Spiking Neural Networks

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Abstract

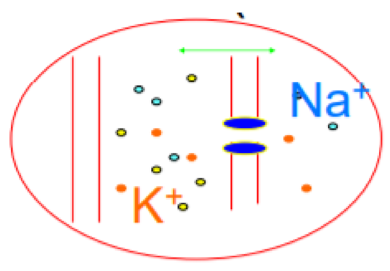
Standard existing models like Integrate-and-fire models and HH models are deterministic and generate the spike train that looks regular when driven by a constant stimulus. But a neuron activity in vivo recording shows a high degree of irregularity . A neuron in sensory cortex in vivo give neural recording different for one trail to another and some neurons emits spontaneous spikes even without any external stimulus due to this there is voltage fluctuations and gives large degree of variation in inter spike intervals. To mimic LIF models as unpredictability of neural recording, we need to add the noise to them. In order to achieve this , we should have proper quantification for variability of neuronal spike trains. There is no unique and well-defined Mean firing Rate. There are there definitions for it which is described in the poster. We use Inter Spike interval (ISI) to study neural variability.

Source of Variability

There are two sources of noise

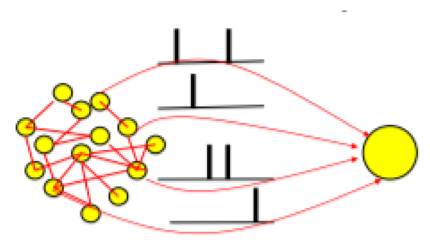
1. **Intrinsic Noise:**

   The noise inside the individual neuron
   - Finite number of channels
   - Finite Temperature.



2. **Extrinsic Noise:**

   The noise that is caused due to spikes arrived from the pre-synaptic neurons which are beyond the scope of experimentalists for prediction
   - Network noise can be minimized by isolation of neuron
   - Spikes that are produced by fluctuation input current is more reliable than step current.



Coefficient of Variation

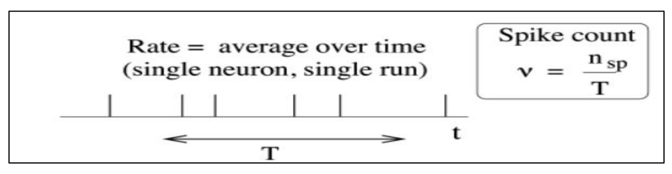
1. It is the probability of spike occurring at an instance based on the distribution of spike over an interval of time
2. Spike rate is dependent on the distribution of spikes over the time interval.
   - In homogeneous Distribution Spike rate is constant
   - In non-homogeneous Distribution Spike rate is dependent on the time instance.
3. Probability of firing at an instance in an homogeneous distribution is
$$P_F = \rho\Delta t$$
4. Probability of firing at an instance in an non-homogeneous distribution is
$$P_F = \rho(t)\Delta t$$
5. Probability of not generating a spike with in the interval chosen between two spikes is called survivor time
6. Survivor time for homogeneous distributions is same for continuous and discrete distribution $S(t) = e^{-\rho t}$
7. Survivor time for non-homogeneous distributions is $S(t/t') = e^{\int_{t'}^{t} -\rho(t)dt'}$
8. The Inter spike Interval distribution is the probability of spike at an instance t' given that there is a spike in the interval t $P\left(\frac{t}{t'}\right) = \rho(t)\, e^{\int_{t'}^{t} -\rho(t')dt'}$
9. The mean interval distribution is the standard probability density function on the given spike train under stationary conditions is
$$< s > = \int_0^\infty s * P_0(s)ds\,.$$
10. The mean firing rate is the inverse of the mean interval distribution
11. In order to quantify the width of the inter spike interval distribution derived from the stationary conditions which is actually sharply peaked coefficient of variation is used. $C_v = \frac{<\Delta s^2>}{<s>^2}$.

Mean Firing Rates

There is know unique and well-defined definition for mean firing rate. But There there most commonly used definitions for it.

1. **Rate as a Spike Count :**

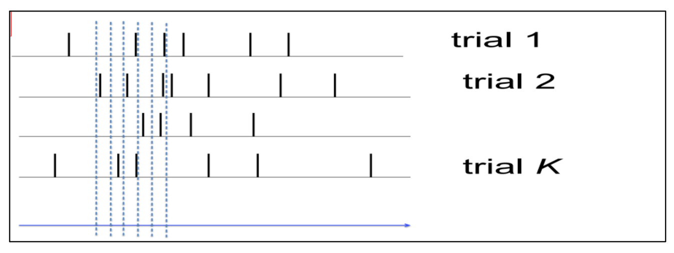
   It is the commonly used definition. Here we define spike rate as number of spike occurred in the duration of time T.
$$v_k = \frac{n^{sp}}{T}$$



   Probability of finding spike in small interval ΔT for homogenous Poisson process is given :
$$P_F(t; t + \Delta t) = v\Delta T.$$
$$v = \lim_{\Delta t \to 0} \frac{P_F(t;t+\Delta t)}{\Delta t}$$
   Expected Number of spikes that occur in in time T for homogenous Poisson process is therefore
$$< n^{sp} > = v.T$$
   Drawback : Very Slow

2. **Rate as a Spike Density and the PSTH:**

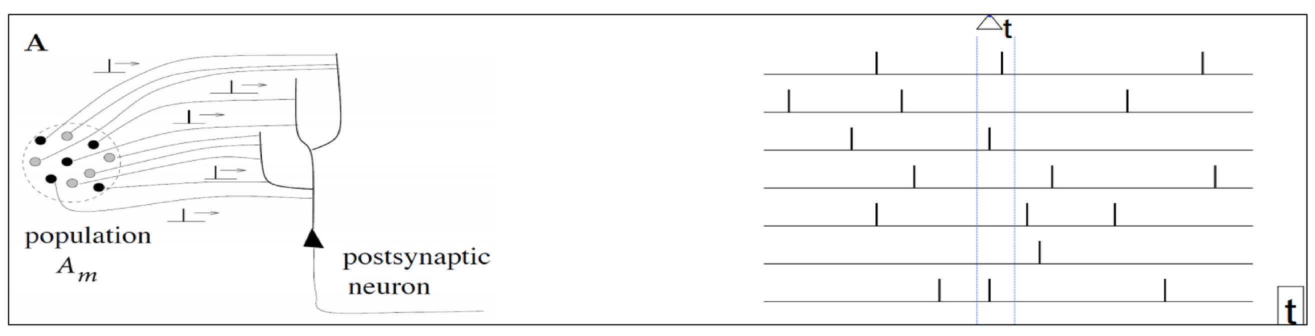
   The same experiment is repeated several times with same input sequence and the neural repones is noted in PSTH with bin width Δt. By using PSTH we can measure the neural activity.



   - Spike Density at time t ($\rho(t)$) $= \frac{1}{K}\frac{P_F(t;t+\Delta t)}{\Delta t}$.
   - Representation of spike train in terms of δ-functions is shown below
$$S(t) = \sum_f \delta(t - t^f)$$
   - In each trail k , we get the spike count in a short time interval Δt by integrating the spike train over time. $n_k^{sp}(t) = \int_t^{t+\Delta t} s_k(t')\mathrm{d}t'$.
   - On averaging the over K trails and dividing by n we get estimation for instantaneous firing rate which represented by below equation.
$$v(t) = \frac{1}{K.\Delta t}\sum_{k=1}^{K} n_k^{sp}(t).$$
   Drawback : Many trails required for decision

3. **Rate as a Population Activity:**

   Instead of many trails on same neuron, We will take population of neurons and perform experiment simultaneously on the neurons.



$$A(t) = \frac{1}{\Delta t}\frac{n_{act}(t; t + \Delta t)}{N} = \frac{1}{\Delta t}\frac{\int_t^{t+\Delta t}\sum_j \sum_f \delta(t - t^f)dt}{N}$$

Auto Correlation and Noise Spectrum

1. Auto correlation is the concept of finding the probability of spike at an instance independent of previous spike received
2. $C_{ii}(S) = < S_i(t), S_i(t + s) >_t$ where $<.>_t$ is the average over the time t
3. Auto correlation function is intimately related to power spectral density of the neural spike train called noise spectrum.
4. The power spectral density of spike train is defined as $P(\omega) = \lim t \to \infty\, P_t\,(\omega)$ where P_t *is* the power of segment of length t in a spike train
5. Power spectrum density of a spike train is the Fourier transform of the auto correlation
$$C_{ii}(\omega) = \int_{-\infty}^{\infty} < S_i(t)\, S_i(t + s) > e^{-i\omega s}\, ds$$

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