Lecture 3

Chapter 29

DC MOTOR

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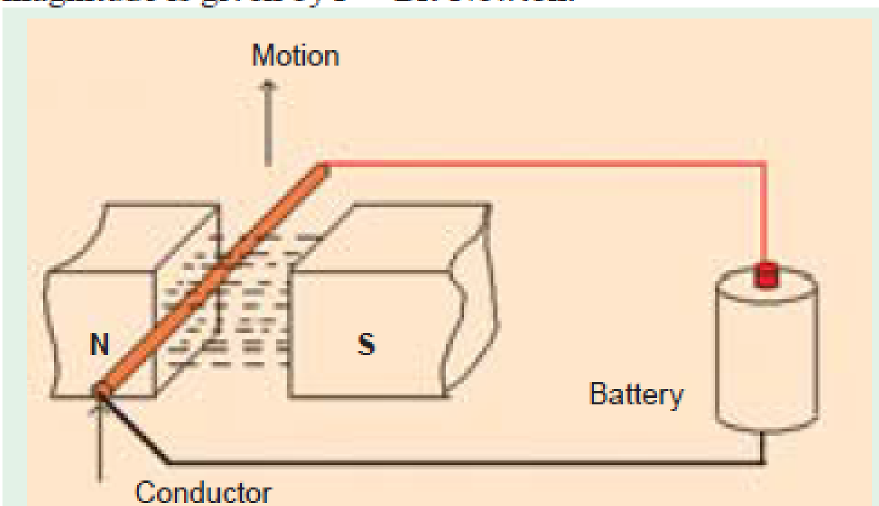
Working Principle

29.1. Motor Principle

An Electric motor is a machine which converts electric energy into mechanical energy. Its action is based on the principle that when a current-carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's Left-hand Rule and whose magnitude is given by $F = BIl$ Newton.



Principle of Motor

Constructionally, there is no basic difference between a d.c. generator and a d.c. motor. In fact, the same d.c. machine can be used interchangeably as a generator or as a motor. D.C. motors are also like generators, shunt-wound or series-wound or compound-wound.

In Fig. 29.1 a part of multipolar d.c. motor is shown. When its field magnets are excited and its armature conductors are supplied with current from the supply mains, they experience a force tending to rotate the armature. Armature conductors under N-pole are assumed to carry current downwards (crosses) and those under S-poles, to carry current upwards (dots). By applying Fleming's Left-hand Rule, the direction of the force on each conductor can be found. It is shown by small arrows placed above each conductor. It will be seen that each conductor can be found. It will be seen that each conductor experiences a force F which tends to rotate the armature in anticlockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

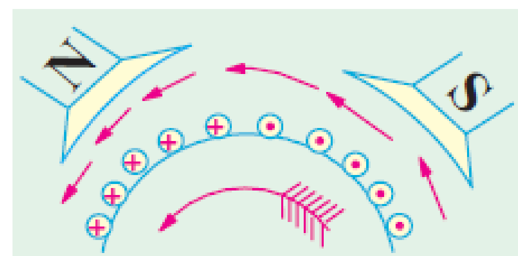


Fig. 29.1

It should be noted that the function of a commutator in the motor is the same as in a generator. By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.

29.3. Significance of the Back e.m.f.

As explained in Art 29.2, when the motor armature rotates, the conductors also rotate and hence cut the flux. In accordance with the laws of electromagnetic induction, e.m.f. is induced in them whose direction, as found by Fleming's Right-hand Rule, is in opposition to the applied voltage (Fig. 29.5). Because of its opposing direction, it is referred to as counter e.m.f. or back e.m.f. E_b. The equivalent circuit of a motor is shown in Fig. 29.6. The rotating armature generating the back e.m.f. E_b is like a battery of e.m.f. E_b put across a supply mains of V volts. Obviously, V has to drive I_a against the opposition of E_b. The power required to overcome this opposition is $E_b I_a$.

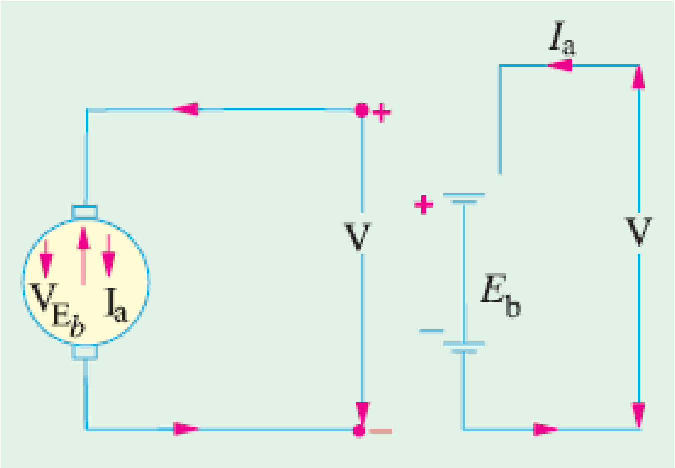


Fig. 29.5

In the case of a cell, this power over an interval of time is converted into chemical energy, but in the present case, it is converted into mechanical energy.

$$\text{It will be seen that } I_a = \frac{\text{Net voltage}}{\text{Resistance}} = \frac{V - V_b}{R_a}$$

where R_a is the resistance of the armature circuit. As pointed out above,

$$E_b = \Phi Z N \times (P/A) \text{ volt where } N \text{ is in r.p.s.}$$

Back e.m.f. depends, among other factors, upon the armature speed. If speed is high, E_b is large, hence armature current I_a, seen from the above equation, is small. If the speed is less, then E_b is less, hence more current flows which develops motor torque (Art 29.7). So, we find that E_b acts like a governor *i.e.*, it makes a motor self-regulating so that it draws as much current as is just necessary.

General Torque Equation

29.6. Torque

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius r metre acted upon by a circumferential force of F Newton which causes it to rotate at N r.p.m. (Fig. 29.10).

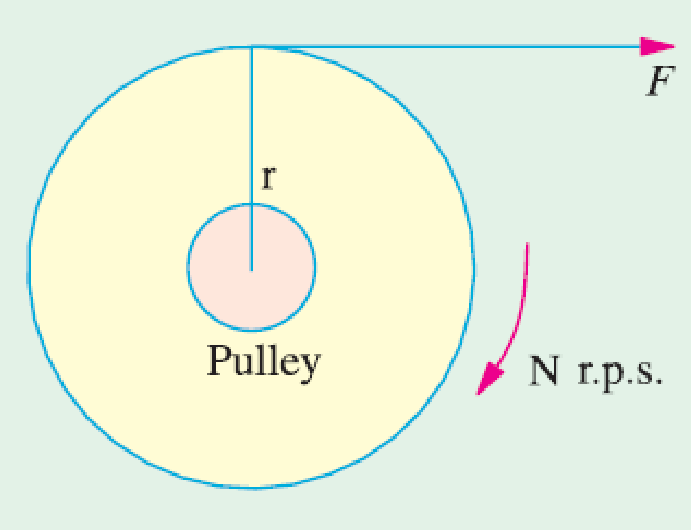


Fig. 29.10

Then torque $T = F \times r$ Newton-metre (N - m)

Work done by this force in one revolution

$= \text{Force} \times \text{distance} = F \times 2\pi r$ Joule

Power developed $= F \times 2\,\pi r \times N$ Joule/second or Watt

$= (F \times r) \times 2\pi\, N$ Watt

Now $2\,\pi N$ = Angular velocity ω in radian/second and $F \times r$ = Torque T

$\therefore$ Power developed $= T \times \omega$ watt or $P = T\,\omega$ Watt

Moreover, if N is in r.p.m., then

$$\omega = 2\,\pi N/60 \text{ rad/}s$$

$$\therefore \quad P = \frac{2\pi N}{60} \times T \quad \text{or} \quad P = \frac{2\pi}{60}\,.\,NT = \frac{NT}{9.55}$$

Armature Torque of Motor

29.7. Armature Torque of a Motor

Let T_a be the torque developed by the armature of a motor running at N r.p.s. If T_a is in N/M, then

power developed $= T_a \times 2\pi\, N$ watt ...(*i*)

We also know that electrical power converted into mechanical power in the armature (Art 29.4)

$$= E_b I_a \text{ watt} \quad \ldots(ii)$$

Equating (*i*) and (*ii*), we get $T_a \times 2\pi N = E_b I_a$ ...(*iii*)

Since $E_b = \Phi Z N \times (P/A)$ volt, we have

$$T_a \times 2\pi N = \Phi\, ZN \left(\frac{P}{A}\right) . I_a \text{ or } T_a = \frac{1}{2\pi} . \Phi Z I_0 \left(\frac{P}{A}\right) N\text{-}m$$

$$= 0.159 \text{ N newton metre}$$

$$\therefore \quad T_a = 0.159\, \Phi Z I_a \times (P/A)\ N\text{-}m$$

As seen from (*iii*) above

$$T_a = \frac{E_b\, I_a}{2\pi\, N} \text{ N - m - N in r.p.s.}$$

If N is in r.p.m., then

$$T_a = \frac{E_b\, I_a}{2\pi\, N/60} = 60\,\frac{E_b\, I_a}{2\pi N} = \frac{60}{2\pi}\,\frac{E_b\, I_a}{N} = 9.55\,\frac{E_b I_a}{N} \text{ N-m}$$

Shaft Torque of Motor

29.8. Shaft Torque (T_{sh})

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as shaft torque T_{sh}. It is so called because it is available at the shaft. The motor output is given by

Output = $T_{sh} \times 2\pi N$ Watt provided T_{sh} is in N-m and N in r.p.s.

$$\therefore \qquad T_{sh} = \frac{\text{Output in watts}}{2\pi N} \text{ N- m} - N \text{ in r.p.s}$$

$$= \frac{\text{Output in watts}}{2\pi N / 60} \text{ N-m} - N \text{ in r.p.m.}$$

$$= \frac{60}{2\pi} \frac{\text{output}}{N} = 9.55 \frac{\text{Output}}{N} \text{ N-m.}$$

MATH

Example 29.9. *Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.*

(Elect. Machine AMIE Sec. B Winter 1991)

Solution. Developed torque or gross torque is the same thing as armature torque.

$\therefore \quad T_a = 0.159\, \Phi\, ZA\, (P/A)$

$= 0.159 \times 25 \times 10^{-3} \times 800 \times 45\ (4/2) = 286.2$ N-m

$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193$ V

Now, $\quad E_b = \Phi\, ZN\, (P/A)$ or $193 = 25 \times 10^{-3} \times 800 \times N \pi \times (4/2)$

$\therefore \quad N = 4.825$ r.p.s.

Also, $\quad 2\pi N T_{sh} =$ output or $2\pi \times 4.825\ T_{sh} = 8200 \qquad \therefore T_{sh} = 270.5$ N-m

Motor Characteristics

29.12. Motor Characteristics

The characteristic curves of a motor are those curves which show relationships between the following quantities.

1. **Torque and armature current** *i.e.* T_a/I_a characteristic. It is known as ***electrical characteristic***.

2. **Speed and armature current** *i.e.* N/I_a characteristic.

3. **Speed and torque** *i.e.* N/T_a characteristic. It is also known as ***mechanical characteristic***. It can be found from (1) and (2) above.

While discussing motor characteristics, the following two relations should always be kept in mind :

$$T_a \propto \Phi I_a \quad \text{and} \quad N \propto \frac{E_b}{\Phi}$$

Series Motor Characteristics

$$T_a \propto \Phi I_a \quad \text{and} \quad N \propto \frac{E_b}{\Phi}$$

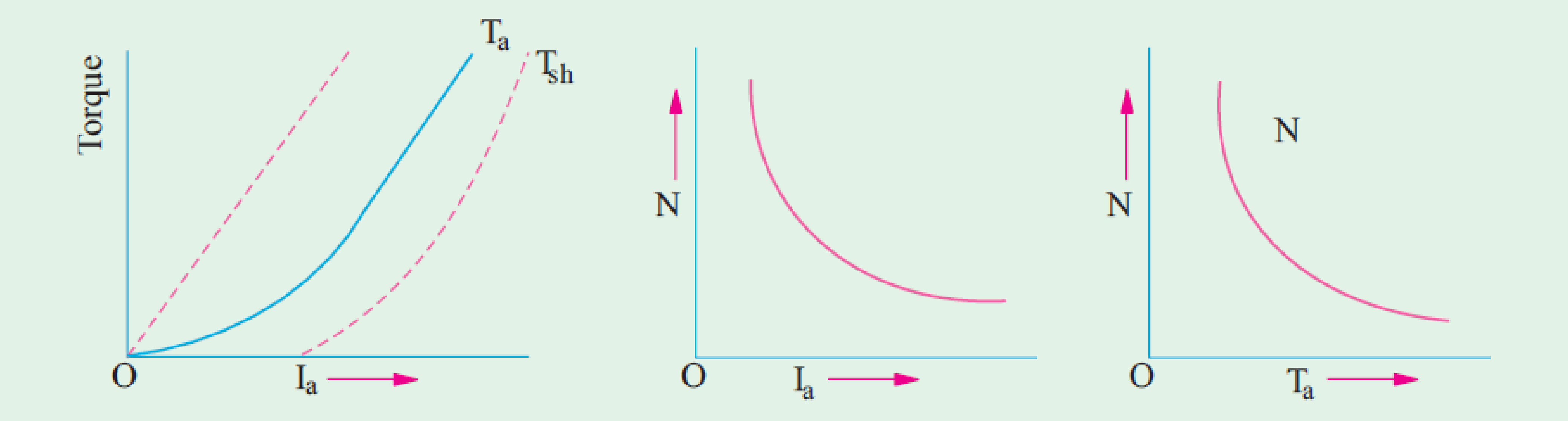


Fig. 29.14 Fig. 29.15 Fig. 29.16

Shunt Motor Characteristics

$$T_a \propto \Phi I_a \quad \text{and} \quad N \propto \frac{E_b}{\Phi}$$

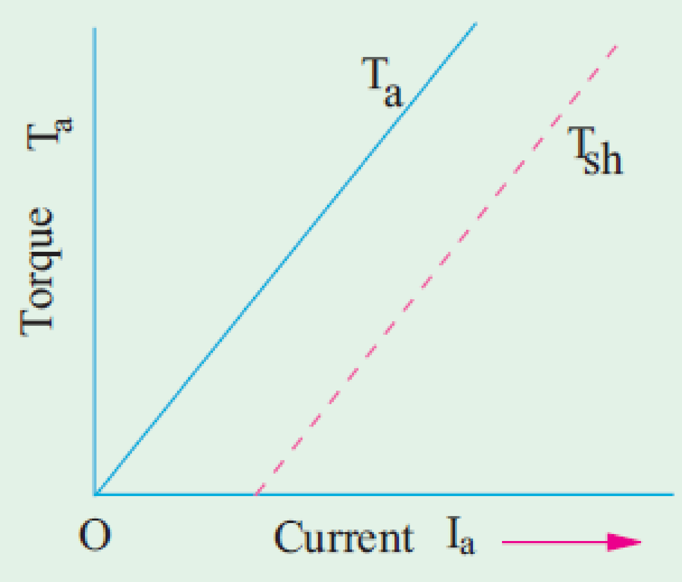


Fig. 29.17

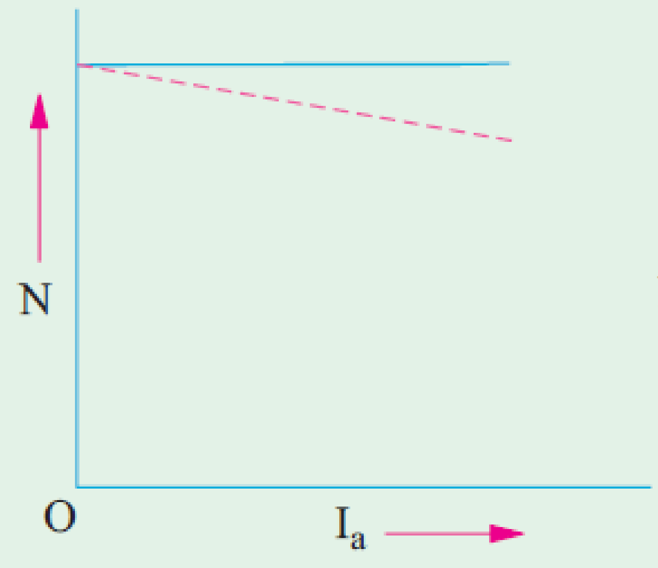


Fig. 29.18

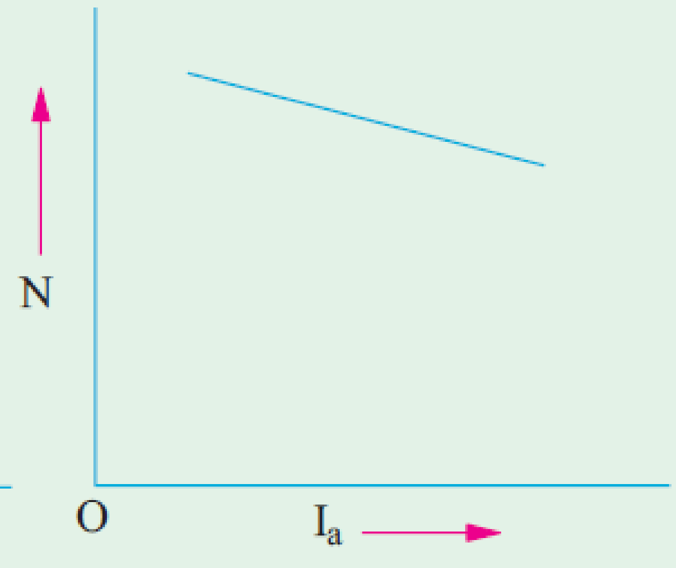


Fig. 29.19