Lecture 4

Chapter 11

AC Power Analysis

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Instantaneous Power

The **instantaneous power** (in watts) is the power at any instant of time.

$$\boxed{p(t) = v(t)i(t)} \qquad (11.1)$$

Let the voltage and current at the terminals of the circuit be

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad (11.2a)$$

$$i(t) = I_m \cos(\omega t + \theta_i) \qquad (11.2b)$$

where V_m and I_m are the amplitudes (or peak values), and θ_v and θ_i are the phase angles of the voltage and current, respectively. The instantaneous power absorbed by the circuit is

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \qquad (11.3)$$

We apply the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)] \qquad (11.4)$$

and express Eq. (11.3) as

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i) \qquad (11.5)$$

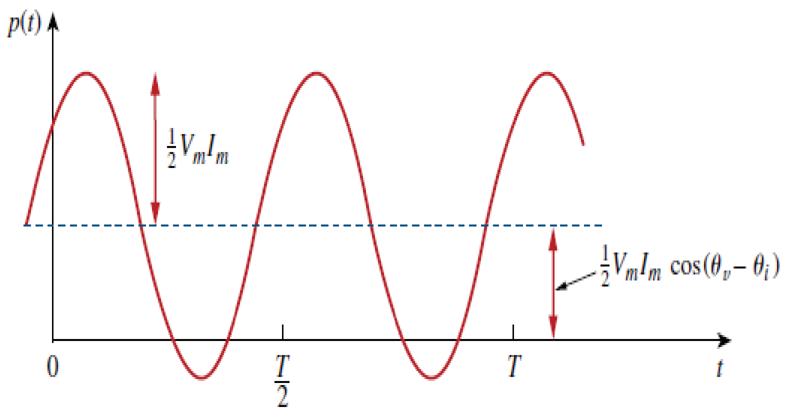


Figure 11.2
The instantaneous power $p(t)$ entering a circuit.

This shows us that the instantaneous power has two parts. The first part is constant or time independent. The second part is a sinusoidal function

Average Power

The **average power**, in watts, is the average of the instantaneous power over one period. Thus, the average power is given by

$$P = \frac{1}{T}\int_0^T p(t)\, dt \qquad \textbf{(11.6)}$$

Substituting $p(t)$ in Eq. (11.5) into Eq. (11.6) gives

$$\begin{aligned} P &= \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\, dt \\ &\quad + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\, dt \\ &= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt \\ &\quad + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\, dt \end{aligned} \qquad \textbf{(11.7)}$$

The first integrand is constant. The second integrand is a sinusoid. We know that the average of a sinusoid over its period is zero. Thus, average power becomes

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) \qquad \textbf{(11.8)}$$

Exercise

Example 11.2

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$ when a voltage $\mathbf{V} = 120\underline{/0^\circ}$ is applied across it.

Solution:

The current through the impedance is

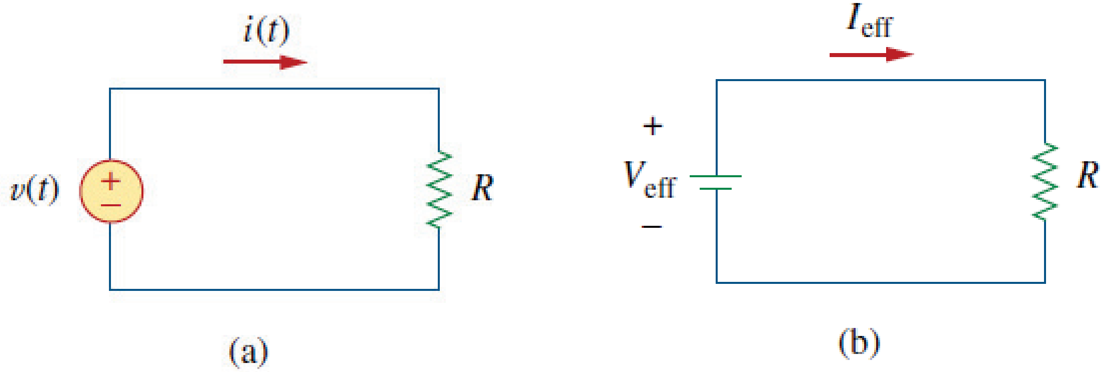
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\underline{/0^\circ}}{30 - j70} = \frac{120\underline{/0^\circ}}{76.16\underline{/-66.8^\circ}} = 1.576\underline{/66.8^\circ}\ \text{A}$$

The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ) = 37.24\ \text{W}$$

Effective or RMS Value

The **effective value** of a periodic current is the ***dc current*** that delivers the ***same average power*** to a resistor as the ***periodic current.***



The circuit in (a) is ac while that of (b) is dc. Our objective is to find ***Ieff*** that will transfer the same power to resistor *R* as the sinusoid *i*.

The average power absorbed by the resistor in the ac circuit is

$$P = \frac{1}{T}\int_0^T i^2 R\, dt = \frac{R}{T}\int_0^T i^2\, dt \qquad \textbf{(11.22)}$$

while the power absorbed by the resistor in the dc circuit is

$$P = I_{\text{eff}}^2 R \qquad \textbf{(11.23)}$$

Equating the expressions in Eqs. (11.22) and (11.23) and solving for I_{eff}, we obtain

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} \qquad \textbf{(11.24)}$$

Eqn 11.24 indicates that the effective value is the (square) *root* of the *mean* (or average) of the *square* of the periodic signal. Thus, the effective value is often known as the ***root-mean-square* value, or *rms*** value for short; and we write

$$I_{\text{eff}} = I_{\text{rms}},$$

Effective or RMS Value

For the sinusoid, $i(t) = I_m \cos\omega t$, The effective or rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2\omega t \, dt}$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}} = 0.707 I_m \qquad \textbf{(11.28)}$$

So, effective value is 70.7% of maximum value.

Exercise

Example 11.8

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 10-Ω resistor.

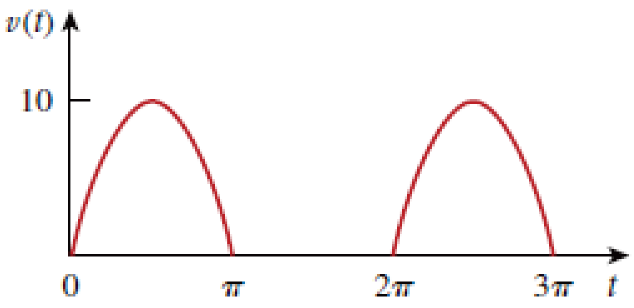


Figure 11.16
For Example 11.8.

Solution:
The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10\sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value is obtained as

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^\pi (10\sin t)^2\,dt + \int_\pi^{2\pi} 0^2\,dt\right]$$

But $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$. Hence,

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^\pi \frac{100}{2}(1 - \cos 2t)\,dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\Bigg|_0^\pi$$

$$= \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25, \qquad V_{\text{rms}} = 5\text{ V}$$

The average power absorbed is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5\text{ W}$$

H.W
Practice problem
11.7