Lecture 5

Chapter 11

AC Power Analysis

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Apparent Power

The average power in Eq. (11.8) can be written in terms of the rms values.

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i) \\ = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) \qquad \textbf{(11.30)}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) = S\cos(\theta_v - \theta_i) \qquad \textbf{(11.34)}$$

We have added a new term to the equation:

$$\boxed{S = V_{\text{rms}} I_{\text{rms}}} \qquad \textbf{(11.35)}$$

The average power is a product of two terms. The product $V_{\text{rms}} I_{\text{rms}}$ is known as the *apparent power* S. The factor $\cos(\theta_v - \theta_i)$ is called the *power factor* (pf).

The apparent power (in VA) is the product of the rms values of voltage and current.

Power Factor

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

$$\boxed{\mathrm{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \underline{/\theta_v}}{I_m \underline{/\theta_i}} = \frac{V_m}{I_m} \underline{/\theta_v - \theta_i}$$

- The power factor is dimensionless
- The angle $\theta_v - \theta_i$ is called the **power factor angle**

Power Factor

power. The value of pf ranges between zero and unity. For a purely resistive load, the voltage and current are in phase, so that $\theta_v - \theta_i = 0$ and pf $= 1$. This implies that the apparent power is equal to the average power. For a purely reactive load, $\theta_v - \theta_i = \pm 90^\circ$ and pf $= 0$. In this case the average power is zero. In between these two extreme cases, pf is said to be *leading* or *lagging*. Leading power factor means that current leads voltage, which implies a capacitive load. Lagging power factor means that current lags voltage, implying an inductive load. Power factor affects the electric bills consumers pay the electric utility companies, as we will see in Section 11.9.2.

Exercise

Example 11.10

Determine the power factor of the entire circuit of Fig. 11.18 as seen by the source. Calculate the average power delivered by the source.

Solution:
The total impedance is

$$\mathbf{Z} = 6 + 4 \parallel (-j2) = 6 + \frac{-j2 \times 4}{4 - j2} = 6.8 - j1.6 = 7\underline{/-13.24^\circ}\ \Omega$$

The power factor is

$$\text{pf} = \cos(-13.24) = 0.9734 \qquad \text{(leading)}$$

since the impedance is capacitive. The rms value of the current is

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{Z}} = \frac{30\underline{/0^\circ}}{7\underline{/-13.24^\circ}} = 4.286\underline{/13.24^\circ}\text{ A}$$

The average power supplied by the source is

$$P = V_{\text{rms}} I_{\text{rms}}\,\text{pf} = (30)(4.286)0.9734 = 125\text{ W}$$

or

$$P = I_{\text{rms}}^2 R = (4.286)^2(6.8) = 125\text{ W}$$

where R is the resistive part of $\mathbf{Z}$.

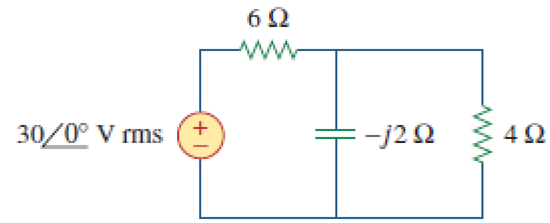

Figure 11.18
For Example 11.10.

Complex Power

Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}}$$

where

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}}\underline{/\theta_v} \tag{11.42}$$

and

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}}\underline{/\theta_i} \tag{11.43}$$

Thus we may write Eq. (11.41) as

$$\begin{aligned} \mathbf{S} &= V_{\text{rms}}I_{\text{rms}}\underline{/\theta_v - \theta_i} \\ &= V_{\text{rms}}I_{\text{rms}}\cos(\theta_v - \theta_i) + jV_{\text{rms}}I_{\text{rms}}\sin(\theta_v - \theta_i) \end{aligned} \tag{11.44}$$

Complex Power

The complex power may be expressed in terms of the load impedance **Z**. From Eq. (11.37), the load impedance **Z** may be written as

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \underline{/\theta_v - \theta_i} \qquad \textbf{(11.45)}$$

Thus, $\mathbf{V}_{\text{rms}} = \mathbf{Z}\mathbf{I}_{\text{rms}}$. Substituting this into Eq. (11.41) gives

$$\boxed{\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*} = \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^*} \qquad \textbf{(11.46)}$$

Since $\mathbf{Z} = R + jX$, Eq. (11.46) becomes

$$\mathbf{S} = I_{\text{rms}}^2 (R + jX) = P + jQ \qquad \textbf{(11.47)}$$

where P and Q are the real and imaginary parts of the complex power; that is,

$$P = \text{Re}(\mathbf{S}) = I_{\text{rms}}^2 R \qquad \textbf{(11.48)}$$

$$Q = \text{Im}(\mathbf{S}) = I_{\text{rms}}^2 X \qquad \textbf{(11.49)}$$

P is the average or real power and it depends on the load's resistance R. Q depends on the load's reactance X and is called the *reactive* (or quadrature) power.

Comparing Eq. (11.44) with Eq. (11.47), we notice that

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i) \qquad \textbf{(11.50)}$$

Power Triangle

It is a standard practice to represent **S**, *P*, and *Q* in the form of a triangle, known as the **power triangle**

Reminder

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

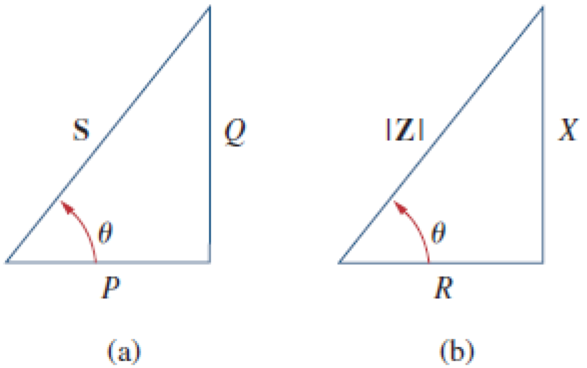

Figure 11.21
(a) Power triangle, (b) impedance triangle.

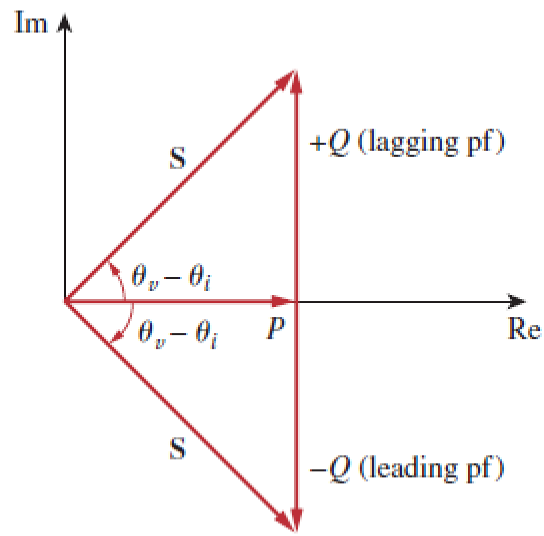

Figure 11.22
Power triangle.

Summary

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$

$$= |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| \underline{/\theta_v - \theta_i}$$

$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$

$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$