Chapter 12

Three-Phase Circuits

Md. Omar Faruque
Lecturer
Department of Electrical and Electronic Engineering
City University
Email – write2faruque@gmail.com

Power in a Balanced System

12.7 Power in a Balanced System

Let us now consider the power in a balanced three-phase system. We begin by examining the instantaneous power absorbed by the load. This requires that the analysis be done in the time domain. For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos \omega t, \qquad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120°)$$
$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120°) \tag{12.41}$$

where the factor $\sqrt{2}$ is necessary because V_p has been defined as the rms value of the phase voltage. If $\mathbf{Z}_Y = Z\underline{/\theta}$, the phase currents lag behind their corresponding phase voltages by θ. Thus,

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \qquad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120°)$$
$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120°) \tag{12.42}$$

where I_p is the rms value of the phase current. The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$\begin{aligned} p &= p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ &= 2V_pI_p[\cos \omega t \cos(\omega t - \theta) \\ &\quad + \cos(\omega t - 120°)\cos(\omega t - \theta - 120°) \\ &\quad + \cos(\omega t + 120°)\cos(\omega t - \theta + 120°)] \end{aligned} \tag{12.43}$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)] \tag{12.44}$$

gives

$$\begin{aligned} p &= V_pI_p[3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240°) \\ &\quad + \cos(2\omega t - \theta + 240°)] \\ &= V_pI_p[3 \cos \theta + \cos \alpha + \cos \alpha \cos 240° + \sin \alpha \sin 240° \\ &\quad + \cos \alpha \cos 240° - \sin \alpha \sin 240°] \\ &\quad \text{where } \alpha = 2\omega t - \theta \\ &= V_pI_p\left[3 \cos \theta + \cos \alpha + 2\left(-\frac{1}{2}\right)\cos \alpha\right] = 3V_pI_p \cos \theta \end{aligned} \tag{12.45}$$

Thus the total instantaneous power in a balanced three-phase system is constant—it does not change with time as the instantaneous power of each phase does. This result is true whether the load is Y- or Δ-connected. This is one important reason for using a three-phase system to generate and distribute power. We will look into another reason a little later.

Since the total instantaneous power is independent of time, the average power per phase P_p for either the Δ-connected load or the Y-connected load is $p/3$, or

$$P_p = V_pI_p \cos\theta \tag{12.46}$$

and the reactive power per phase is

$$Q_p = V_pI_p \sin\theta \tag{12.47}$$

The apparent power per phase is

$$S_p = V_pI_p \tag{12.48}$$

The complex power per phase is

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p\mathbf{I}_p^* \tag{12.49}$$

where $\mathbf{V}_p$ and $\mathbf{I}_p$ are the phase voltage and phase current with magnitudes V_p and I_p, respectively. The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_pI_p \cos\theta = \sqrt{3}V_LI_L \cos\theta \tag{12.50}$$

For a Y-connected load, $I_L = I_p$ but $V_L = \sqrt{3}V_p$, whereas for a Δ-connected load, $I_L = \sqrt{3}I_p$ but $V_L = V_p$. Thus, Eq. (12.50) applies for both Y-connected and Δ-connected loads. Similarly, the total reactive power is

$$Q = 3V_pI_p \sin\theta = 3Q_p = \sqrt{3}V_LI_L \sin\theta \tag{12.51}$$

and the total complex power is

$$\boxed{\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p\mathbf{I}_p^* = 3I_p^2\mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*}} \tag{12.52}$$

where $\mathbf{Z}_p = Z_p\underline{/\theta}$ is the load impedance per phase. ($\mathbf{Z}_p$ could be $\mathbf{Z}_Y$ or $\mathbf{Z}_\Delta$.) Alternatively, we may write Eq. (12.52) as

$$\boxed{\mathbf{S} = P + jQ = \sqrt{3}V_LI_L\underline{/\theta}} \tag{12.53}$$

Remember that V_p, I_p, V_L, and I_L are all rms values and that θ is the angle of the load impedance or the angle between the phase voltage and the phase current.

Example 12.7

A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

Solution:

The apparent power is

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13 \text{ VA}$$

Since the real power is

$$P = S\cos\theta = 5600 \text{ W}$$

the power factor is

$$\text{pf} = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075$$

H.W

Practice problem

12.7

Chapter 14

Frequency Response

Series Resonance

Resonance is a condition in an *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

- ❖ Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective. They are used in many applications such as selecting the desired stations in radio and TV receivers.

Consider the series *RLC* circuit shown in Fig. 14.21 in the frequency domain. The input impedance is

$$\mathbf{Z} = \mathbf{H}(\omega) = \frac{\mathbf{V}_s}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C} \tag{14.22}$$

or

$$\mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{14.23}$$

Resonance results when the imaginary part of the transfer function is zero, or

$$\mathrm{Im}(\mathbf{Z}) = \omega L - \frac{1}{\omega C} = 0 \tag{14.24}$$

The value of ω that satisfies this condition is called the *resonant frequency* ω_0. Thus, the resonance condition is

$$\omega_0 L = \frac{1}{\omega_0 C} \tag{14.25}$$

or

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}} \tag{14.26}$$

Since $\omega_0 = 2\pi f_0$,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \tag{14.27}$$

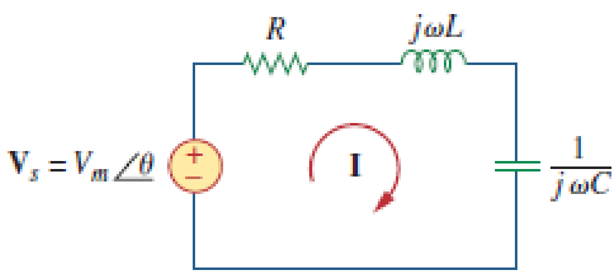

Figure 14.21
The series resonant circuit.

The frequency response of the circuit's current magnitude

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \tag{14.28}$$

Series Resonance

is shown in Fig. 14.22; the plot only shows the symmetry illustrated in this graph when the frequency axis is a logarithm. The average power dissipated by the *RLC* circuit is

$$P(\omega) = \frac{1}{2} I^2 R \qquad (14.29)$$

The highest power dissipated occurs at resonance, when $I = V_m/R$, so that

$$P(\omega_0) = \frac{1}{2}\frac{V_m^2}{R} \qquad (14.30)$$

At certain frequencies $\omega = \omega_1, \omega_2$, the dissipated power is half the maximum value; that is,

$$P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R} \qquad (14.31)$$

Hence, ω_1 and ω_2 are called the *half-power frequencies.*

The half-power frequencies are obtained by setting Z equal to $\sqrt{2}R$, and writing

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R \qquad (14.32)$$

Solving for ω, we obtain

$$\boxed{\begin{aligned} \omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \omega_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{aligned}} \qquad (14.33)$$

We can relate the half-power frequencies with the resonant frequency. From Eqs. (14.26) and (14.33),

$$\omega_0 = \sqrt{\omega_1 \omega_2} \qquad (14.34)$$

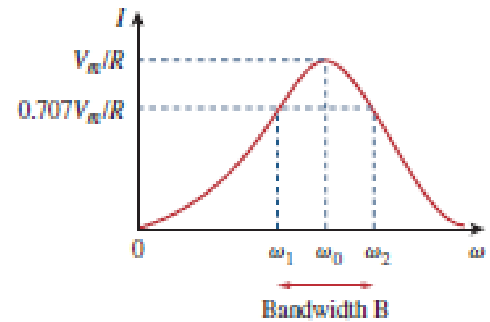

Figure 14.22
The current amplitude versus frequency for the series resonant circuit of Fig. 14.21.

Series Resonance

The quality factor relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation:

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} \tag{14.36}$$

It is also regarded as a measure of the energy storage property of a circuit in relation to its energy dissipation property. In the series RLC circuit, the peak energy stored is $\frac{1}{2}LI^2$, while the energy dissipated in one period is $\frac{1}{2}(I^2R)(1/f_0)$. Hence,

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f_0)} = \frac{2\pi f_0 L}{R} \tag{14.37}$$

or

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}} \tag{14.38}$$

Notice that the quality factor is dimensionless. The relationship between the bandwidth B and the quality factor Q is obtained by substituting Eq. (14.33) into Eq. (14.35) and utilizing Eq. (14.38).

$$\boxed{B = \frac{R}{L} = \frac{\omega_0}{Q}} \tag{14.39}$$

or $B = \omega_0^2 CR$. Thus

The **quality factor** of a resonant circuit is the ratio of its resonant frequency to its bandwidth.

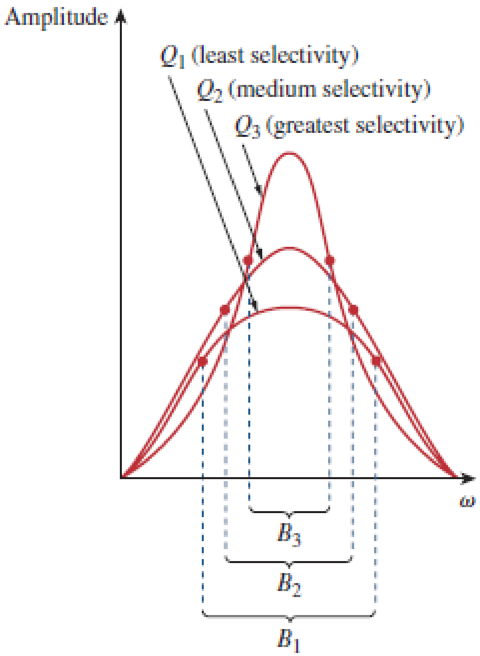

Figure 14.23
The higher the circuit Q, the smaller the bandwidth.

The quality factor is a measure of the selectivity (or "sharpness" of resonance) of the circuit.

MATH

Example 14.7

In the circuit of Fig. 14.24, $R = 2\,\Omega$, $L = 1$ mH, and $C = 0.4\,\mu$F. (a) Find the resonant frequency and the half-power frequencies. (b) Calculate the quality factor and bandwidth. (c) Determine the amplitude of the current at ω_0, ω_1, and ω_2.

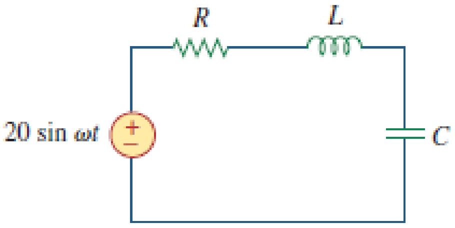

Figure 14.24
For Example 14.7.

Solution:

(a) The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}} = 50 \text{ krad/s}$$

METHOD 1 The lower half-power frequency is

$$\begin{aligned}\omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\\ &= -\frac{2}{2 \times 10^{-3}} + \sqrt{(10^3)^2 + (50 \times 10^3)^2}\\ &= -1 + \sqrt{1 + 2500} \text{ krad/s} = 49 \text{ krad/s}\end{aligned}$$

Similarly, the upper half-power frequency is

$$\omega_2 = 1 + \sqrt{1 + 2500} \text{ krad/s} = 51 \text{ krad/s}$$

(b) The bandwidth is

$$B = \omega_2 - \omega_1 = 2 \text{ krad/s}$$

or

$$B = \frac{R}{L} = \frac{2}{10^{-3}} = 2 \text{ krad/s}$$

The quality factor is

$$Q = \frac{\omega_0}{B} = \frac{50}{2} = 25$$

Parallel Resonance

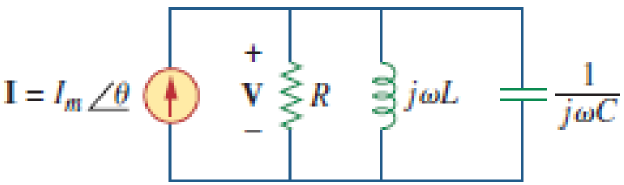

Figure 14.25
The parallel resonant circuit.

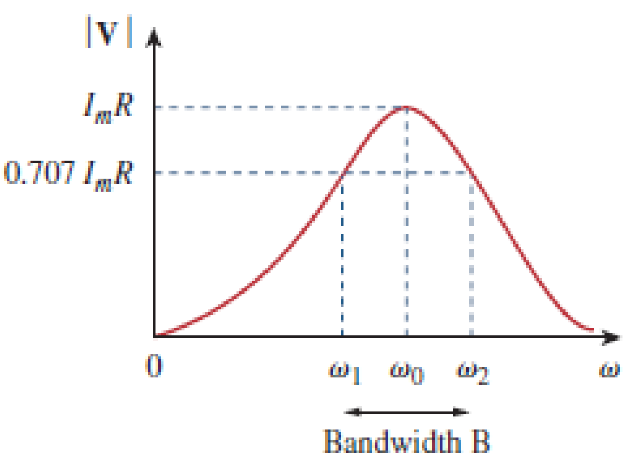

Figure 14.26
The current amplitude versus frequency for the series resonant circuit of Fig. 14.25.

14.6 Parallel Resonance

The parallel *RLC* circuit in Fig. 14.25 is the dual of the series *RLC* circuit. So we will avoid needless repetition. The admittance is

$$\mathbf{Y} = H(\omega) = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \tag{14.41}$$

or

$$\mathbf{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \tag{14.42}$$

Resonance occurs when the imaginary part of **Y** is zero,

$$\omega C - \frac{1}{\omega L} = 0 \tag{14.43}$$

or

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}} \tag{14.44}$$

Parallel Resonance

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$
$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \qquad \textbf{(14.45)}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC} \qquad \textbf{(14.46)}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L} \qquad \textbf{(14.47)}$$

It should be noted that Eqs. (14.45) to (14.47) apply only to a parallel RLC circuit. Using Eqs. (14.45) and (14.47), we can express the half-power frequencies in terms of the quality factor. The result is

$$\omega_1 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}, \qquad \omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q} \qquad \textbf{(14.48)}$$

Again, for high-Q circuits ($Q \geq 10$)

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2} \qquad \textbf{(14.49)}$$

Table 14.4 presents a summary of the characteristics of the series and parallel resonant circuits. Besides the series and parallel RLC considered here, other resonant circuits exist. Example 14.9 treats a typical example.

Example 14.8

In the parallel RLC circuit of Fig. 14.27, let $R = 8$ kΩ, $L = 0.2$ mH, and $C = 8\ \mu$F. (a) Calculate ω_0, Q, and B. (b) Find ω_1 and ω_2. (c) Determine the power dissipated at ω_0, ω_1, and ω_2.

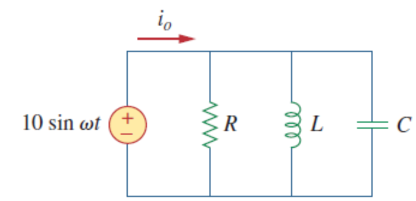

Figure 14.27
For Example 14.8.

Solution:

(a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 8 \times 10^{-6}}} = \frac{10^5}{4} = 25 \text{ krad/s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{8 \times 10^3}{25 \times 10^3 \times 0.2 \times 10^{-3}} = 1{,}600$$

$$B = \frac{\omega_0}{Q} = 15.625 \text{ rad/s}$$

(b) Due to the high value of Q, we can regard this as a high-Q circuit. Hence,

$$\omega_1 = \omega_0 - \frac{B}{2} = 25{,}000 - 7.812 = 24{,}992 \text{ rad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 25{,}000 + 7.812 = 25{,}008 \text{ rad/s}$$

(c) At $\omega = \omega_0$, $\mathbf{Y} = 1/R$ or $\mathbf{Z} = R = 8$ kΩ. Then

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{10\underline{/-90^\circ}}{8{,}000} = 1.25\underline{/-90^\circ} \text{ mA}$$

Since the entire current flows through R at resonance, the average power dissipated at $\omega = \omega_0$ is

$$P = \frac{1}{2}|\mathbf{I}_o|^2 R = \frac{1}{2}(1.25 \times 10^{-3})^2(8 \times 10^3) = 6.25 \text{ mW}$$

or

$$P = \frac{V_m^2}{2R} = \frac{100}{2 \times 8 \times 10^3} = 6.25 \text{ mW}$$

At $\omega = \omega_1, \omega_2$,

$$P = \frac{V_m^2}{4R} = 3.125 \text{ mW}$$

Summary

Summary of the characteristics of resonant *RLC* circuits.

Characteristic	Series circuit	Parallel circuit
Resonant frequency, ω_0	$\dfrac{1}{\sqrt{LC}}$	$\dfrac{1}{\sqrt{LC}}$
Quality factor, Q	$\dfrac{\omega_0 L}{R}$ or $\dfrac{1}{\omega_0 RC}$	$\dfrac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, B	$\dfrac{\omega_0}{Q}$	$\dfrac{\omega_0}{Q}$
Half-power frequencies, ω_1, ω_2	$\omega_0\sqrt{1+\left(\dfrac{1}{2Q}\right)^2} \pm \dfrac{\omega_0}{2Q}$	$\omega_0\sqrt{1+\left(\dfrac{1}{2Q}\right)^2} \pm \dfrac{\omega_0}{2Q}$
For $Q \geq 10$, ω_1, ω_2	$\omega_0 \pm \dfrac{B}{2}$	$\omega_0 \pm \dfrac{B}{2}$

Passive Filters

A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

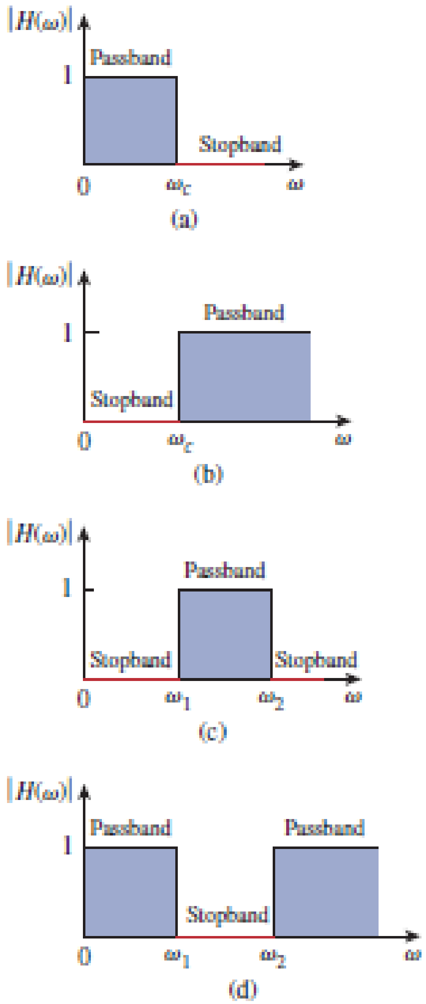

Figure 14.30
Ideal frequency response of four types of filters: (a) lowpass filter, (b) highpass filter, (c) bandpass filter, (d) bandstop filter.

As shown in Fig. 14.30, there are four types of filters whether passive or active:

1. A *lowpass filter* passes low frequencies and stops high frequencies, as shown ideally in Fig. 14.30(a).
2. A *highpass filter* passes high frequencies and rejects low frequencies, as shown ideally in Fig. 14.30(b).
3. A *bandpass filter* passes frequencies within a frequency band and blocks or attenuates frequencies outside the band, as shown ideally in Fig. 14.30(c).
4. A *bandstop filter* passes frequencies outside a frequency band and blocks or attenuates frequencies within the band, as shown ideally in Fig. 14.30(d).

Table 14.5 presents a summary of the characteristics of these filters. Be aware that the characteristics in Table 14.5 are only valid for first- or second-order filters—but one should not have the impression that only these kinds of filter exist. We now consider typical circuits for realizing the filters shown in Table 14.5.

TABLE 14.5

Summary of the characteristics of ideal filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.