Unstructured environmental mapping using low cost sensors

Samuel B. Lazarus, Antonios Tsourdos, Rafał Żbikowski, and Brian A. White

***Abstract*— Achieving an innovative and intelligent low cost sensor system for an autonomous mobile robot to navigate and map the unstructured environment is the main aim of this paper. The environment or the map is unstructured in the sense that, the obstacles in the environment are unknown. There are various approaches to this problem, but the focused approach here can give robust performance of low cost sensor based localisation and mapping. The robust performance is quantified by explicit bounds of the position estimate of a mobile robot and to make it to build the environmental map of its surroundings. The mobile robots generally carry dead reckoning sensors such as wheel encoders and inertial sensors (INS), accelerometers and gyroscopes, to measure the acceleration and the angular rate, while the obstacle detection and map-making is done with the low cost sensor such as time-of-flight ultrasonic sensors. The localisation task is achieved by combining the different source of sensor measurements which are complementary and therefore compensate for each other's limitations, so that the resulting performance of the sensor system is better than of its individual components which provides high richness in the mapping task.**

***Index Terms*— map building, robot localisation, EKF, robust estimation, Hough Transform.**

I. Introduction

Autonomous mobile robot localisation and mapping is the task of guiding a mobile robot to a desired destination, or along a pre-specified path, in an unstructured environment or in an unknown environment which consists of landmarks, obstacles, etc, and simultaneously localise the robot and to recognise the unknown obstacles and add them to the map. In order to achieve this objective the robot needs to be equipped with sensors suitable for localisation and mapping throughout the path it followed. The localisation of the mobile robots is generally done with the sensors such as wheel encoders and inertial sensors (INS). So the use of multiple sensors also has other advantages such as,

- Measurement errors or failure from one of the sensors will not have a catastrophic effect on the system, since the same information will be available from other sensors as well.
- The selection of sensors can be flexible as more than one sensors can be employed to measure the same parameter.

The obstacle detection and the map-making is done with the time-of-flight ultrasonic sensors. By using the ultrasonic sensors the mapping task is achieved by implementing the pattern recognition and data association method using the Hough transform (HT) techniques. In the localisation task, the GPS measurements are aided with inertial sensors (INS) to exploit the bounds in the errors in the inertial sensors which, in turn, gives a better estimate of the mobile robot position so that, the accuracy and the richness in the mapping problem is relatively increased. Hence, a major limitation of the inertial sensor measurements are compensated by an external measurements from the GPS.

The first contribution of this paper is the development of an integrated approach of the localisation of the autonomous vehicle and the mapping of an unstructured environment with a low cost sensor such as ultrasonic sensors. The second contribution of the paper is the development of a novel method to identify the unknown obstacle in an unstructured or in an unknown environment by implementing a robust technique of the pattern recognition and data association approach based on the Hough transform and the principles of the interval analysis. The proposed approach for the mapping is resulting in an interval bounded localisation of the unknown obstacles.

This paper also illustrates the localisation of the mobile robot by aiding the GPS measurements with the INS measurements with the use of a nonlinear extended Kalman filter. Since we get the fused estimated robust position of the mobile robot, the accuracy in the mapping task using the Hough Transform will be relatively increased. Hence, the proposed approach for the mapping of an unstructured environment is a computationally attractive approach and also results in an accurate map of the unknown obstacles in an unknown environment and guaranteed bounded errors for the fused estimated position of the autonomous mobile robot.

II. Prior work in the field

Sensor fusion [1] is the combining of sensory data or data derived from sensory data from environment sources such that the resulting information is more accurate, more complete, or more dependable and reliable or refer to the result of an emerging view. Sensor fusion involves the combination of information from different sensors [2]. Many researches have proposed their algorithms by combining different sources of sensors [3], [4], [5].

In [6], the statistical framework using EKF for solving simultaneously the mapping problem and the robot localisation relative to its growing map was proposed. It was found that the estimation errors of landmark locations are necessarily correlated, since they are based on relative measurements taken from the same uncertain robot position. It was proved recently in [7] that for the linear case, in the limit, the

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landmark estimates are fully correlated. This implies that the EKF should maintain the full state covariance matrix so that, the map consistency can be maintained. FastSLAM was proposed in [8] and [9] based on a particle filter with some suboptimal assumptions. A critical issue in the particle filter methods is the re-sampling technique, because improper re-sampling tends to produce over-confident estimation results. The mapping of an indoor environment using an advanced sonar is presented in [10], but in practical implementations of the stochastic mapping algorithms such as EKF, a key limitation is the low number of features it can deal with, as for every feature observed a new state is added in the EKF state equations. In [11] an attempt was made to reduce the correlation between the robot poses and the features to a level where they can be neglected. In the same way in [12], given the building, a segment based map without using the pose information is produced. In [13] an algorithm has been developed that selects and binds neighbouring pairs among sensed points in detection of unknown obstacles where a simple directed sensing strategy is employed during the mapping but in reality the mapping task is not robust against the outliers.

In [14] navigation and planning in an unknown environment using vision sensors and a cognitive approach is described. There a vision based mapping in an unknown environment extracts an unspecified land marks and the proposed transition is merged in a neural field. The limitation of robustness of this approach rely on the underlying competition mechanism where the neuron for the higher value needs to be selected.

Bryson and Sukkarieh [15] presents their work in localisation in an unknown environments using inertial SLAM. In this work, the authors consider the information-based measures of performance such as, entropy which describe the level of uncertainty of estimates in the SLAM navigation process. Similarly, [16], [17] and [18] presents a behavior and terrain based approaches for an autonomous airborne vehicle. In fact, the accuracy and the robustness of these approaches while using the vehicle depends upon the estimates, which are simultaneously coupled with the action taken by the vehicle, in terms of the vehicle's motion and the different types of observed features at different interval time period. Suppose, If an aerial vehicle operates in a large environment such as urban environments, the uncertainty in the vehicle pose estimates gradually get increased when the vehicle does not come to a known obstacle features. If the estimated value of uncertainty is very large, which will affect the stability of the aerial vehicle, which in turn provides the inaccurate feed back to the system would cause a dangerous platform motion of the vehicle which leads to the catastrophic failure of the vehicle.

Given such a large background, this shows that a very few efficient results have been produced in building the map using low cost sensors. So, an efficient and flexible method to build the map of the unstructured environment which will use a low cost sensor with a maximum efficiency is required.

III. EKF BASED ROBOT LOCALISATION USING GPS AIDED INS

An GPS aided INS with a nonlinear extended Kalman filter is used to estimate the heading angle and the X & Y position of the robot. The INS measurements are yaw angle rate from the rate gyroscope and X & Y axis accelerations from the accelerometers.

The robot is assumed to have GPS measurements. Therefore, the GPS measurements can be used to bound and estimate the errors in the INS estimates. To estimate the errors in the INS states a Kalman filter is used, which utilises the measurements from the GPS (X & Y position). The Kalman filter uses an INS error model which gives the optimal Kalman gain. This Kalman gain is used with the innovations (X & Y position and X & Y velocities from the GPS measurement) to estimate the errors in the INS estimates.

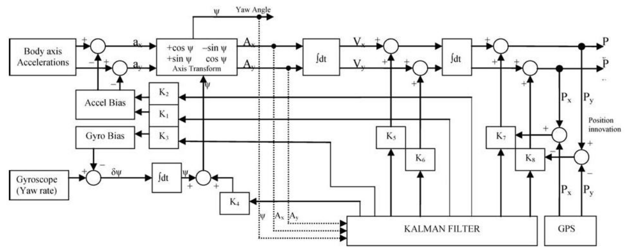

Fig. 1. The block diagram of Kalman filter based localisation using GPS aided INS

As shown in figure 1, the heading angle measurement is obtained by integrating the gyroscope output; the X & Y accelerations of the body frame are measured by the accelerometers. Obviously the measurements taken from the X & Y acclerometers are in the body frame axis, but the X & Y velocities are in the navigational frame axis so, in order to make the relationship between the body axis frame and the navigational axis frame the direction cosine matrix is used. Then, the navigation axis accelerations are integrated twice to obtain the X & Y velocities and positions, but there are errors in the INS estimates due to noise, bias and drift in the measurements. So, the errors in INS are estimated using the measurements from the GPS (i.e., X & Y Position).

$$\begin{bmatrix} \triangle \dot{a_x} \\ \triangle \dot{a_y} \\ \triangle \dot{\omega} \\ \delta \dot{\psi} \\ \delta \dot{V_x} \\ \delta \dot{V_y} \\ \delta \dot{P_x} \\ \delta \dot{P_y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \cos\psi & \sin\psi & 0 & -a_x & 0 & 0 & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & a_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \triangle a_x \\ \triangle a_y \\ \triangle \omega \\ \delta \psi \\ \delta V_x \\ \delta V_y \\ \delta P_x \\ \delta P_y \end{bmatrix} \tag{1}$$

The above state matrix **A** of the Kalman filter shows that the Kalman filter requires the INS model input (a_x, a_y), where, $-a_x, a_y$ are the measurements from the accelerometers in the INS state. The Kalman gain is used with the innovations to estimate the bias errors and the errors in the yaw angle, the X & Y axis velocities and positions. These errors are then used to update the INS states.

$$\triangle fX_{k+1|k+1} = \triangle fX_{k+1|k} + K_1\ \mu_{k+1} \tag{2a}$$
$$\triangle fY_{k+1|k+1} = \triangle fY_{k+1|k} + K_2\ \mu_{k+1} \tag{2b}$$
$$\triangle\omega_{k+1|k+1} = \triangle\omega_{k+1|k} + K_3\ \mu_{k+1} \tag{2c}$$
$$\psi_{k+1|k+1} = \psi_{k+1|k} + K_4\ \mu_{k+1} \tag{2d}$$
$$\begin{bmatrix} v_x \\ v_y \end{bmatrix}_{k+1|k+1} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}_{k+1|k} + \begin{bmatrix} K_5 & 0 \\ 0 & K_6 \end{bmatrix}_{\mu_k+1} \tag{2e}$$
$$\begin{bmatrix} p_x \\ p_y \end{bmatrix}_{k+1|k+1} = \begin{bmatrix} p_x \\ p_y \end{bmatrix}_{k+1|k} + \begin{bmatrix} K_7 & 0 \\ 0 & K_8 \end{bmatrix}_{\mu_k+1} \tag{2f}$$

A. Error estimation in Inertial navigation systems(INS)

Inertial navigation systems (INS) are self contained, non-radiating, non-jammable, dead-reckoning navigation systems, which provide dynamic information through direct measurements. Gyroscopes provide angular rate information and accelerometers provide velocity rate information, and together they form an inertial navigation system. Since the rate information is only available, it must be integrated once to provide absolute measurements of orientation and velocity, and integrated twice for position. But even a very small error in the information provided by inertial sensors causes an unbounded growth in the error of integrated measurement. Therefore, the INS by itself is characterised by position errors that grow with time and distance travelled. This problem can be overcome if the inertial sensors can be corrected periodically with other sensing mechanisms, so that the accumulated error can be reduced. In ground vehicle applications, a number of systems have been developed which use some form of absolute sensing mechanisms for guidance in order to eliminate or bound this error.

B. The Bias Error Estimation

The Instrumentation error plays a major role in the Inertial sensors; basically the two major errors present in the INS sensors are the rate gyro bias error from the gyroscope measurements and the X and Y accelerometer bias errors from the accelerometer measurements. In order to estimate this, these bias errors are added in the Kalman filter states. But in reality the Kalman filter is only a noise filter; so the bias and drift in the measurements cannot be estimated accurately and, over time, the errors due to bias and drift will increase.

C. The Gyro Bias Error Estimation

The rate gyroscope uses Coriolis effect of sensor to sense the speed of rotation (i.e, the rate of turn). So, in all of the gyroscopic measurements it will have an output signal bias which is the observed signal when no input is present. In sequence of time period this gyro bias ($\triangle\omega$) will result in major angular error($\delta\psi$). This rate gyro bias of 0.01 $rad.s^{-1}$ gives the angle error as flows:

- 0.001 rad after 1 second,
- 0.06 rad after 1 minute, and
- 3.6 rad after 1 hour.

So, the gyro bias ($\triangle\omega$) gives the angle error as:

$$\delta\psi = \int_0^t \triangle\omega\ dt = \triangle\omega\ t \tag{3a}$$

D. The Accelerometer Bias Error Estimation

All accelerometers have an output signal bias which is the observed signal when no input is present. In this case the input signal is not easily obtained since it is constantly subjected to one g of acceleration due to gravity. Thus in the case of the accelerometers, the bias would be the difference between the observed output signal and one g. Obviously the accelerometer must be oriented with the axis being studied vertical and the X and Y accelerometer measurements are integrated twice to get the X and Y velocities and X and Y positions respectively. So, the accelerometer bias ($\triangle_{a_{xy}}$) will result in a major velocity and position errors ($\delta V_{xy}, \delta P_{xy}$). This accelerometer bias of 1mg gives the positions errors as flows:

- 5mm after 1 second,
- 18mm after 1 minute,
- 1.8km after 10 minutes, and
- 65km after 1 hour.

So, the accelerometer bias ($\triangle a_{xy}$) gives the velocity and position error as:

$$\delta V_{xy} = \int_0^t \triangle a_{xy}\ dt = \triangle a_{xy}\ t \tag{4a}$$
$$\delta P_{xy} = \frac{1}{2}\triangle a_{xy}\ t^2 \tag{4b}$$

IV. The Robot and Ultrasonic Sensor Data Modelling

The mobile robot moves in a 2D environment and its motion is planned with respect to a set of obstacles and landmarks which are unknown. These obstacles and landmarks define the world reference frame W and a body frame R which is defined with respect to the body of the robot. The origin c of R is the middle of the robot. Its coordinates in W are x_c and y_c and θ is the heading angle of the robot, defined as the angle between the body frame R and the world frame W. In the world frame W the points and their coordinates are represented by lower-case letters and by tilded lower-case letters in R.

$$\mathbf{m} = \begin{pmatrix} x_c \\ y_c \end{pmatrix} + \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \widetilde{x} \\ \widetilde{y} \end{pmatrix} \tag{5}$$

The robot's position is described by the parameters x_c, y_c and θ, which form the configuration vector $\mathbf{p} = (x_c, y_c, \theta)^T$.

A. Ultrasonic Time-To-Flight (TOF) system

Ultrasonic TOF ranging is today the most common technique employed on mobile robotic systems, primarily due to ready availability of such low-cost systems and ease of their interface. One of the earliest applications of ultrasonic sensing is associated with the development of Polaroid

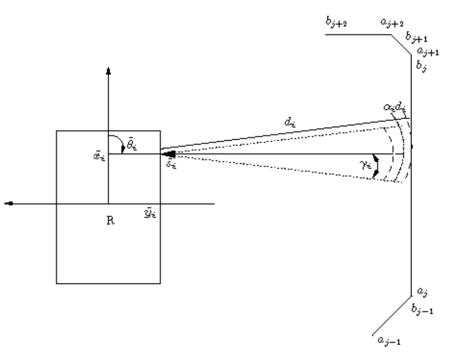

Fig. 2. Robot and Sensor model

automatic focusing camera in 1980. In this application, as in all other ultrasonic applications, the round trip time of flight measurement of an ultrasonic pulse to and from a target is used to determine the distance using the simple formula,

$$d = \frac{ct}{2} \tag{6}$$

where d the target distance, c the velocity of sound and t the TOF of the ultrasonic pulse.

The most important part of these sensors is the transducer, which converts an electrical pulse or train of pulses into ultrasonic vibrations that travel through air interrogating the area for targets, which also converts the received ultrasonic energy back into an electrical signal for getting the target distance. Most of these ultrasonic transducers are constructed from a piezoelectric material.

When there are measurements from the ultrasonic sensors, this can be used to build a map. Usually when a measurement is received from an ultrasonic sensor it is assumed to be just a point. However, this gives rise to angular ambiguity, because the sensor measurement signal may have been returned from any point within the sensor emission cone. This sensor angular ambiguity can also be modelled explicitly, if the sensor measurement return is represented as an arc instead of just a point. For example, if the obstacle is a line feature such as a wall, a set of sensor measurement returns will be a set of coherent arcs which are tangent to the wall, or it will be a set of intersecting arcs, if the obstacle is a point feature such as corners or edges. The outliers can be easily identified as they will appear as a set of incoherent arcs. In this paper, the land-marks and the obstacles are represented by a set of 2D line features. So, a line feature (i.e., a wall) which is smooth will only produce a sensor measurements return from a sensor S_j, when the sensor emission cone contains the orthogonal to the line feature (i.e., a wall) as given below:

$$\theta_{S_j} - \frac{\beta}{2} \leq \theta_k^B \leq \theta_{S_j} + \frac{\beta}{2} \tag{7}$$

where, β is the angle of incidence of the sensor signal on the walls. The distance actually given by the sensor ρ^{S_j} will correspond to the perpendicular distance from the sensor to the wall and it is shown in figure 3. Thus with the sensor model given above, associating sensor measurement returns to the line features (i.e., a wall) can be described as identifying the set of sensor measurement arcs which are tangent to the same line features (i.e.,a wall). In the presence of a large amount of outliers in the sensor data, robust techniques such as Hough transform can be used to identify the line features.

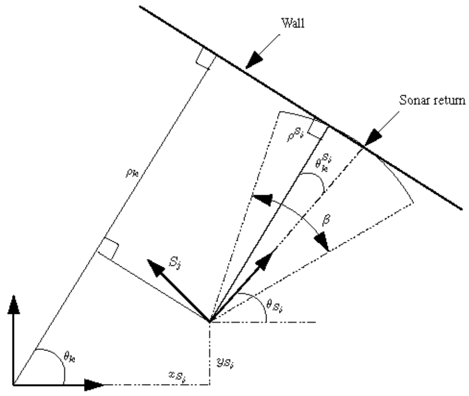

Fig. 3. Model of sonar sensor for line features

V. Mapping the unknown obstacles using Hough Transform

The Hough transform (HT) is a well known technique in computer vision as a shape detection (i.e., pattern recognition) method. In general, it detects the parametric curves within sets of primitive feature points. The HT also has the advantage of been robust enough to handle the presence of outliers and noise. Therefore HT can be used to detect straight line segments such as walls.

The HT is basically a voting scheme where each piece of sensor information accumulates evidence about the presence of certain features, that is compatible with the actual measurements [19]. In this paper, since the obstacles are represented as line features, the voting algorithm is performed in a discredited space called the HT space, which represents all the possible feature locations. The voting algorithm in the HT space can be seen as a measure of strength of each point in the HT space.

The straight line in the HT is based on the principle that all the points in a straight line are collinear. A straight line in the two dimensional $x - y$ coordinate plane (input space) can be described by the general equation;

$$\rho = x \ \cos\theta + y \ \sin\theta \tag{8}$$

where, ρ is the distance to the origin, and θ is the direction at right angles to the line as given in figure.
4.

For example, if a single point x_i, y_i is considered, then all the lines passing through the point can be obtained by varying ρ and θ on the condition that:

$$x_i \ \cos\theta + y_i \ \sin\theta = \rho \tag{9}$$

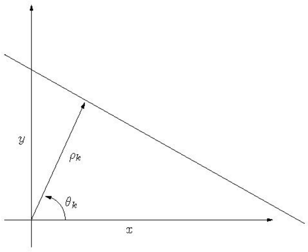

Fig. 4. Parameters of a line

for i := 1 to number of positions do
  for j :=1 to number of sensors do
    compute sensor location $x_{S_j} = (x_{S_j}, y_{S_j}, \theta_{S_j})^T$
    for $\theta_k^{S_j} := -\beta/2$ to $\beta/2$ step δ_θ do
      compute the line parameters and vote
      $\theta_k := \theta_{S_j} + \theta_k^{S_j}$;
      $\rho_k = \rho_{S_j} + x_{S_j}\cos(\theta_k^{S_j}) + y_{S_j}\sin(\theta_k^{S_j})$;
      vote(θ_k, ρ_k,i, j);
    end
  end
end

Fig. 5. The Basic Hough Transform voting algorithm for lines using ultrasonic sensors

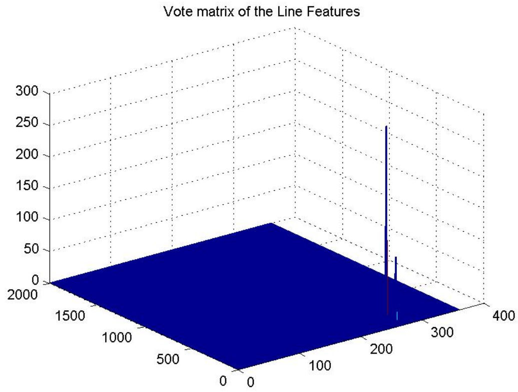

Fig. 6. Vote matrix of the line features using Hough Transform voting algorithm

Therefore, if ρ and θ are considered as a new variables, the point x_i, y_i that corresponds to a curve in $\theta - \rho$ parameter plane is called the Hough Transform.

It is also assumed that the origin of the $x - y$ coordinate system is in the center of the input space and the line angle θ can be limited to $0 \leq \theta < \pi$ if the values of ρ is negative. So, the $\theta - \rho$ parameter plane is also called the HT space. Since all the points in a straight line are collinear, all the points from a straight line in the input $x - y$ space will map to a single point in the HT space. The implementation of the Hough transform is based on parameter discretisation. If the values of ρ and θ are discrete, an accumulator array $A(\theta, \rho)$ can be formed whose elements are initialised to zero. The parameter θ and ρ are quantised in values θ_k with $k = 1, ..., n$ such that, $\theta_k - \theta_{k-1} = \Delta\theta$ and ρ_k with $k = 1, ..., m$ such that, $\rho_k - \rho_{k-1} = \Delta\rho$.

The Hough transform of a feature point x_i, y_i is performed by computing ρ using equation (8) for all n values of θ_k, and the corresponding cells $A(\theta_k, \rho_k)$ are incremented. This procedure is repeated for all feature points and all the collinear feature points will now show up as peaks in the accumulator array $A(\theta_k, \rho_k)$ as shown in figure 6. One of the key issues of its practical implementation is choosing the parameters defining the Hough space and their quantization. The lines are represented in a base reference that is located in the center of the mobile robot using parameters θ and ρ defining the line orientation and its distance to the origin.

The above HT is implemented for the ultrasonic sensor measurement data along with the EKF estimated robot positions by using the two dimensional distance readings as feature points and the voting algorithm for the line features using the HT. This is summarised in figure 5.

Finally, the voting table is searched for the local maxima having a number of votes above a certain threshold value. Then the feature points, that form the local maxima in the Hough space i.e., the cell in $A(\theta_k, \rho_k)$ are processed to form the coordinates of the line features i.e., the wall in the global reference frame in order to build the local map of the line feature and the position of the unknown obstacle can be employed using the principles of the interval analysis [20]. The figure 6 shows the initial result of the vote matrix for the line feature, while applying the HT algorithm.

VI. Discussion and Results

In order to achieve a low cost sensor system to build the map of an unknown environment and overcome the uncertainty in the system model and sensor noise statistics we proposed this novel algorithm for the localisation of the unknown obstacle and therefore mapping the environment.

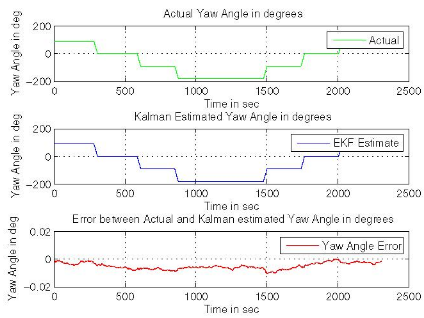

Fig. 7. EKF estimated robot angle

The fused position of the mobile robot from the EKF and the measurements from the ultrasonic sensors are used to extract the line features at every sequence of 50 positions by using the pattern recognition and data association method Hough transform technique. At each iteration we get the local map of the surrounding unknown obstacles by localising them within an interval. The EKF estimated yaw angle is shown in figure 7. The HT results while mapping the unknown obstacles are shown in figures 8 & 9. Finally, the updated global map of the unknown environment with the EKF

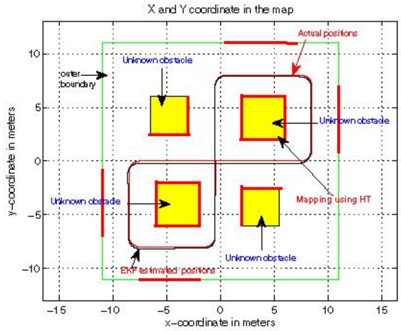

Fig. 8. Mapping the unknown environment using Hough Transform

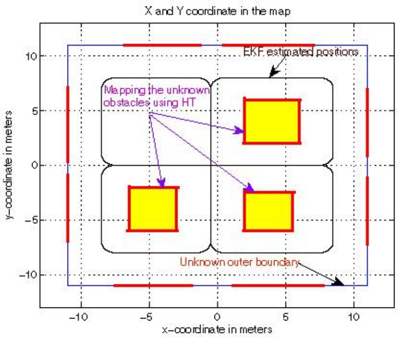

Fig. 9. Mapping the unknown environment using Hough Transform

estimated X & Y position of the mobile robot are shown in figures 10 & 11.

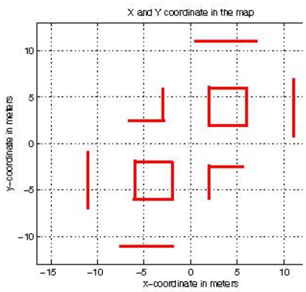

Fig. 10. The global updated map of the obstacles

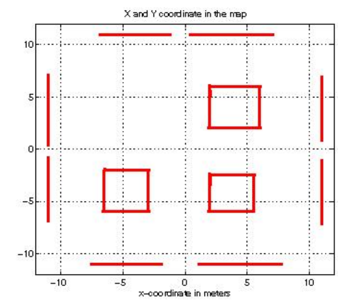

Fig. 11. The global updated map of the obstacles

VII. Conclusion

The paper presents firstly the navigation of the autonomous mobile robot with the use of an integrated GPS and INS using multiple sensors. Secondly a novel approach of mapping an unstructured environment by localisation within interval position for the unknown obstacles is presented. This proposed approach based on Hough transform and the principles of the interval analysis provides a high richness and robust against sensor uncertainty in mapping the unknown obstacles. It is also proposed to extend the work for the localisation and mapping of the unconstricted environment using multiple vehicle would make the resulting performance of the sensor system much better than of its individual components. Towards this objective the authors of this paper intend to investigate the feasibility of applying this algorithm for a swarm of vehicles.

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