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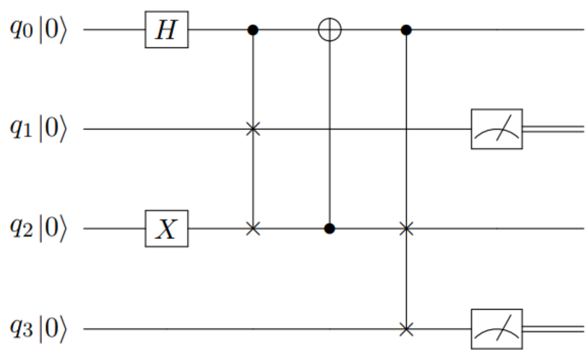
Implementation of a "Universal Statistical Simulator" with a Quantum Galton Board

1. Introduction

This project is based on a paper by Carney and Varcoe titled "Universal Quantum Simulator" (arXiv:2202.01735 **[quant-ph]**, https://doi.org/10.48550/arXiv.2202.01735). Based on the classical Galton board, the authors present a quantum circuit implementation. In a classical Galton board, a ball is dropped with a probability of 50% at each peg of going to the left or to the right. At the bottom of the board a binomial/Gaussian distribution can be observed, when many balls are dropped into the board.

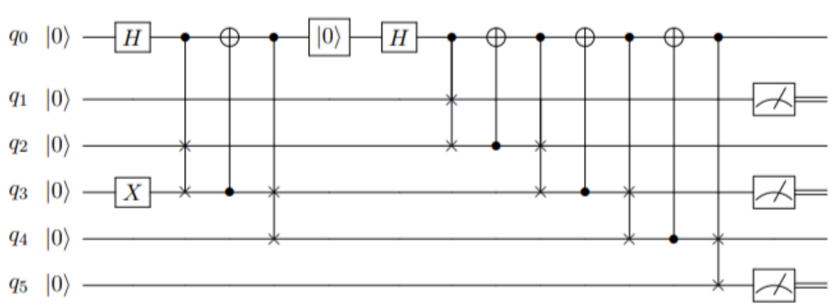
2. The quantum peg – a 1-level QGB

The authors use five quantum gates to construct a quantum peg, the equivalent of a classical peg in a classical Galton board. These gates include a Hadamard, an X, a CNOT and two SWAP gates. One peg consists of 2 CSWAPS with an intervening CNOT. Together these gates make up a 1-level QGB. One qubit (q0) functions as a quantum coin (thanks to the Hadamard gate). The ball is initially on qubit q2 (|1⟩), while qubits q1 and q3 are in the state |0⟩. Based on our Hadamard coin on q0, the |1⟩ is moved to either q1 or q3 with a probability of 50% either way, reflecting the behavior of a ball at a classical peg. Q1 and q3 are measured to determine the location of the "ball" (|1⟩). The following figure taken from the original publication serves to illustrate this smallest possible Galton board.



3. N-level QGB

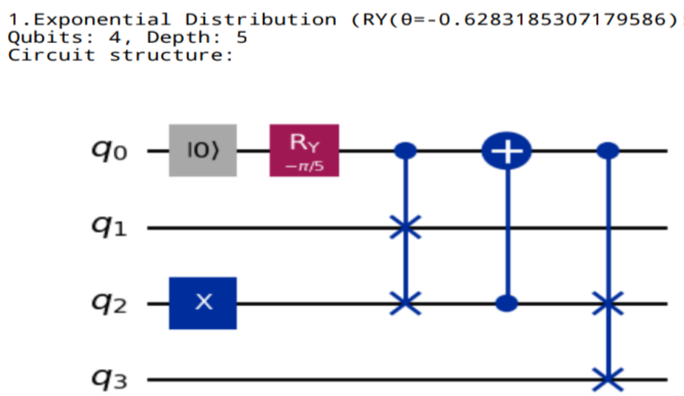
This 1-level QGB can be extended to a 2-level one by adding 2 more qubits and 8 additional gates (one of which is a Hadamard gate). Now three measurements are performed on qubits q1, q3, and q5. Prior to the second Hadamard gate, q0 has to be reset to |0⟩.



Based on this architecture, an n-level GB can be constructed. With these gates (RESET, Ry or H, SWAP, CNOT) one arrives at a total gate count of approximately $2n^2 + 5n + 2$. $n + 1$ measurement operations are performed and the number of qubits is equal to $2n$ ($2n - 1$ working qubits + 1 control qubit). Q0 has to be reset before every Hadamard gate.
2^n trajectories are possible using $O(n^2)$ resources.
Such a QGB can be modified by removing some peg positions and using different probabilities. This then becomes a universal statistical simulator.

4. Generating non-binomial distributions with a biased QGB

A Biased QGB (B-QGB) can be constructed by replacing the Hadamard gate with a $Ry(\theta)$ rotation gate. This allows for the generation of skewed or non-normal distributions due to the modified probabilities of the “quantum coin” on q0. Setting $\theta=-\pi/5$ gives an exponential distribution, while $\theta=2.8$ with proper post-processing (mirroring counts in bin k to bin n-k) produces an inverted Gaussian, i.e. a Hadamard quantum walk. Again, a reset gate is required to ensure pegs are initialized correctly for controlled rotations. This is how a modified quantum peg looks like:



With this modification, it was possible to produce non-binomial distributions in this project.

5. Noise models and error mitigation in n-level QGBs

For various levels of n, several simulations were performed:

- Noiseless
- Noisy without circuit optimization
- Noisy with circuit optimization
- Noisy with circuit optimization and error mitigation

The resulting distributions were compared by using the “total variation distance”.

6. Conclusion andOutlook

The authors succeeded in creating a QGB with minimal circuit depth, which in its biased version can be used as a Universal Statistical Simulator.