MATH 420 **Name:**
Homework 1
September 20, 2023

Submit your work on separate paper.

1. Find a certificate of infeasibility for the system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, $\boldsymbol{x} \geq \boldsymbol{0}$ given by

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 1 & -1 & 0 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

2. Show that the following linear program is *unbounded*:

$$\begin{aligned} &\max \begin{bmatrix} 0 & 0 & 3 & -1 \end{bmatrix} \boldsymbol{x} \\ &\text{subject to } \begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 1 & -8 & 4 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 6 \\ 4 \end{bmatrix} \\ &\boldsymbol{x} \geq \boldsymbol{0}. \end{aligned}$$

   Find a feasible solution having objective value exactly 10000.

3. A portfolio manager for a bank has $\$10$ million to invest. The securities available for purchase, as well as their respective quality ratings, maturities, and yields are shown in the following table:

Bond name	Bond Type	Bank's Rating	Years to Maturity	After-tax Yield
A	Municipal	9	2	4.3%
B	Agency	2	15	2.7
C	Government	1	4	2.5
D	Government	1	3	2.2
E	Municipal	5	2	4.5

   The bank places the following policy limitations on the portfolio manager's actions:

   (a) Government and agency bonds must total at least $\$4$ million.

   (b) The *average* quality of a portfolio cannot exceed 1.4 on the bank's quality scale. (A low number on this scale means a high-quality bond.)

   (c) The average years to maturity of the portfolio must not exceed 5 years.

   Write a linear program to maximize after-tax earnings. The optimal solution of the LP should indicate to the portfolio manager how any dollars to invest in each bond. *Do not solve your linear program.*

4. Convert the following linear program into standard form.

$$\text{Minimize: } 2x_1 - x_2 + 4x_3 + 2x_4 + 4x_5$$

$$\text{Subject to: } \begin{bmatrix} 1 & 2 & 4 & 7 & 3 \\ 2 & 8 & 9 & 0 & 0 \\ 1 & 1 & 0 & 2 & 6 \\ -3 & 4 & 3 & 1 & -1 \end{bmatrix} \boldsymbol{x} \begin{matrix} \leq \\ = \\ \geq \\ \geq \end{matrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$x_1 \geq 0, x_2 \geq 1, x_3 \geq 0, x_4 \geq 0$$

5. The following linear program is in standard form:

$$\text{Maximize: } \begin{bmatrix} 1 & -2 & 0 & 1 & 3 \end{bmatrix} \boldsymbol{x}$$

$$\text{Subject to: } \begin{bmatrix} 1 & -1 & 2 & -1 & 0 \\ 2 & 0 & 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\boldsymbol{x} \geq \vec{0}$$

   (a) Compute the canonical form of the linear program with respect to the basis C_1, C_4.

   (b) Compute the canonical form of the linear program with respect to the basis C_3, C_5.

   (c) In each case, compute the corresponding basic solution and determine if it is feasible or not.