Plummer Sphere:

Basic information:

Developed in 1911 by Henry Plummer (1875-1946), the Plummer sphere, also referred to as the Plummer potential, was initially a means to model globular clusters and spherically symmetric galaxies. Close approach of massive bodies at high velocities caused complications in force measurements, and so this model was designed to soften the force at short distances.

Using spherically symmetric structures as the primary objects of interest for this model means that the density and potential equations derived for it are relatively simple, and solely dependent on the separation distance.

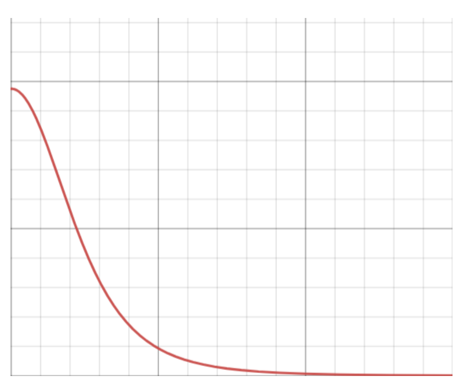
With more accurate and versatile models having been developed since, the Plummer model is no longer used to study/simulate these systems for major research purposes. It is however employed as a toy model in many studies, as a starting point for more complex systems, or to introduce N-body simulations of stellar systems and globular clusters to those new to the topic. This was also the model we used in our Techniques course for force softening in N-body simulations.

Many papers today also focus on either improving the Plummer model, or just simply showing its drawbacks. Most applications however, use it as a simple force softener and not much beyond that.

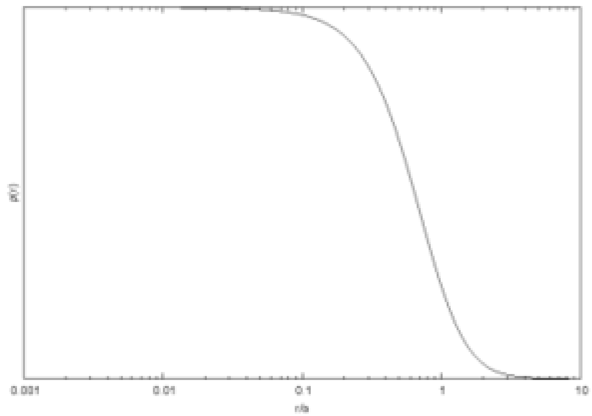
Equations and Plots:

3 Dimensional Density Profile: with M_0 = total cluster mass, a = Plummer radius (scale parameter of total radius)

$$\rho_p(r) = \frac{3M_0}{4\pi a^3}\left(1 + \frac{r^2}{a^2}\right)^{-5/2}$$



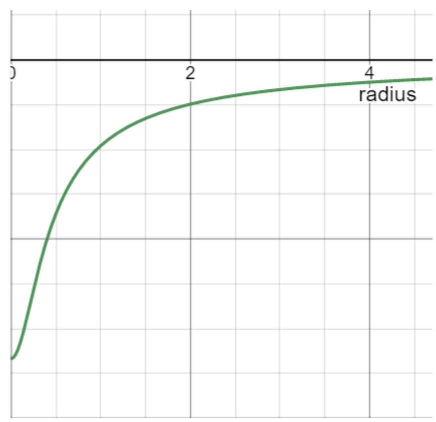
Separation vs Density(from Desmos)



<u>Separation vs Density (Wikipedia)</u>

<u>Plummer potential:</u>

$$\Phi_p(r) = \frac{-GM_0}{\sqrt{r^2+a^2}}$$

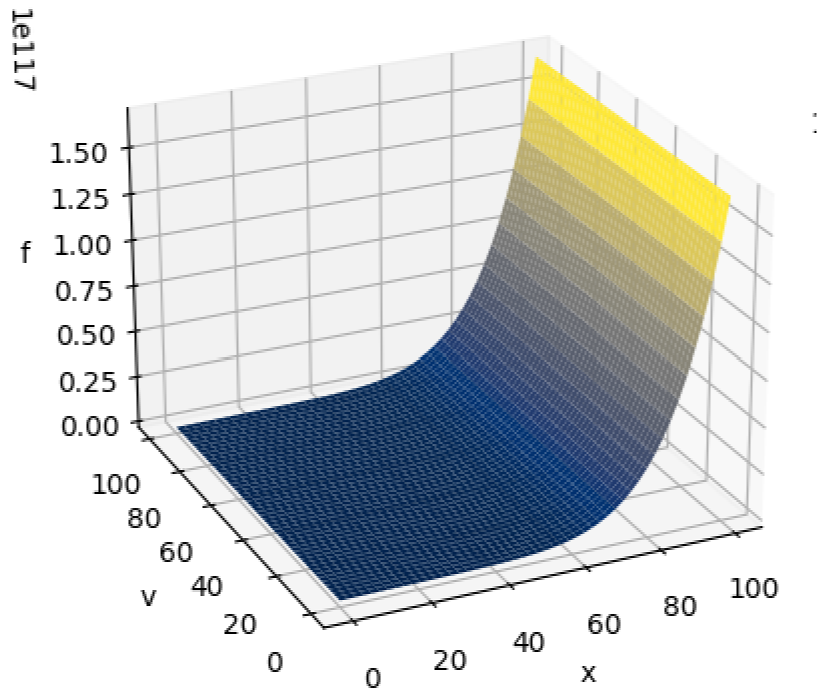


<u>Separation vs Potential (Desmos)</u>

The constant ‘a’ causes the force softening and prevents the potential from reaching an infinite value at close distances.

Isotropic Distribution Function: if E<0, f =0, otherwise E = $\frac{1}{2}v^2 + \Phi_p(r)$ where E is the specific energy (J/kg)

$$f(x, v) = \frac{24\sqrt{2}}{7\pi^3}\frac{a^2}{G^5 M_0^4}(-\ E(x, v))^{7/2}$$



Distribution function f(x,v)

As far as I can tell, this is a distribution of the energy, but having done dimensional analysis, the units at the end are really strange and I'm not sure how to interpret it.

Velocity Dispersion:

Statistical dispersion of radial velocities in the cluster. This, along with the Plummer potential can be used to estimate the total mass of the cluster.

Aside: statistical dispersion- how much a distribution stretches away from its mean value.

$$\sigma_p^2(r) = \frac{GM_0}{6\sqrt{r^2+a^2}}$$

Articles:

[1]J. Bovy, “Detecting disruption of Dark Matter Halos with Stellar Streams,” *APS*, Mar. 2016. https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.116.121301#fulltext

[2]C. Dyer and P. Ip, “Softening in N body Simulations of Collisionless Systems,” *Astrophysics Data System*, May 1993. https://adsabs.harvard.edu/full/1993ApJ...409...60D

[3]W. Dehnen, “Towards Optimal Softening in 3D N-body codes I: Minimizing the Force Error ,” *Oxford Academic*, Jun. 2001. https://academic.oup.com/mnras/article/324/2/273/1020633

[4]K. Long and C. Murali, “Analytical Potentials for Barred Galaxies,” *Astrophysics Data System*, Sep. 1992. https://articles.adsabs.harvard.edu//full/1992ApJ...397...44L/0000044.000.html