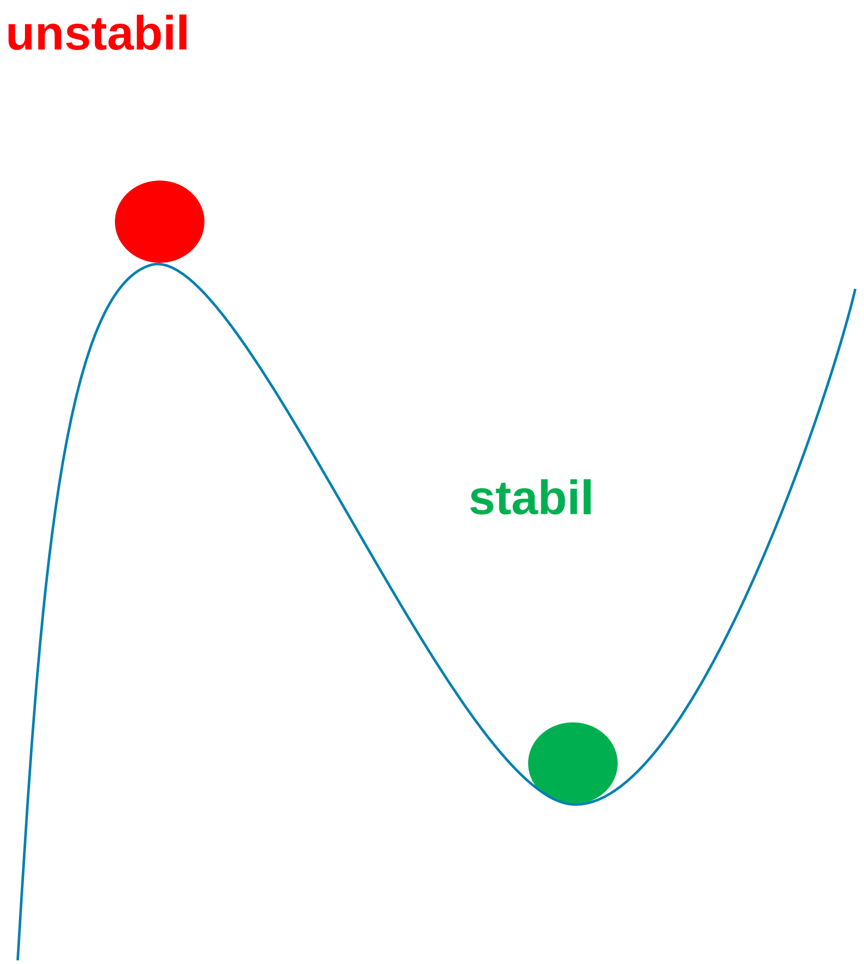
HOCHSCHULE RUHR WEST
UNIVERSITY OF APPLIED SCIENCES

Die Hochschule Ruhr West

NEVER STOP
GROWING

Stabilität

der Zustand, in dem eine geringe Störung in einem System keine störenden Auswirkungen auf dieses System hat.



Lösung einer Differentialgleichung

asymptotisch stabil

$$\lim_{x \to \infty} F_1(x) - F_2(x) = 0$$

Beispiel:

$$\frac{dy}{dx} = -y(x)$$

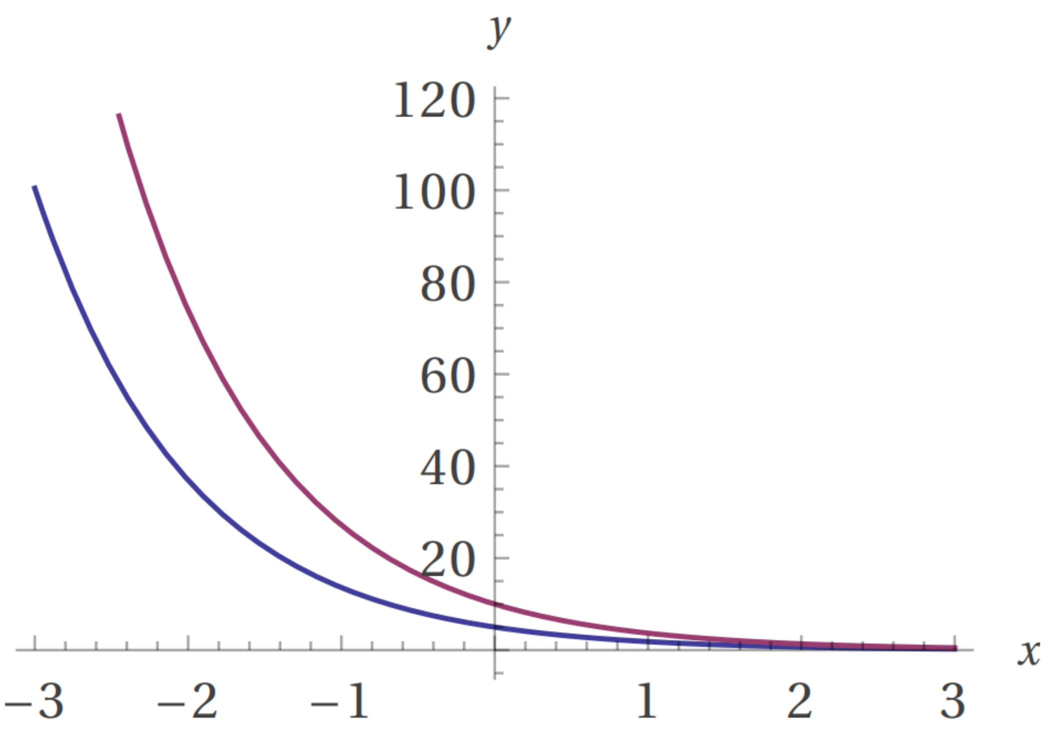
Asymptotisch stabil

$$\frac{dy}{dx} = -y(x)$$

$$F_1(x) = c_1 e^{-x} \quad F_2(x) = c_2 e^{-x}$$

$$\lim_{x\to\infty} F_1(x) - F_2(x) = \lim_{x\to\infty} c_1 e^{-x} - c_2 e^{-x} =$$

$$= (c_1 - c_2) \lim_{x\to\infty} e^{-x} = 0$$



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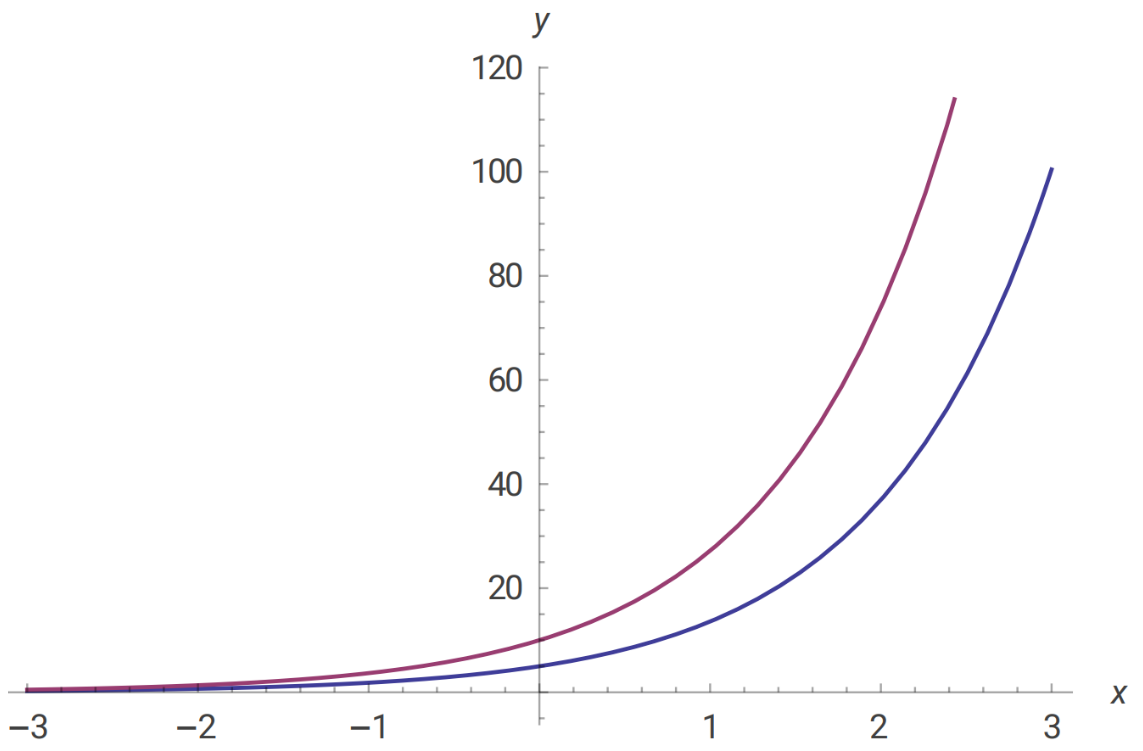
Unstabil

Beispiel:

$$\frac{dy}{dx} = y(x)$$

$$F_1(x) = c_1 e^x \qquad F_2(x) = c_2 e^x$$

$$\lim_{x\to\infty} F_1(x) - F_2(x) = (c_1 - c_2)\lim_{x\to\infty} e^x = \infty$$



Lösung eines Anfangswertproblem

$(\boldsymbol{\varepsilon}\,, \boldsymbol{K})$ unstabil

$$\exists \boldsymbol{x}^*$$
$$\|\boldsymbol{F_1}(\mathbf{0}) - \boldsymbol{F_2}(\mathbf{0})\| < \varepsilon$$
$$\|\boldsymbol{F_1}(\boldsymbol{x}^*) - \boldsymbol{F_2}(\boldsymbol{x}^*)\| > \mathbf{K}$$

In Praxis:

$$\varepsilon = \boldsymbol{Rundungsfehler}$$

$$\mathbf{K} = \ \mathbf{max}\|\boldsymbol{F}(\boldsymbol{x})\| \quad a \leq x \leq b$$

Schießverfahren

Randwertproblem → Anfangswertproblem

Stabiles Randwertproblem → Unstabiles Anfangswertproblem

Beispiel

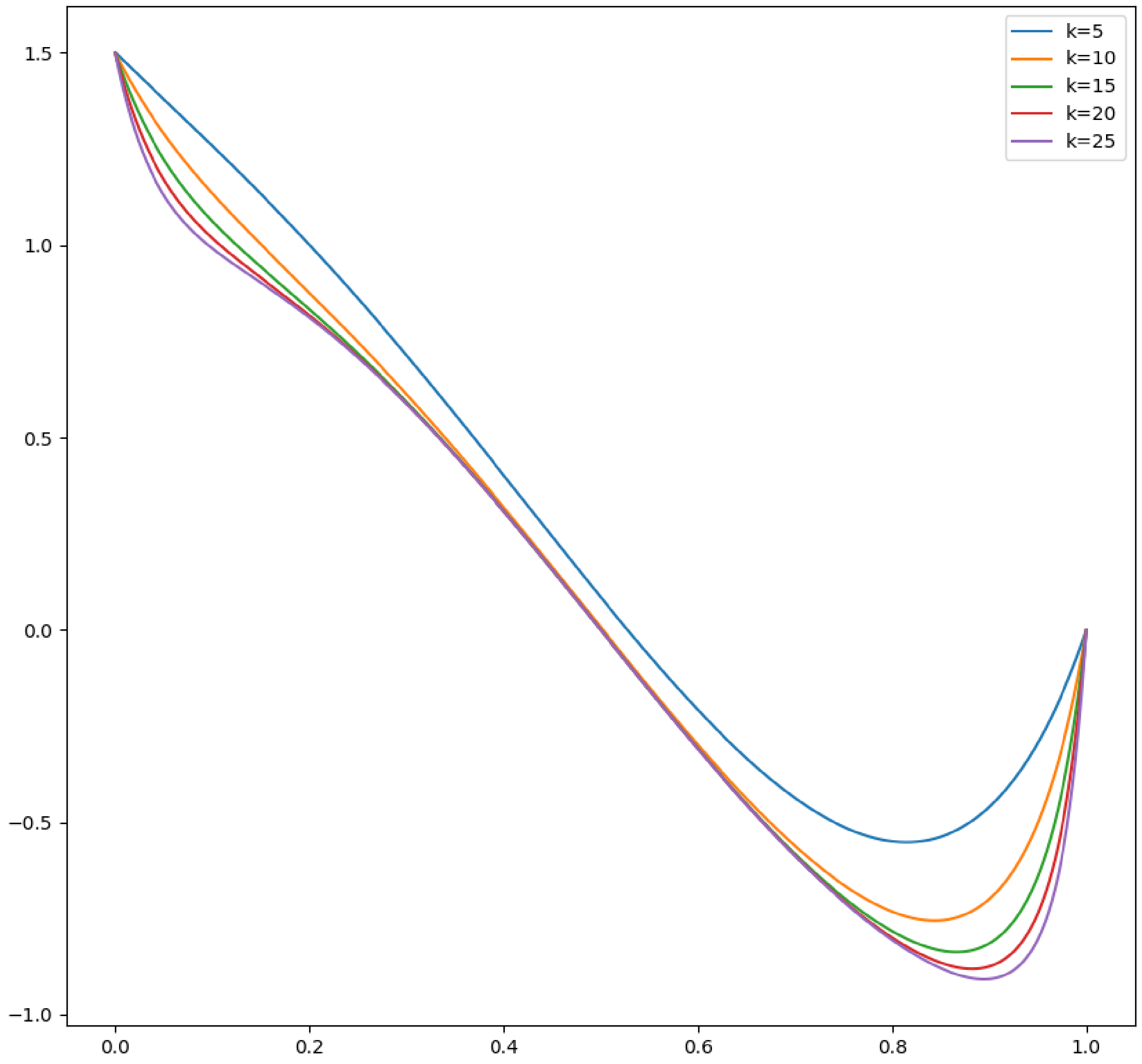
$$\frac{d^3y}{dx^3} = 2k\frac{d^2y}{dx^2} + k^2\frac{dy}{dx} - 2k^3y(x) + \ (k^2+\pi^2)(2k\cos(\pi x) + \pi\sin(\pi x))$$

$$y(0) = \frac{e^{-k}(3e^{2k} + 2e^{k} + 1)}{2 + e^{-k}}$$

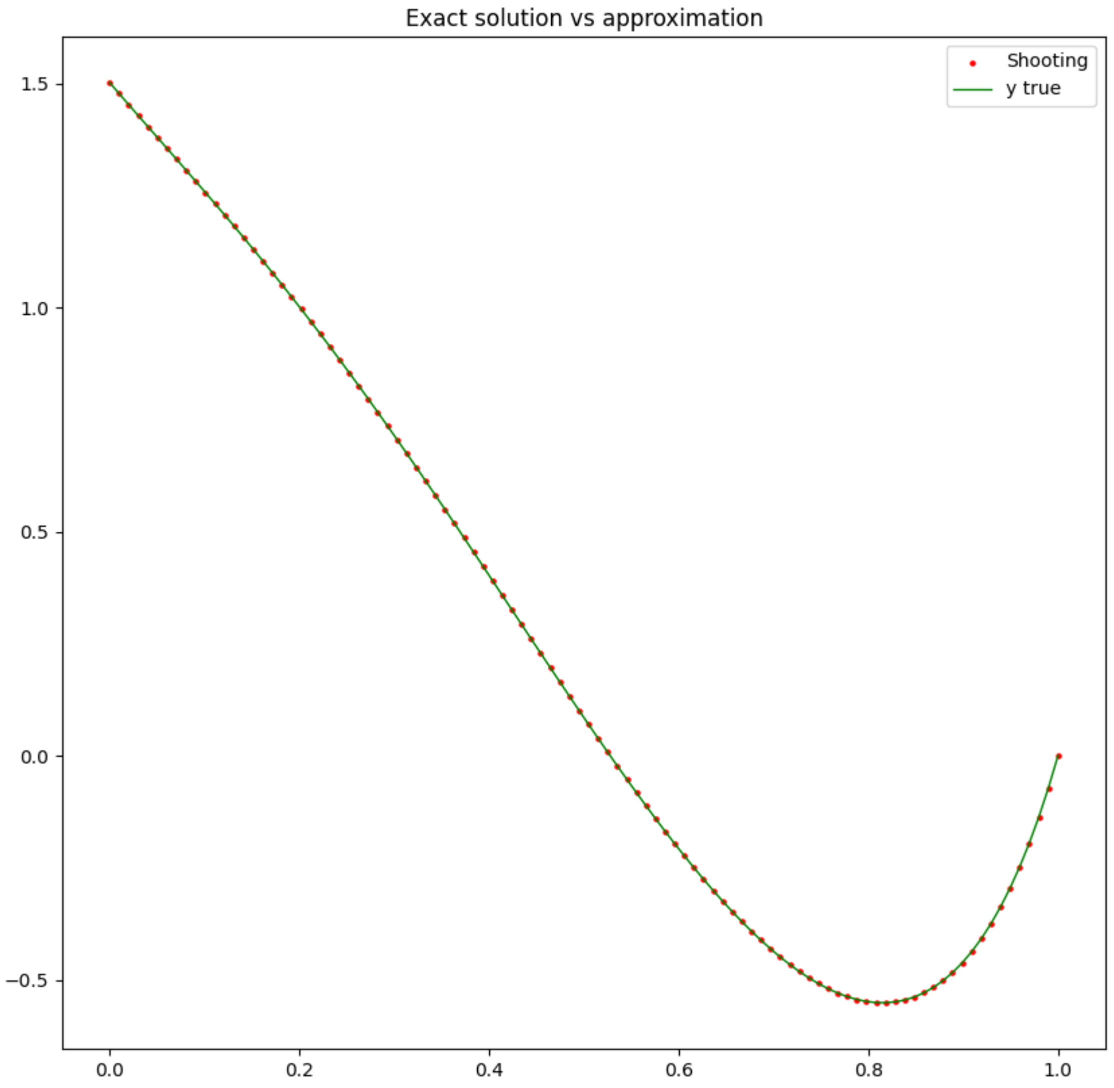
$$y(1) = \frac{k(2e^{-2k} + e^{-k} - 1)}{2 + e^{-k}}$$

$$y'(0) = 0$$

$$y(x) = \frac{e^{k(x-1)} + e^{2k(x-1)} + e^{-kx}}{2 + e^{-k}} + \cos(\pi x)$$

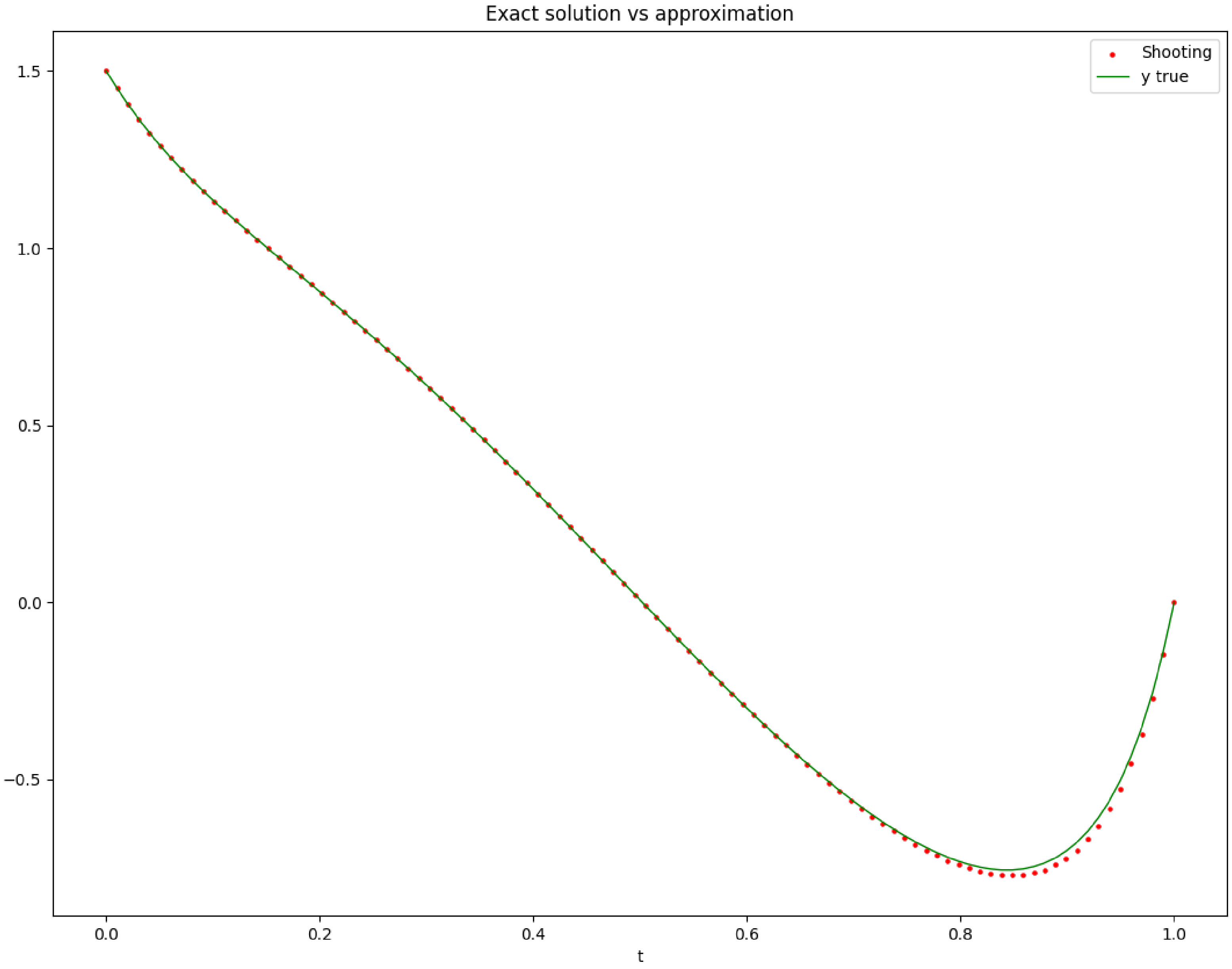


K=5



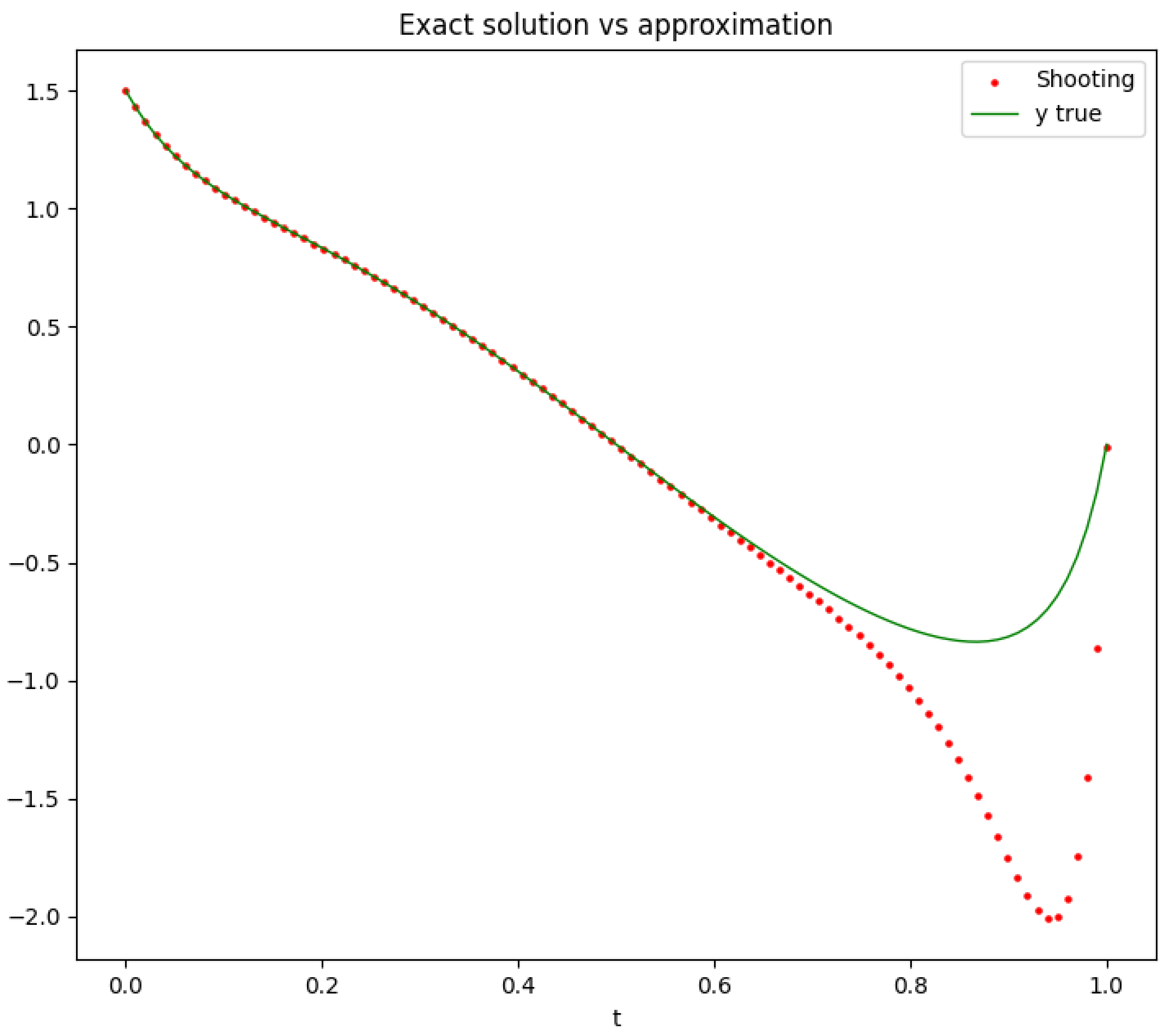
Exact solution vs approximation
Shooting
y true
1.5
1.0
0.5
0.0
−0.5
0.0
0.2
0.4
0.6
0.8
1.0

K=10

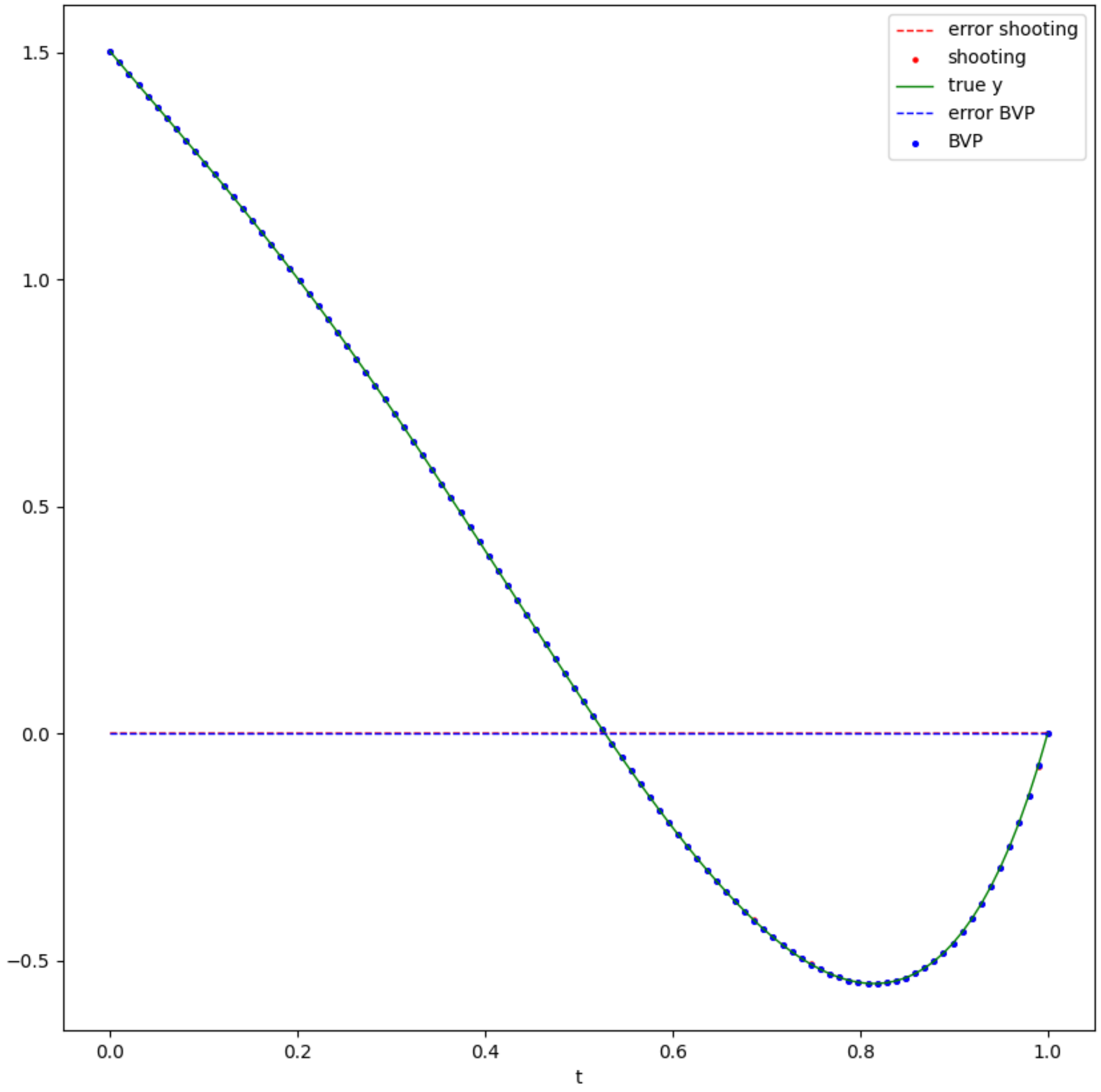


Exact solution vs approximation
Shooting
y true
1.5
1.0
0.5
0.0
−0.5
0.0
0.2
0.4
0.6
0.8
1.0
t

K=15


Exact solution vs approximation
Shooting
y true
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.2
0.4
0.6
0.8
1.0
t

K=5



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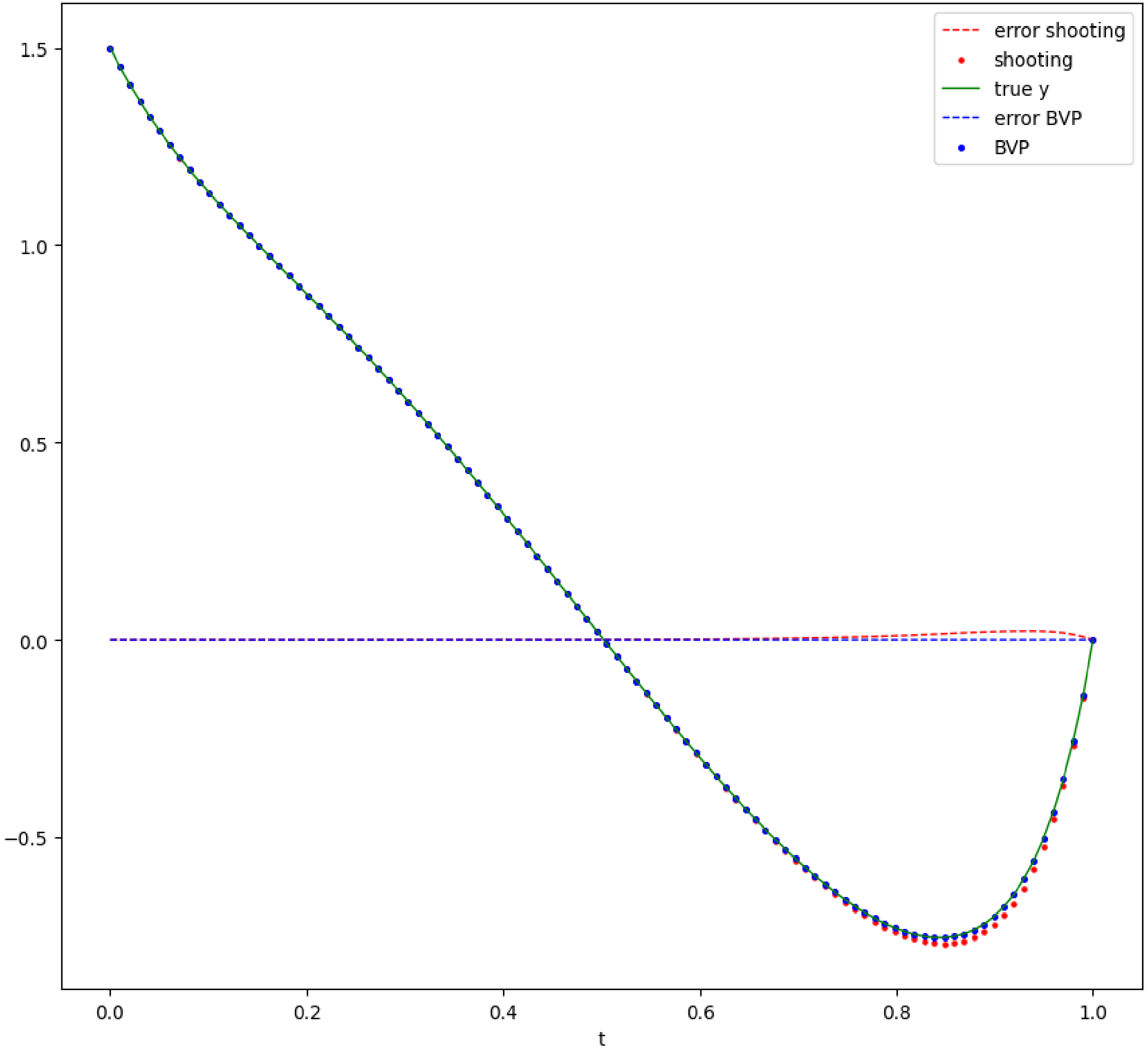
K=10



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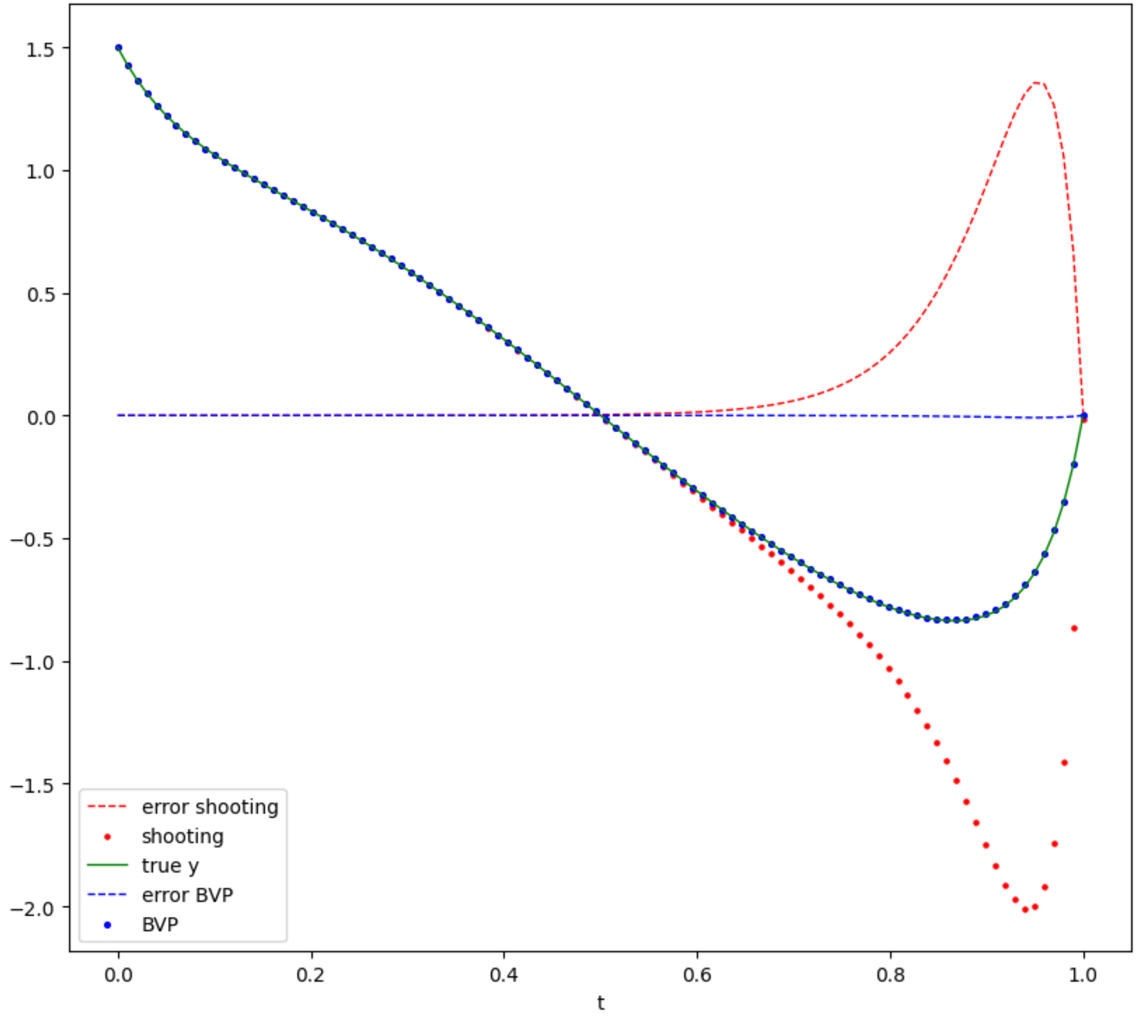
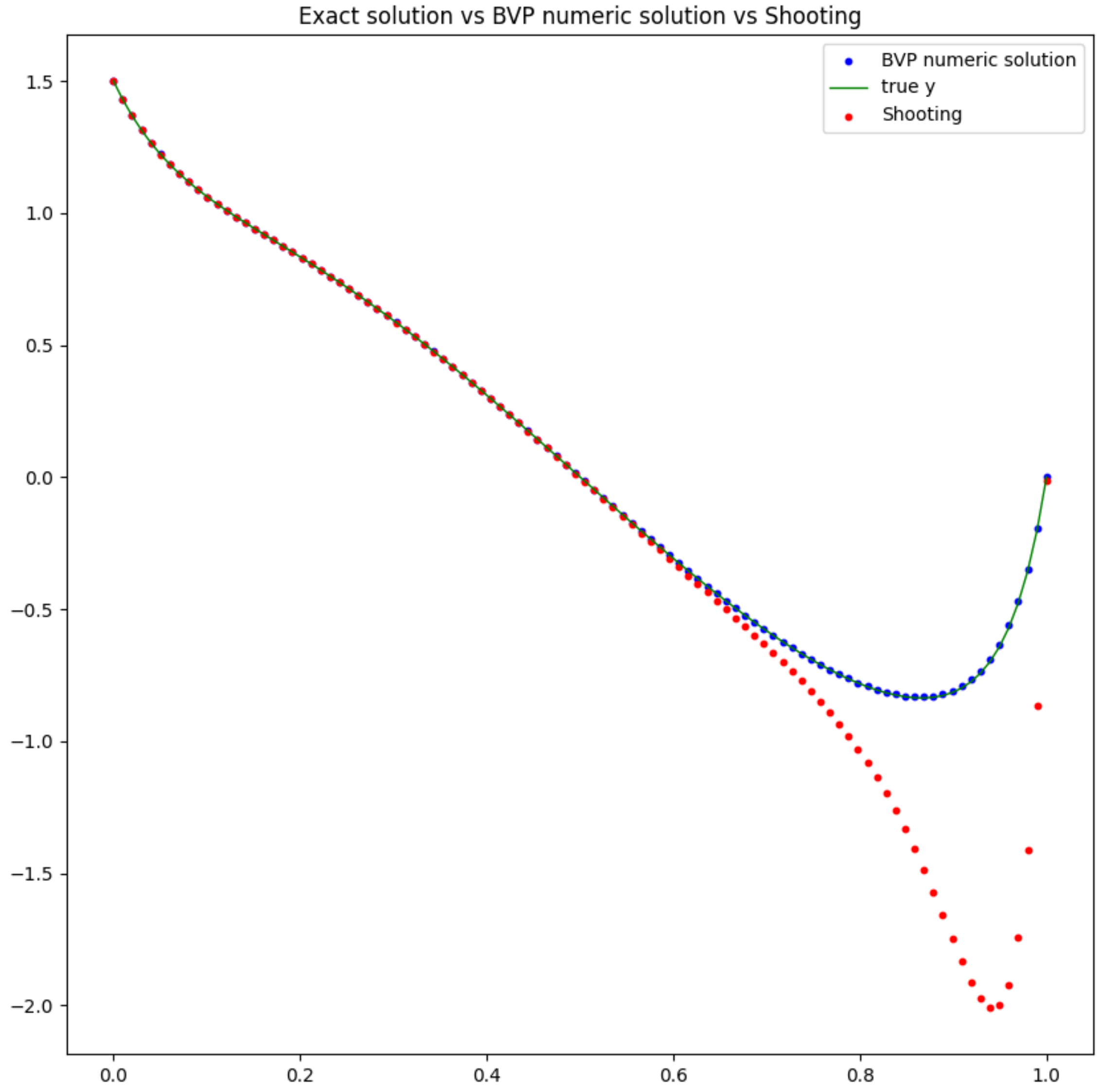
K=15



1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.2
0.4
0.6
0.8
1.0
t
error shooting
shooting
true y
error BVP
BVP

K=15



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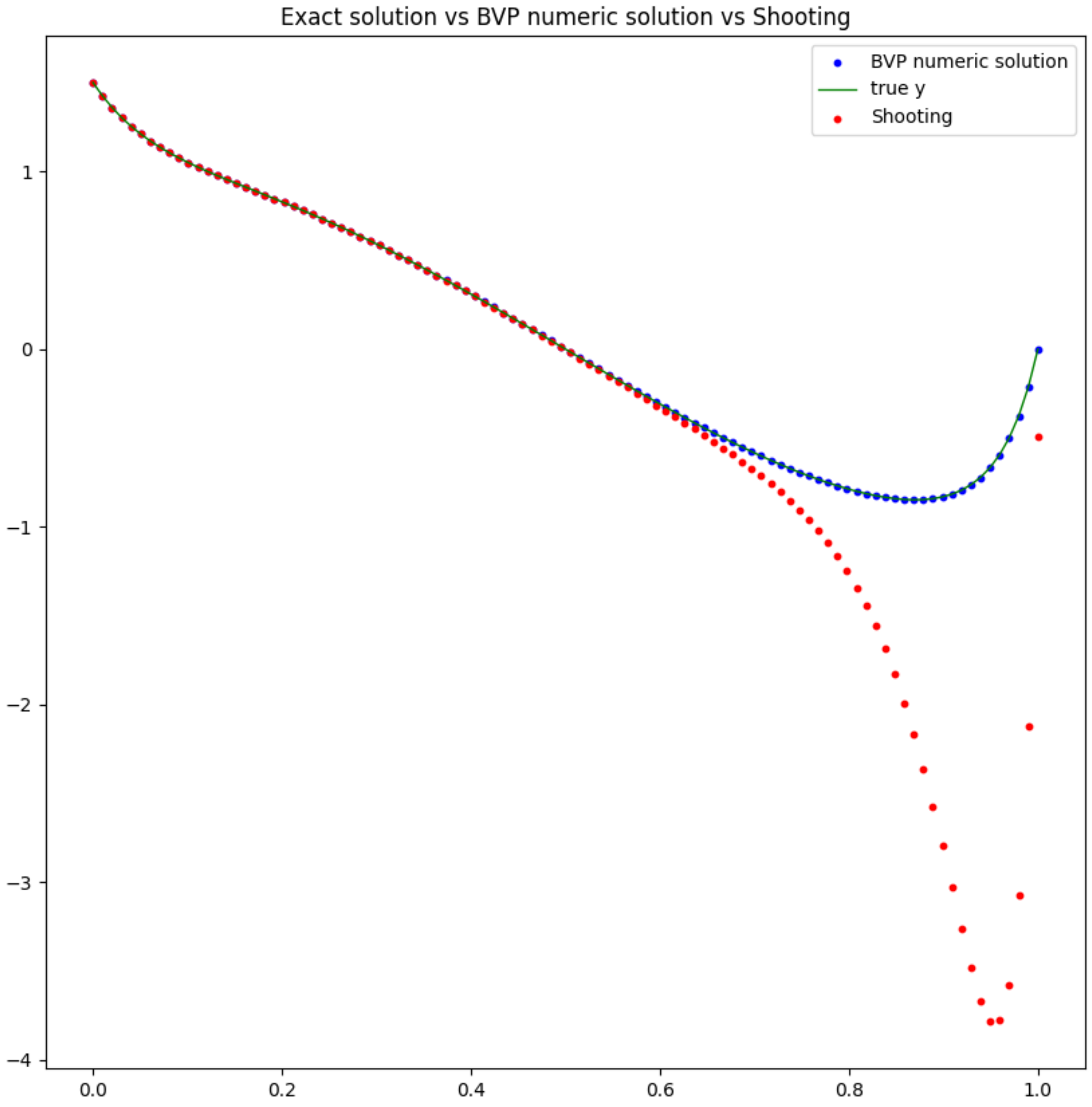
K=16



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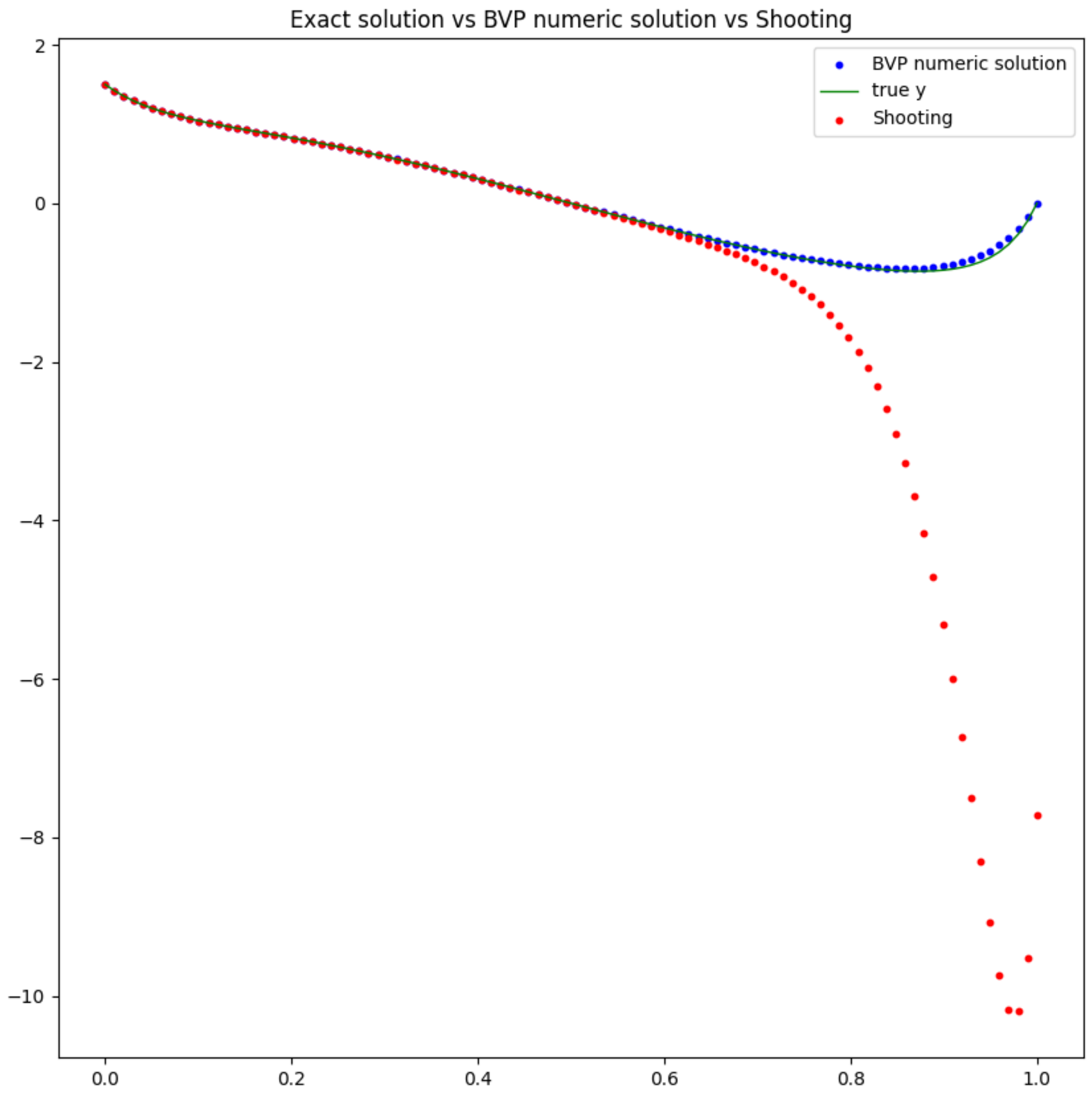
HRW

K=17



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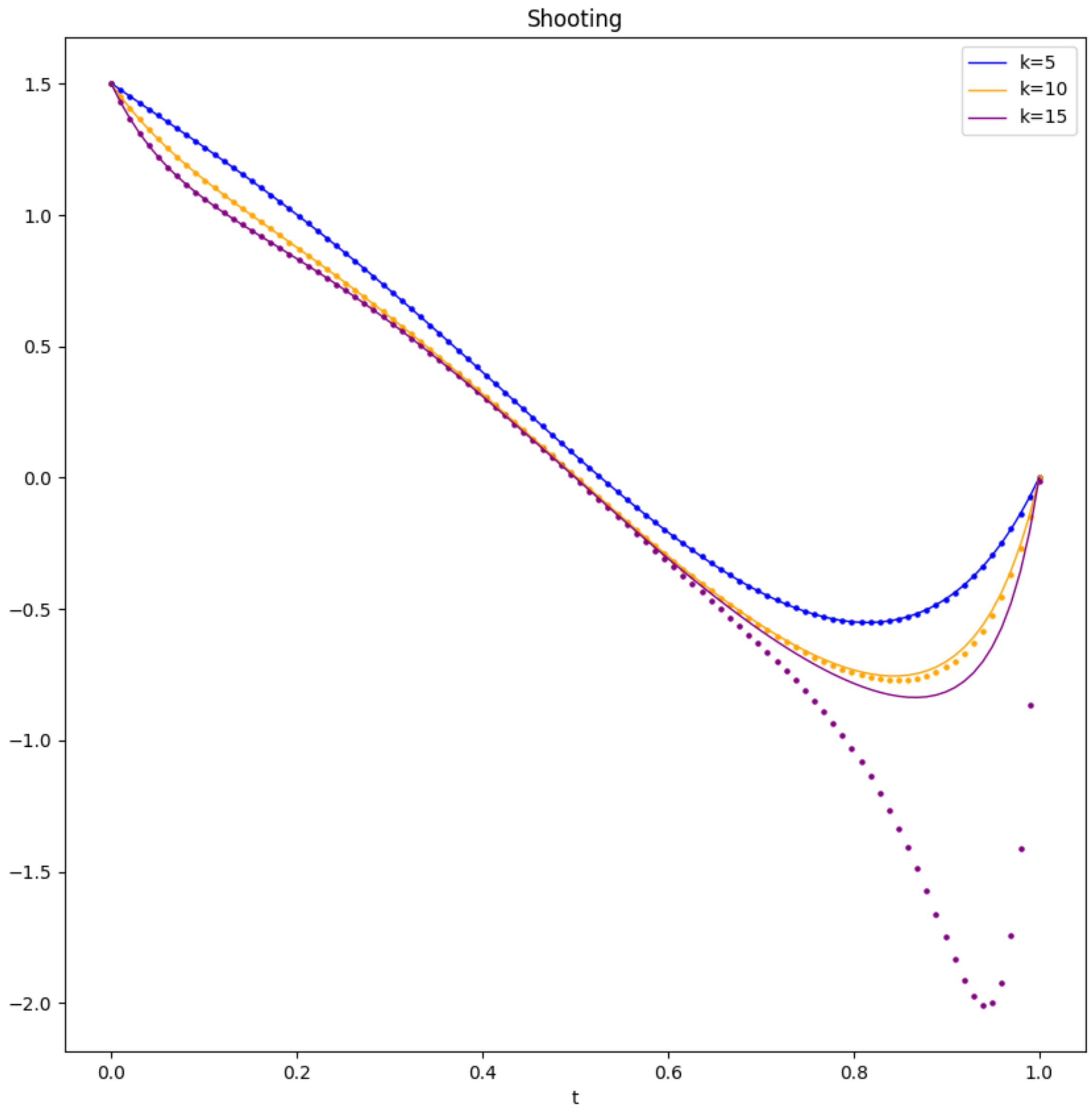
Schießverfahren



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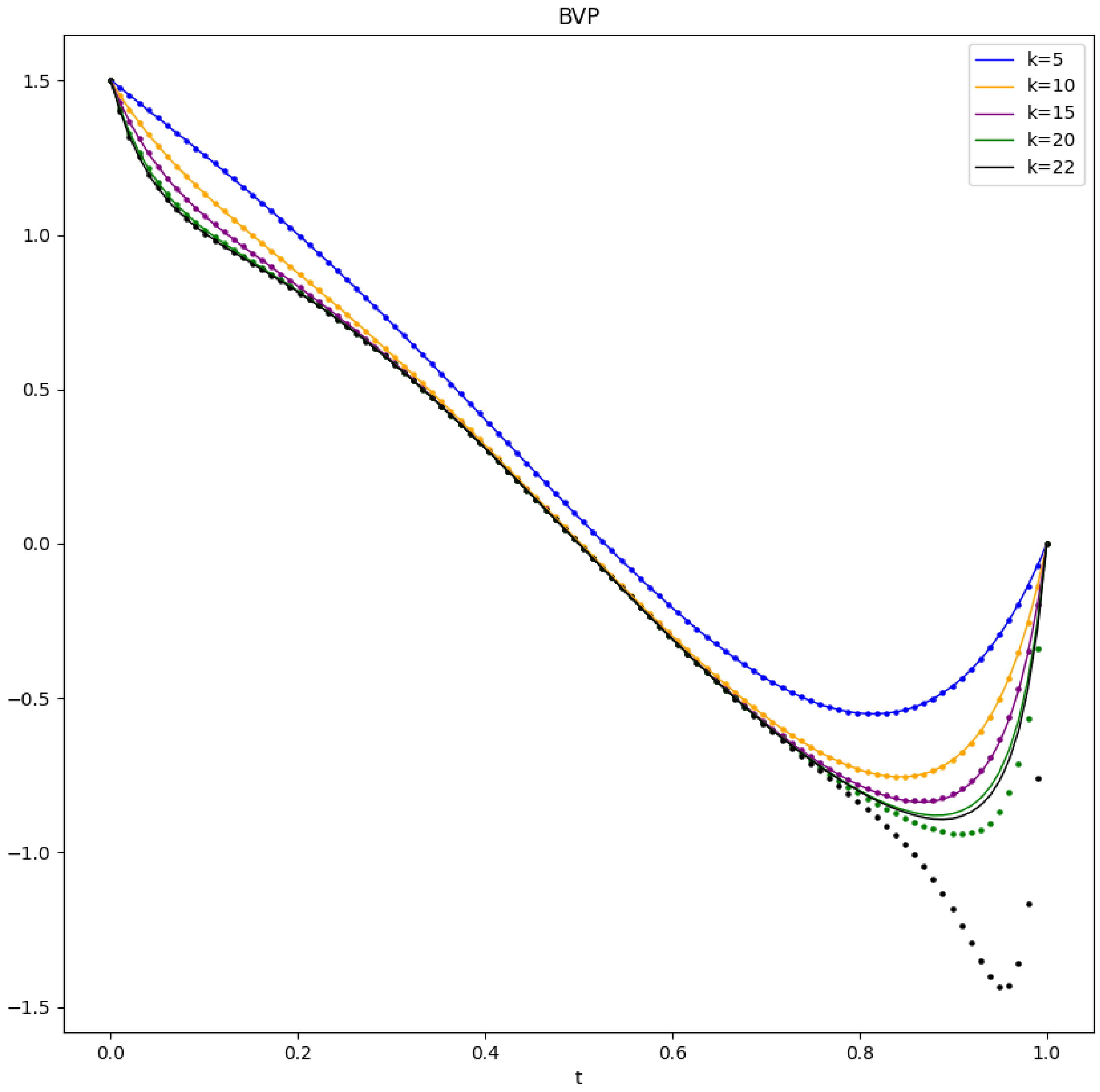
BVP



References

[1] Britannica, The Editors of Encyclopaedia. "stability". Encyclopedia Britannica, 5 May. 2016, [https://www.britannica.com/science/stability-solution-of-equations. Accessed 10 December 2022](https://www.britannica.com/science/stability-solution-of-equations).

[2] U.M. Ascher and L.R. Petzold, Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations, Society for Industrial and Applied Mathematics, 1998.

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Herzlichen Dank für Ihre Aufmerksamkeit

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