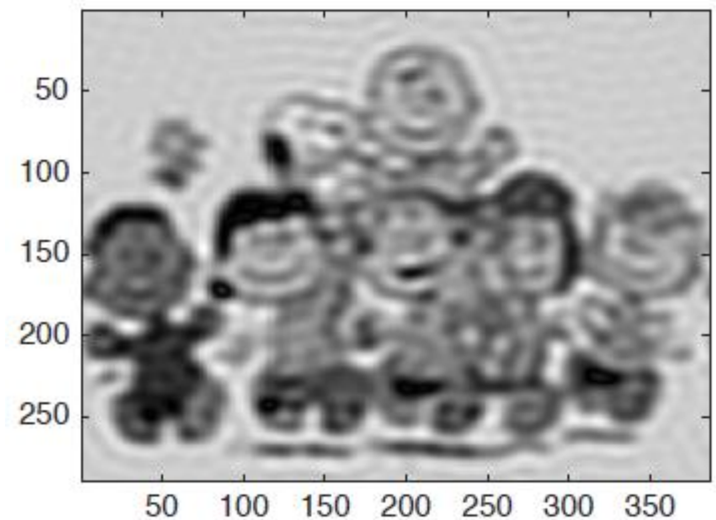
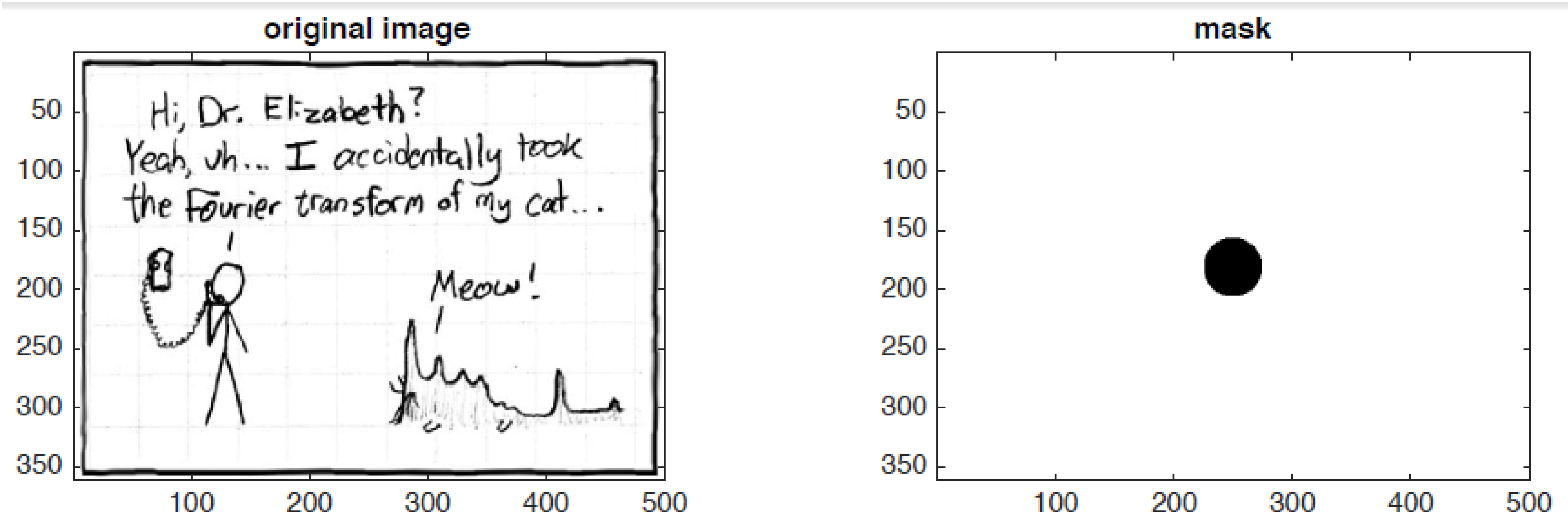
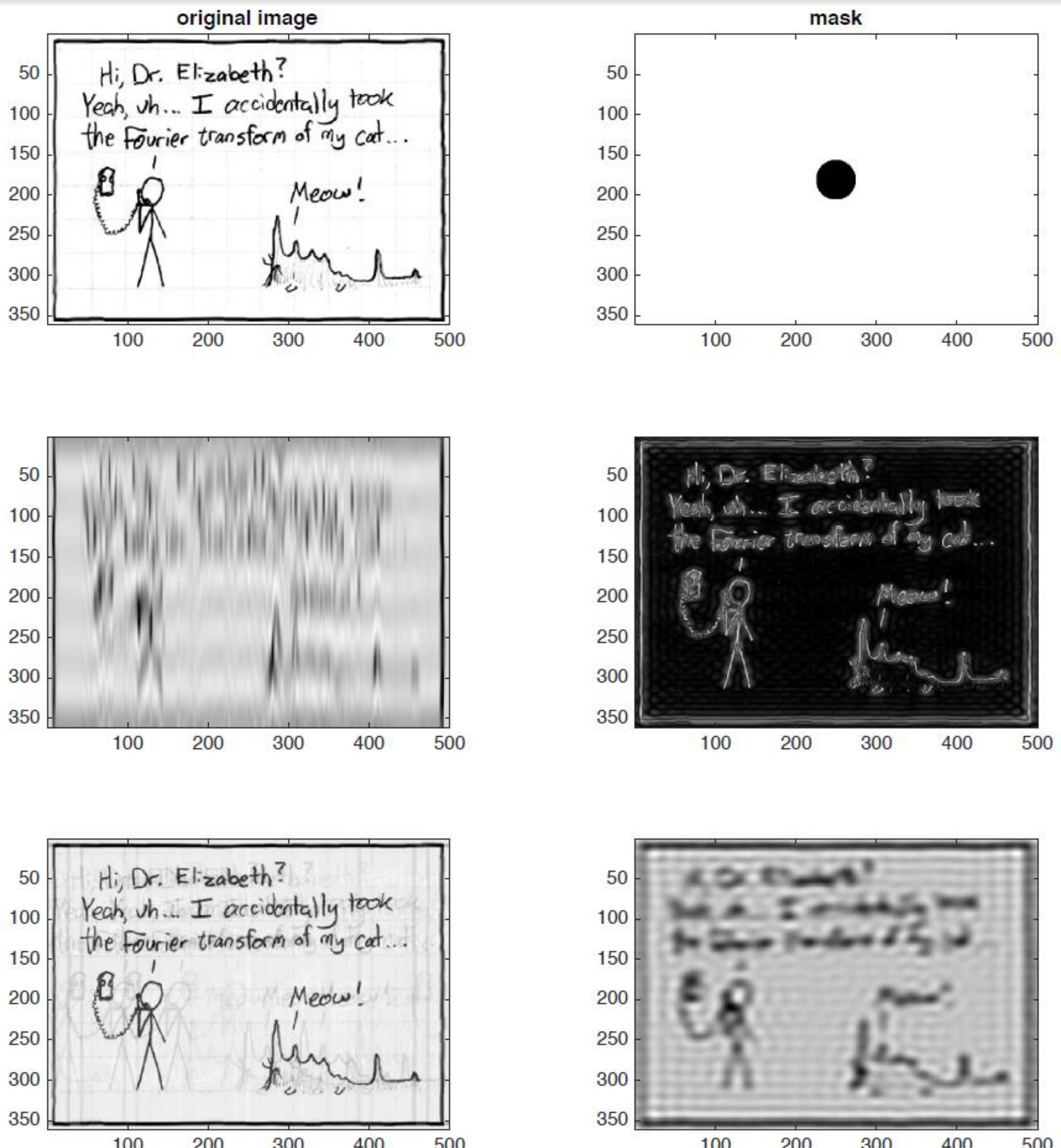
Fourier Filters (By Eye):3



Fourier Filters (By Eye): 4




original image
mask
Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took
the Fourier transform of my cat...
Meow!
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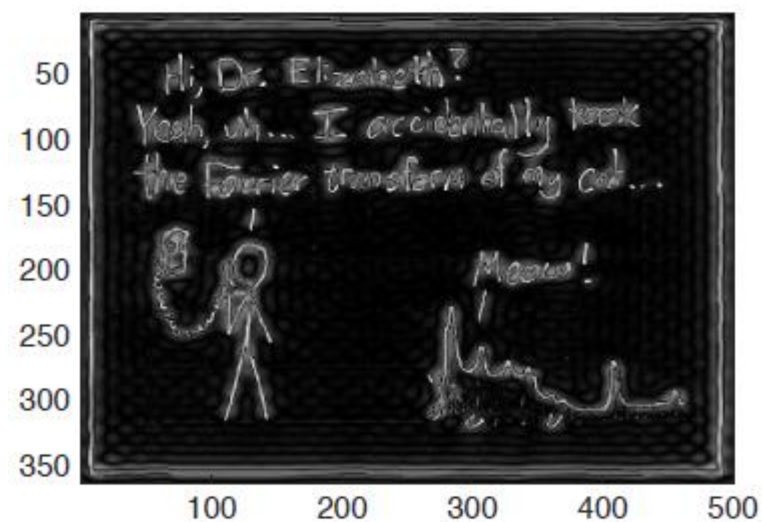
400
500

Fourier Filters (By Eye):4



Fourier Filters Bonus Question:1

- Why did I use cartoons for all of these examples?

Gaussian Beams:1

What equation does the "Gaussian Beam" satisfy?

Gaussian Beams:1

Paraxial Helmholtz Equation

Gaussian Beams:2

What is the “rayleigh range" defined as?

Gaussian Beams:2

$$z_o = \frac{\pi\, w_o^2}{\lambda}$$

Gaussian Beams:3

If I want a waist of w_o, and I place a lens of f at a location where the radius of curvature is infinite, and the beam has a waist of w':

1. Where will the minimum waist be located?
2. What f should the lens be?

Gaussian Beams:3

$$w' = w_o\sqrt{1+\left(\frac{z}{z_o}\right)^2}$$

$$\left(\frac{w'^2}{w_o^2}-1\right)z_o^2 = z^2$$

Gaussian Beams:3

$$f = R(z) = z + \frac{z_o^2}{z}$$

Cavities

Given a cavity with two mirrors of equal magnitude of curvature (the sign you can adjust based on placement) separated by $6z_o$, what must the radius of curvatures be?

1. Consider the case where the mirrors are placed symmetrically on either side of the minimum waist.
2. Consider the case where the mirrors are placed both on one side of the minimum waist.

Cavities

- Do on board

$$R = z + \frac{z_0^2}{z}$$

$$0 = z^2 - Rz + z_0^2 \rightarrow 0 = \frac{z}{z_0}^{2} - \frac{R}{z_0}\frac{z}{z_0} + 1$$

Cavities

- Do on board

So now absorb the z_o into all the z and R factors

$$z_{+,-} = \frac{R}{2} \pm \sqrt{\left(\frac{R}{2}\right)^2 - 1}$$

This will give a z_+ and a z_-. Then set the difference of these to the mirror separation of $6z_0$.

$$z_+ - z_- = d = 6 = 2\sqrt{\left(\frac{R}{2}\right)^2 - 1}$$

$$36/4 = \left(\frac{R}{2}\right)^2 - 1 \rightarrow R = \sqrt{40}$$

To get back into real units we can stick the z_o back in.

$R = \sqrt{40} z_o$

Polarization

Using the Jones Vector notation, how do we find if another polarization is orthogonal?

Polarization

- Do on board

$$\vec{J}_i^\dagger \vec{J}_j = 0$$

Polarization

- Do on board

Ex:

$$J_1 \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

So the orthogonal vector is:

$$J_2 \rightarrow \frac{-1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix}$$

We can check this by taking the dot product of the two vectors

$$J_1^\dagger J_2 = \frac{-1}{2}\left[(-i)(1) + (i)(1)\right] = 0$$

Polarization

What does a $\lambda/2$ wave plate do? What are the "fast" and "slow" axes in the form of its matrix representation?

Polarization

- Write matrix on board

$$\phi = k_o \cdot d(\delta n)$$

Polarization

How can I rotate the polarization from

$$J_1 \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow J_2 \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with a $\lambda/2$ wave plate?

Use the rotation matrices!

The matrix for the $\lambda/2$ plate is just given by :

$$\begin{pmatrix} 1 & 0 \\ 0 & exp(i\phi) \end{pmatrix}$$

where $\phi = \pi$. Which gives:

$$S_\pi = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Transforming this matrix by the rotation matrix $R(\theta)S_\pi R^\dagger(\theta)$

$$R(\theta)S_\pi R^{dagger}(\theta) = \begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ -\sin(2\theta) & -\cos(2\theta) \end{pmatrix}$$

This acting on the initial vector produces:

$$\begin{pmatrix} \cos(2\theta) \\ -\sin(2\theta) \end{pmatrix}$$

So this means for an angle of $\theta = \pi/4$ of the wave plate, we'd completely exchange the initial vector to being y-polarized!

Conceptual questions (TF):1

1. A diverging lens makes an object appear farther than it actually is. **T / F**

Conceptual questions (TF):2

2. The focal length of a glass focusing lens underwater is typically shorter than in air. **T / F**

Conceptual questions (TF):3

3. If a laser is collimated on earth and directed into space, the beam will eventually begin to diverge (neglect atmospheric effects). **T / F**

Conceptual questions (TF):4

4. When a plano-convex lens is used to focus parallel rays, the curved side should face the parallel rays, to suppress aberrations. **T / F**

Conceptual questions (TF):5

5. Two lossless mirrors can be arranged in a way such that 100% of light of two distinct frequencies is being transmitted. **T / F**

Conceptual questions (TF):6

6. Two lossless mirrors can be arranged in a way such that 100% of white light is being transmitted. **T / F**

Conceptual questions (TF):7

7. Two lossless mirrors can be arranged in a way such that 100% of light of a particular frequency is reflected. **T/F**

Conceptual questions (TF):8

8. The Free Spectral Range (FSR) is purely determined by the cavity length.

Conceptual questions (TF):9

9. The Finesse of the cavity is determined by both the cavity length and the reflectivity.

Conceptual questions (TF):10

10. Metals make good lossless ideal mirrors.

Done!

- Good luck on Friday!