Lecture 2

Chapter 9

Sinusoids and Phasors

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Phasor

A **phasor** is a complex number that represents the amplitude and phase of a sinusoid.

Phasors provide a simple means of analyzing linear circuits excited by sinusoidal sources

A complex number z can be written in rectangular form as

$$z = x + jy \qquad \textbf{(9.14a)}$$

$z = x + jy$	Rectangular form	
$z = r\angle\phi$	Polar form	(9.15)
$z = re^{j\phi}$	Exponential form	

Phasor Transformation

By suppressing the time factor, we transform the **sinusoid** from the **time domain** to the **phasor domain**. This transformation is summarized as follows:

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \underline{/\phi} \tag{9.25}$$

(Time-domain representation) (Phasor-domain representation)

TABLE 9.1

Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \underline{/\phi}$
$V_m \sin(\omega t + \phi)$	$V_m \underline{/\phi - 90^\circ}$
$I_m \cos(\omega t + \theta)$	$I_m \underline{/\theta}$
$I_m \sin(\omega t + \theta)$	$I_m \underline{/\theta - 90^\circ}$

Exercise

Example 9.6

Given $i_1(t) = 4\cos(\omega t + 30^\circ)$ A and $i_2(t) = 5\sin(\omega t - 20^\circ)$ A, find their sum.

Solution:

Here is an important use of phasors—for summing sinusoids of the same frequency. Current $i_1(t)$ is in the standard form. Its phasor is

$$\mathbf{I}_1 = 4\underline{/30^\circ}$$

We need to express $i_2(t)$ in cosine form. The rule for converting sine to cosine is to subtract 90°. Hence,

$$i_2 = 5\cos(\omega t - 20^\circ - 90^\circ) = 5\cos(\omega t - 110^\circ)$$

and its phasor is

$$\mathbf{I}_2 = 5\underline{/-110^\circ}$$

If we let $i = i_1 + i_2$, then

$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 &= 4\underline{/30^\circ} + 5\underline{/-110^\circ}\\ &= 3.464 + j2 - 1.71 - j4.698 = 1.754 - j2.698\\ &= 3.218\underline{/-56.97^\circ}\text{ A}\end{aligned}$$

Transforming this to the time domain, we get

$$i(t) = 3.218\cos(\omega t - 56.97^\circ)\text{ A}$$

Of course, we can find $i_1 + i_2$ using Eq. (9.9), but that is the hard way.

H.W
Practice problem
9.6

Phasor Relationships for Circuit Elements

TABLE 9.2

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Impedance and Admittance

The **impedance Z** of a circuit is the ratio of the phasor voltage **V** to the phasor current **I**, measured in ohms (Ω).

In the preceding section, we obtained the voltage-current relations for the three passive elements as

$$\mathbf{V} = R\mathbf{I}, \qquad \mathbf{V} = j\omega L\mathbf{I}, \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C} \tag{9.38}$$

These equations may be written in terms of the ratio of the phasor voltage to the phasor current as

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C} \tag{9.39}$$

From these three expressions, we obtain Ohm's law in phasor form for any type of element as

$$\boxed{\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \qquad \text{or} \qquad \mathbf{V} = \mathbf{Z}\mathbf{I}} \tag{9.40}$$

where $\mathbf{Z}$ is a frequency-dependent quantity known as *impedance*, measured in ohms.

Impedance and Admittance

TABLE 9.3

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \dfrac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \dfrac{1}{j\omega L}$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

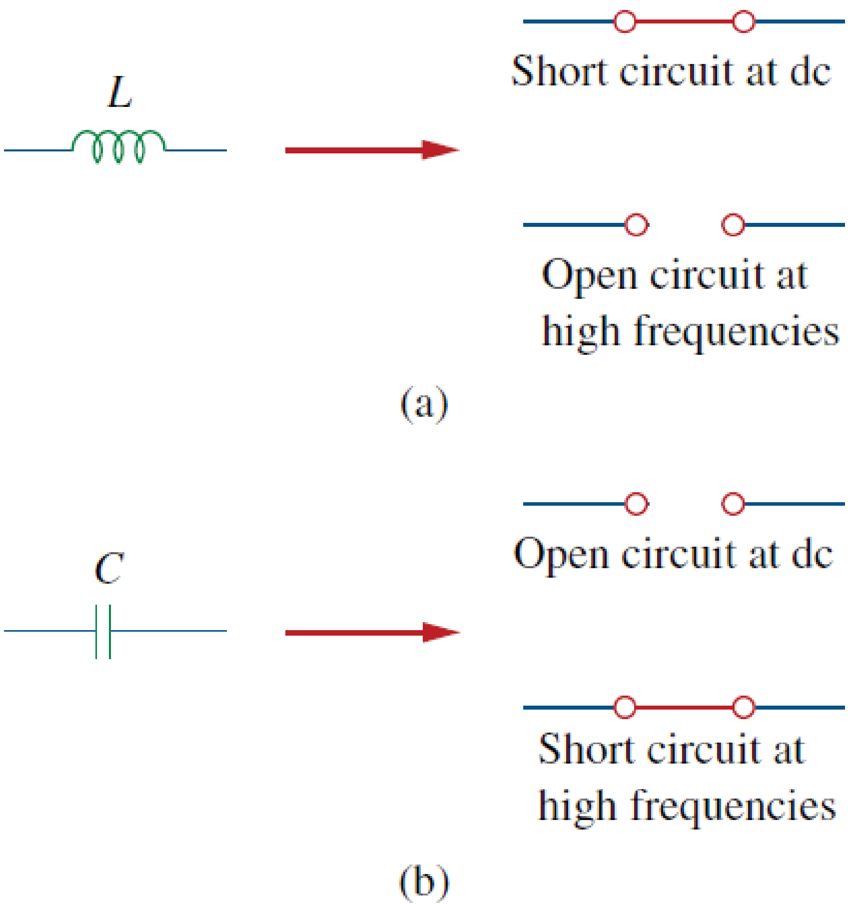

Figure 9.15
Equivalent circuits at dc and high frequencies: (a) inductor, (b) capacitor.

Exercise

Find $v(t)$ and $i(t)$ in the circuit shown in Fig. 9.16.

Example 9.9

Solution:
From the voltage source $10 \cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10\underline{/0^\circ} \text{ V}$$

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

Hence the current

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\underline{/0^\circ}}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2} \qquad (9.9.1)$$
$$= 1.6 + j0.8 = 1.789\underline{/26.57^\circ} \text{ A}$$

The voltage across the capacitor is

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789\underline{/26.57^\circ}}{j4 \times 0.1} \qquad (9.9.2)$$
$$= \frac{1.789\underline{/26.57^\circ}}{0.4\underline{/90^\circ}} = 4.47\underline{/-63.43^\circ} \text{ V}$$

Converting $\mathbf{I}$ and $\mathbf{V}$ in Eqs. (9.9.1) and (9.9.2) to the time domain, we get

$$i(t) = 1.789 \cos(4t + 26.57^\circ) \text{ A}$$
$$v(t) = 4.47 \cos(4t - 63.43^\circ) \text{ V}$$

Notice that $i(t)$ leads $v(t)$ by 90° as expected.

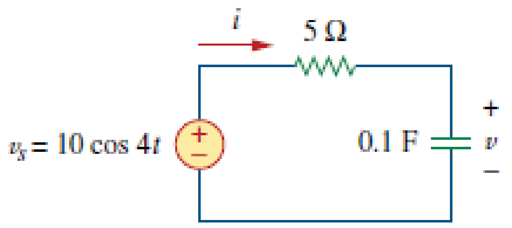

Figure 9.16
For Example 9.9.

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Practice problem
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Exercise

Example 9.10

Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

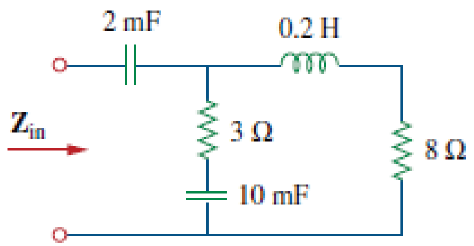

Figure 9.23
For Example 9.10.

Solution:
Let

$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor
$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the10-mF capacitor
$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

Then

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Thus,

$$\mathbf{Z}_{in} = 3.22 - j11.07\ \Omega$$

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Practice problem
9.10