Lecture 1

Chapter 9

Sinusoids and Phasors

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Sinusoids

A sinusoid is a signal that has the form of the sine or cosine function.

A sinusoidal current is usually referred to as ***alternating current*** (***ac***). Circuits driven by sinusoidal current or voltage sources are called ***ac circuits.***

Sinusoids

Consider the sinusoidal voltage

$$v(t) = V_m \sin \omega t \tag{9.1}$$

where

V_m = the *amplitude* of the sinusoid
ω = the *angular frequency* in radians/s
ωt = the *argument* of the sinusoid

The sinusoid is shown in Fig. 9.1(a) as a function of its argument and in Fig. 9.1(b) as a function of time. It is evident that the sinusoid repeats itself every T seconds; thus, T is called the *period* of the sinu-soid. From the two plots in Fig. 9.1, we observe that $\omega T = 2\pi$,

$$\boxed{T = \frac{2\pi}{\omega}} \tag{9.2}$$

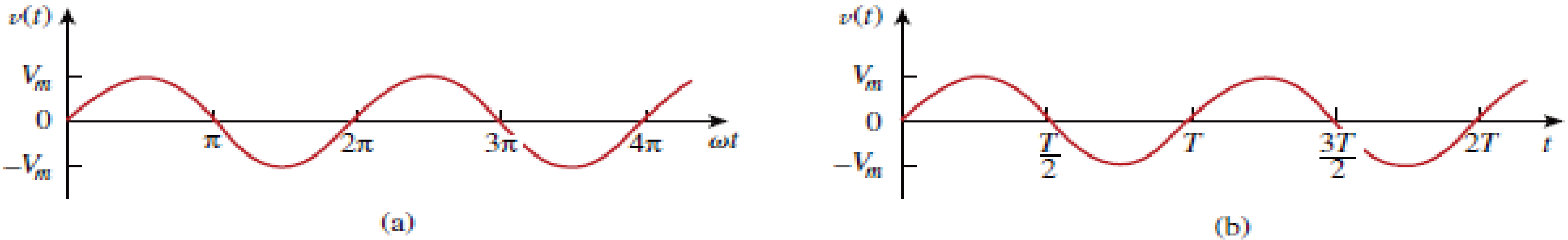

Figure 9.1
A sketch of $V_m \sin \omega t$: (a) as a function of ωt, (b) as a function of t.

Periodic Function

A **periodic function** is one that satisfies $f(t) = f(t + nT)$, for all t and for all integers n.

The fact that $v(t)$ repeats itself every T seconds is shown by replacing t by $t + T$ in Eq. (9.1). We get

$$\begin{aligned} v(t + T) &= V_m \sin\omega(t + T) = V_m \sin\omega\left(t + \frac{2\pi}{\omega}\right) \\ &= V_m \sin(\omega t + 2\pi) = V_m \sin\omega t = v(t) \end{aligned} \qquad \textbf{(9.3)}$$

Hence,

$$\boxed{v(t + T) = v(t)} \qquad \textbf{(9.4)}$$

that is, v has the same value at $t + T$ as it does at t and $v(t)$ is said to be *periodic*. In general,

Concept of Lead & Leg

Let us now consider a more general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi) \qquad \textbf{(9.7)}$$

where $(\omega t + \phi)$ is the argument and ϕ is the *phase*. Both argument and phase can be in radians or degrees.

Let us examine the two sinusoids

$$v_1(t) = V_m \sin \omega t \qquad \text{and} \qquad v_2(t) = V_m \sin(\omega t + \phi) \qquad \textbf{(9.8)}$$

shown in Fig. 9.2. The starting point of v_2 in Fig. 9.2 occurs first in time. Therefore, we say that v_2 *leads* v_1 by ϕ or that v_1 *lags* v_2 by ϕ. If $\phi \neq 0$, we also say that v_1 and v_2 are *out of phase*. If $\phi = 0$, then v_1 and v_2 are said to be *in phase*; they reach their minima and maxima at exactly the same time. We can compare v_1 and v_2 in this manner because they operate at the same frequency; they do not need to have the same amplitude.

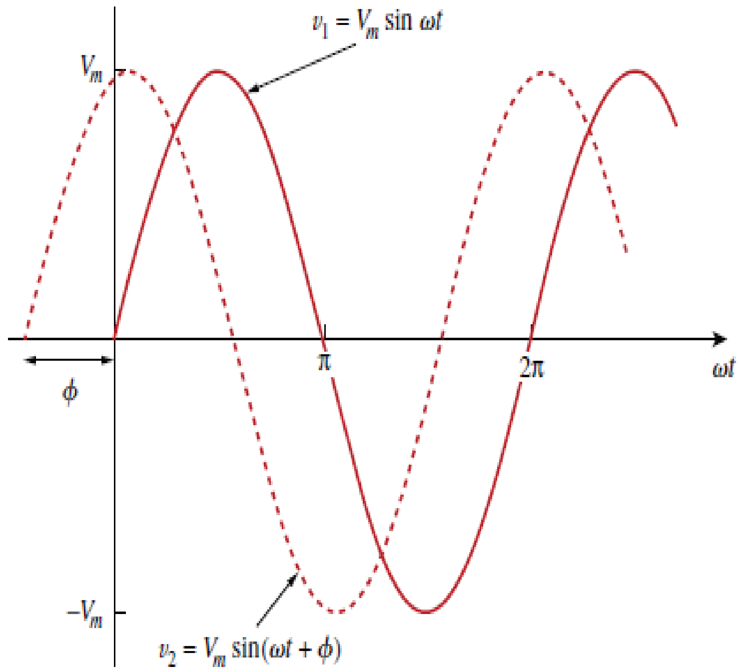

Figure 9.2
Two sinusoids with different phases.

Exercise

Find the amplitude, phase, period, and frequency of the sinusoid

Example 9.1

$$v(t) = 12\cos(50t + 10°)$$

Solution:

The amplitude is $V_m = 12$ V.
The phase is $\phi = 10°$.
The angular frequency is $\omega = 50$ rad/s.
The period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50} = 0.1257$ s.
The frequency is $f = \dfrac{1}{T} = 7.958$ Hz.

H.W
Practice problem
9.1

Exercise

Example 9.2

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

Solution:

Let us calculate the phase in three ways. The first two methods use trigonometric identities, while the third method uses the graphical approach.

METHOD 1 In order to compare v_1 and v_2, we must express them in the same form. If we express them in cosine form with positive amplitudes,

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50^\circ - 180^\circ)$$

$$v_1 = 10\cos(\omega t - 130^\circ) \quad \text{or} \quad v_1 = 10\cos(\omega t + 230^\circ) \qquad \textbf{(9.2.1)}$$

and

$$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$$

$$v_2 = 12\cos(\omega t - 100^\circ) \qquad \textbf{(9.2.2)}$$

It can be deduced from Eqs. (9.2.1) and (9.2.2) that the phase difference between v_1 and v_2 is 30°. We can write v_2 as

$$v_2 = 12\cos(\omega t - 130^\circ + 30^\circ) \quad \text{or} \quad v_2 = 12\cos(\omega t + 260^\circ) \qquad \textbf{(9.2.3)}$$

Comparing Eqs. (9.2.1) and (9.2.3) shows clearly that v_2 leads v_1 by 30°.

Formulas

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$

$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

H.W
Practice problem
9.2