Part I : Plummer Sphere

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May 2023

1 Introduction

The first 3 weeks of the project were dedicated to basic background research and introduction to one of the simplest N-body systems; the Plummer sphere.

Considering that one of the main goals of the project is to understand how the energy of the system evolves, two diagnostics (density profile, and mean radial velocity profile) are observed.

The theoretical model is compared with the simulated system to ensure that the n-body system(s) we create for all subsequent simulations will follow the theory as accurately as possible for this type of system.

2 Background Theory

The primary sources for the theoretical basis for this model are Aarseth's 1974 paper[2], and the project supervisor's thesis [1].

To ease calculations and derivations, we will let $G = M = R = 1$ where the G, M, and R represent the gravitational constant, the total mass of the cluster, and a dimensional parameter for the cluster respectively.

The density profile is then given by

$$\rho(\mathbf{r}) = \frac{(3/4\pi)}{[1+(r/R)^2]^{5/2}} \tag{1}$$

and the initial distribution function is given by

$$f(\mathbf{r}, \mathbf{V}) = \frac{24\sqrt{2}}{7\pi^3}(-E)^{7/2} \tag{2}$$

These two equations are enough to analyze the first diagnostic (density profile). For the second, we need to find the mean velocity moments given by [1]

$$< v_r^m v_t^n >= \frac{4\pi}{\rho} \int_{\Phi(r)}^{\Phi(\infty)} dE \int_0^{J_{max}} \frac{JdJ}{r^2 v_r} f(E, J) v_r^m v_t^n \tag{3}$$

which for the purpose of this portion of the project, reduces to

$$< v_r^2 >= \frac{4\pi}{\rho} \int_{\Phi(r)}^{0} f(E)dE \int_0^{J_{max}} \frac{JdJ}{r^2} v_r. \tag{4}$$

Solving this integral gives

$$< v_r^2 >= \frac{1}{6}(-\Phi) = \frac{1}{6(1+\frac{r^2}{a^2})^{1/2}} \tag{5}$$

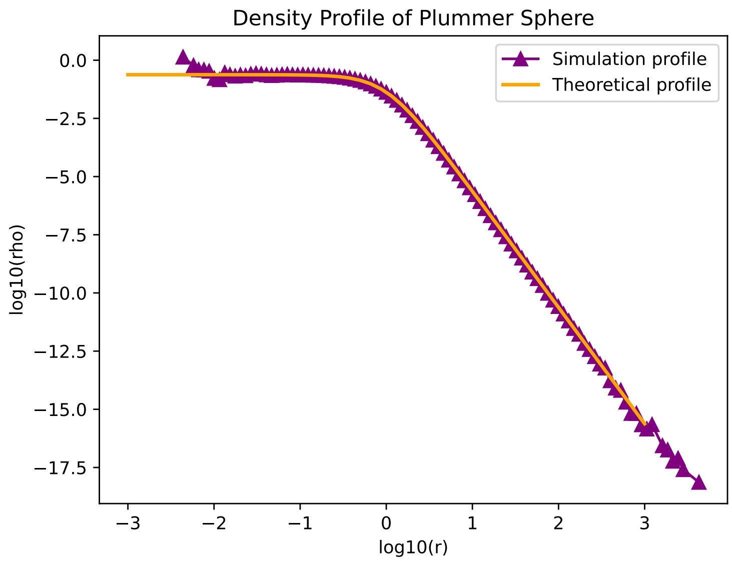
Since we're considering an isotropic, spherical system, it follows that the transverse velocity consists of the components of velocity that are transverse to the radial, so we expect that $< v_t^2 >$ will simply be twice that of $< v_r^2 >$, or more explicitly

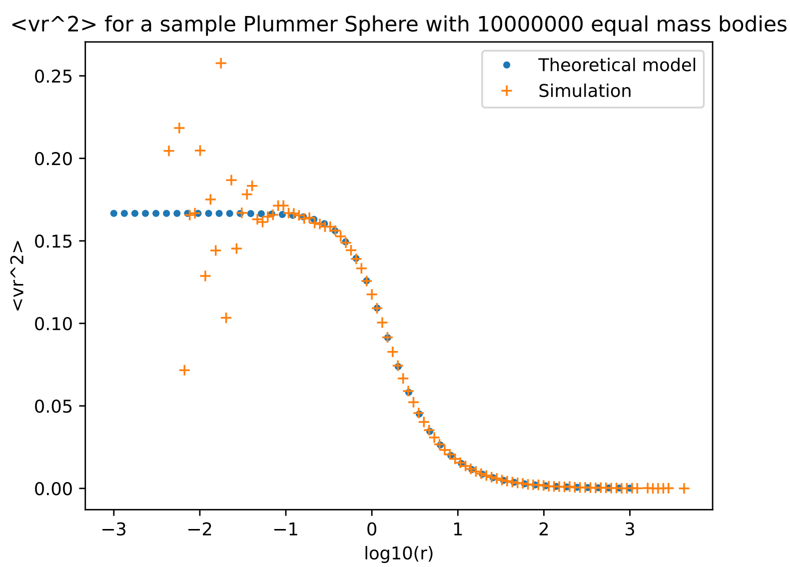
$$< v_t^2 >=< v_\theta^2 > + < v_\phi^2 >= \frac{1}{3}(-\Phi) = \frac{1}{3(1+\frac{r^2}{a^2})^{1/2}} \tag{6}$$

where we assume that $< v_r^2 >=< v_\theta^2 >=< v_\phi^2 >$.

3 Comparison of Theoretical and Simulated Models

The code for the system(s) created for this part of the project was written by directly following the steps highlighted in the appendix of Aarseth's paper [2]. The following figures compare the diagnostic profiles of the simulated system created using this paper, and the theoretical model shown by directly plotting the main diagnostic equations discussed in the previous section.





References

[1] Joseph MacMillan. ***Adiabatic Growth of Black Holes***. National Library of Canada= Bibliotheque nationale du Canada, Ottawa, 2002.

[2] M.Henon S.J. Aarseth and R.Wielen. **A Comparison of Numerical Methods for the Study of Star Cluster Dynamics**. *Astronomy and Astrophysics*, 37:183–187, 1974.