SENSOR TILT VIA CONIC SECTIONS

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FotoNation Galway

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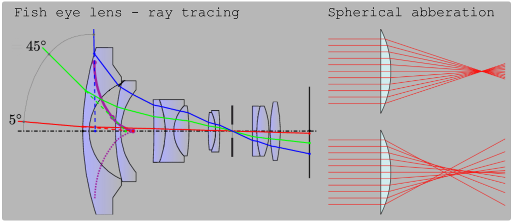
FotoNation XPERI

Camera calibration • some assumptions

$$\hat{X} \xrightarrow{\text{extrinsics}} \hat{P} \xrightarrow{\text{lens}} \hat{p} \xrightarrow{\text{sensor}} \hat{p}' \xrightarrow{\text{intrinsics}} \hat{x}$$

- chromatic aberration is negligible
- spherical aberration is negligible
- Light entering the lens body has a field angle described relative to a common point
- Light impacting the ccd imaging sensor emanates from a common point - the "projection centre"

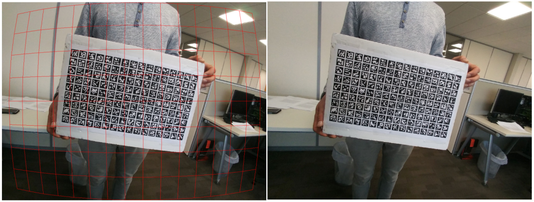


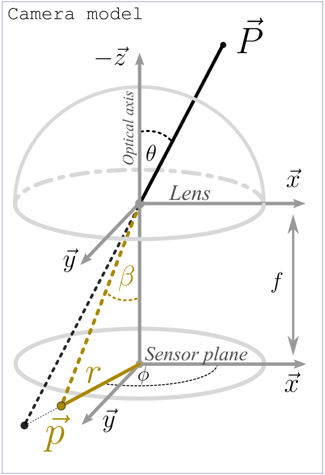
Sensor tilt • camera calibration

$$\hat{X} \xrightarrow{\text{extrinsics}} \hat{P} \xrightarrow{\text{lens}} \hat{p} \xrightarrow{\text{sensor}} \hat{p}' \xrightarrow{\text{intrinsics}} \hat{x}$$

Camera calibration:

- process of determining the geometric properties of a camera
- lens distortion correction
- improve image quality
- maps real world coordinates to pixels



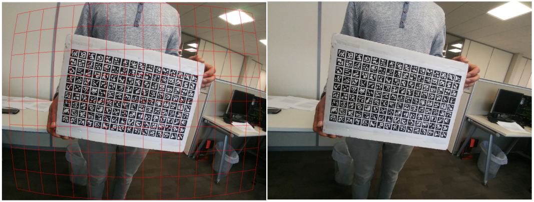


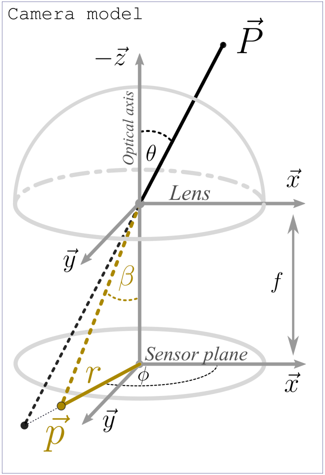
Sensor tilt • camera calibration

$$\hat{X} \xmapsto{\text{extrinsics}} \hat{P} \xmapsto{\text{lens}} \hat{p} \xmapsto{\text{sensor}} \hat{p}' \xmapsto{\text{intrinsics}} \hat{x}$$

Camera calibration:

- makes use of a calibration target
- typically a 1 or 3 plane checkerboard
- geometric properties are known
- detector is used to extract the pixel coordinates of the target corners





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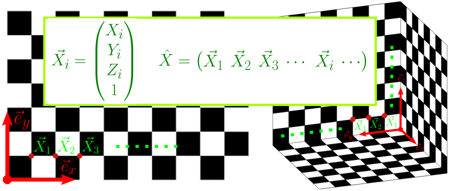
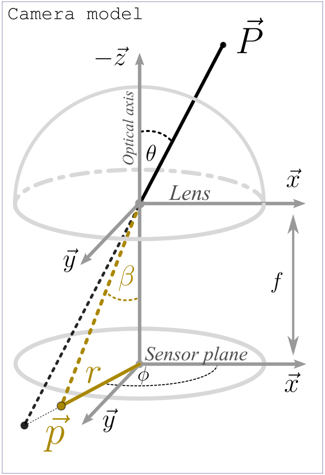


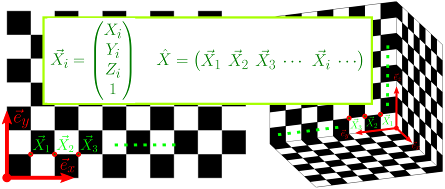


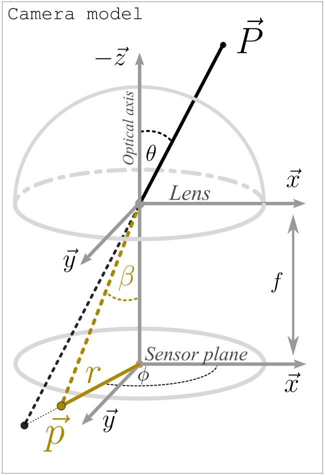
Sensor tilt • world coordinates

$$\hat{X} \xrightarrow{\text{extrinsics}} \hat{P} \xrightarrow{\text{lens}} \hat{p} \xrightarrow{\text{sensor}} \hat{p}' \xrightarrow{\text{intrinsics}} \hat{x}$$

World coordinates:

- $\hat{P}$ are the target coordinates expressed in the world coordinate system
- $\hat{P} = [\,\hat{R}\,|\,\vec{t}\,]\hat{X}$
- $\hat{R} \in SO(3) \qquad \vec{t} \in \mathbb{R}^3 \qquad \vec{P}_i \in \mathbb{R}^3$
- $\hat{P} = (\; \vec{P}_1 \;\; \vec{P}_2 \;\; \vec{P}_3 \;\; \cdots \;\; \vec{P}_i \;\; \cdots \;)$





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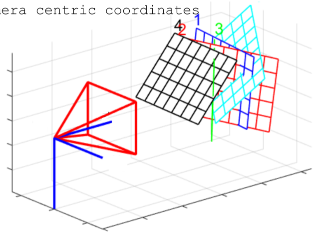
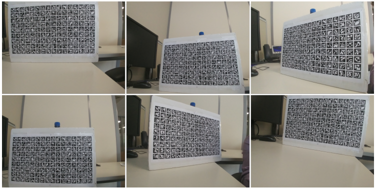
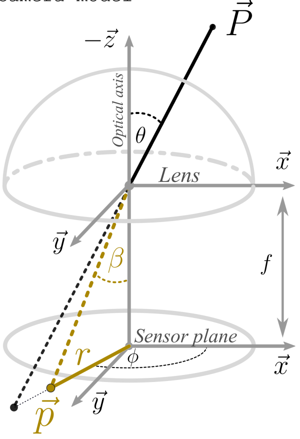
camera centric coordinates



calibration target images



Camera model



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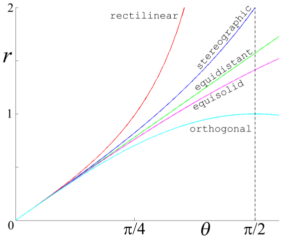
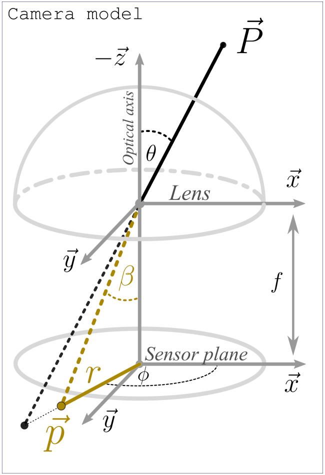
Fish-eye lens models:

1. stereographic: $r(\theta) = 2f\tan(\frac{\theta}{2})$
2. equidistant: $r(\theta) = f\,\theta$
3. equisolid: $r(\theta) = 2f\sin(\frac{\theta}{2})$
4. orthogonal: $r(\theta) = f\sin(\theta)$





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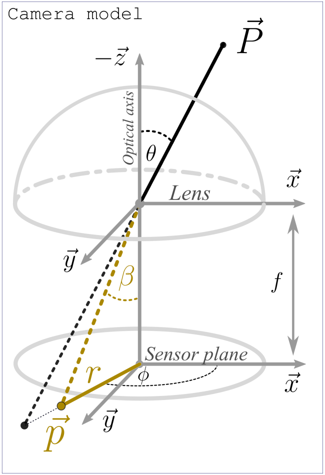
Taylor series expansion ($f = 1$):

$$r(\theta) = \theta + \kappa_1\theta^3 + \kappa_2\theta^5 + \kappa_3\theta^7 + \cdots$$



Sensor tilt • world coordinates

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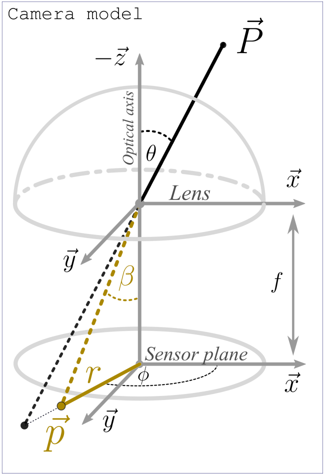
Fish-eye lens models:

1. $(\kappa_1, \kappa_2, \kappa_3, \cdots) = (\frac{1}{12}, \frac{1}{120}, \frac{17}{20160}, \cdots)$
2. $(\kappa_1, \kappa_2, \kappa_3, \cdots) = (0, 0, 0, \cdots)$
3. $(\kappa_1, \kappa_2, \kappa_3, \cdots) = (-\frac{1}{24}, \frac{1}{1920}, -\frac{1}{322560}, \cdots)$
4. $(\kappa_1, \kappa_2, \kappa_3, \cdots) = (-\frac{1}{6}, \frac{1}{120}, -\frac{1}{5040}, \cdots)$

Taylor series expansion ($f = 1$):

$$r(\theta) = \theta + \kappa_1\theta^3 + \kappa_2\theta^5 + \kappa_3\theta^7 + \cdots$$



Sensor tilt • world coordinates

$$\hat{X} \xmapsto{\text{extrinsics}} \hat{P} \xmapsto{\text{lens}} \hat{p} \xmapsto{\text{sensor}} \hat{p}' \xmapsto{\text{intrinsics}} \hat{x}$$

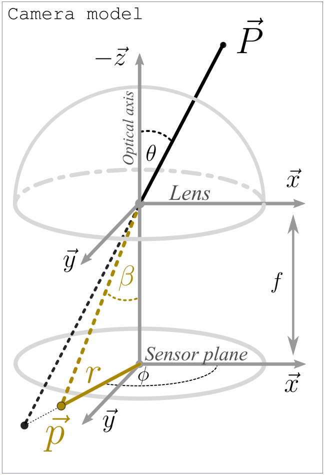
Lens distorted coordinates:

$$\theta = \tan^{-1}\left(\frac{\sqrt{P_x^2 + P_y^2}}{P_z}\right)$$

$$\varphi = \tan^{-1}\left(\frac{P_y}{P_x}\right)$$

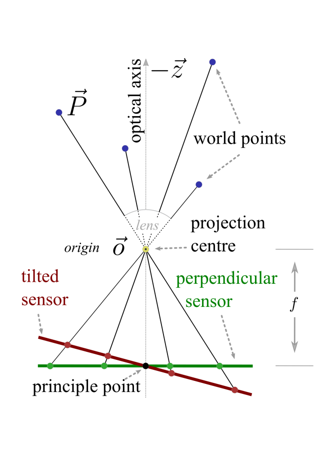
$$\vec{p} = \begin{pmatrix} p_x \\ p_y \\ 1 \end{pmatrix} = \begin{pmatrix} r(\theta)\cos(\varphi) \\ r(\theta)\sin(\varphi) \\ 1 \end{pmatrix}$$

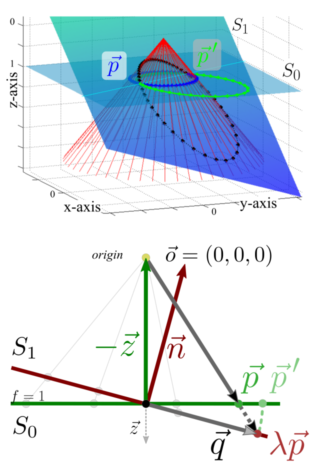
$$\hat{p} = \begin{pmatrix} \vec{p}_1 & \vec{p}_2 & \vec{p}_3 & \cdots & \vec{p}_i & \cdots & \end{pmatrix}$$



Sensor tilt • tilt shifted coordinates

$$\hat{X} \xmapsto{\text{extrinsics}} \hat{P} \xmapsto{\text{lens}} \hat{p} \xmapsto{\text{sensor}} \hat{p}' \xmapsto{\text{intrinsics}} \hat{x}$$





Sensor tilt • tilt shifted coordinates

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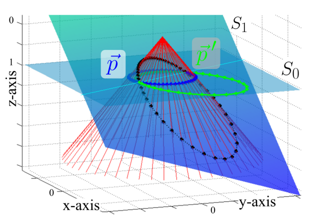
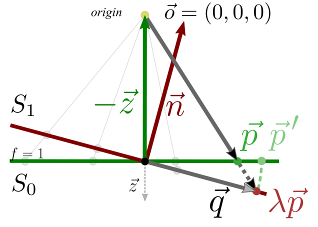
The sensor plane has 2 positions:

- S_0 perpendicular sensor
- S_1 tilted sensor

Photons emanating from the world point(s) $\vec{P}$ are projected through the lens elements to impact the perpendicular sensor S_0 at the point(s) $\vec{p}$.

$$\vec{z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \vec{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} \qquad \vec{p} = \begin{pmatrix} p_x \\ p_y \\ 1 \end{pmatrix}$$





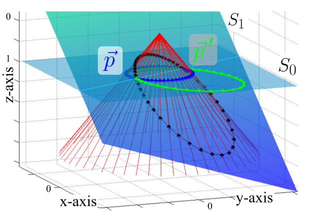
Sensor tilt • tilt shifted coordinates

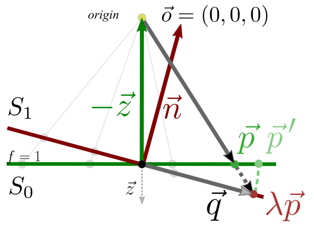
$$\hat{X} \xmapsto{\text{extrinsics}} \hat{P} \xmapsto{\text{lens}} \hat{p} \xmapsto{\text{sensor}} \hat{p}' \xmapsto{\text{intrinsics}} \hat{x}$$

The projected point forms a vector $\lambda\vec{p}$ touching the tilted plane S_1, connecting with $\vec{q}$ in the local coordinates of S_1.

$$\vec{q} = \lambda\vec{p} - \vec{z} \qquad \rightarrow \qquad \lambda = \frac{\vec{n}\cdot\vec{z}}{\vec{n}\cdot\vec{p}}$$

$$\lambda = \frac{n_z}{n_x p_x + n_y p_y + n_z}$$





Sensor tilt • tilt shifted coordinates

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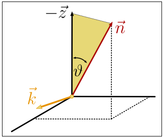


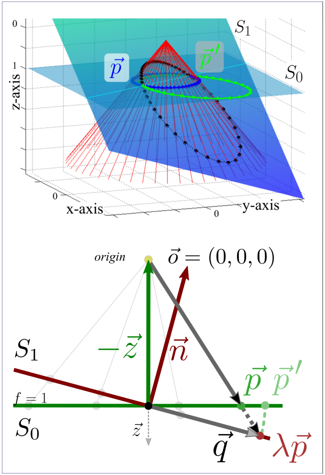
Rotation axis, $\vec{k} = -\vec{n} \times \vec{z}$

$$\vec{k} = \begin{pmatrix} -n_y \\ n_x \\ 0 \end{pmatrix}$$

Rodrigues' rotation matrix:

$$\hat{R} = \hat{\sigma}_1 + \hat{k} + \hat{k}^2 \frac{1}{1+\cos(\vartheta)}$$

$$\hat{R} = \begin{pmatrix} 1 + \frac{n_x^2}{n_z - 1} & \frac{n_x n_y}{n_z - 1} & n_x \\ \frac{n_x n_y}{n_z - 1} & 1 + \frac{n_y^2}{n_z - 1} & n_y \\ -n_x & -n_y & -n_z \end{pmatrix}$$



Sensor tilt • tilt shifted coordinates

$$\hat{X} \xmapsto{\text{extrinsics}} \hat{P} \xmapsto{\text{lens}} \hat{p} \xmapsto{\text{sensor}} \hat{p}' \xmapsto{\text{intrinsics}} \hat{x}$$

To perform a rotation around the projective plane origin:

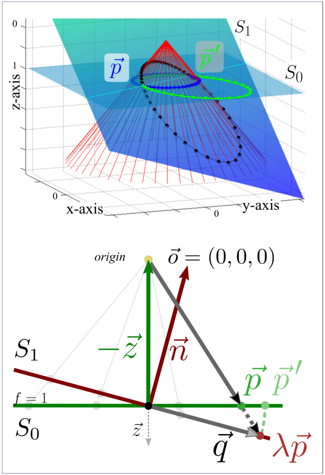
1. the points are shifted to the origin
2. rotated
3. shifted back

$$\vec{p}\,' = \hat{R}\left(\lambda\vec{p} - \vec{z}\right) + \vec{z}$$

The tilt shifted map $\vec{p} \mapsto \vec{p}\,'$:

$$p_x{}' = \lambda \frac{(n_x^2 + n_z(n_z - 1))p_x + n_x n_y p_y}{n_z - 1}$$

$$p_y{}' = \lambda \frac{(n_y^2 + n_z(n_z - 1))p_y + n_x n_y p_x}{n_z - 1}$$



Sensor tilt • pixel coordinates

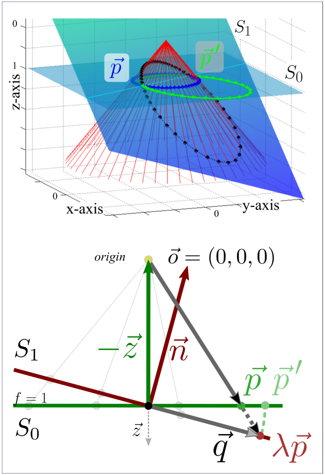
$$\hat{X} \xrightarrow{\text{extrinsics}} \hat{P} \xrightarrow{\text{lens}} \hat{p} \xrightarrow{\text{sensor}} \hat{p}' \xrightarrow{\text{intrinsics}} \hat{x}$$

The mapping from homogeneous camera coordinates to pixel coordinates is the linear transformation

$$\hat{x} = \hat{K}\hat{p}'$$

$\hat{K}$ is the camera matrix containing the (f_x, f_y) focal lengths & optical centre (x_c, y_c).

$$\hat{K} = \begin{pmatrix} f_x & 0 & x_c \\ 0 & f_y & y_c \\ 0 & 0 & 1 \end{pmatrix}$$



Sensor tilt • tilt corrected coordinates

$$\hat{x} \xrightarrow{\text{intrinsics}} \hat{p}' \xrightarrow{\text{sensor}} \hat{p} \xrightarrow{\text{lens}} \hat{P} \xrightarrow{\text{extrinsics}} \hat{X}$$

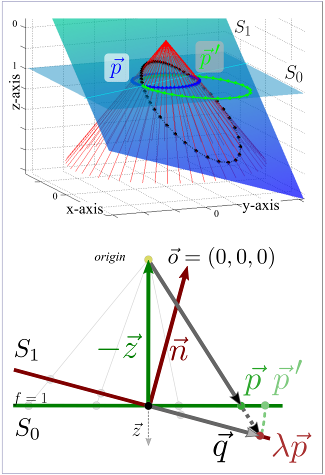
The tilt compensated coordinates $\vec{p}$ are obtained by:

1. shift the points to the origin
2. perform the inverse rotation $\hat{R}^\dagger$
3. shift back
4. rescale by dividing by the z component

$$\vec{p} = \lambda' \left(\hat{R}^\dagger \left(\vec{p}\,' - \vec{z} \right) + \vec{z} \right)$$

the scale factor λ' is the pinhole projection,

$$\lambda' = \left(n_x p'_x + n_y p'_y + 1 \right)^{-1}$$



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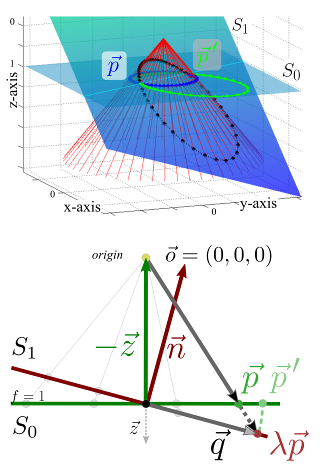
$$\hat{x} \xmapsto{\text{intrinsics}} \hat{p}' \xmapsto{\text{sensor}} \hat{p} \xmapsto{\text{lens}} \hat{P} \xmapsto{\text{extrinsics}} \hat{X}$$

The tilt corrected map $\vec{p}\,' \mapsto \vec{p}$:

$$p_x = \lambda' \frac{\left(n_x^2 + n_z - 1\right) p_x{}' + n_x n_y p_y{}'}{n_z - 1}$$

$$p_y = \lambda' \frac{\left(n_y^2 + n_z - 1\right) p_y{}' + n_x n_y p_x{}'}{n_z - 1}$$



Sensor tilt • summary

The tilt shifted map $\vec{p} \mapsto \vec{p}\,'$:

$$
\begin{aligned}
{p_x}' &= \left(\left(n_x^2 + n_z\,(n_z - 1)\right) p_x + n_x n_y p_y\right) \left(n_x p_x + n_y p_y + n_z\right)^{-1} (n_z - 1)^{-1} \\
{p_y}' &= \left(\left(n_y^2 + n_z\,(n_z - 1)\right) p_y + n_x n_y p_x\right) \left(n_x p_x + n_y p_y + n_z\right)^{-1} (n_z - 1)^{-1}
\end{aligned}
$$

The tilt corrected map $\vec{p}\,' \mapsto \vec{p}$:

$$
\begin{aligned}
p_x &= \left(\left(n_x^2 + n_z - 1\right) {p_x}' + n_x n_y {p_y}'\right) \left(n_x {p_x}' + n_y {p_y}' + 1\right)^{-1} (n_z - 1)^{-1} \\
p_y &= \left(\left(n_y^2 + n_z - 1\right) {p_y}' + n_x n_y {p_x}'\right) \left(n_x {p_x}' + n_y {p_y}' + 1\right)^{-1} (n_z - 1)^{-1}
\end{aligned}
$$