Single-Carrier Frequency Domain Equalization over MIMO Frequency Selective Fading Channels

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Abstract—In this article, first the structure and performance of single carrier frequency domain equalization (SC-FDE) system are analyzed, and the minimum mean square error (MMSE) equalization algorithm is discussed for frequency selective fading channel. Next, the application of spatial modulation (SM) technology is introduced in SC-FDE system, and simulations are made to compare the bit error rate (BER) performance of SC-FDE systems with Alamouti space time coding, V-BLAST and SM, respectively. Finally, Trellis Coded Modulation (TCM) is considered to use in SM-SC-FDE system to suppress the influence of channel correlation on the system and improve the system performance.

Keywords—Spatial modulation; SC-FDE; Channel correlation; MMSE; TCM

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technique can address the conflict between high data rates and the limited spectrum resources for wireless communications. However, in the frequency selective channel, the time domain equalization is very complicated due to the presence of inter symbol interference (ISI) and multiple access interference (MAI). In order to reduce the complexity of the receiver equalizer, orthogonal frequency division multiplexing (OFDM) and SC-FDE technique are proposed [1, 2]. In some papers it is shown that SC-FDE can obtain superior performance when compared with OFDM system [3, 4], such as lower sensitivity of carrier frequency offset and phase noise, and smaller peak-to-average power ratio (PAPR). The latter means that a smaller linear range is required for the power amplifier of an SC transmitter to support a given average power.

In MIMO system, space time block code (STBC) and Vertical-Bell Lab Layered Space-Time (V-BLAST) are used to gain additional diversity and multiplexing gain [4, 5], respectively. However, inter antenna synchronization (IAS) and inter channel interference (ICI) are inevitable for MIMO system. Especially, for frequency selective fading channel, ICI is relatively serious, and IAS requires a lot of hardware overhead at the receiver. In order to address the issues of ICI and IAS, R.MeSleh first proposed the concept of spatial modulation (SM) in [6]. SM technique can achieve high speed data transmission with low complexity, and reduce hardware cost of the system. Compared with V-BLAST, the number of receive antennas (RAs) is not need to be more than the number of transmit antennas (TAs). The combination of SM and SC-

FDE has also been studied in [7, 8], and it makes full use of the advantages of the two techniques. Then, the combination of STBC and SM scheme is proposed in SC-FDE system, to further improve the reliability of the system [9, 10].

In previous studies and analysis for SC-FDE with SM system (SM-SC-FDE), the influence of channel correlation is not considered. However, the rank of the channel matrix can be reduced when the channel is correlated, which leads to the decrease of the spatial degrees of freedom, and the overall BER performance would be reduced for the SM-SC-FDE system. This paper will study the performance of the SM-SC-FDE system under the condition of multipath correlation channel. For the sake of reducing the impact of multipath fading and channel correlation, [11] introduces Trellis Coded Modulation (TCM) and interleaving for MIMO system. In this paper, we consider the application of TCM coding and interleaving for SM-SC-FDE system to reduce the impact of the channel correlation.

The rest of this letter is organized as follows. Section II introduces the SC-FDE system model, and the MMSE-FDE is especially explained. In Section III, the basic principle of spatial modulation is introduced. SM-SC transmitter and receiver structure are described, and frequency domain equalization and detection algorithms are explored for SM-SC system. In Section IV, TCM is used in SM-SC-FDE system. The BER performance of the system is simulated under the condition of frequency selective correlation channel. Simulation results of BER for SM-SC-FDE and TCM-SM-SC-FDE are presented. Finally, In Section V, some conclusions are given.

II. MIMO-SC-FDE SYSTEM DESCRIPTIONS

Assuming that a MIMO-SC-FDE system has N_t TAs and N_r RAs, the channel is quasi-static within the duration of a data block, and each bit block of length N is appended with a length- $N_{\rm cp}$ cyclic prefix (CP). The length of channel memory is denoted by L. The length of the CP should satisfy $N_{\rm cp} \geq L$ to eliminate the interference between the data blocks caused by the multipath effect. For simplicity, we can assume the perfect synchronization and channel sate information (CSI). After the removal of CP, the signals received at j-th antenna can be formulated as:

$$\mathbf{y}_{(j)} = \sum_{i=1}^{N_t} \mathbf{h}_{(j,i)} \mathbf{x}_{(i)} + \mathbf{n}_{(j)}$$
 (1)

where

$$\boldsymbol{h}_{(j,i)} = \begin{bmatrix} h_{(j,i)}^{1} & 0 & \cdots & h_{(j,i)}^{L} & h_{(j,i)}^{L-1} & \cdots & h_{(j,i)}^{3} & h_{(j,i)}^{2} \\ h_{(j,i)}^{2} & h_{(j,i)}^{1} & \cdots & 0 & h_{(j,i)}^{L} & \cdots & h_{(j,i)}^{4} & h_{(j,i)}^{3} \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots \\ h_{(j,i)}^{L} & h_{(j,i)}^{L-1} & \cdots & \ddots & \cdots & 0 & 0 \\ 0 & h_{(j,i)}^{L} & \cdots & & \ddots & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & h_{(j,i)}^{L-2} & h_{(j,i)}^{L-3} & \cdots & h_{(j,i)}^{1} & 0 \\ 0 & 0 & \cdots & h_{(j,i)}^{L-1} & h_{(j,i)}^{L-2} & \cdots & h_{(j,i)}^{2} & h_{(j,i)}^{1} \end{bmatrix}_{N \times N}$$

By fast Fourier transform (FFT), the received signal vector is transformed to frequency domain (FD). The FD signal vector **Y** is given by

$$Y = HX + N \tag{3}$$

where

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_{(1,1)} & \boldsymbol{H}_{(1,2)} & \cdots & \boldsymbol{H}_{(1,N_t)} \\ \boldsymbol{H}_{(2,1)} & \boldsymbol{H}_{(2,2)} & \cdots & \boldsymbol{H}_{(2,N_t)} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{H}_{(N_t,1)} & \boldsymbol{H}_{(N_t,2)} & \cdots & \boldsymbol{H}_{(N_t,N_t)} \end{bmatrix}_{(NN_t \times NN_t)}$$
(4)

 $X = [X_{(1)}^T, X_{(2)}^T, \dots, X_{(N_t)}^T]^T$ denotes the symbols at all the TAs. The symbol vector matrix of the RAs is denoted as $Y = [Y_{(1)}^T, Y_{(2)}^T, \dots, Y_{(N_t)}^T]^T$. $N = [N_{(1)}^T, N_{(2)}^T, \dots, N_{(N_t)}^T]^T$ is FD noise vector matrix. $H_{(j, t)}$ is $N \times N$ diagonal matrix and each diagonal element corresponds to the FD value of $h_{(j, t)}$ [4]

$$\boldsymbol{H}_{(j, \ i)} = \sum_{l=0}^{L-1} h_{(j, \ i)}^{l} e^{\left(-j\frac{2\pi kl}{N}\right)}, \quad k = 0, 1, \dots N-1$$
 (5)

The linear MMSE detection is given for MIMO-SC-FDE system. The output symbol can be computed as

$$\hat{X} = CY = C(HX + N) \tag{6}$$

where

$$C = H^H (HH^H + \frac{1}{SNR}I)^{-1}$$
 (7)

III. SM-SC-FDE SYSTEM MODEL

Spatial modulation (SM) technology is distinguished from the traditional time frequency modulation technique by introducing the position of the TA as the third dimension. The position of the TA is also considered as a new mapping resource for SM technique. Since only a single TA is activated at any time instant in SM system, its transmitted signal is sparse in the spatial domain, so IAS and ICI are completely avoided. The advantages of SM contain its low-dimensional signal representation as well as low radio frequency (RF) hardware complexity, cost and size [12]. In this section, we consider a broadband SM-SC system employing N_r TAs as well as N_r RAs and communicating over a frequency selective fading channel. The system model is depicted in Fig.1.

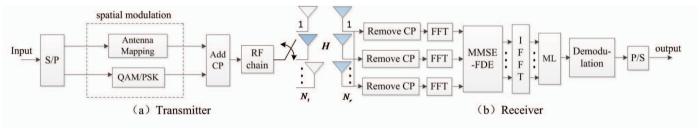


Fig.1 SM-SC-FDE system model

A. The SM-SC-FDE Transmitter

M denotes the cardinality of the signal constellation diagram. Spatial multiplexing, transmit diversity and SM are described in Fig.2 (N_t =2, M=2). Spatial multiplexing is clearly explained in Fig.2 (a), two QAM symbols are simultaneously transmitted from two TAs in one channel use. And Fig.2 (b) describes Alamouti coding system, two QAM symbols are firstly encoded, and then transmitted at the same time from a pair of TAs in two channel. In Fig.2 (c), only one out of two symbols is explicitly transmitted, while the other

symbol is implicitly transmitted by determining the index of the active TA in each channel use [12]. Compared with V-BLAST, the design of SM receiver does not need other algorithms to eliminate the interference between antennas. In addition, the SM is also applicable to the large scale MIMO channel with asymmetric number of uplink and downlink antenna. However, SM technique has a high requirement for the number of TAs. At least two transmit antennas to achieve SM, and the number of TAs must be set at the index of 2.

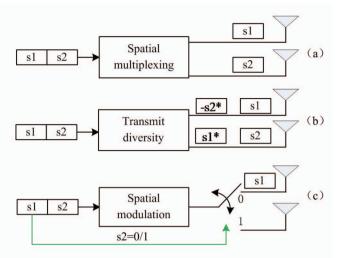


Fig.2 three important MIMO concepts: (a) spatial multiplexing; (b) transmit diversity; and (c) SM.

Table 1 the SM-mapping rules for $N_r=4$, BPSK

Input Bits	Antenna selection	Transmit Symbol		
000	1	-1		
001	1	+1		
010	2	-1		
011	2	+1		
100	3	-1		
101	3	+1		
110	4	-1		
111	4	+1		

Table 1 illustrates the mapping rules for $N_{c}=4$, BPSK. For instance, the input bit sequence $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$ is mapped by using the SM mapping rules. The binary 0 is mapped to the BPSK symbol -1. The front two bits $\begin{bmatrix} 1 \\ 1 \end{bmatrix}^T$ is mapped to antenna index, and the fourth transmit antenna is selected. And only the 4th antenna transmits symbol at this time slot. Thus, the corresponding column vector in x is $[0, 0, 0, -1]^T$ Afterwards, CP is added, and the *i-th* row x_i of x is sent by the *i-th* TA. $B = \log_2 N_i + \log_2 M$ is the total number of bits for each symbol. The spatial domain of the SM three-dimensional constellation can provide an additional multiplexing gain which is proportional to the logarithm of the TAs number. Compared with the traditional single antenna, the spectral efficiency is improved, the amount of increase is $\log_2 N_i$; by using SM it is able to obtain higher channel capacity than the MIMO space diversity scheme due to the additional multiplexing gain.

At the transmitter, a SM symbol is expressed as

$$\mathbf{s}(k) = [\underbrace{0, \cdots, 0}_{m(k)-1}, s_l(k), \underbrace{0, \cdots, 0}_{N_t - m(k)}]^{\mathrm{T}} \in \mathbb{C}^{N_t \times 1}.$$
 (8)

And there, $s_l(k)$ is a PSK/QAM symbol, k is the symbol index value, and the antenna index of the k time interval is recorded as m(k). The transfer data block of SM contained K symbols is denoted as

$$\mathbf{s} = [\mathbf{s}(1), \cdots, \mathbf{s}(K)] \in \mathbb{C}^{N_t \times K} \tag{9}$$

After removal of the length N_{cp} CP, the received symbol is denoted as

$$\mathbf{y} = [y_1(1), \dots, y_1(K), \dots, y_{N_r}(1), \dots, y_{N_r}(K)]^T \in \mathbb{C}^{NrK \times 1}$$

$$= h\overline{\mathbf{s}} + \mathbf{n}$$
(10)

where $\overline{s} \in \mathbb{C}^{N_rK \times 1}$ is stack operation of s, n is additive Gaussian white noise, and meets the conditions of independent and identically distributed (i.i.d.). $h \in \mathbb{C}^{N_rK \times N_rK}$ is denoted as

$$\boldsymbol{h} = \begin{bmatrix} \boldsymbol{h}_{11} & \cdots & \boldsymbol{h}_{1N_t} \\ \vdots & \ddots & \vdots \\ \boldsymbol{h}_{N_t 1} & \cdots & \boldsymbol{h}_{N_t N_t} \end{bmatrix}$$
(11)

where the sub matrix h_{nm} is a cyclic matrix containing the number of L paths.

B. The SM-SC-FDE Receiver

Under the condition of FFT, h_{nm} is decomposed as

$$\boldsymbol{h}_{nm} = \boldsymbol{Q}^{-1} \boldsymbol{\Lambda}_{nm} \boldsymbol{Q} \tag{12}$$

 $\Lambda_{nm} \in \mathbb{C}^{K \times K}$ is a *diagonal* matrix whose (k, k) entry is equal to the *k-th* FFT coefficient of h_{nm} . The elements at the *k-th* row and *l-th* column of Q are expressed as

$$[\mathbf{Q}]_{kl} = (1/\sqrt{K}) \exp[-2\pi j(k-1)(l-1)/K]$$
 (13)

By applying the FFT, the received y is transformed to the frequency domain. The frequency domain signal vectors Y_f is given by

$$\mathbf{Y}_f = \mathbf{\Lambda} \mathbf{S}_f + \mathbf{N}_f \tag{14}$$

and

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}_{11} & \cdots & \boldsymbol{\Lambda}_{1N_t} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Lambda}_{N_t 1} & \cdots & \boldsymbol{\Lambda}_{N_r N_t} \end{bmatrix} \in \mathbb{C}^{N_r K \times N_t K}$$

$$\boldsymbol{S}_f = (\boldsymbol{I}_{N_t} \otimes \boldsymbol{Q}) \overline{\boldsymbol{s}} \in \mathbb{C}^{N_t K \times 1}$$

$$N_f = (I_{N_-} \otimes Q)n \tag{15}$$

where \otimes means Kronecker product. The weight of the MMSE equalizer is calculated as

$$\boldsymbol{W} = (\boldsymbol{R}_{yy})^{-1} \boldsymbol{R}_{ys} = \left(\frac{\boldsymbol{\Lambda} \boldsymbol{\Lambda}^{H}}{N_{t}} + N_{0} \boldsymbol{I}_{N_{t}K}\right)^{-1} \frac{\boldsymbol{\Lambda}}{N_{t}}$$
(16)

where

$$\mathbf{R}_{yy} = E\left[\mathbf{Y}_{f}\mathbf{Y}_{f}^{\mathrm{H}}\right] = \frac{\mathbf{\Lambda}\mathbf{\Lambda}^{\mathrm{H}}}{N_{t}} + N_{0}\mathbf{I}_{N_{r}K}$$

$$\mathbf{R}_{ys} = E\left[\mathbf{Y}_{f}\mathbf{S}_{f}^{\mathrm{H}}\right] = \frac{\mathbf{\Lambda}}{N}$$
(17)

Transform frequency domain equalization signal into time domain

$$\hat{\overline{s}} = (I_{N_f} \otimes Q^1) W^T Y_f \tag{18}$$

After data reorganization, the estimated value of SM-SC is obtained as

$$\hat{\mathbf{S}} = [\hat{\mathbf{s}}(1), \cdots, \hat{\mathbf{s}}(K)]^{\mathrm{T}}$$
 (19)

At the receiver, maximum likelihood (ML) detection can keep relative low complexity, because only one antenna transmits data at any time instant. ML algorithm is used to obtain the TA index and the transmitted signal [13], which can be formulated as

$$\langle \hat{\boldsymbol{m}}(k), \hat{l}(k) \rangle = \arg\min_{m,l} || \hat{\boldsymbol{s}}(k) - \boldsymbol{s}_{m,l} ||^2$$
 (20)

where

$$\mathbf{s}_{m,l} = [\underbrace{0, \cdots, 0}_{m-1}, s_l, \underbrace{0, \cdots, 0}_{N_r - m}]^{\mathrm{T}} \in \mathbb{C}^{N_t \times 1}$$
 (21)

IV. SIMULATION RESULTS

The BER performance of V-BLAST, SM and Alamouti coding in SC-FDE system is simulated according to the simulation parameters given in Table 2. Multipath Rayleigh fading channels are assumed for simulations. The BER performance of different systems is evaluated by Monte Carlo simulations and the running times is set to 10⁴. The different modulation methods are applied to keep same transmission rate. Different detection algorithms are used for three schemes to ensure similar computational complexity. Fig.3 shows simulation results.

Table 2 Simulation Parameters

Transmission rate	MIMO scheme	Antenna number	Modulation	Detection algorithm
6bit/Hz/s	V-BLAST	2×4	8QAM	SIC-Sort
	Alamouti	2×4	64QAM	MMSE
	SM	2×4	32QAM	ML
	SM	4×4	16QAM	ML

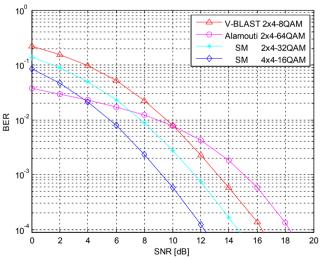


Fig.3 the BER performance of V-BLAST, Alamouti and SM

In Fig.3, the BER performance is given for Alamouti, SM and V-BLAST in multipath Rayleigh fading channel. In the case of the same transmission rate, the BER performance of SM is better than that of Alamouti at high SNR. As compared

with the V-BLAST system, SM always has a better BER performance. Therefore, SM transmission technology can effectively improve the BER performance of MIMO system. For SM scheme, the BER of 2×4 with 32QAM scheme is higher than that of 4×4 with 16QAM scheme (about 2 dB), because the higher modulation order, the worse the BER performance.

The channel correlation between transmit and receive antennas is caused by inadequate antenna spacing and the presence of local scatters. The correlated multipath Rayleigh fading channel matrix is written as $\mathbf{H}_{cor} = \mathbf{R}_t^{1/2} \mathbf{H} \mathbf{R}_r^{1/2}$, where $\mathbf{R}_t = \begin{bmatrix} r_{ij} \end{bmatrix}_{N_t \times N_t}$ and $\mathbf{R}_r = \begin{bmatrix} r_{ij} \end{bmatrix}_{N_r \times N_r}$ are the spatial correlation matrices at the transmitter and the receiver, respectively. The exponential correlation matrix model is applied in simulation, thus $r_{ij} = r_{ji}^* = r^{j-i}$ ($i \le j$), and r is the correlation coefficient of the neighboring TAs and RAs.

Fig.4 illustrates the influence of channel correlation on the SC-FDE system, r is set to 0 and 0.5 respectively with 8QPSK modulation. FFT length is 128, and CP length is 32. The same transmission rate (5 bit/Hz/s) is applied for simulations. From the Fig.4 it can be seen that the performance with MMSE-FDE is better than that with ZF equalization. However, when there is a correlation between the channels, whether the MMSE or ZF is applied, the BER performance will decrease with the increase of the correlation, if not specially treated. And the channel correlation has much severer impact on the system at high SNR.

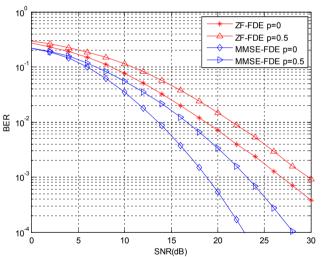


Fig.4 the BER performance of MMSE-FDE and ZF-FDE for 4×4 system

As we all know, TCM is a combination of coding and modulation techniques. It can increase the effective distance between the antennas by using TCM coding in MIMO-OFDM system. Therefore it is also applicable to the SM-SC-FDE system. The Viterbi decoder needs to be added after MMSE-FDE at the receiver. The system parameters are given in Table 3. The simulation is made for SM-SC-FDE. TCM with 16QAM modulation is used with the transmission efficiency of 5 bit/Hz/s. TCM coding efficiency is 1/2. Simulation results are shown in Fig.5.

As seen in Fig.5, when the correlation coefficient is reduced, the bit error rate of TCM coding or uncoded scheme is

enhanced. Compared with the uncoded system, the TCM coded system obtains SNR gain of 2dB at 10⁻³ BER, and TCM coding picks up still about 2dB gain at 10⁻⁴ BER. Therefore,

the simulation results show that the performance of the system can be improved to a certain extent by the addition of TCM coding.

Table 3 The system simulation parameters

TCM	$N_t \times I$	V_r	FF	T length	CP lengtl	h	Modulation		Simulation times
Yes	4×4	1		128	32		16QAM		10 ⁴
No	4×4	ļ		128	32		8QAM		104
Transmission efficiency tap number I		Relative time delay (ns) Aver		verage power (dB)		Correlation coefficient			
5bit	/Hz/s	3		[0 10	0 400]		[0 -6 -12]		0 / 0.5 / 0.75

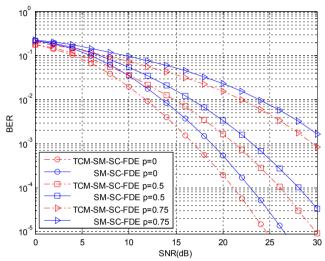


Fig.5 the BER performance of TCM-SM--SC-FDE and SM-SC-FDE

V. CONCLUTION

Aiming at the problem of ICI and IAS in traditional MIMO transmission technology (STBC and V-BLAST), the paper introduced SM technique in SC-FDE system. SM-SC-FDE system only requires one single-RF link at the transmitter, therefore IAS would not be required. Only one antenna transmits data at each time slot to avoid ICI. On the premise of the same transmission efficiency and approximate detection complexity, the performance of SC-FDE with SM is better than that with V-BLAST. Accordingly, SM has relatively broad application prospect in low and medium speed transmission system. In addition, the influence of channel correlation is relatively severe in frequency selective fading channel, and it can be effectively reduced by using TCM coding in SM-SC-FDE system. Our future work will mainly concentrate on the application of spatial modulation and SC-FDE techniques in large-scale MIMO systems.

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