Uncorrelated Multisymbol Signals for MIMO System Identification

Mahmoud El-Fandi, Ian A. Henderson, Joseph McGhee, and Phillip McGlone

***Abstract*—This paper presents a new approach to the system identification of a two-input two-output multivariable system. In a similar manner to that used for the single-input/single-output system, the multi-input multi-output (MIMO) system response may be obtained directly from measured input output frequency responses. An investigation of compact multifrequency data measurement signals, which have been extensively cataloged by the authors, has provided compact multifrequency ternary and quaternary input signals that have uncorrelated spectra. They are powerful multisymbol, multilevel computer generated measurement signals whose signal power is concentrated in either two or three dominant harmonics. As the signals, which are given in this paper, have six, seven, or eight symbols in their measurement codes, both the computation time for the frequency estimates and the experimental time are minimized. Two multifrequency quaternary signals with uncorrelated spectra are used to identify a simulated distillation column. It is shown that the cross coupling terms between the measurement channels may be removed by these digital measurement signals with the same number of symbols but different measurement codes.**

***Index Terms*—Digital measurements, frequency domain analysis, identification, multi-input multi-output (MIMO) systems, signals.**

I. Introduction

MULTIVARIABLE systems, which are also known as multi-input multi-output (MIMO) systems, have more than one input and one or more outputs [1]. In the single input single output system there is only one measurement channel. For this case a frequency response test and identification is well known. As well as the required measurement channels, MIMO systems have many other channels which exist between any input and each output. Obviously, a suitable test signal may be used to obtain offline frequency responses from the same number of separate experiments as there are inputs. However, if these experiments were combined, the output for each of the separate experiments would most likely be affected by the dynamic interaction of the other inputs. This is unsatisfactory when designing an online control strategy.

Test signals are required which remove the cross coupling terms between the measurement channels during system identification. This is performed by [2] for identification in the time domain by using two perturbations signals with zero cross-correlation. An immediate simplification of this major test signal problem is obtained by restricting the multivariable system to two inputs and two outputs. As previously shown by Godfrey [2], the test signals should give a cross-correlation spectrum of zero when the identification is based on frequency response analysis .

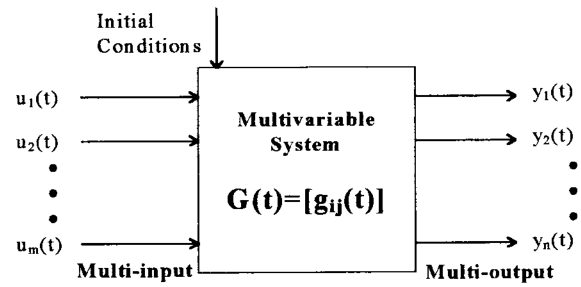

Fig. 1. Deterministic representation channel MIMO system.

Reference [3] shows how to generate two signals with zero cross-correlation by using a pseudo-random binary sequence (PRBS) signal. A PRBS test signal is used with two periods for one measurement channel and a modified version of the PRBS signal, which is called an n-sequence PRBS, in the other measurement channel. It may be generated by inverting the second period of a PRBS. Reference [2] also shows how to generate these two signals by using inverted PRBS. The same approach is extended for more than two channels. A major disadvantage of this technique is the increase in the number of bits or symbols in the measurement code, which is needed to generate these signals. Twice and four times the number of PRBS symbols are required, for the system identification of two MIMO and three MIMO systems, respectively. Choosing this test signal leads to an increase in the cost, the experimental time and the instability of the statistical behavior of the system.

Compact multifrequency data measurement (MDM) signals, which are used in multifrequency testing [4], [5] and analysis [6], [7], have been extensively studied by the authors. Frequency response data and online signal processing are used in system identification, autotuning, and adaptive frequency control. All multifrequency binary sequence (MBS) signals with up to sixteen symbols, multifrequency ternary sequence (MTS) signals with up to nine symbols, and multifrequency quaternary sequence (MQaS) signals with up to six symbols have been cataloged in [5]. These are discrete interval signals which concentrate their power in a few dominant harmonics and allow a high signal to noise ratio (SNR) for noise contaminated measurements at these dominant harmonics.

A search of the catalog in [5] has obtained pairs of compact multifrequency ternary and quaternary signals which have un-

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The authors are with the Industrial Control Center, University of Strathclyde, Glasgow G11QE, U.K. (e-mail: joe@icu.strath.ac.uk; http://www.icc.strath.ac.uk).

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correlated spectra. In this paper, the multifrequency quaternary signals with six symbols are used to identify a simulated distillation column with a linear multivariable model [8]. It is shown that the cross coupling terms between the measurement channels may be removed from the output signals. This paper emphasizes the importance of using these MDM signals in multivariable system identification.

II. Multivariable Systems

A. Modeling a MIMO System

The model of a MIMO system, which is given in Fig. 1, must account for the cross coupling between the measurement channels [1]. Every output from each equivalent single input single output measurement channel has additional terms due to the dynamic interaction from the other inputs. These may be modeled by cross coupled channels from each of the other inputs to each output of the equivalent single input single output measurement channel. Hence, the linear MIMO system can be represented in continuous time by a weighting function matrix [9]

$$G(t) = [g_{ij}(t)] \quad \text{where} \quad i = \quad 1, 2, \cdots, m$$

and

$$j = 1, 2, \cdots, n. \tag{1}$$

The system has m inputs and n outputs. Each output, y_j is connected to all inputs through the weighting function $g_{ij}(t)$. This may be written as

$$y_i(t) = \sum_{j=1}^{n} \int_0^t g_{ij}(t-\tau) u_j(\tau)\, d\tau \qquad i = 1, 2, \cdots, m. \tag{2}$$

The cross coupling between the measurement channels and the corresponding system identification problem [1] is best illustrated by the simpler two input two output MIMO system of Fig. 2. In this block diagram the two equivalent single input single output measurement channels are represented by the direct paths through the frequency functions $G_{11}(j\omega)$ and $G_{22}(j\omega)$. Because of the cross coupling between these channels, output Y_1 has an additional term due to input U_2 through the frequency function path, $G_{12}(j\omega)$. Output Y_2 also has an additional term due to input U_1 through the frequency function path, $G_{21}(j\omega)$. The outputs $Y_1(j\omega)$ and $Y_2(j\omega)$ may be given by two equations

$$Y_1(j\omega) = G_{11}(j\omega)U_1(j\omega) + G_{12}(j\omega)U_2(j\omega) \tag{3}$$

$$Y_2(j\omega) = G_{21}(j\omega)U_1(j\omega) + G_{22}(j\omega)U_2(j\omega). \tag{4}$$

When this is given in its matrix form it becomes

$$\begin{pmatrix} Y_1(j\omega) \\ Y_2(j\omega) \end{pmatrix} = \begin{pmatrix} G_{11}(j\omega) & G_{12}(j\omega) \\ G_{21}(j\omega) & G_{22}(j\omega) \end{pmatrix} \begin{pmatrix} U_1(j\omega) \\ U_2(j\omega) \end{pmatrix}. \tag{5}$$

If periodic MDM signals are used to identify the MIMO system, in general the signal power is concentrated in a number of harmonics $k = 1, 2, \cdots, n$. Equations (3) and (4) may be rewritten as

$$Y_1(j\omega_k) = G_{11}(j\omega_k)U_1(j\omega_k) + G_{12}(j\omega_k)U_2(j\omega_k) \tag{6}$$

$$Y_2(j\omega_k) = G_{21}(j\omega_k)U_1(j\omega_k) + G_{22}(j\omega_k)U_2(j\omega_k). \tag{7}$$

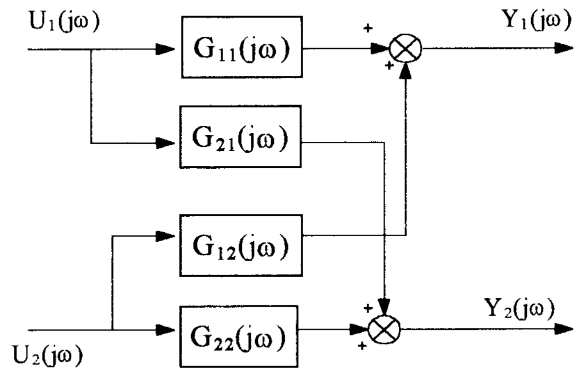


Fig. 2. An interacting representation of two for MIMO system.

Let the input signal U_1 concentrate its power in harmonic numbers m, n and input signal U_2 concentrate its power in harmonic numbers p, q. Hence, (6) and (7) become

$$\begin{aligned} Y_1(j\omega_{m,\,n}) = {} & G_{11}(j\omega_{m,\,n})U_1(j\omega_{m,\,n}) \\ & + [G_{12}(j\omega_{p,\,q})U_2(j\omega_{p,\,q})] \end{aligned} \tag{8}$$

$$\begin{aligned} Y_2(j\omega_{p,\,q}) = {} & G_{22}(j\omega_{p,\,q})U_2(j\omega_{p,\,q}) \\ & + [G_{21}(j\omega_{m,\,n})U_1(j\omega_{m,\,n})]. \end{aligned} \tag{9}$$

When the two input signals are chosen to concentrate their signal power in different harmonic numbers, the output $Y_1(j\omega_{m,n})$ is simplified so that (8) becomes a single input single output equation in terms of m and n even although the terms for p and q are present. Similarly, the output $Y_2(j\omega_{p,q})$ is simplified to make (9) a single input single output equation in terms of the harmonic numbers p and q. The cross coupling terms in (8) and (9) do not contribute to the measurement at harmonic numbers m and n in (8) and harmonic numbers p and q in (9).

B. Model of a Distillation Column

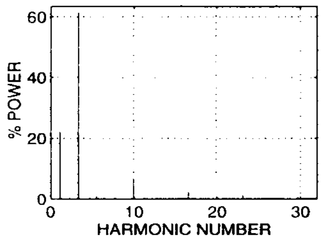
Distillation uses the processes of evaporation and condensation to separate substances. The distillation column is the most commonly used example of a two input two output MIMO system. It is used extensively in the chemical industry [8]. The main application is in an oil refinery where crude oil can be separated into many fractions such as gas, oil, lubricating oils, heavy fuels, and asphalt.

The distillation column consists of two inputs and two outputs. Outputs Y_1 and Y_2 are the overhead and bottom product compositions, respectively, while inputs, U_1 and U_2, are the reflex and vapor flow rates, respectively. A linear model of a distillation column [8] is used in a simulation study to examine the system identification by MDM signals of a two MIMO system. Its matrix equation is given by

$$\begin{bmatrix} Y_1(j\omega) \\ \\ Y_2(j\omega) \end{bmatrix} = \begin{bmatrix} \dfrac{4.533e^{-0.5j\omega}}{7j\omega + 1} & \dfrac{41.344e^{-0.5j\omega}}{4.5j\omega + 1} \\ \dfrac{0.1086e^{-0.5j\omega}}{7j\omega + 1} & \dfrac{0.844e^{-0.5j\omega}}{4.5j\omega + 1} \end{bmatrix} \begin{bmatrix} U_1(j\omega) \\ \\ U_2(j\omega) \end{bmatrix}. \tag{10}$$

TABLE I
PAIRS OF TEST SIGNALS (≤8 SYMBOLS) WITH UNCORRELATED SPECTRUM FOR THE SYSTEM IDENTIFICATION OF TWO-INPUT TWO-OUTPUT MULTIVARIABLE SYSTEMS

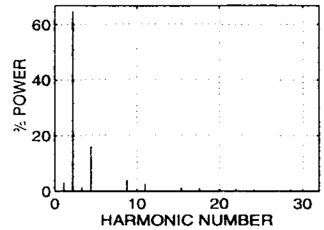
Name:-S6Qa22 DM Code: Aaed



HAR	E(k)	PHASE	P(k)
1	+0.424	−30.000	+22.109
3	+0.707	−90.000	+61.404
9	+0.236	−90.000	+6.823

SELECTED POWER = 90.336 %
PARSEVAL POWER = 0.407 W

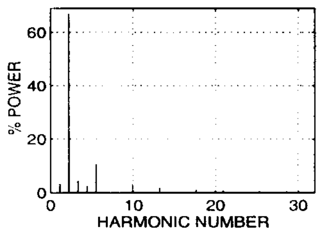
Name:-S6Qa23 DM Code: abeabd



HAR	E(k)	PHASE	P(k)
2	+0.955	−30.000	+64.793
4	+0.477	−150.000	+16.198

SELECTED POWER = 80.991 %
PARSEVAL POWER = 0.704 W

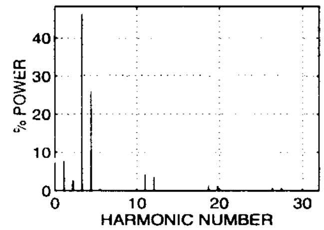
Name:-S7T34 DM Code: acbcacb or $Oa^2c^2b^2c$



HAR	E(k)	PHASE	P(k)
2	+0.874	−90.000	+66.899
5	+0.350	−90.000	+10.704

SELECTED POWER = 77.603 %
PARSEVAL POWER = 0.571 W

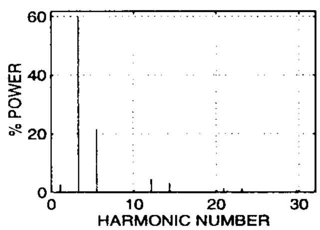
Name:-S7T24 DM Code: abc^3ab or $Oa^2b^2c^3$



HAR	E(k)	PHASE	P(k)
1	+0.299	+90.000	+7.817
3	+0.727	−90.000	+46.233
4	+0.545	−90.000	+26.006

SELECTED POWER = 80.057 %
PARSEVAL POWER = 0.571 W

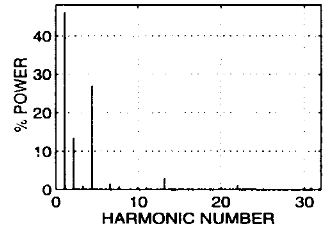
Name:-S8T92 DM Code: Aabca



HAR	E(k)	PHASE	P(k)
3	+0.947	−67.500	+59.740
5	+0.568	−112.500	+21.507

SELECTED POWER = 81.247 %
PARSEVAL POWER = 0.750 W

Name:-S8T77 DM Code: $cacab^3a$



HAR	E(k)	PHASE	P(k)
1	+0.832	−67.500	+46.124
2	+0.450	+45.000	+13.509
4	+0.637	+90.000	+27.019

SELECTED POWER = 86.653 %
PARSEVAL POWER = 0.750 W

When the cross coupling between the measurement channels is removed (10) becomes

$$Y_1(j\omega) = \frac{4.533e^{-0.5j\omega}}{7j\omega + 1} U_1(j\omega) \tag{11}$$

$$Y_2(j\omega) = \frac{0.844e^{-.05j\omega}}{4.5j\omega + 1} U_2(j\omega) \tag{12}$$

where the corresponding transfer functions are given by

$$G_{11}(s) = \frac{4.533e^{-0.5s}}{7s + 1} \tag{13}$$

$$G_{22}(s) = \frac{0.844e^{-0.5s}}{4.5s + 1}. \tag{14}$$

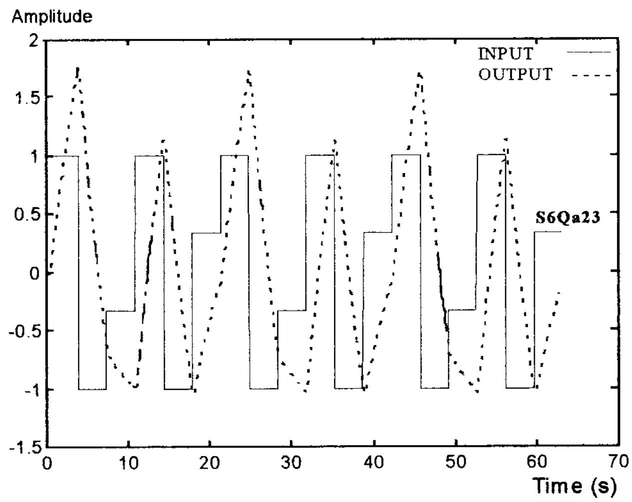
All the measurement and cross coupled channel, frequency functions are represented by an approximately linear model of a first order system with time delay. The first measurement channel to be identified is represented by a first order system with a time delay of 0.5 s. The input of this channel is reflex flow rate while the output is overhead product composition. There is also a large cross coupling from the next channel input which is represented by a first order system with the same time delay and added to the first output signal. The second measurement channel is also a first order system with a 0.5 s delay. The input of this channel is the vapor boil-up rate and the output is the bottom product composition. The corresponding output is affected by a much smaller cross coupling from the first input through another first order system with the same time delay.

III. MULTIFREQUENCY DATA MEASUREMENT SIGNALS

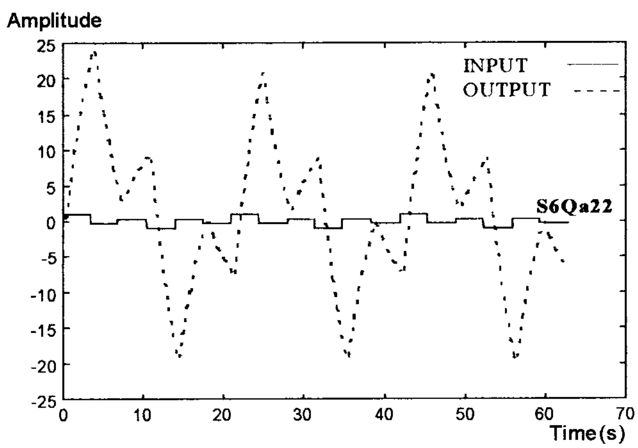
Multifrequency data measurement (MDM) signals [5], [10] are described by measurement codes in terms of alphabetical symbols. Complementary a, $b \cdots d$, $e \cdots f$, g, etc. symbols represent different ± voltage levels while the c symbol represents zero volts from a DAC. They have a number of important features [5] which facilitate multifrequency response testing with one experiment.

- They are based on the identification source and channel coding theorems [5].
- They are periodic and may be described and designed using descriptive language techniques [4], [5].
- They are zero mean signals which ensures zero off set when testing linear systems [2].
- Their total signal power is concentrated in a few harmonics [2].
- They are compact signals whi
- ch allow a variety of designs using shift keyed modulation [11]–[13].
- They exhibit different spectral signatures with either a narrow or a broadband spectrum.
- They may have an odd, even, octave, or decade harmonic power distribution [5].
- They are multilevel, multisymbol computer test signals, which are generated by step functions, and are easy to use in modern instrumentation [10].

There are 375 MBS signals, 195 MTS signals, and 34 MQaS signals listed in [5]. This catalog allows a search of the many



(a)



(b)

Fig. 3. Distillation column model time response (a) channel 1, G_{11}, without cross coupling and (b) cross coupling of channel 2 into 1, G_{12}.

compact MDM test signals for a specific application. Using the code S6Qa22 as an example, the DM codes are named as follows:

- S—Strathclyde (named after the university);
- 6—total number of symbols in the basic code;
- Qa—quaternary (4 alphabetical symbols, i.e., a, $b \pm 1$ V and d, $e \pm 1/3$ V, respectively, in the catalog);
- 22—number associated with the discovery of each unique DM code.

For this paper, the catalog [5] was examined for signal pairs which have the same number of symbols but exhibit cross-correlation spectra with zero values. In comparison with the number of signals in the catalog, there are only a few pairs of signals which can be used to identify a two MIMO system. However, they are compact and have the total power concentrated in either two or three dominant harmonics which can provide high precision, robust, multifrequency measurements.

In this investigation, only compact signals with up to eight symbols in their measurement codes were examined. No MBS pairs, one seven-symbol MTS pair, one eight-symbol MTS pair, and one six-symbol MQaS pair were found. These three signal pairs are given in Table I. The S7T34, S8T92, and S6Qa23 are special since their harmonic power distribution is unique but may be combined with other possibilities from the catalog. Further information may be obtained by a full inspection of the MDM signals in [5].

The MQaS pair was chosen for the system identification of the distillation column. As one signal has only even harmonics while the other has only odd harmonics, they provide the ideal overlap in the dominant harmonics for MIMO system identification. The number of possible MDM signal pairs for the same number of total symbols increases with additional alphabetical symbols. As this increases the number of voltage levels from the DAC with a corresponding greater possible distribution of the signal power, the choice of the MQaS pair is not unexpected.

IV. Simulation Study of Distillation Column

Matlab and Simulink are used to simulate the distillation column model. The data measurement (DM) toolbox [5] is extended to allow the required input test signals to be generated for a study of the system identification of the distillation column. The pair of signals should have two properties. First, the cross spectrum between them must be zero if the cross coupling terms at the outputs of the single input single output measurement channels are to be uncorrelated. Second, as each channel is represented by a first order system, a test signal whose signal power is concentrated in two or three harmonics will be sufficient to identify the distillation column.

From Table I, the signal S6Qa23 is applied to the first measurement channel. This signal has its power concentrated in the second and fourth harmonics. The signal S6Qa22 which has its power mainly in the first and third harmonics is applied to the second measurement channel. The ninth harmonic is not used. Any other harmonic in the two signals has less than 5% of the total signal power.

The period of the two signals is chosen to give a fundamental frequency of 0.3 Hz. Two harmonics of frequency 0.6 and 1.2 Hz are applied to the first channel while two harmonics of frequency 0.3 and 0.9 Hz are applied to the second channel. Two periods of the signals are applied before the signal processing data is taken from the third and last period. To avoid aliasing problems, 64 samples are taken during the time for one symbol of each signal. The waveforms of each output of the measurement channels without any cross coupling are given in Fig. 3(a) and 4(a). The waveforms showing the cross coupling between the channels are given in Fig. 3(b) and 4(b). As expected, Fig. 3(b) shows very little cross coupling of measurement channel 1 into measurement channel 2.

The two frequency estimates for the two measurement channels for the two signals are given in Table II. The calculated values for the magnitude and the phase of each measurement channel without an interaction are also given in the table. If the interaction between the channels is removed by the chosen signals, the deviation between the measured and calculated results for both magnitude and phase should be negligible. As

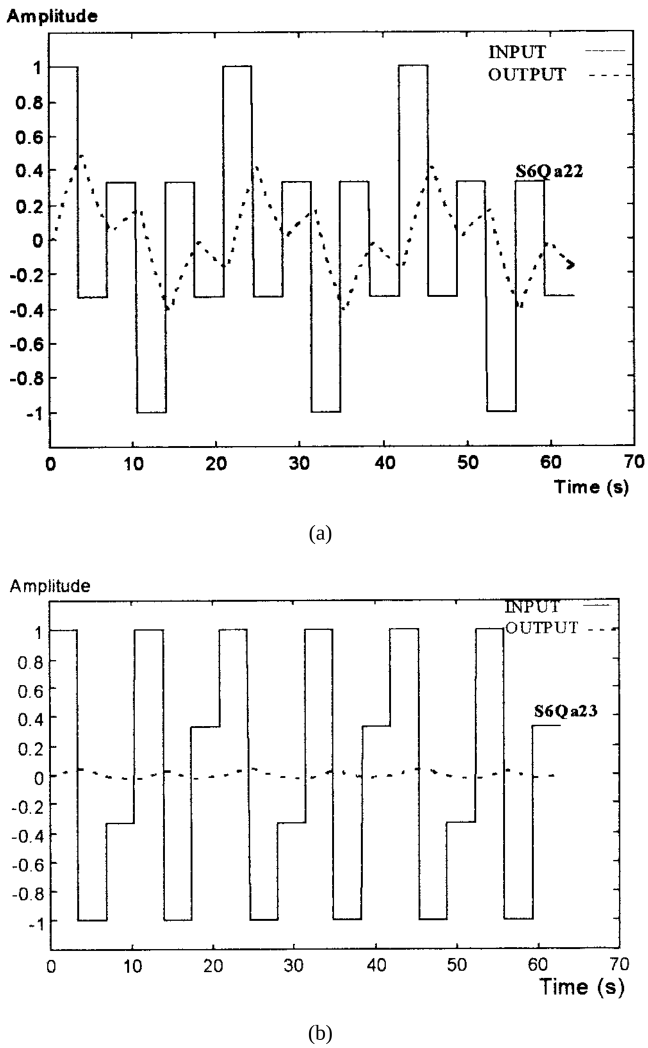


Fig. 4. Distillation column model time response (a) channel 2, G_{22}, without cross coupling and (b) cross coupling of channel 1 into 2, G_{21}.

this is seen to be true, it is concluded that this pair of signals has given an excellent identification of the distillation column.

To make a comparison, an experiment with the same conditions is repeated using a seven bit PRBS signal, which is applied to both inputs. Because of the negligible cross coupling of measurement channel 1 into measurement channel 2, there is no appreciable difference between the measured and calculated results for measurement channel 2. However a large difference is obtained between the two measured and calculated magnitudes for measurement channel 1. In particular, the magnitude deviates by 2.31 dB at a frequency of 0.6 Hz.

V. Conclusion

A new approach to the system identification of a two-input two-output multivariable system has been demonstrated. These new pairs of compact MDM signals have uncorrelated spectra and concentrate the signal power in either two or three dominant harmonics. Each signal must have the same number of total symbols in the measurement code. The compact nature of their measurement codes and the few dominant harmonics involved ensures a minimum time for an experiment and for offline or online computation. In the identification of frequency response parameters, this gives a superiority over the use of PRBS signals. As they are digital signals, they allow a high precision system identification of multivariable systems.

TABLE II
Frequency Data for the Two Measurement Channels of Distillation Column

Measurement Channel 1 Frequency (Hz)	Estimates by S6Qa23 Magnitude	Phase (°)	Calculated Values Magnitude	Phase (°)
[2] 0.60	0.422	-94.73	0.423	-93.80
[4] 1.20	-5.42	-119.5	-5.42	-117.6
Measurement Channel 2 Frequency (Hz)	**Estimates by S6Qa22 Magnitude**	**Phase (°)**	**Calculated Values Magnitude**	**Phase (°)**
[1] 0.30	28.46	-59.22	28.45	-58.79
[3] 0.90	20.87	-101.7	20.88	-100.3

A simulation of a two input two output MIMO distillation column has proved that these pairs of MDM signals remove the cross coupling terms during the measurement of the frequency response with one experiment. They cause the distillation column to behave as two separate single input single output channels allowing a simple identification of the parameters of the two measurement channels. This has important implications for autotuning and adaptive control of multivariable processes. A deeper search of all compact MDM codes, which necessitates further work to extend the catalog of these important signals, is required.

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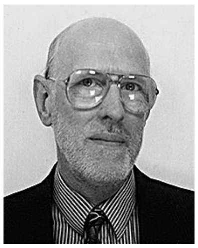
Mahmoud El-Fandi was born in Tripoli, Libya, in 1960. He received the B.Sc. degree in computer electronic engineering from the University of Al-fateh, Tripoli, in 1983. In 1993, he received the M.Phil. degree and in 1997 the Ph.D. degree in control engineering from the University of Strathclyde, Glasgow, U.K.

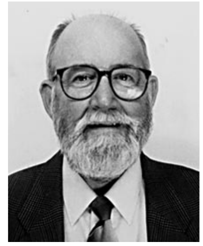
From 1984 to 1989, he was a Computer Technical Engineer at the Ministry of Energy, Tripoli. Currently he is doing postdoctoral work at the Industrial Control Center, University of Strathclyde. His research interests include digital signal processing applied to multifrequency signals, data measurement, and control engineering.



Ian A. Henderson was born in Muirhead, U.K., in 1937. He received the B.Sc. and Ph.D. degrees from the University of Strathclyde, Glasgow, U.K., in 1959 and 1982, respectively.

Since 1967, he has been a Lecturer in the Department of Electronic and Electrical Engineering, University of Strathclyde. As a member of the Industrial Control Center, he has published more than 100 papers. His research interests include data measurement, system identification, simulation modeling, and engineering education.

Dr. Henderson is a Chartered Engineer, a member of the Institution of Electrical Engineers (U.K.), and a member of the TC4 IMEKO Committee.



Joseph McGhee was born in Lanarkshire, U.K., in 1940. He graduated with first class honors in electrical engineering from the University of Strathclyde, Glasgow, U.K., in 1966.

Since 1968, he has been with the University of Strathclyde where he is a Reader in measurement and instrumentation in the Industrial Control Center, Department of Electronic and Electrical Engineering. Since 1994, he has been a Visiting Lecturer and an Examiner at the International Faculty of Engineering, Technical University of Lodz, Lodz, Poland. His research interests include measurement and control, temperature measurement, measurement education, and system identification. He is the coauthor of three books and more than 100 papers.

Mr. McGhee is a Chartered Engineer, a member of IMEKO TC-12 on thermal and temperature measurement, a member of the Institution of Electrical Engineers, and a member of the Institute of Measurement and Control.

Phillip McGlone, photograph and biography not available at the time of publication.