THESIS PROPOSAL

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1. Introduction

1.1. **Fourier Continuation.** Fourier Continuation (FC) is an approximation method used to extend the computational abilities of a Fourier Series to non-periodic functions.

1.2. **Past Work.**

2. Current Work

2.1. **Application to the Heat Equation.** We study the Boundary Value Problem $(I-\alpha\frac{\partial^2}{\partial x^2})u = f$ with boundary values $u(a) = u_0$ and $u(b) = u_1$. As a motivating example, consider $f(x) = x$ on $[0, 1]$. Let $\mathcal{L} = (I - \alpha\frac{\partial^2}{\partial x^2}$. When we solve $u = \mathcal{L}^{-1}f$ using Fourier Continuation approximations, some energy from the continuation domain can be spread back into the system. Since we know analytically for $\alpha > 0$ the system is stable, using the Fourier Continuation approximation can be a computationally inaccurate approach. Our goal is to find an additional constraint that would preserve the accuracy of the Fourier Continuation approximation while ensuring the stability of the operation.

2.2. **Green's Functions.**

2.3. **Results.** For the individual Gram Polynomials, we saw accuracy of $\mathcal{O}(10^{-14})$. In the following figure, we see the accuracy as a function of parameter α

3. Future Work

3.1. **Computational Work.** We are going to use this to put as a time step of the heat equation and solve that PDE. Our goal is to show that we have a stable approximation that can be used.

3.2. **Analytical Work.** The result that yields the same Fourier coefficients for any given Gram polynomial independent of choice of α is unexpected. Our goal is to develop an analytic proof that justifies this result in general.