Chapter 10

Sinusoidal Steady-State Analysis

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Example 10.1

Find i_x in the circuit of Fig. 10.1 using nodal analysis.

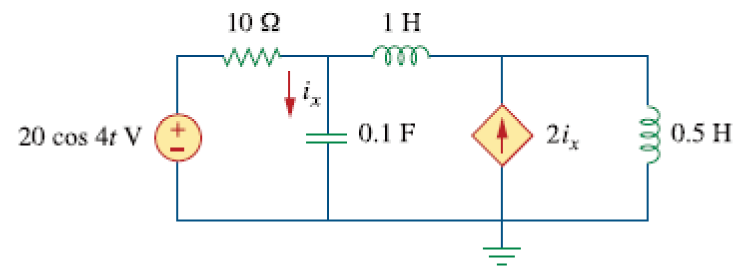


Figure 10.1
For Example 10.1.

Solution:

We first convert the circuit to the frequency domain:

$$20\cos 4t \quad \Rightarrow \quad 20\angle 0^\circ, \qquad \omega = 4 \text{ rad/s}$$
$$1\text{ H} \quad \Rightarrow \quad j\omega L = j4$$
$$0.5\text{ H} \quad \Rightarrow \quad j\omega L = j2$$
$$0.1\text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2.5$$

Thus, the frequency domain equivalent circuit is as shown in Fig. 10.2.

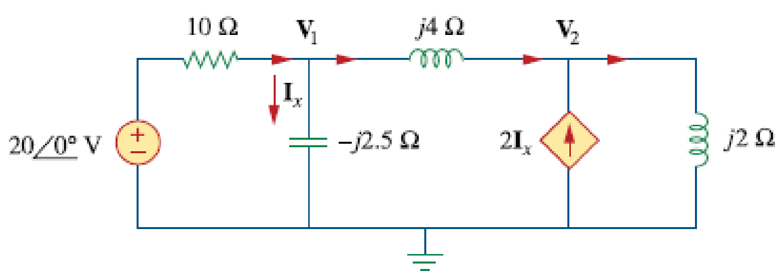


Figure 10.2
Frequency domain equivalent of the circuit in Fig. 10.1.

Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \qquad \textbf{(10.1.1)}$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \qquad \textbf{(10.1.2)}$$

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\angle 18.43^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\angle 198.3^\circ \text{ V}$$

Nodal Analysis

The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

H.W
Practice problem
10.1

Super Nodal Analysis

Solution:
Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

or

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \qquad \textbf{(10.2.1)}$$

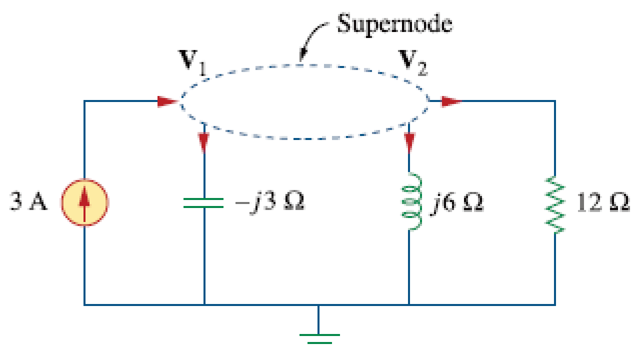


Figure 10.5
A supernode in the circuit of Fig. 10.4.

But a voltage source is connected between nodes 1 and 2, so that

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45^\circ \qquad \textbf{(10.2.2)}$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\angle 135^\circ = (1 + j2)\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{V}_2 = 31.41\angle -87.18^\circ \text{ V}$$

From Eq. (10.2.2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\angle 45^\circ = 25.78\angle -70.48^\circ \text{ V}$$

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.4.

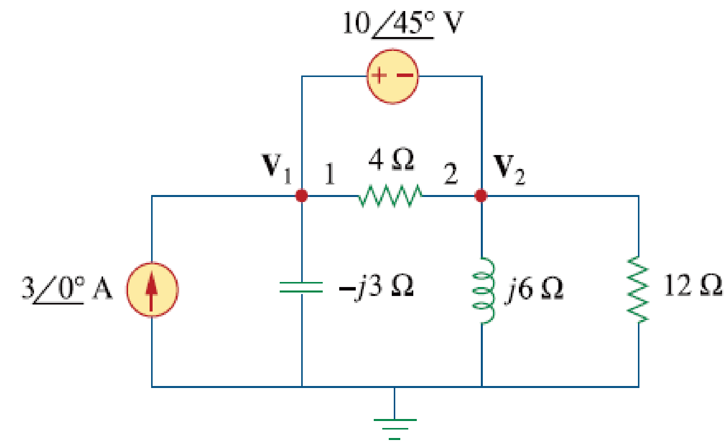


Figure 10.4
For Example 10.2.

Example 10.2

H.W
Practice problem
10.2

Mesh Analysis

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.

Example 10.3

Solution:

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad \textbf{(10.3.1)}$$

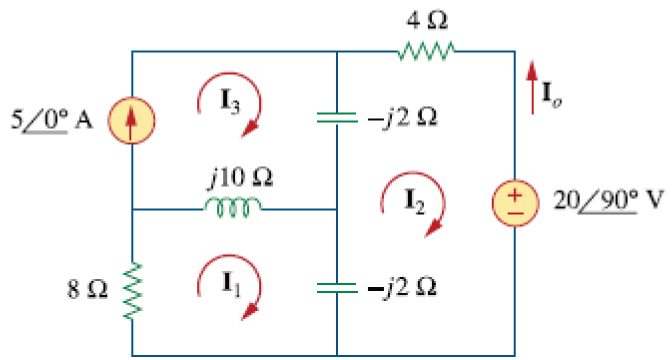


Figure 10.7
For Example 10.3.

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90^\circ} = 0 \qquad \textbf{(10.3.2)}$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad \textbf{(10.3.3)}$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad \textbf{(10.3.4)}$$

Equations (10.3.3) and (10.3.4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\underline{/-35.22^\circ}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\underline{/-35.22^\circ}}{68} = 6.12\underline{/-35.22^\circ}\text{ A}$$

The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78^\circ}\text{ A}$$

H.W
Practice problem
10.3

Super Mesh Analysis

Solution:
As shown in Fig. 10.10, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

or

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10 \qquad \textbf{(10.4.1)}$$

For mesh 2,

$$\mathbf{I}_2 = -3 \qquad \textbf{(10.4.2)}$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0 \qquad \textbf{(10.4.3)}$$

Due to the current source between meshes 3 and 4, at node A,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4 \qquad \textbf{(10.4.4)}$$

Example 10.4

Solve for $\mathbf{V}_o$ in the circuit of Fig. 10.9 using mesh analysis.

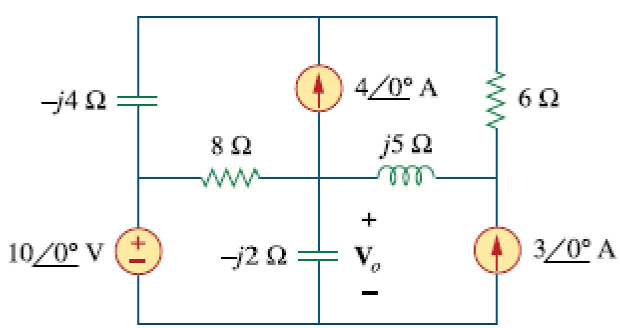


Figure 10.9
For Example 10.4.

METHOD 1 Instead of solving the above four equations, we reduce them to two by elimination.

Combining Eqs. (10.4.1) and (10.4.2),

$$(8 - j2)\mathbf{I}_1 - 8\mathbf{I}_3 = 10 + j6 \qquad \textbf{(10.4.5)}$$

Combining Eqs. (10.4.2) to (10.4.4),

$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35 \qquad \textbf{(10.4.6)}$$

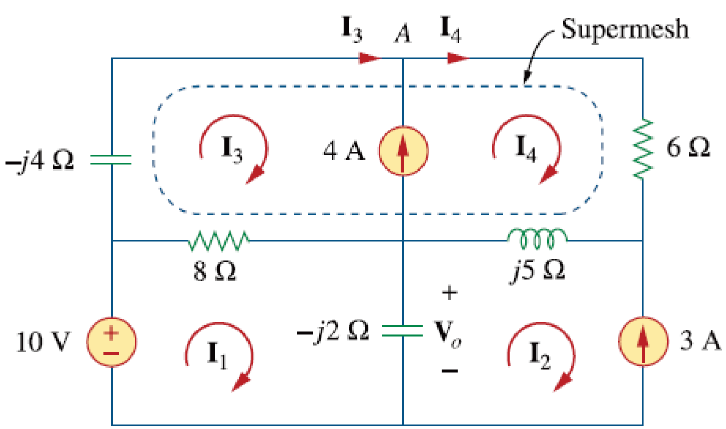


Figure 10.10
Analysis of the circuit in Fig. 10.9.

Super Mesh Analysis

From Eqs. (10.4.5) and (10.4.6), we obtain the matrix equation

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

We obtain the following determinants

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$
$$= -58 - j186$$

Current $\mathbf{I}_1$ is obtained as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\underline{/274.5^\circ} \text{ A}$$

The required voltage $\mathbf{V}_0$ is

$$\mathbf{V}_o = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618\underline{/274.5^\circ} + 3)$$
$$= -7.2134 - j6.568 = 9.756\underline{/222.32^\circ} \text{ V}$$

H.W
Practice problem
10.4

Superposition Theorem

Use the superposition theorem to find $\mathbf{I}_o$ in the circuit in Fig. 10.7.

Example 10.5

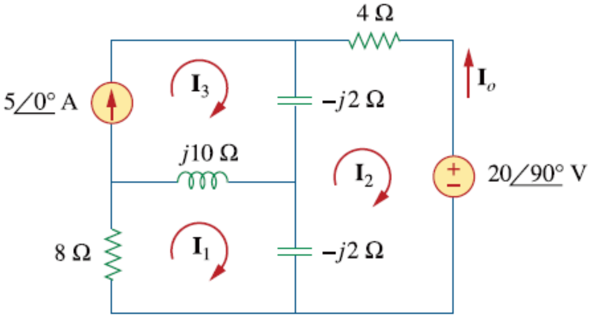


Figure 10.7
For Example 10.3.

Solution:
Let

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o \tag{10.5.1}$$

where $\mathbf{I}'_o$ and $\mathbf{I}''_o$ are due to the voltage and current sources, respectively. To find $\mathbf{I}'_o$, consider the circuit in Fig. 10.12(a). If we let $\mathbf{Z}$ be the parallel combination of $-j2$ and $8 + j10$, then

$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

and current $\mathbf{I}'_o$ is

$$\mathbf{I}'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

or

$$\mathbf{I}'_o = -2.353 + j2.353 \tag{10.5.2}$$

To get $\mathbf{I}''_o$, consider the circuit in Fig. 10.12(b). For mesh 1,

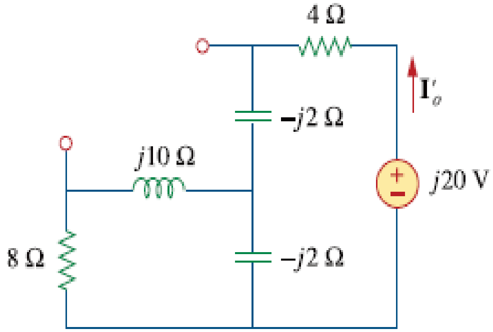
$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0 \tag{10.5.3}$$

For mesh 2,

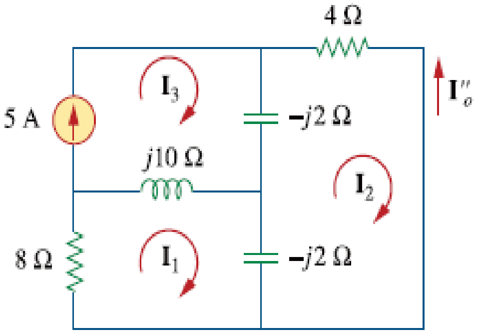
$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0 \tag{10.5.4}$$

For mesh 3,

$$\mathbf{I}_3 = 5 \tag{10.5.5}$$



(a)



(b)

Figure 10.12
Solution of Example 10.5.

Superposition Theorem

From Eqs. (10.5.4) and (10.5.5),

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5 \qquad \textbf{(10.5.6)}$$

Substituting Eqs. (10.5.5) and (10.5.6) into Eq. (10.5.3), we get

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $\mathbf{I}_o''$ is obtained as

$$\mathbf{I}_o'' = -\mathbf{I}_2 = -2.647 + j1.176 \qquad \textbf{(10.5.7)}$$

From Eqs. (10.5.2) and (10.5.7), we write

$$\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = -5 + j3.529 = 6.12\angle 144.78^\circ \text{ A}$$

H.W
Practice problem
10.5