Hierarchical Latent Variable Models for Neural Data Analysis

Charlie Gillet cgillet@ucsd.edu
Mohamed Elshazly melshazly@ucsd.edu
Nathen Lee nal008@ucsd.edu
Mikio Aoi maoi@ucsd.edu

Abstract

All human experience is mediated by the brain via the coordinated activity of neurons across multiple regions, with their firings encoding the feedback and ideas associated. The field of neural encoding and decoding aims to identify suitable representations that can describe the relationship between external stimuli or behaviors and the neural activity observed through electrodes or larger-scale brain imaging techniques. Uncovering these representations has numerous potential applications, including improved diagnosis of brain diseases, the development of brain-computer interfaces (BCIs) that enable thought-controlled devices/prosthetics, and the restoration of functions lost from brain damage. As data collection technologies mature, we aim to build on established techniques to develop new methods for inferring neural firing rates between multiple brain regions with a focus on interpretability, ultimately uncovering relationships between them.

Code: `https://github.com/mo-mo35/DSC180A-Quarter1/tree/main`

1 Introduction

As methods for recording neural data advance rapidly in both volume and speed (Buccino, Garcia and Yger 2022), neurophysiologists face increasing challenges in developing innovative techniques to assess and sort incoming spike signals (neuron firing rates) and inferring relationships between neural activities across different brain regions. We seek to create and utilize a method for extracting shared and independent latent features that accurately represent interpretable neural population dynamics across distinct brain regions.

This will be accomplished by utilizing the Auto-Encoding Variational Bayes framework introduced by researchers Kingma and Welling (2013) who have worked to combine variational inference— a distribution approximation method— with neural networks in order to introduce a probabilistic effect to auto-encoders in the latent space. In addition, we apply the work done by Bach and Jordan (2006) on Probabilistic Canonical Correlation Analysis (PCCA) to the latent space in order to infer relationships between the latent features across regions.

Finally, in order to give the latent variables the flexibility to evolve over time (e.g. to represent how firing varies over time) we make the assumption that they will each be generated by a Gaussian Process (GP) with a Radial Basis Function (RBF) kernel rather than a typical parametric form. Additionally, to ensure non-negativity of firing rates, we apply an exponentiation step to transform the latent representations back to observed space. We will aim to combine the three to provide an interpretable approach for neural population dynamics.

2 Methods

2.1 Overview

The model we wish to construct builds on four main core components that serve as the foundation, allowing it to operate as we desire it to. These four components are: the FA framework, AEVB framework, PCCA, and GPs. By using these core principles, our latent variable model will be able to extend the principles of factor analysis to extract latent representations within neural data to provide more interpretable firing rate dynamics than other current methods.

2.2 Factor Analysis

Factor Analysis models are built off of three core objectives: latent variables, dimensionality reduction, and noise filtering. The model's key idea is to draw correlations in the data and represent them in fewer variables or dimensions (Rubin and Thayer 1982). Factor Analysis is performed utilizing the Expectation-Maximization algorithm (EM) as it is essential in learning and optimizing the parameters that will be used in the model. The E-step requires computing the expected log-likelihood with respect to the latent variables while the M-step requires finding the parameters that maximize it (Ghahramani, Hinton et al. 1996). These steps are then constantly executed over a set number of iterations or until it converges.

2.3 Variational Inference and Auto-Encoding Variational Bayes

Variational Autoencoders (VAEs) are probabilistic models that learn a probabilistic representation of data from an encoder-decoder process. The encoder takes observed data, x, and maps it into a latent space represented by a probability. Given the data, it then approximates the probability of a latent variable, z, which is known as the posterior distribution p(z | x). Meanwhile, the decoder takes the latent variable sampled from the prior distribution p(z), and reconstructs the data by mapping it back onto the observed data based on the maximum likelihood p(x | z) (Kingma and Welling 2013).

Variational Inference is a critical piece in the encoding process because directly computing the true posterior distribution is intractable. By assuming our likelihood to be a Poisson distribution to reflect the biological dynamics of the brain, we no longer have a conjugate pair of prior and likelihood distributions and cannot use EM for our parameters. Variational inference introduces a way to approximate the true posterior with a simpler distribution. To measure how well the approximation aligns with the true posterior, the evidence lower bound (ELBO) is computed, and the greater the value is, the closer the approximation is to the true posterior. Therefore, we look to maximize the ELBO to optimize the model.

1–3.

$$\mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p(\boldsymbol{x}, \boldsymbol{z})}{q_\phi(\boldsymbol{z} \mid \boldsymbol{x})}\right] = \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p_\theta(\boldsymbol{x} \mid \boldsymbol{z}) p(\boldsymbol{z})}{q_\phi(\boldsymbol{z} \mid \boldsymbol{x})}\right] \tag{1}$$

$$= \mathbb{E}_{q_\phi(z|x)}[\log p_\theta(\boldsymbol{x} \mid \boldsymbol{z})] + \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p(\boldsymbol{z})}{q_\phi(\boldsymbol{z} \mid \boldsymbol{x})}\right] \tag{2}$$

$$= \underbrace{\mathbb{E}_{q_\phi(z|x)}[\log p_\theta(\boldsymbol{x} \mid \boldsymbol{z})]}_{\text{reconstruction term}} - \underbrace{\mathscr{D}_{\mathrm{KL}}(q_\phi(\boldsymbol{z} \mid \boldsymbol{x}) \,||\, p(\boldsymbol{z}))}_{\text{prior matching term}} \tag{3}$$

The ELBO is composed of the reconstruction or evidence of our likelihood and the other is the Kullback–Leibler (KL) divergence between our approximate posterior and the latent marginal (Kingma and Welling 2013). The KL divergence is a representation of the distance between two distributions and is useful for approximating. Because of this, we wish to minimize this value, thus maximizing the ELBO.

It is worth noting that the ELBO can be seen to be made of two parts, the reconstruction term which pushes the lower bound to match the data, and the KL divergence which seeks to force the shape of the approximate posterior to our GP prior. This regularizes the latent space and prevents overfitting.

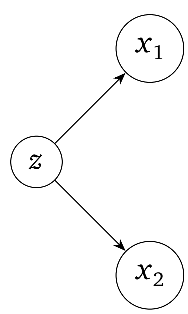
In practice, computing the reconstruction's integral is extremely costly in high dimensions and difficult to solve explicitly. Therefore in order to bypass this, we use Monte Carlo sampling for approximating the expectation. However, directly sampling is a major issue since we cannot optimize our parameters through backpropagation. An easy fix is to use the reparameterization trick to transform points sampled from a simpler distribution (e.g. standard Gaussian) to latent space using the parameters from our approximate posterior (Kingma and Welling 2013). Doing this allows for both an easier integration of the expectation for the gradient of our lower bound and allows us to still sample from our approximate posterior.

$$\tilde{L}^A(\theta, \phi; x^{(i)}) = \frac{1}{L}\sum_{l=1}^{L} \log p_\theta(x^{(i)}, z^{(i,l)}) - \log q_\phi(z^{(i,l)}|x^{(i)})$$

$$z = \mu_\phi(x) + \sigma_\phi(x) \cdot \epsilon$$
$$\text{where } \epsilon \sim \mathcal{N}(0, 1)$$

2.4 Probabilistic Canonical Correlation Analysis

Probabilistic Canonical Correlation Analysis (PCCA) is an extension of canonical correlation analysis (CCA) that reaches into probability to model relationships between two sets of multivariate data while taking noise into account. This method assumes that the two separate observed datasets, X1 and X2, share the same latent variable, Z. PCCA utilizes a probabilistic framework to find pairs of linear transformations that reveal correlation in the latent space between two sets of data and the EM algorithm is used to optimize this process (Bach and Jordan 2006).



2.5 Gaussian Processes

Gaussian processes (GPs) are stochastic processes where any finite set of points will have a joint multivariate Gaussian distribution (Williams and Rasmussen 2006). GPs are made up of a mean function and covariance function and aim to capture relationships between points by taking a prior distribution and using the observed data to generate a posterior distribution while also addressing and handling any noise.

$$y_i \sim GP\big(\sigma, k_\theta(\mathbf{x}, \mathbf{x}')\big) + \epsilon_i$$

Due to its ability to deal with noise, GPs are also useful in smoothing data, which is very helpful in constructing latent variable models. Smooth data improves the mapping between latent variables and observed data causing a reduction in the influence of noise and ultimately improving model performance.

3 Results and Discussion

3.1 Simulated Data

In order to partially validate our assumptions we simulated decoding from the latent space of our framework back into estimated spike trains that would resemble electrophysiology data from the brain.

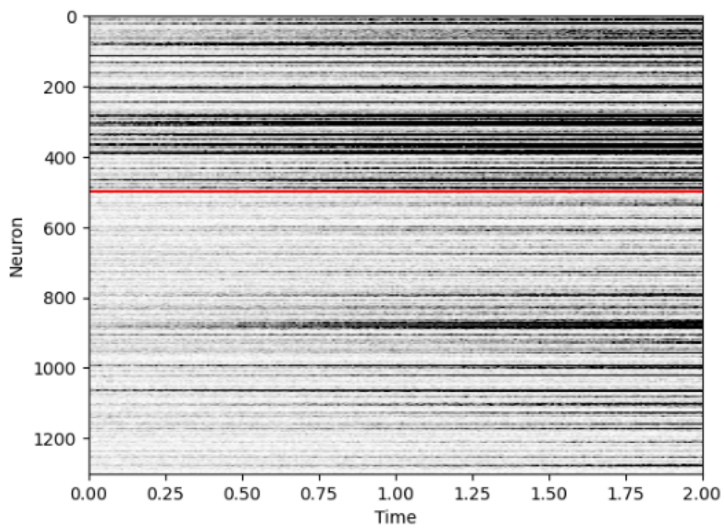


Figure 1: The figure shows a raster plot of these simulated spike trains, with the first region having a neural population of 500 and the second a population of 800. We can see how some areas share the same firing rate across regions; while other areas fire or stay inactive independently.

We generated these spike trains by randomly generating the underlying GP for our latent variables, then transformed them back into the observed space via a block matrix with factor loadings for each latent variable vector. Additionally, we exponentiated our generated observed variables to ensure they were non-negative before passing them into a Poisson distribution to create our spikes.

3.2 Limitations and Future Work

The main limitation we faced while constructing our latent variable model lies within the data. As the data we worked with in this project is simulated, the data does not accurately represent and reflect real neural data. Additionally, there is far less simulated data we worked with compared to the amount of real data we will be working with in the future. Because of this, while our model is functional, it is only guaranteed to generate results for the simulated data we are working with. Moving forward, there will be changes made to push past these limitations and ensure our model will perform up to standard.

As previously stated, we will begin incorporating real data instead of our simulated data as our model continues through development. We plan on using data collected from the International Brain Laboratory (IBL) from their experiments on mice. These experiments have mice undergo tasks that involve a reward if completed successfully. The IBL database contains large amounts of neural data collected from various different mice over multiple experiment trials. Due to the large amount of data we will be working with when we incorporate the real data, addressing noise and dimensionality reduction will be far more significant in ensuring our model performs to the best of its ability. As we look to further advance our model through optimization and fine tuning, our model will slowly adapt to working with real data and generate the results we are looking for.

4 Conclusion

This model integrates FA, AEVB, PCCA, and GPs to extract and interpret neural population dynamics, addressing key challenges in dimensionality reduction and noise influence. Results from working on simulated data have shown steady progress and future work using IBL datasets will refine model performance and ultimately be thoroughly applicable in neuroscience research. Our model offers potential for advancing our means of analysis on neural data by providing better representations that keep pace with the constantly evolving data collection methods.

References

Bach, Francis R., and Michael I. Jordan. 2006. "A Probabilistic Interpretation of Canonical Correlation Analysis." Technical Report 688, University of California, Berkeley, Department of Statistics, Berkeley, CA, USA. [Link]

Buccino, Alessio P, Samuel Garcia, and Pierre Yger. 2022. "Spike sorting: new trends and challenges of the era of high-density probes." *Progress in Biomedical Engineering* 4 (2), p. 022005

Ghahramani, Zoubin, Geoffrey E Hinton et al. 1996. "The EM algorithm for mixtures of factor analyzers." Technical Report, Technical Report CRG-TR-96-1, University of Toronto

Kingma, Diederik P, and Max Welling. 2013. "Auto-encoding variational bayes." *arXiv preprint arXiv:1312.6114*

Rubin, Donald B, and Dorothy T Thayer. 1982. "EM algorithms for ML factor analysis." *Psychometrika* 47 : 69–76

Williams, Christopher KI, and Carl Edward Rasmussen. 2006. *Gaussian processes for machine learning*. 2 MIT press Cambridge, MA