An Improved QRD-M Algorithm in MIMO Communications

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Abstract—**QR decomposition based M algorithm (QRD-M) is a type of maximum likelihood signal detecting (MLSD) scheme to provide a tradeoff detecting method between the system performance and complexity in multi-input multi-output (MIMO) communications. But for the conventional QRD-M algorithm, only one parameter, the maximum number of the survived branches, is used to make tradeoff, which cannot provide more valuable tradeoff options with better performance to complexity ratio. In this paper, an improved QRD-M algorithm with two independent parameters is proposed. It can provide more and better tradeoff options, which is proved through complexity analysis and numerical simulations.**

Keywords: MIMO, Signal Detecting, MLSD, QRD-M, OFDM

I. INTRODUCTION

MIMO system can provide significant performance gain on the system capacity over the traditional single antenna system (SISO) [1]. Therefore it becomes a favorite solution to support higher data rate transmission in wireless communications. However, because of the presence of the multi-stream interference (MSI) in MIMO transmit antennas, the conventional linear equalizations, such as zero forcing and linear minimum mean square error equalizations, can not achieve good performance.

There are many proposals about how to improve the system performance in MIMO communications, such as serial interference cancellation, parallel interference cancellation in [2,3,4]. Among all the MIMO signal detecting methods, MLSD is the optimal solution in the sense of minimizing the error probability when the transmitted signals are uniformly distributed in their finite alphabets. But unfortunately its complexity will dramatically increase as the number of transmit antenna or the constellation size of the transmitted signal size increases. As a complexity reduced MLSD algorithm, sphere decoding algorithm has also been proposed in MIMO communications [6, 7], which tries to resolve the problem through the deep-firstly tree traversal method. The main disadvantage of sphere decoding is that its complexity cannot keep constant. When signal-to-noise ratio (SNR) is high it pays very low average complexity, but when SNR is low its average complexity will also increase to a higher level [8]. Furthermore, even in high SNR area its complexity is still not stable which may grows to as much as MLSD [9] in the worst case.

QRD-M algorithm is an implementation of the breadth-first tree traversal in MIMO signal detection which can provide very stable complexity [12, 13, 16]. In the conventional QRD-M algorithm, the parameter M is used as the limitation of the number of maximum survived branches in its breadth-first tree traversal. By setting M equal to different values, it can provide different tradeoff solutions between the system performance and complexity [10, 11]. The larger M is set, the better performance is obtained and the more complexity is required. Because the conventional QRD-M algorithm only makes tradeoff through different M values, it cannot provide more and better tradeoff options in practical.

In this paper, the QRD-M algorithm is decomposed into an equivalent model with two concatenated detecting modules and an improved QRD-M algorithm is proposed based on the model. By feeding two independent parameters to these two modules, the improved algorithm can provide more tradeoff options than that of the conventional one and through complexity analysis and numerical simulations it is proved that tradeoff options with better performance to complexity ratio can be found.

The rest of the paper is organized as follows. In section II, the V-BLAST structure MIMO-OFDM system model and the conventional QRD-M algorithm are briefly introduced. In section III, the equivalent model and the improved QRD-M algorithm are presented. Finally, the complexity analysis, numerical comparisons and conclusions are given in section IV and V, respectively.

II. V-BLAST STRUCTURE MIMO-OFDM SYSTEM & THE CONVENTIONAL QRD-M ALGORITHM

A. V-BLAST Structure MIMO-OFDM System

After introduced in [5], V-BLAST becomes a popular spatial multiplexing technique in MIMO communications. The system model of the well-known V-BLAST structure MIMO-OFDM system is shown in figure 1 [14], where the signal pre-coding and channel estimation modules are not considered.

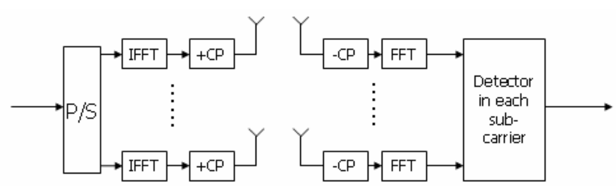

Fig.1 V-BLAST structure MIMO-OFDM block diagram

The above MIMO-OFDM system considered in figure 1 has N_t transmit and N_r receive antennas and $N_t \le N_r$. Assuming perfect timing and frequency synchronization, the received signal at each sub-carrier can be formulated in (1), where the sub-carrier index is ignored.

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$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{1}$$

Where $\mathbf{y}$ and $\mathbf{n}$ denote the N_r-size received signal vector and the additive white Gaussian noise (AWGN) vector with power σ^2, respectively. $\mathbf{x}$ denotes the N_t-size transmitted signal vector and $\mathbf{H}$ denotes $N_\mathrm{r} \times N_\mathrm{t}$-size MIMO channel matrix in currently sub-carrier defined in (2).

$$\mathbf{H} = \begin{bmatrix} h_{0,0} & h_{0,1} & \cdots & h_{0,N_\mathrm{t}-1} \\ h_{1,0} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ h_{N_\mathrm{r}-1,0} & \cdots & \cdots & h_{N_\mathrm{r}-1,N_\mathrm{t}-1} \end{bmatrix} \tag{2}$$

Where $h_{i,j}$ denotes the channel fading factor in current sub-carrier between the j^{th} transmit antenna and the i^{th} receive antenna.

B. MLSD and the Conventaionl QRD-M Algorithm

In the sense of minimizing the error probability, MLSD is the optimal. It performs signal detection based on (3).

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left\{ \|\mathbf{y} - \mathbf{Hx}\|^2 \right\} \tag{3}$$

Since the whole alphabet size of vector $\mathbf{x}$ depends on the constellation size of each signal and the number of transmit antennas, an exhaustive searching over the whole alphabet has to be done when performing MLSD, which is infeasible in practical if either the constellation size or the signal dimension size is very large.

For $\mathbf{y}$, $\mathbf{n}$, $\mathbf{x}$ and $\mathbf{H}$ are complex in practical, they can be transformed into real number.

$$\mathbf{r} = \begin{bmatrix} \mathbf{y_R} \\ \mathbf{y_I} \end{bmatrix}, \boldsymbol{\eta} = \begin{bmatrix} \mathbf{n_R} \\ \mathbf{n_I} \end{bmatrix}$$
$$\mathbf{s} = \begin{bmatrix} \mathbf{x_R} \\ \mathbf{x_I} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{H_R} & \mathbf{H_I} \\ -\mathbf{H_I} & \mathbf{H_R} \end{bmatrix} \tag{4}$$

Where $(\cdot)_\mathrm{R}$ and $(\cdot)_\mathrm{I}$ denote the real and image parts of $(\cdot)$, respectively. Using (4), (1) can be rewritten into

$$\mathbf{r} = \mathbf{Bs} + \boldsymbol{\eta} \tag{5}$$

It is noticed the following QR decomposition can be performed on channel matrix $\mathbf{B}$.

$$\mathbf{B} = \mathbf{QR} \tag{6}$$

Where $\mathbf{Q}$ is a $2N_\mathrm{r} \times 2N_\mathrm{r}$ size unitary matrix and $\mathbf{R}$ is $2N_\mathrm{r} \times 2N_\mathrm{t}$ size matrix defined in (7).

$$\mathbf{R} = \begin{bmatrix} \mathbf{T} \\ \mathbf{0}_{2N_\mathrm{r}-2N_\mathrm{t},2N_\mathrm{t}} \end{bmatrix} \tag{7}$$

Where $\mathbf{T}$ is a $2N_\mathrm{t} \times 2N_\mathrm{t}$ size up-triangle matrix. Let $(\cdot)^*$ denote the conjugation transposition of $(\cdot)$. After left-multiplied by $\mathbf{Q}^*$, (5) can be transformed into (8).

$$\begin{aligned} \mathbf{Q}^*\mathbf{r} &= \mathbf{Q}^*\mathbf{QRs} + \mathbf{Q}^*\mathbf{n} \\ &= \mathbf{Rs} + \mathbf{Q}^*\mathbf{n} \end{aligned} \tag{8}$$

And then ignoring the zero part at the bottom of $\mathbf{R}$, (8) can be rewritten into

$$\tilde{\mathbf{r}} = \mathbf{Ts} + \tilde{\mathbf{n}} \tag{9}$$

Where $\tilde{\mathbf{r}}$ and $\tilde{\mathbf{n}}$ denote the vectors including the first $2N_\mathrm{t}$ rows of $\mathbf{Q}^*\mathbf{r}$ and $\mathbf{Q}^*\mathbf{n}$, respectively. Therefore, the MLSD problem defined in (3) can be reformulated as

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \left\{ \|\tilde{\mathbf{r}} - \mathbf{Ts}\|^2 \right\} \tag{10}$$

Because $\mathbf{T}$ is an up-triangle matrix, (10) can be resolved by tree traversal methods in graph theory.

QRD-M is a breadth-first tree traversal algorithm [13, 16], which reduces system complexity by keeping only M candidates with the lest accumulated metrics at each tree searching stage. Define $N = 2N_\mathrm{t}$, w_i the metric of each extended branch at stage i can be calculated through (11).

$$w_i = \sum_{k=i-1}^{N-1} \left(\tilde{r}_k - \sum_{l=k}^{N-1} t_{k,l} s_l \right)^2, \quad i = N, N-1, \cdots, 1 \tag{11}$$

Where $\tilde{r}_k$, $t_{k,l}$ and s_l denote the elements from $\tilde{\mathbf{r}}$, $\mathbf{T}$ and $\mathbf{s}$ corresponding to their subscripts.

Starting from the last signal of $\mathbf{s}$, the flowchart of the conventional QRD-M algorithm is summarized in figure 2.

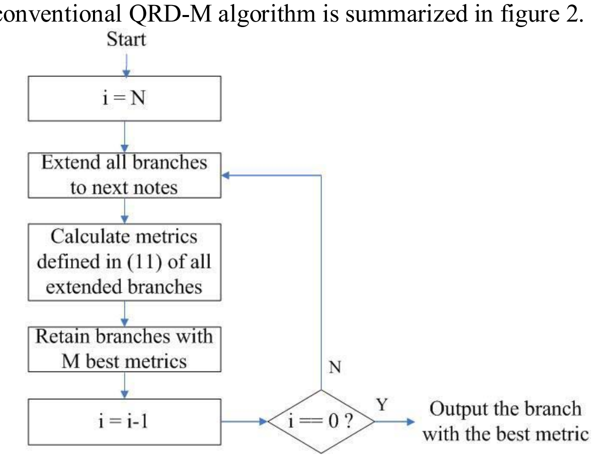

Fig.2 The conventional QRD-M algorithm

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For easy understanding, an example of the conventional QRD-M algorithm is illustrated in figure 3. We assume the transmitted signals are QPSK modulated and MIMO antenna configuration is 2×2, therefore the number of the stages in the signal tree is 4 after signal transformation from complex to real. In figure 3 the number of survived branches M is equal to 2, and the dash line, solid line and bold line denote the abandoned branch, the searched branch and the survived branch respectively.

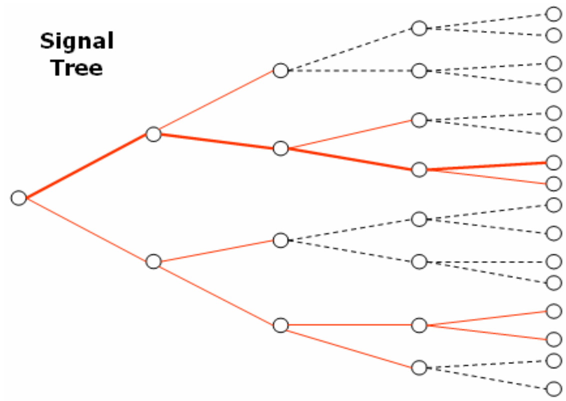

Fig.3 An example of the conventional QRD-M algorithm

III. The Improved QRD-M Algorithm

A. The equivalent model with two concatenated detection modules

It is noticed from the example in figure 3 that the conventional QRD-M algorithm consists of two parts. In first two layers, it does a conventional MLSD and outputs 2 branches. In last two layers, it selects 2 branches from 4 branches as survivors. Therefore, the conventional QRD-M algorithm can be modeled by two concatenated detection modules in figure 4.

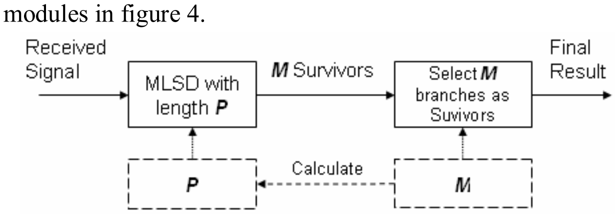

Fig.4 The equivalent model of the conventional QRD-M algorithm

In this model, parameter P decides how many signals should be included in MLSD and parameter M is the number of maximum survived branches.

For the conventional QRD-M algorithm in figure 4, P is determined by M. For instance, $M = 2$ also determines $P = 2$ in figure 3. But actually it may reduces the number of performance to complexity tradeoff options if M and P are dependent.

B. The improved QRD-M algorithml

The improved QRD-M algorithm can also be modeled by the concatenated detection modules in figure 5, but P and M will be pre-set independently.

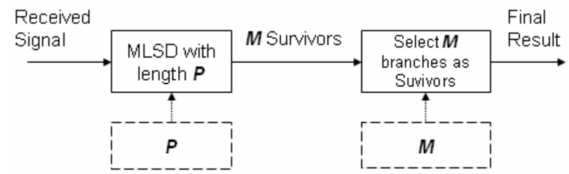

Fig.5 The equivalent model of the improved QRD-M algorithm

Based on the modified model, the flowchart of the improved QRD-M algorithm is summarized in figure 6.

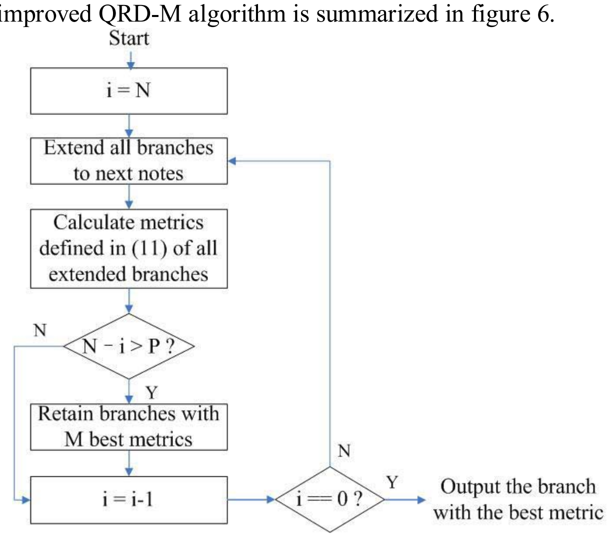

Fig.6 The conventional QRD-M algorithm

Assuming $P = 3$, $M = 1$ and the other parameters are the same as that in figure 3, an example of the improved QRD-M algorithm is also illustrated in figure 7 for easy understanding.

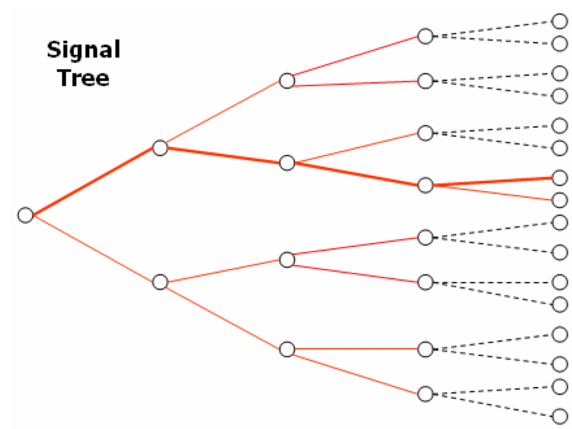

Fig.7 An example of the improved QRD-M algorithm

IV. Complexity Analysis And Numerical Simulations

A. *Complexity analysis*

Define D_i as the finite alphabet size of the i^{th} signal in $\mathbf{s}$. For instance, because all signals are QPSK modulated in the example in figure 7, the D_i of their real or image parts in $\mathbf{s}$ are equal to 2. We assume all possible calculation results of $t_{k,l}s_l$ in (11) are already calculated and buffered before tree searching. With the help of the definitions of M, P, D_i and N, the number of the multiplications in the QRD-M algorithm and the number of the buffers required are presented in the table I, in which

TABLE I. Complexity Analysis of the QRD-M Algorithm

Complexities (The number of multiplications required)	$C = \sum_{j=1}^{P} \prod_{i=1}^{j} D_i + \sum_{i=P+1}^{N} MD_i$
The number of buffers to save all possible $t_{k,l}s_l$	$B = \frac{1}{2}\sum_{i=1}^{N} iD_i$

Assuming the antenna configuration of MIMO system is 4×4 and the transmitted signals are QPSK/16QAM modulated, the above calculation results with different M and P are listed in table II and III, respectively. When the parameter pairs (M, P) are equal to those in the conventional QRD-M, they are marked with Conv. while the others are marked with Prop. in the tables and the following numerical simulations.

TABLE II. Calculation Results in QPSK Modulation

Conv.		**Prop.**		B
(M,P)	C	(M,P)	C	
(2, 2)	30	(2, 3)	34	36
(4, 3)	54	(4, 4)	62	
(8, 4)	94	(8, 5)	110	

TABLE III. Calculation Results in 16QAM Modulation

Conv.		**Prop.**		B
(M,P)	C	(M,P)	C	
(2, 1)	60	(2, 2)	68	72
(4, 2)	116	(4, 3)	164	
(8, 2)	212	(8, 3)	244	

B. *Numerical simulations*

The bit error rates (BER) of Conv. and Prop. In table II and III are compared by numerical simulations. The environment specification is listed in table IV.

TABLE IV. Simulation Enviroment Parameters

Systems	*MIMO-OFDM*
Sampling Rate	*3.84 M*
CP Length	*18 symbols*
FFT Size	*256 symbols*
Carrier Frequency	*2.6 G Hz*
Modulation	*QPSK & 16QAM*
Channel Profile	*ITU-VA*
Channel Estimation	*Perfect*

Figure 8 presents the BER comparisons between Conv. and Prop. in QPSK modulation with different M. It is noticed that in 10^{-2} BER level Prop. with M equal to 2 outperforms Conv. with the same M more than 2 dB, and it has less than 0.2 dB performance difference compared with Conv. with M equal to 4. According to their complexity comparisons in Table II, obviously this is a better tradeoff option than others.

Figure 9 presents the BER comparison results in 16QAM modulation. Although Conv. with M equal to 4 outperforms Prop. with M equal to 2 about 1 dB in 10^{-2} BER level, it has to pay much more complexity according to Table III. It also notice that Prop. with $M = 4$ pays much less complexity than that of Conv. with $M = 8$ but Prop. performs as well as Conv. in low Eb/N0 and outperforms Conv. in high Eb/N0. In conclusion, the tradeoff in the improved QRD-M algorithm is more flexible than that of the conventional one and it can provide better performance to complexity tradeoff options in practical.

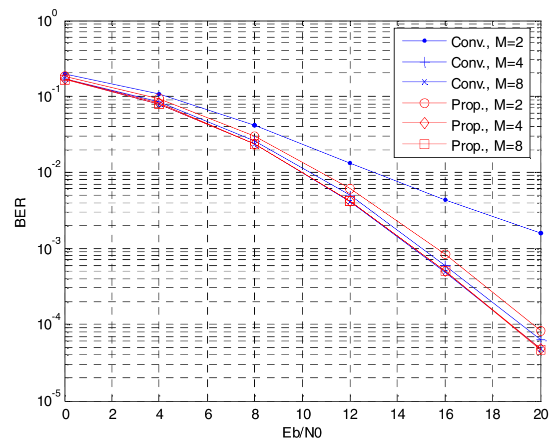

Fig.8 BER comparison in QPSK modulation

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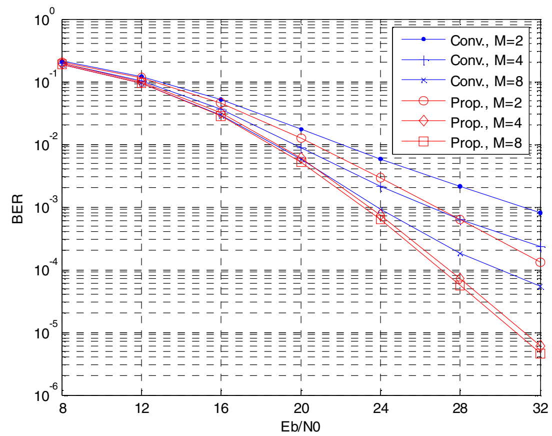

Fig.9 BER comparison in 16QAM modulation

V. CONCLUSIONS

Based on an equivalent model with two concatenated detection modules, the conventional QRD-M algorithm is analyzed and improved in this paper. Through complexity analysis and numerical simulations, it is proved that our proposal can provide more flexible tradeoff options with better performance to complexity ratio.

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