Chapter 1 - The Reals

1.2 - Basic Set Theory Definitions

Definition 1.1

- A set is an unordered collection of elements.
- x in S is written as $x \in S$, where x is an element of S.
- Set builder notation:

$$S = \{\text{elements} \quad : \quad \text{conditions to be in the element}\}$$

  For example, $\{1, 4, 9, 16, \dots\} = \{n^2 \,:\, n \in \mathbb{N}\}$.
- The empty set $\emptyset$ is the set with no elements.
- $A \subseteq B$ means that A is a subset of B. This means that for all x, if $x \in A$, then $x \in B$.
- $A \cap B = \{x \,:\, x \in A \text{ and } x \in B\}$
- $A \cup B = \{x \,:\, x \in A \text{ or } x \in B\}$
- $A - B = \{x \,:\, x \in A \text{ and } x \notin B\}$
- A and B are *disjoint* if $A \cap B = \emptyset$
- If $A \subseteq U$, where U is the universal set, then the *complement* of A in U is $A^c = U - A$
- The cartesian product $A \times B = \{(a, b) \,:\, a \in A \text{ and } b \in B\}$
- The power set of a set A is $\mathcal{P}(A) = \{X \,:\, X \subseteq A\}$
- If $A_1, A_2, A_3, \dots, A_n$ are all sets, then the union of all of them is

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$$

- If $A_1, A_2, A_3, \dots, A_n$ are all sets, then the intersection of all of them is

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

- The cardinality of A is the number of elements or size of the set, denoted by $|A|$