Lecture 8

Chapter 13
Magnetically Coupled Circuits

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Mutual Inductance

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as *mutual inductance*.

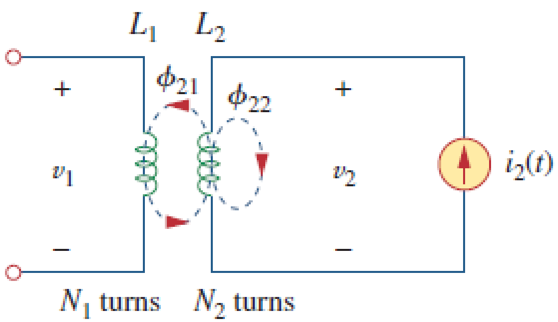


Figure 13.3
Mutual inductance M_{12} of coil 1 with respect to coil 2.

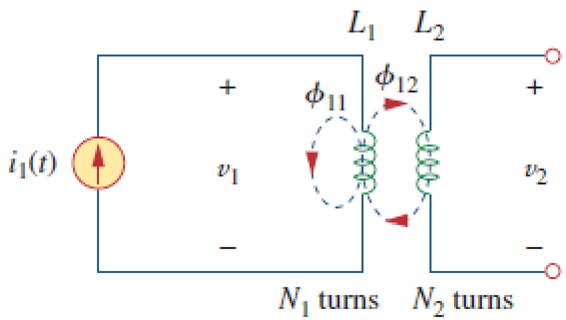


Figure 13.2
Mutual inductance M_{21} of coil 2 with respect to coil 1.

open-circuit *mutual voltage* $\boxed{v_1 = M_{12}\frac{di_2}{dt}}$

$$\boxed{v_2 = M_{21}\frac{di_1}{dt}}$$

$$M_{12} = M_{21} = M \qquad (13.17)$$

and we refer to M as the mutual inductance between the two coils. Like self-inductance L, mutual inductance M is measured in henrys (H).

Dot Convention

The dots are used along with the dot convention to determine the polarity of the mutual voltage. The dot convention is stated as follows:

If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

Alternatively,

If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.

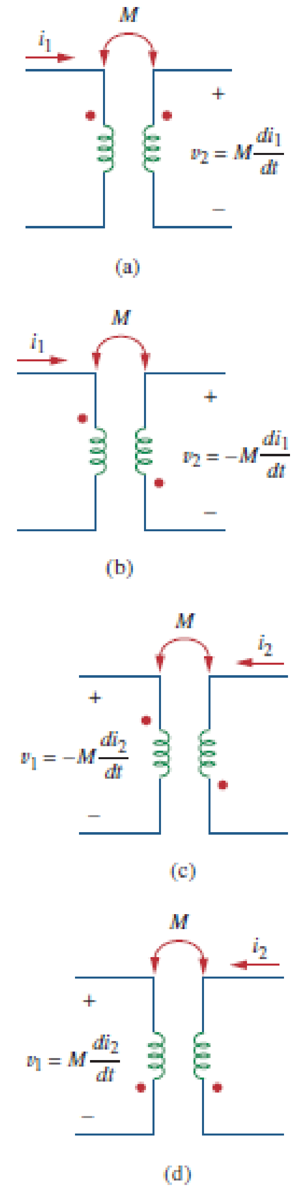


Figure 13.5
Examples illustrating how to apply the dot convention.

Analysis of mutually coupled Circuits

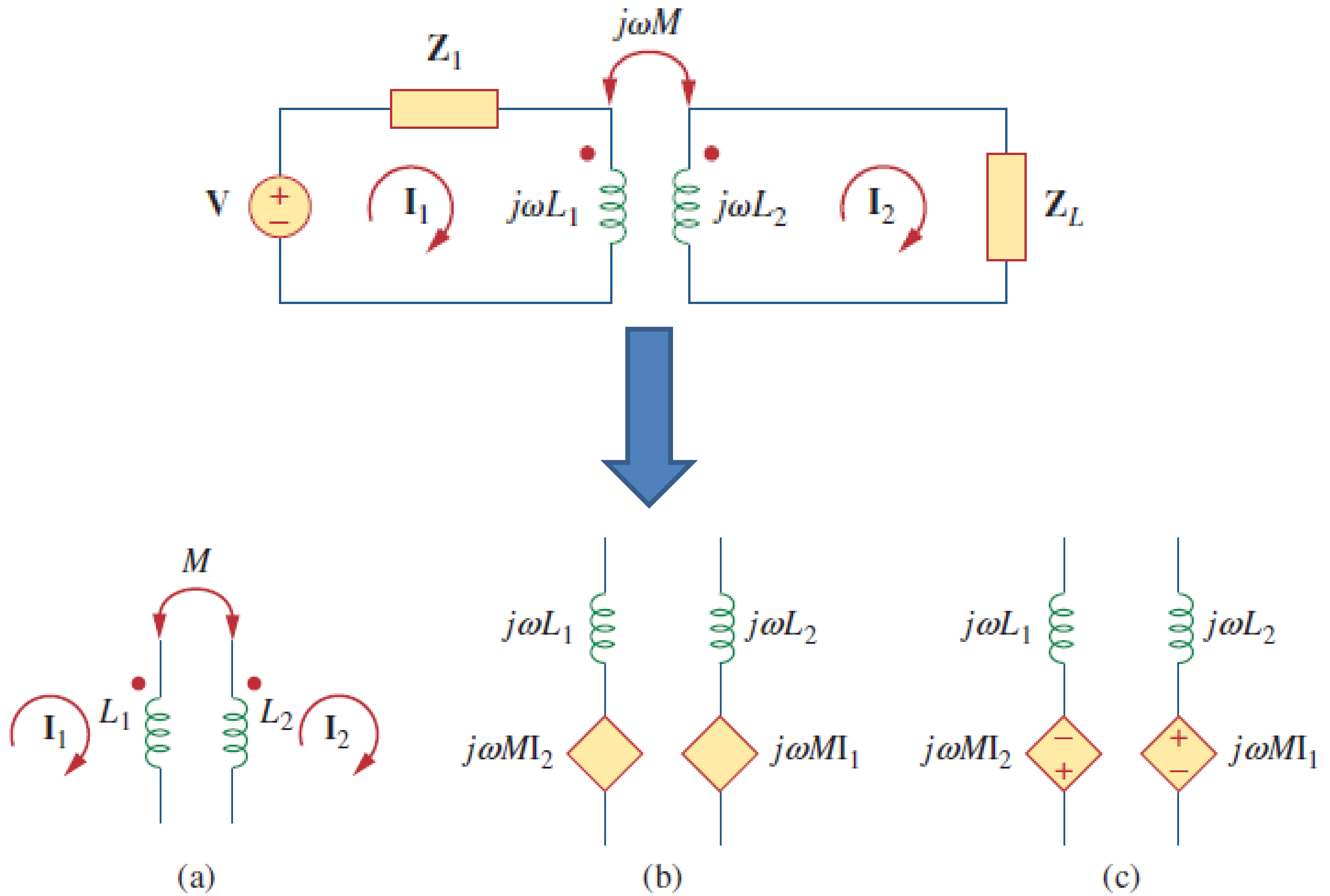
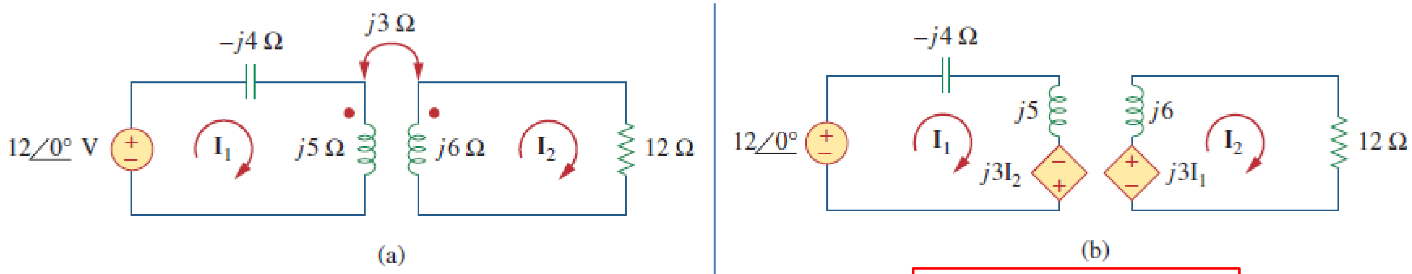


Figure 13.8
Model that makes analysis of mutually coupled easier to solve.

MATH

Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.9.

Example 13.1



Simplified circuit

Solution:

For loop 1, KVL gives

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

or

$$j\mathbf{I}_1 - j3\mathbf{I}_2 = 12 \qquad \textbf{(13.1.1)}$$

For loop 2, KVL gives

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = \frac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2 \qquad \textbf{(13.1.2)}$$

Substituting this in Eq. (13.1.1), we get

$$(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12$$

or

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\underline{/14.04^\circ} \text{ A} \qquad \textbf{(13.1.3)}$$

From Eqs. (13.1.2) and (13.1.3),

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\underline{/-63.43^\circ})(2.91\underline{/14.04^\circ})$$
$$= 13.01\underline{/-49.39^\circ} \text{ A}$$

H.W
Practice problem
13.1

Linear transformer

A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.

A linear transformer may also be regarded as one whose flux is proportional to the currents in its windings.

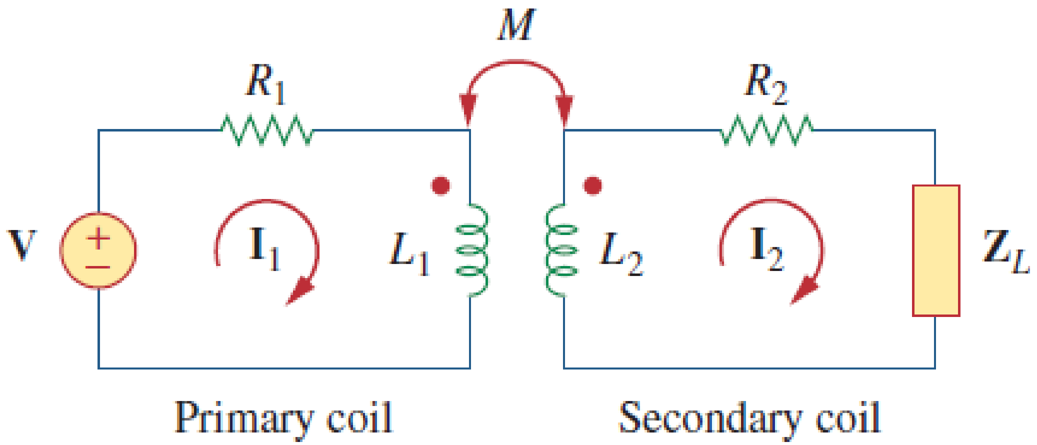


Figure 13.19
A linear transformer.

Equivalent circuit of a Linear Transformer

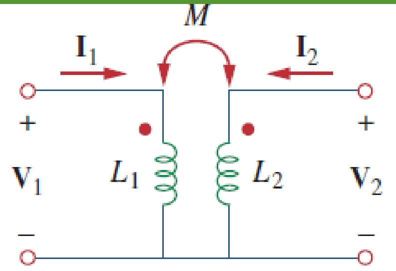


Figure 13.21
Determining the equivalent circuit of a linear transformer.

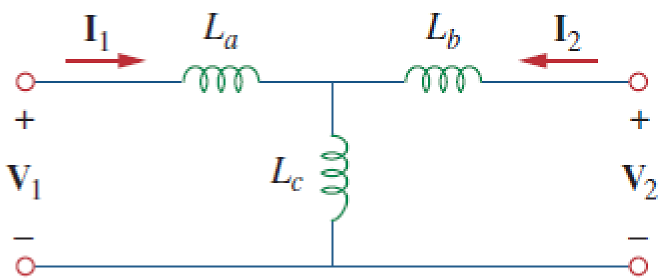
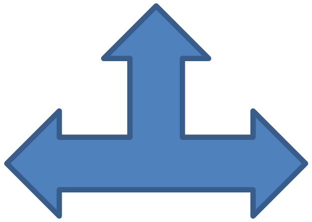


Figure 13.22
An equivalent T circuit.



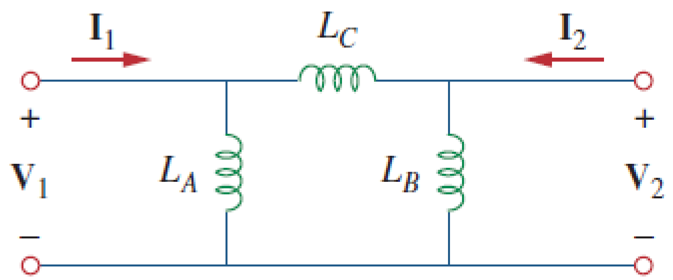


Figure 13.23
An equivalent Π circuit.

$$L_a = L_1 - M, \qquad L_b = L_2 - M, \qquad L_c = M$$

$$L_A = \frac{L_1L_2 - M^2}{L_2 - M}, \qquad L_B = \frac{L_1L_2 - M^2}{L_1 - M}$$

$$L_C = \frac{L_1L_2 - M^2}{M}$$

Example 13.5

Determine the T-equivalent circuit of the linear transformer in Fig. 13.26(a).

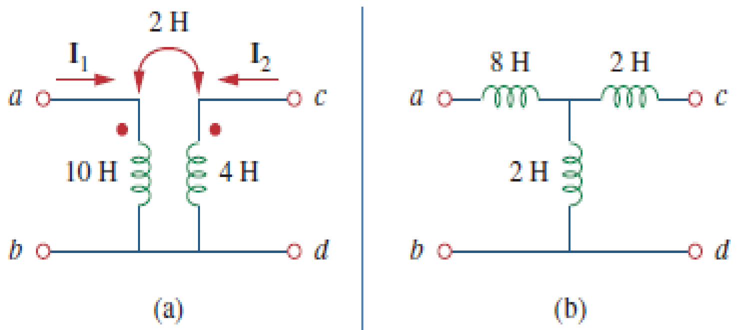


Figure 13.26
For Example 13.5: (a) a linear transformer, (b) its T-equivalent circuit.

H.W
Practice problem
13.5

H.W
Example
13.6

Solution:
Given that $L_1 = 10$, $L_2 = 4$, and $M = 2$, the T-equivalent network has the following parameters:

$$L_a = L_1 - M = 10 - 2 = 8\text{ H}$$

$$L_b = L_2 - M = 4 - 2 = 2\text{ H}, \qquad L_c = M = 2\text{ H}$$