How Big is a Quasar?

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Abstract

In this project, we solve for the non-relativsitic and relativistic diameters (D_{NR} and D_R) of a QSO from the *MAST* archival repository's Light curve file which is processed using Python 3.8. The variability in the Light curve of the QSO $3C$ 273 is used as a parameter which governs the Δt in our formulation of the formulae. We get a very desired result in the form of the $D_R \sim 10 D_{NR}$.

1 Theoretical Background

The Relativistic Beaming Effect (RBE, as we shall call it for the rest of the report) is caused when a body (in our project's case, a Quasar is a type 1 AGN) expands by any mechanism, generally due to accretion at relativistic rates, which forces us to take into account for the various relativistic effects. In general a Quasar expands a relativistic rate, so to assume, as the Super Massive Black hole that is hypothised to be present at the center of an AGN is the source which causes the QSO in our case to expand. The expansion is supposed to take place relativistically since the accretion rate is very high in the case of Black holes and hence the modelling of a QSO should take into account the relativistic effect (which is also caused by other factors which shall be discussed in details later).

When the radio Quasars were obtained as an observation. people shifted their observations to the $U-B$ regions of ultraviolet as well to see the optical properties of the QSOs and they found that a lot of QSOs emitted in these wavelengths very brightly.

The reason behind the relativistic treatment of the QSO is the fact that my doing a survey on a large number of Quasars, we found that the continuum of a Quasar can be described with the aid of a Power-law spectrum:

$$S_v = v^{-\alpha} \tag{1}$$

where, α is the spectral index for the Quasars. The radiation then is characterised by the synchotron emission frequency,

$$\nu_c = \frac{3\gamma^2 eB}{4\pi m_e c} \sim \frac{4.2 \times 10^6 \gamma^2 B}{1G} Hz.$$

where, γ is the relativistic factor

$$(2)$$

Seeing from 2, we get to know that the radio emission taking place in the Quasars at such extent is a result of the relativistic motion of the electrons in the inner region of the Quasar, since, for $\nu \sim 10^9$ Hz., we need to have $B \sim 10^{-4}G$, which corresponds to $\gamma \sim 10^5$, which is a highly relativistic case. Hence, while studying the spectra/light curve of a Quasar (or any other AGN for that matter), we need to take the relativistic case into consideration.

1.1 Abberation

In the case of a particle moving with relativistic velocity, the transformation of the velocities follow the Lorentz Transformation,

$$\left.\begin{aligned} u_{\parallel} &= \frac{u'_{\parallel} + \nu}{1 + \nu\frac{u'_{\parallel}}{c^2}} \\ u_{\perp} &= \frac{u'_{\perp}}{\gamma(1 + \frac{\nu u'_{\parallel}}{c^2})} \end{aligned}\right\} \quad (3)$$

From these two set of eqautions for the transformation between the S and the S' (rest) frame, where ν is the relative velocity of the S frame w.r.t. to the S' frame and $u_{\parallel}$ and $u_{\perp}$ are the parallel and the perpendicular velocity of the photons in the rest frame and the coressponding primed transofrmed variables are the values of the same in the moving frame of reference of the observer on the ground.

eqn.(3) is the basis of the relativsitic aberration that takes palce in the relativsitic astrophysical phenomena such as the case of our project of Quasar emission. The two directions of parallel and perpendicular are related by the aberration formula,

$$\begin{aligned} \tan\theta = \frac{u_{\perp}}{u_{\parallel}} &= \frac{u'_{\perp}}{\gamma(u'_{\parallel} + \nu)} \\ &\implies \frac{u' \sin\theta'}{\gamma(1 + \cos\theta' + \nu)} \end{aligned} \quad (4)$$

In this cases when we take the case where the sources are photons (implying that they are travelling at the speed of ligght c), we get an interesting case,

$$\tan\theta = \frac{sin\theta'}{\gamma(\cos\theta' + \frac{\nu}{c})} \tag{5}$$

Assuming that the angles are perpendicular to the observing site, we can simplify the expression further to $\tan\theta = \frac{c}{\gamma\nu}$. This expression for the very highly relativistioc cases becomes,

$\theta \sim \dfrac{1}{\gamma}$

due to the fact that , γ 1, (highly relativistic expansion rates of the Quasars due to the accretion of the SMBH can cause this phenomena to physically hapepen for the strong Quasars as the one considered in the project here.)

(6)

This effect is called the aberration and this is very closely related to the relativistic beaming effect as we will discuss later now.

2 Statement of the Problem

In this project we study the Quasar Light curve obtained from the *MAST* archival repository and plot and analyse it using Python 3.8 with the help of some packages viz. *lightkurve* and *astropy.* After doign this the purpose would be to dtudy the variability demonstrated by the QSO in the Optical region and from that calculate the relativistic and the non-relativistic diameter of the QSO from the analytical calculations that i have shown.

3 Details of the methods of study

This project is based on the problem 4.1 of [1] and the solution of the problem relies heavily on the topics discussed in the theoretical background of this report. We have data in the S' frames of our observatories and the data which is observed in the form of light curve is plotted, analysed and comments upon the diameter of the Quasar are made in the end from all these processes.

The optical data of QSO is obtained from the *Mikulski Archive for Space Telescopes* in the form of a *FITS* file which is processed further using the *Pandas Dataframe* on Python 3.8 and further the csv file is read by using module called as *Lightkurve* as well read by the normal *AstroPy* to fit curves to the data etc. broadly.

3.1 The Problem with the Relativistic expansion of the QSO

the thing with the Quasar observations is that:

1. the flux (apparent) obtained from the studies is slightly higher than the actual flux
2. the unaccountability of beaming effect in such relativistic sources gives us the wrong impression of the dimensions of the objects (in this case the QSO) which can be rectified by a formulation

Now while the 1st point is out of scope of this project as of now, but, I will try to explain the second point as clearly as I can from here on.

We know from section 1 that the QSOs are expanding relativistically with the help of various observations (out of scope of this project), and hence, we need to account for these effects which present themselves cloaked in the form of relativistic aberration, red-shift etc.

Now, since the chosen QSO has a redshift $z \sim 0.159$, we can use the red-shift - (peculiar) velocity relation for the first order (since z ¡¡ 1):

$$\nu = H_0 d \tag{7}$$

where, the distance d is the proper distance taken to be in Mpc and the other variables has the likewise dimensions.

Now, the model of the QSO emission is such that the photons are assumed to be radiated isotropically with energy E in the rest frame S' and the when its obtained in the form of Flux in the observer's frame S.

Assuming that the source of the photons is expanding, again isotropically, then emitting photons. Now, assuming a non-relativistic speeds of the signals, we can assume that the time taken for the photons at the 'back' to reach the front of the screen would be around Δt and hence since the photons travelling speed is c, we would have the diameter of the Quasar (D_{NR}) in such a case to be,

$$D_{NR} = c\Delta t \tag{8}$$

On the other hand, for the non-relativistic case, we have to consider the case of the beaming effect taking place because of which some changes take place in the form of the 'shape' of the source.

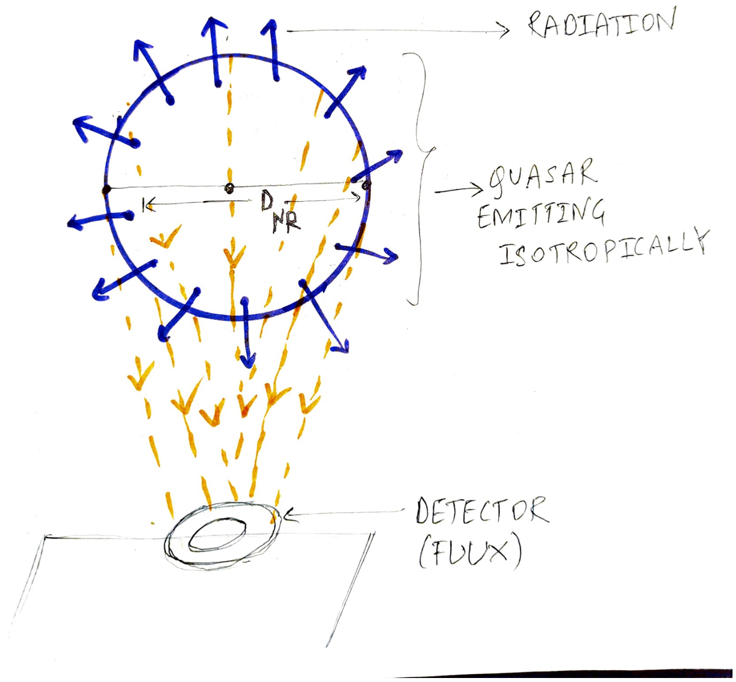

Figure 1: This image demonstrates the isotropic expansions and in the case where the S as well as the S' frame both are related to each other at non-relativistic speeds to each other and hence for the observer/recorder the source looks spherically symmetric still and does not change it's shape. The expression here for the diameter is given by eq. (8).

For the derivation we refer to the figure and the labels used in that,

$$\tan\theta = \frac{D_R}{2d} \implies \frac{D_R}{2c\Delta t}$$

We know that the angular separation can be attributed tot he e3effect of aberration in the media and hence can be treated with the equation 6 and hence,

$$//\gamma = \frac{D_R}{2c\Delta t} \implies D_R \sim 2\gamma c\Delta t$$

this completes the derivation for the non-relativistic case of the beaming effect QSO

Now here, the Δt is the time that is the variability time that we will be studying how to calculate using the $MAST$ archival repository data in the next sections.

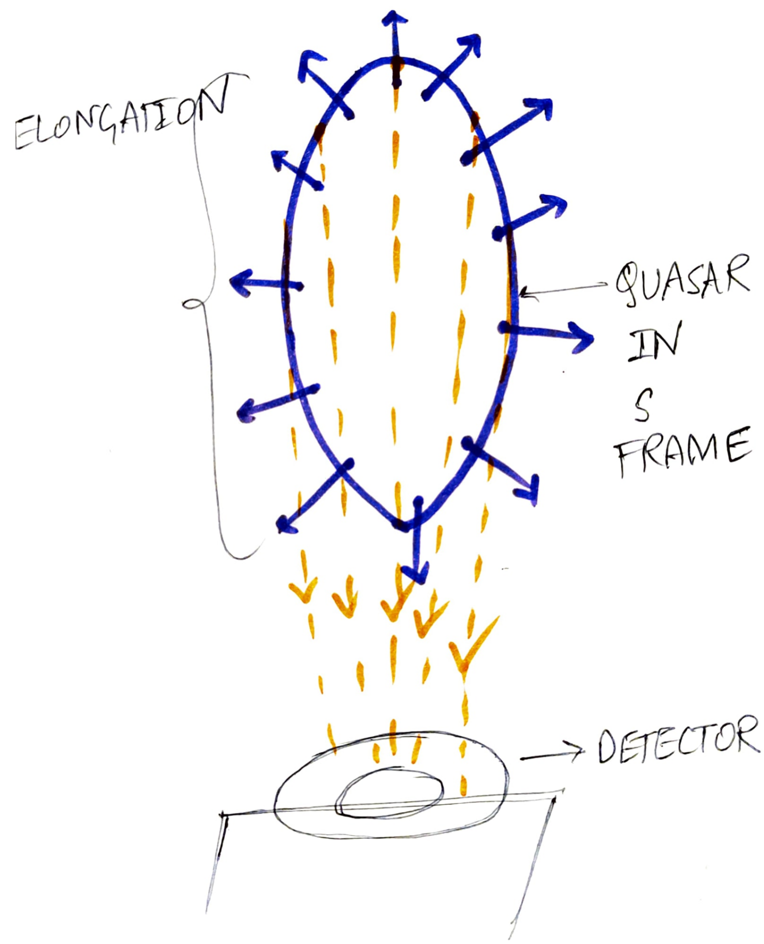

Figure 2: The relativistic case can be thought of this way due to the *lag* the signal faces because of the time taken for the end of the source to come to front increases.)

Using this relation we shall comment upon he relativistic as well as the non-relativistic case of the diameter of the QSO and then clear our doubts regarding its size.

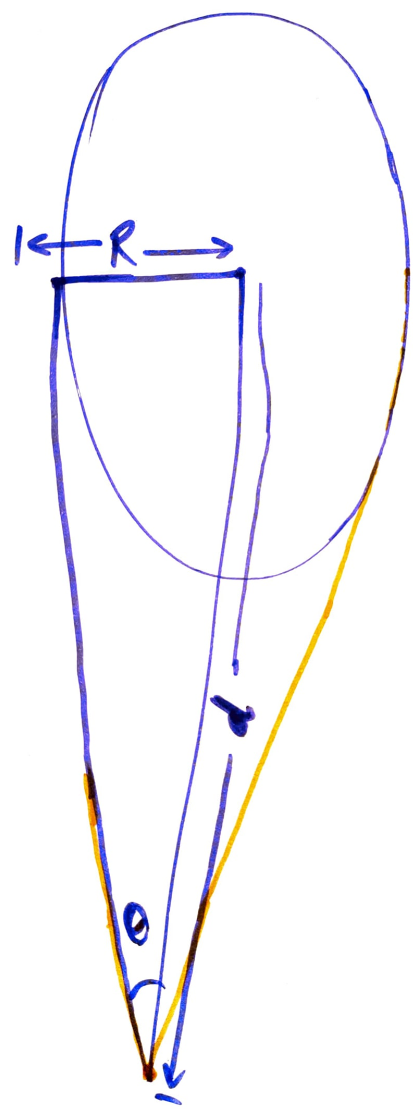

Figure 3: The relativistic case can be thought of this way due to the *lag* the signal faces because of the time taken for the end of the source to come to front increases.)

3.2 Loading data from the *MAST* archive

Being a popular Quasar from a popular cluster, I chose the 3C 273 Quasar from the Virgo cluster, since a lot of data like , d_L, z etc. are already well known and registered in the archival data of the object.

The light curve of the object is obtained as the processed *FITS* file of the time-series of the QSO from which the relevant files are extracted by constructing a *Pandas Dataframe* and plotting the relevant columns using *Lightkurve* package in Python.

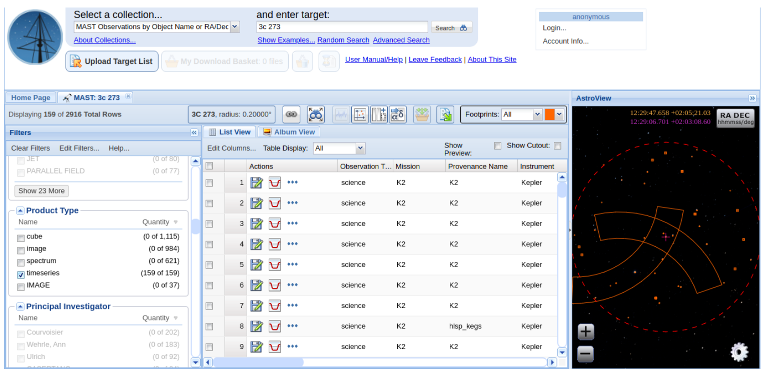

Figure 4: The screenshot of the *MAST* archive page and the options selected to get the processed *FITS* file of the Light curve of 3*C* 273

3.3 CODE

3.3.1 Conversion of *FITS* into CSV

The following is the code written for the conversion of the light curve file to a csv:

```
import sys
import csv ##############Since the file is very large

csv.field_size_limit(sys.maxsize)

hdu_list = fits.open(file_pth)
hdu_list.info()

from astropy.io import ascii
ascii.write(data, 'values.csv', format='csv', fast_writer=False)


with fits.open(file_pth) as hdu:   #####DATA MAKER FROM THE PROCESSED FITS FILE
    asn_table = Table(hdu[1].data)
data = asn_table
print(data)
```

```
import pandas as pd

# making dataframe
df = pd.read_csv("values.csv")

# output the dataframe
print(df)
new_df = df.replace([np.inf, -np.inf], np.nan).dropna(axis=0)

from astropy.io import ascii      #############TO CONVERT DF TO CSV
ascii.write(new_df, 'values2.csv', format='csv', fast_writer=False)
from google.colab import files
files.download('values2.csv') ###############TO DOWNLOAD THE CSV
```

3.3.2 Plotting of The Lightcurve

The following code was implemented to plot the lightcurve file and using speculation, the avg. time period $\Delta t_{min.}$ of the Quasar Lightcurve was obtained.

```
import pandas as pd

# making dataframe
df = pd.read_csv("values.csv")

# output the dataframe
print(df)
new_df = df.replace([np.inf, -np.inf], np.nan).dropna(axis=0)

from astropy.io import ascii
ascii.write(new_df, 'values2.csv', format='csv', fast_writer=False)
from google.colab import files
files.download('values2.csv')

x = new_df['TIME']
y = new_df['PDCSAP_FLUX']

plt.plot(x, y)

'''from scipy.optimize import curve_fit as cf

def SQfunction(x,a,b):
    return a*np.sin(b*x)
constants = cf(SQfunction,x,y)
a_fit = constants[0][0]
b_fit = constants[0][1]
```

```
fit = []
for i in x:
  fit.append(SQfunction(i, a_fit, b_fit))



plt.plot(x,fit,c='k',label="Fit")
plt.legend()
plt.xlabel("z")
plt.ylabel("$d_L$")
plt.title("Fitting the data using SciPy for $ax^2 + b$")
plt.show()
print(constants)
constants = op, cov'''

import lightkurve as lk
lc = lk.KeplerLightCurveFile(file_pth)
lc.plot()
lc.flux = lc['sap_flux']

ax = lc.plot(column='pdcsap_flux', label='PDCSAP Flux', normalize=True)
lc.plot(column='sap_flux', label='SAP Flux', normalize=True, ax=ax)
plt.xlim(2780, 2790)
plt.show()
```

After the light curve has been plotted, the problem still remains of finding the interval numerically and then taking its average for better results by putting it into the equation for both the cases.

3.3.3 Getting the variabillity of the Quasar through the Δt finding

```
def get_slope(x_data, y_data):
    x1_list = []
    x2_list = []
    y1_list = []
    y2_list = []
    slopes = []

   # the length of both the x and y data must be same.
    for i in range(1,len(x_data)):
        x1 = x_data.values[i-1]
        x2 = x_data.values[i]
        y1 = y_data.values[i-1]
        y2 = y_data.values[i]
        slope = (y2-y1)/(x2-x1)
```

```
        # saves values:
        slopes.append(slope)
        x1_list.append(x1)
        x2_list.append(x2)
        y1_list.append(y1)
        y2_list.append(y2)
    slopes = np.where(np.isnan(np.array(slopes)),0,np.array(slopes))
    return slopes,np.array(x1_list),np.array(x2_list),np.array(y1_list),np.array

fig, axes = plt.subplots(nrows=3, ncols=1,figsize=(15,10))
axes[0].set_title('Slope variation')
axes[0].boxplot(slopes,vert=False)
axes[0].grid()

slope_threshold = 15000
axes[1].plot(x1,slopes,label='Slopes w.r.t time')
axes[1].plot(x1,np.repeat(slope_threshold,len(x1)),label=f'slope = {slope_thresh
axes[2].plot(x1,y1,label='Data')

axes[1].legend()
axes[2].legend()
plt.show()

plt.figure(figsize=(15,10))
plt.plot(x1,y1)

selective_x = []
selective_y = []

for i in range(1,len(slopes)):
    if (slopes[i]-slopes[i-1])>slope_threshold:
        selective_x.append(x1[i])
        selective_y.append(y1[i])
        plt.scatter(x1[i],y1[i], s=100)

# Get the final time intervals:
time_intervals = []
for i in range(1,len(selective_x)):
    time_diff = selective_x[i]-selective_x[i-1]
    time_intervals.append(time_diff)

time_intervals = np.array(time_intervals)

time_intervals.mean()
```

This program makes use of the regression points and uses the points wherte the slope decreases or increases in slope marks the points around which the slope is taken. Then, these differences are taken the averages of and the values of the frequency is obtained which in turn makes us find the diameter of the QSO.

4 Results and Discussions

To open the file, we use the code in the section 3.3.1 and obtain these files: As

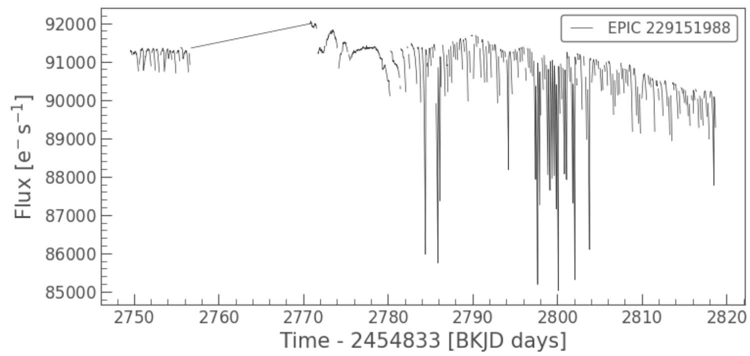

Figure 5: Opening the file using *lightkurve* using the CSV file mad from the processed *FITS* file from the *MAST* archival repository.

we can see this file is the light curve of a QSo since the intervals of time are in BKJD and hence mathces the deciption of that of a QSO.

We try and open this file along with a closed view and further selecting the part of the spectrum we need to comment about the data of the Diameter (both relativistic and non-relativistic). Even this data cannot be used fully, so we

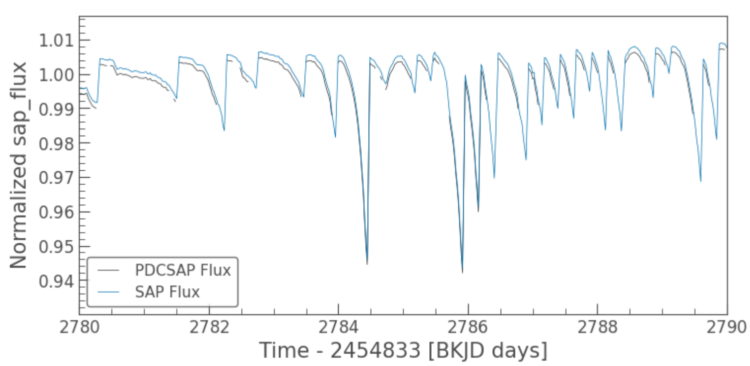

Figure 6: The file zoomed in and both the Fluxes type plotted on single plot.

obtain a plot which fits the points using the program in the section 3.3.3 and thus get the final results. From the given code, we found that the $\Delta t \sim 0.364$

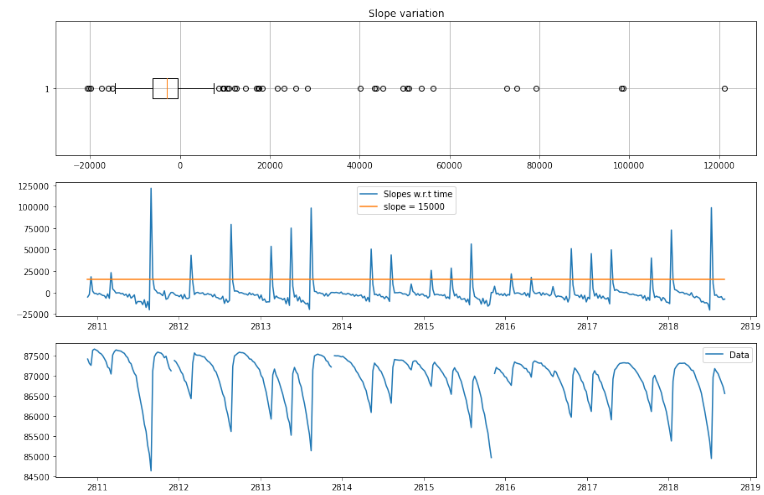

Figure 7: The points obtainied and the time period obtained between two adjacent points in the single curve region of a small part of the ligth curve gives us the average value of the Δt which we shall use to calculate the D_{NR} and D_R (non-relativistic and relativistic cases simulataneously).

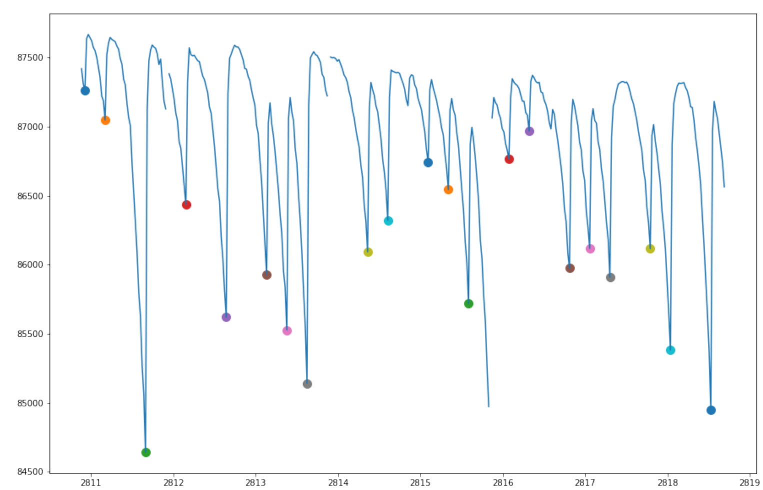

Figure 8: The dots represented the points detetcted by the program around which the Δt is calculated. In the end, the loop of the program takes all these Δt_i and takes the avergae of it to produce a reliable avergae Δt which is used for the final calculations and we shall it to produce impeccable results.

from which we get the values of Diamters using the derived formulae 8 as:

$$D_{NR} \sim 7.775 \times 10^9 km \tag{9}$$

which is approximately ten times the Sun-jupiter distance.

This looks very small for an AGN which is despite being $\sim$ 100 Mpc away the brightest object in the sky. If we now take all of the datat presented into this project into account and finally calculate the relativistic distance traking into account the relativistic beaming effect, we get around ten times the answer!

$$D_R \sim 8.0 \times 10^{10} km \tag{10}$$

which sounds still considerable and apt for an object that generates such kind of energy.

5 Relevance and Limitations

- I developed the Python codes which can analyze a Light curve taken from any processed *FITS* file of the archival depositories and plot it by making a proper CSV file from scratch.
- This project also encompasses the main subject of the calculation of the QSO's size from its variability as seen by an observer in the moving fram w.r.t. Quasar (in the relativistic case).
- I showed that the perception of a Quasar being a small, energetic source of radiation in the universe is mostly wrong since they are way bigger if we realise for their relativistic expansion rates.
- I also formulated the formulae correctly which tells us (just from the information of the redshift z, variability time period Δt) the Diameters of the QSOs. (both relativistic and non-relativistic).
- Due to the theoretical formulation limiting the case to the first order of the Hubble's laww, we cannot use this theory for $z \sim 1$. But we can, after some modifications done to it.
- Program for calculations of the time-difference average can be applied to a lot of astrophysical observations for a greater accuracy.

References

[1] *Radiative Processes in Astrophysics* - George B. Rybicki, Alan P. Lightman

[2] `https://mast.stsci.edu/portal/Mashup/Clients/Mast/Portal.html`

[3] *Extragalactic Astronomy and Cosmology* - Peter Schneider

[4] `https://en.wikipedia.org/wiki/3C_273`