



A problem for which KF should be a good solution

- It is very early in the specialization, but it is still possible to work through an example—without going into all the details—to illustrate how a KF works.
- Consider a car traveling in a straight line. We would like to estimate its position.
- We use GPS as a position sensor—but GPS estimates have measurement uncertainty and we wonder whether blending our knowledge of the car's dynamics with GPS might help give a better position estimate. (This is a navigation problem.)
- We might think, “Maybe we could use a Kalman filter to estimate the car's position better than simply using GPS.”
 - We would be right!



Designing the Kalman filter: The model

- To design the KF, we require a discrete-time state-space model of the dynamics of the car.
- We adopt the model of 1-d motion of a rigid object from the prior lesson.
 - The state comprises position p_k and velocity (speed) s_k :

$$\underbrace{\begin{bmatrix} p_k \\ s_k \end{bmatrix}}_{x_k} = \underbrace{\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} p_{k-1} \\ s_{k-1} \end{bmatrix}}_{x_{k-1}} + \underbrace{\begin{bmatrix} 0 \\ \Delta t \end{bmatrix}}_B u_{k-1} + w_{k-1},$$

where Δt is the time interval between iterations $k - 1$ and k .

- u_k is equal to force divided by mass; w_k is a vector that perturbs both p_k and s_k .
- The measurement is a noisy GPS position estimate:

$$z_k = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_C \begin{bmatrix} p_k \\ s_k \end{bmatrix} + v_k.$$



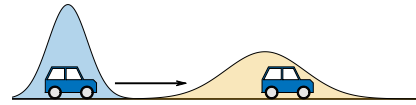
Designing the Kalman filter: The uncertainties

- We also need to describe what we know about the uncertainties of the scenario.
 - What do we know about the initial state of the car, x_0 ?
 - What do we know about process noise w_k (wind) affecting the state's evolution?
 - What do we know about the measurement noise v_k (GPS measurement error)?
- For most of the specialization, we will assume that these uncertainties are described by Gaussian (normal) distributions, which can be specified if we know the means and standard deviations of the probability density functions. We assume:
 - That GPS measurement error has zero mean and standard deviation of 5 ft (about 1.5 m). The overall confidence is then about ± 15 ft (4.5 m).
 - That x_0 is initialized via a GPS measurement, so has zero mean and standard deviation of 5 ft.
 - That w_k has zero mean and a standard deviation of 0.1 ft s^{-1} on the velocity state.



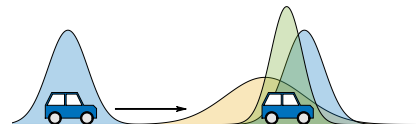
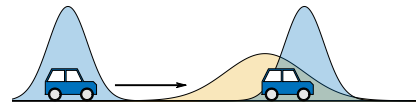
Visualizing the Kalman-filter process: Prediction

- We first initialize the KF state estimate for iteration $k = 0$.
- x_0 is uncertain since it is estimated from a GPS measurement.
- We draw this uncertainty as a shaded blue pdf.
- We don't know the car's position exactly; but we do know where we expect it to be (at the measurement location) and we know the range of likely true locations (about ± 15 ft).
- One time-step later, the car has moved.
- The KF uses the model plus knowledge of u_k to predict where the car will be, drawn as the shaded yellow pdf.
- Since we do not know w_k , the uncertainty of the position estimate has increased.



Visualizing the Kalman-filter process: Estimation

- At this point, we make a GPS measurement of the car's new position, remembering that the measurement has uncertainty.
- Again, we show this as a shaded blue pdf.
- We now have an uncertain prediction of the position from the model (yellow) and an uncertain measurement (blue).
 - We need to combine these.
- The KF takes into consideration the prediction and the measurement and their uncertainties to compute an estimate of the car's location.
- This estimate (green) will be optimal in some sense, and will have less uncertainty than either the prediction or the measurement.



Properties of the Kalman-filter solution

- Repeating from the prior lesson, the KF repeatedly takes two steps: prediction and correction.
- The prediction step uses the known dynamics of the state and known characteristics of the process noise w_k .
- The correction step combines the prediction and the measurement and their uncertainties to make a state estimate valid for this time step.
- The output of the KF at every time step comprises two quantities: the state estimate and confidence bounds on this estimate.¹
- The KF never “knows” the true system state exactly and does not converge to the true state (i.e., with vanishingly small confidence bounds) over time, since process noise w_k continuously modifies the state in unknown ways.
- But, the confidence bounds allow us to interpret the output of the KF appropriately.

¹This is somewhat imprecisely stated, but we will be more rigorous later.



Summary

- This lesson has illustrated KF operation for a specific example.
- You learned that we will need to specify a model of the system and the uncertainties of the signals in the model.
- We will need to initialize the KF with an estimate of x_0 .
- Then, every measurement interval we perform a prediction and a correction step.
- Prediction increases uncertainty and correction decreases uncertainty. The KF “fuses” the prediction with the measurement to make an optimal estimate.
- The output of the KF is the state estimate as well as its confidence bounds.
- The estimate will always contain some randomness due to w_k , so the confidence bounds will not decay to zero width.
- The confidence bounds are necessary so we know the estimate's accuracy level.