

勉強したことのまとめ

大豆生田 幹

Contents

1	07/08/2024	2
1.1	光の測地線方程式	2
1.2	積分	3
2	28/08/2024	6
2.1	$P \gg M$	6
2.2	$P \approx 3M(1 + \epsilon)$	7
3	23/09/2024	9
3.1	3 次式の解	9
4	02/12/2024	11
4.1	Buchdahl 時空における光の軌道	11
4.2	パラメータの制約	12
5	16/12/2024	15
5.1	Buchdahl 時空の曲率	15

Chapter 1

07/08/2024

1.1 光の測地線方程式

粒子の場を表す方程式である、クラインゴルドン方程式

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \left(\frac{mc}{\hbar} \right)^2 \right) \phi(\mathbf{x}, t) = 0$$

質量のない光では

$$\begin{aligned} 0 &= \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \phi(\mathbf{x}, t) \\ &= \square \phi(\mathbf{x}, t) \end{aligned}$$

これを一般の時空に拡張すると

$$g^{ij} \nabla_i \nabla_j \phi = 0$$

ここで、

$$\phi = C e^{\frac{is}{\epsilon}}$$

と書くと

$$\begin{aligned} \nabla_j \phi &= \nabla \left(C e^{\frac{is}{\epsilon}} \right) \\ &= (\nabla_j C) e^{\frac{is}{\epsilon}} + \frac{iC}{\epsilon} e^{\frac{is}{\epsilon}} (\nabla_j S) \\ \nabla_i \nabla_j \phi &= \nabla_i \left((\nabla_j C) e^{\frac{is}{\epsilon}} + \frac{iC}{\epsilon} e^{\frac{is}{\epsilon}} (\nabla_j S) \right) \\ &= (\nabla_i \nabla_j C) e^{\frac{is}{\epsilon}} + (\nabla_i S) (\nabla_j C) \frac{2i}{\epsilon} e^{\frac{is}{\epsilon}} + (\nabla_i \nabla_j S) \frac{iC}{\epsilon} e^{\frac{is}{\epsilon}} - (\nabla_i S) (\nabla_j S) \frac{1}{\epsilon^2} e^{\frac{2is}{\epsilon}} \\ g^{ij} \nabla_i \nabla_j \phi &= g^{ij} (\nabla_i \nabla_j C) e^{\frac{is}{\epsilon}} + g^{ij} (\nabla_i S) (\nabla_j C) \frac{2i}{\epsilon} e^{\frac{is}{\epsilon}} + g^{ij} (\nabla_i \nabla_j S) \frac{iC}{\epsilon} e^{\frac{is}{\epsilon}} - g^{ij} (\nabla_i S) (\nabla_j S) \frac{1}{\epsilon^2} e^{\frac{2is}{\epsilon}} \\ &= (\square C) e^{\frac{is}{\epsilon}} + (\nabla_i S) (\nabla^i C) \frac{2i}{\epsilon} e^{\frac{is}{\epsilon}} + (\square S) \frac{iC}{\epsilon} e^{\frac{is}{\epsilon}} - (\nabla_i S) (\nabla^i S) \frac{1}{\epsilon^2} e^{\frac{2is}{\epsilon}} \\ &\begin{cases} O(\epsilon^{-2}) : (\nabla_i S) (\nabla^i S) = 0 \\ O(\epsilon^{-1}) : 2 (\nabla_i S) (\nabla^i C) + C (\square S) = 0 \\ O(0) : \square C = 0 \end{cases} \end{aligned} \tag{1.1}$$

ここで、

$$(\nabla_i S) (\nabla^i S) = 0$$

は光の測地線方程式を表している。

$$\nabla_i S = k_i$$

とおけば

$$\begin{aligned}
0 &= (\nabla_i S) (\nabla^i S) \\
&= k_i k^i \\
&= \nabla_j (k_i k^i) \\
&= \nabla_j (k_i g^{ij} k_l) \\
&= g^{il} (\nabla_j k_l) k_i + g^{il} (\nabla_j k_i) k_l \\
&= 2 (\nabla_j k_i) k^i \\
&= 2 g^{lj} (\nabla_j \nabla_i S) k^i \\
&= 2 (\nabla_i g^{lj} k_j) k^i \\
&= 2 (\nabla_i k^l) k^i \\
&= 2 \left(\frac{\partial}{\partial x^i} \left(\frac{dx^l}{d\lambda} \right) + \Gamma^l_{im} \left(\frac{dx^m}{d\lambda} \right) \right) \frac{dx^i}{d\lambda}
\end{aligned}$$

光の測地線方程式

$$0 = \left(\frac{d^2 x^l}{d\lambda^2} \right) + \Gamma^l_{im} \left(\frac{dx^i}{d\lambda} \frac{dx^m}{d\lambda} \right)$$

1.2 積分

$$\begin{aligned}
\left(\frac{d\mu}{d\phi} \right)^2 &= 2MG(\mu) \\
\frac{d\mu}{d\phi} &= -\sqrt{2MG(\mu)} \\
\begin{cases} \mu_1 = -\frac{Q+2M-P}{4MP} \\ \mu_2 = -\frac{1}{P} \\ \mu_3 = -\frac{Q-2M+P}{4MP} \end{cases} & \quad (1.2)
\end{aligned}$$

$$\begin{aligned}
\int_0^{\phi_\infty} d\phi &= \int_{\mu_2}^0 \frac{d\mu}{-\sqrt{2MG(\mu)}} \\
\phi_\infty &= \frac{1}{\sqrt{2M}} \int_0^{\mu_2} d\mu ((\mu - \mu_1)(\mu - \mu_2)(\mu - \mu_3))^{-\frac{1}{2}}
\end{aligned}$$

ここで、

$$\begin{aligned}
\chi &= \sqrt{\mu - \mu_1} \\
d\chi &= \frac{1}{2} (\mu - \mu_1)^{-\frac{1}{2}} d\mu
\end{aligned}$$

とおくと

$$\phi_\infty = \frac{1}{\sqrt{2M}} \int_{\sqrt{-\mu_1}}^{\sqrt{\mu_2 - \mu_1}} \frac{2\chi d\chi}{\sqrt{\chi^2(\chi^2 + \mu_1 - \mu_2)(\chi^2 + \mu - \mu_3)}}$$

ここで、

$$\begin{aligned}\chi &= \sqrt{\mu_2 - \mu_1} \eta \\ d\chi &= \sqrt{\mu_2 - \mu_1} d\eta\end{aligned}$$

とおくと

$$\phi_\infty = \frac{2}{\sqrt{2M}} \int_{\sqrt{\frac{\mu_1}{\mu_1 - \mu_2}}}^1 \frac{\sqrt{\mu_2 - \mu_1} d\eta}{\sqrt{-(\mu_2 - \mu_1)(1 - \eta^2)((\mu_2 - \mu_1)\eta^2 - \mu_1 + \mu_3)}}$$

ここで、

$$\begin{aligned}\eta &= \sin x \\ d\eta &= dx \cos x\end{aligned}$$

とおくと

$$\begin{aligned}\phi_\infty &= \frac{2}{\sqrt{2M}} \int_{\sin^{-1} \sqrt{\frac{\mu_1}{\mu_1 - \mu_2}}}^{\frac{\pi}{2}} \frac{\cos x dx}{\sqrt{\cos^2 x \sqrt{-(\mu_2 - \mu_1) \sin^2 x ((\mu_2 - \mu_1)\eta^2 - \mu_1 + \mu_3)}}} \\ &= \frac{2}{\sqrt{2M}} \int_{\sin^{-1} \sqrt{\frac{\mu_1}{\mu_1 - \mu_2}}}^{\frac{\pi}{2}} \frac{dx}{\sqrt{\mu_3 - \mu_1} \sqrt{-\frac{\mu_2 - \mu_1}{\mu_3 - \mu_1} \sin^2 x + 1}} \\ &\quad \begin{cases} \mu_3 - \mu_1 = \frac{Q}{2MP} \\ \mu_2 - \mu_1 = \frac{Q + 6M - P}{4MP} \end{cases} \end{aligned} \tag{1.3}$$

$$\begin{cases} \frac{\mu_2 - \mu_1}{\mu_3 - \mu_1} = \frac{Q + 6M - P}{2Q} \\ \frac{\mu_1}{\mu_1 - \mu_2} = \frac{Q + 2M - P}{Q + 6M - P} \end{cases} \tag{1.4}$$

$$\phi_\infty = \frac{2}{\sqrt{2M}} \int_{\sin^{-1} \sqrt{\frac{Q+2M-P}{Q+6M-P}}}^{\frac{\pi}{2}} \frac{dx}{\sqrt{\frac{Q}{2MP}} \sqrt{1 - \frac{Q+6M-P}{2Q} \sin^2 x}}$$

ここで、

$$\begin{aligned}\kappa &= \sqrt{\frac{Q + 6M - P}{2Q}} \\ \zeta &= \sin^{-1} \sqrt{\frac{Q + 2M - P}{Q + 6M - P}}\end{aligned}$$

さらに

$$\begin{aligned}K(\kappa) &= \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - \kappa^2 \sin^2 x}} \\ F(\zeta, \kappa) &= \int_0^\zeta \frac{dx}{\sqrt{1 - \kappa^2 \sin^2 x}}\end{aligned}$$

とおくと

ϕ_∞ についての式

$$\begin{aligned}\phi_\infty &= 2\sqrt{\frac{P}{Q}} \int_\zeta^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - \kappa^2 \sin^2 x}} \\ &= 2\sqrt{\frac{P}{Q}} (K(\kappa) - F(\zeta, \kappa))\end{aligned}$$

Chapter 2

28/08/2024

2.1 $P \gg M$

$$\begin{cases} Q \approx P \left(1 + 2 \left(\frac{M}{P} \right) - 8 \left(\frac{M}{P} \right)^2 \right) \\ \kappa^2 \approx 4 \left(\frac{M}{P} \right) \\ \zeta \approx \frac{\pi}{4} - \frac{1}{2} \left(\frac{M}{P} \right) \end{cases}$$

以上を考慮して ϕ_∞ を計算する

$$\begin{aligned} \phi_\infty &= 2 \sqrt{\frac{P}{Q}} \int_{\zeta}^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - \kappa^2 \sin^2 x}} \\ &\approx 2 \sqrt{\frac{P}{Q}} \int_{\zeta}^{\frac{\pi}{2}} \left(1 + \frac{1}{2} \kappa^2 \sin^2 x \right) dx \\ &= 2 \sqrt{\frac{P}{Q}} \int_{\zeta}^{\frac{\pi}{2}} \left(1 + \frac{1}{2} \kappa^2 - \frac{1}{4} \kappa^2 \cos 2x \right) dx \\ &\approx 2 \left(1 - \frac{M}{P} \right) \int_{\frac{\pi}{4} - \frac{M}{2P}}^{\frac{\pi}{2}} \left(1 + 2 \frac{M}{P} - \frac{M}{P} \cos 2x \right) dx \\ &= 2 \left(1 - \frac{M}{P} \right) \left[x + \frac{M}{P} x - \frac{M}{2P} \sin 2x \right]_{\frac{\pi}{4} - \frac{M}{2P}}^{\frac{\pi}{2}} \\ &= \frac{\pi}{2} + \frac{2M}{P} \end{aligned}$$

$$2.2 \quad P \approx 3M(1 + \epsilon)$$

$$\left\{ \begin{array}{l} Q \approx 3M + 5M\epsilon \\ \kappa \approx 1 - \frac{2}{3}\epsilon \\ \zeta \approx \sin^{-1} \sqrt{\frac{1}{3}} + \frac{\sqrt{3}}{9 \cos \left(\sin^{-1} \sqrt{\frac{1}{3}} \right)} \epsilon \\ \sqrt{\frac{P}{Q}} \approx 1 - \frac{1}{3}\epsilon \\ \sqrt{1 - \kappa^2} \approx \sqrt{\frac{4}{3}}\epsilon \\ b \approx b_c \left(1 + \frac{3}{2}\epsilon^2 \right) \end{array} \right.$$

$\kappa \approx 1$ での公式

$$K(\kappa) = \ln \frac{4}{\kappa'} + \left(\frac{1}{2} \right)^2 \left(\ln \frac{4}{\kappa'} - \frac{2}{1 \cdot 2} \right) \kappa'^2 + \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 \left(\ln \frac{4}{\kappa'} - \frac{2}{1 \cdot 2} - \frac{2}{3 \cdot 4} \right) \kappa'^4 + \dots$$

$$F(\zeta, \kappa) = \frac{2}{\pi} \mathbf{K}' \ln \tan \left(\frac{\zeta}{2} + \frac{\pi}{4} \right) - \frac{\tan \zeta}{\cos \zeta} \left(a'_0 - \frac{2}{3} a'_1 \tan^2 \zeta + \dots \right)$$

$$\left\{ \begin{array}{l} a'_0 = \frac{2}{\pi} \mathbf{K}' - 1 \\ a'_n = a'_{n-1} - \left[\frac{(2n-1)!!}{2^n n!} \right]^2 \kappa'^{2n} \end{array} \right.$$

$$\mathbf{K}' = \frac{\pi}{2} \left(1 + \left(\frac{1}{2} \right)^2 \kappa'^2 + \dots \right)$$

$$\kappa' = \sqrt{1 - \kappa^2}$$

以上を考慮して b を計算する

$$K(\kappa) \approx \ln \frac{4}{\kappa'}$$

$$F(\zeta, \kappa) \approx \ln \tan \left(\frac{\zeta}{2} + \frac{\pi}{4} \right)$$

$$\begin{aligned}
\phi_\infty &= 2\sqrt{\frac{P}{Q}}(K(\kappa) - F(\zeta, \kappa)) \\
&\approx 2\left(1 - \frac{\epsilon}{3}\right)\left(\ln \frac{4}{\kappa'} - \ln \tan\left(\frac{\zeta}{2} + \frac{\pi}{4}\right)\right) \\
&= \left(1 - \frac{\epsilon}{3}\right)\ln\left(\frac{\frac{12}{\epsilon}}{\tan^2\left(\frac{\sin^{-1}\sqrt{\frac{1}{3}}}{2} + \frac{\pi}{4}\right)}\right) \\
&\approx \ln\left(\frac{12}{\tan^2\left(\frac{\sin^{-1}\sqrt{\frac{1}{3}}}{2} + \frac{\pi}{4}\right)\epsilon}\right) = \ln \frac{\alpha}{\epsilon}
\end{aligned}$$

ここで、

$$\phi_\infty = \frac{1}{2}\pi + \frac{1}{2}\mu$$

より、

$$\begin{aligned}
\frac{\alpha}{\epsilon} &= \exp\left(\frac{\pi}{2} + \frac{\mu}{2}\right) \\
\epsilon &= \frac{\alpha}{\exp\left(\frac{\pi}{2} + \frac{\mu}{2}\right)}
\end{aligned}$$

したがって

$$\begin{aligned}
b &\approx b_c\left(1 + \frac{3}{2}\epsilon^2\right) \\
&= b_c\left(1 + \frac{3}{2}\left(\frac{\alpha}{\exp\left(\frac{\pi}{2} + \frac{\mu}{2}\right)}\right)^2\right) \\
&= b_c\left(1 + \frac{3}{2}\alpha^2 e^{-\mu} e^{-\pi}\right) \\
&\approx 5.19615M + 3.48228M e^{-\mu}
\end{aligned}$$

Chapter 3

23/09/2024

3.1 3次式の解

$$b^2 = \frac{P^3}{P - 2M}$$

この P を b について解きたい

$$\begin{cases} P = yM \\ b = xM \end{cases}$$

と置き、 y を x について解く問題を考える。

$$0 = P^3 - Pb^2 + 2Mb^2$$

$$0 = y^3M^3 - yx^2M^3 + 2x^2M^3$$

$$0 = y^3 - yx^2 + 2x^2$$

$$y = u + v$$

とおくと

$$0 = (u + v)^3 - (u + v)x^2 + 2x^2$$

$$0 = (u^3 + v^3 + 2x^2) + (3uv - x^2)(u + v)$$

$$u + v \neq 0$$

と仮定すると

$$\begin{cases} u^3 + v^3 + 2x^2 = 0 \\ 3uv - x^2 = 0 \end{cases} \quad (3.1)$$

以上より、片方の符号を採用すると

$$\begin{cases} u^3 = -x^2 + x^2 \sqrt{1 - \left(\frac{x}{3\sqrt{3}}\right)^2} \\ v^3 = -x^2 - x^2 \sqrt{1 - \left(\frac{x}{3\sqrt{3}}\right)^2} \end{cases} \quad (3.2)$$

ここで (u, v) の3乗根の主値をそれぞれ (α, β) とおくと

$$\alpha = \left(-x^2 + x^2 \sqrt{1 - \left(\frac{x}{3\sqrt{3}} \right)^2} \right)^{\frac{1}{3}}$$

$$\beta = \left(-x^2 - x^2 \sqrt{1 - \left(\frac{x}{3\sqrt{3}} \right)^2} \right)^{\frac{1}{3}}$$

$$\omega = \exp \left(\frac{2}{3} \pi i \right)$$

(1) $D < 0$ では $(x > 3\sqrt{3})$

$$(u, v) = \begin{cases} (\alpha, \beta) \\ (\omega\alpha, \omega^2\beta) \\ (\omega^2\alpha, \omega\beta) \end{cases}$$

(2) $D = 0$ では $(x = 3\sqrt{3})$

$$(u, v) = \begin{cases} (\omega\alpha, \omega\beta) \\ (\alpha, \omega^2\beta) \\ (\omega^2\alpha, \beta) \end{cases}$$

(3) $D > 0$ では $(x < 3\sqrt{3})$

$$(u, v) = \begin{cases} (\omega\alpha, \omega\beta) \\ (\alpha, \omega^2\beta) \\ (\omega^2\alpha, \beta) \end{cases}$$

Chapter 4

02/12/2024

4.1 Buchdahl 時空における光の軌道

Buchdahl 時空の計量は以下のように与えられる。

$$g_{ij} = \text{diag} \left(-\frac{(1-f(r))^2}{(1+f(r))^2}, (1+f(r))^4, r^2(1+f(r))^4, r^2(1+f(r))^4 \sin^2 \theta \right)$$
$$f(r) = \frac{a}{2\sqrt{1+kr^2}}$$

測地線を考えるには、赤道面 $\theta = \frac{\pi}{2}$ に注目すれば

$$S = \int \mathcal{L} d\tau$$
$$= \int \left(-\frac{(1-f(r))^2}{(1+f(r))^2} \dot{t}^2 + (1+f(r))^4 \dot{r}^2 + r^2(1+f(r))^4 \dot{\phi}^2 \right) d\tau$$

この変分を計算すれば良い。 \mathcal{L} は t, ϕ をあらわには含まないので、オイラーラグランジュ方程式より

$$0 = \left(\frac{d\mathcal{L}}{d\dot{t}} \right) = \frac{-2(1-f(r))^2}{(1+f(r))^2} \dot{t}$$
$$\therefore E = \frac{(1-f(r))^2}{(1+f(r))^2} \dot{t}$$
$$0 = \left(\frac{d\mathcal{L}}{d\dot{\phi}} \right) = 2r^2(1+f(r))^4 \dot{\phi}$$
$$\therefore L = r^2(1+f(r))^4 \dot{\phi}$$

また、null の条件 $0 = g_{ij} \dot{x}^i \dot{x}^j$ より

$$0 = -\frac{(1-f(r))^2}{(1+f(r))^2} \dot{t}^2 + (1+f(r))^4 \dot{r}^2 + r^2(1+f(r))^4 \dot{\phi}^2$$
$$\therefore \left(\frac{E}{L} \right)^2 = \frac{(1-f(r))^2}{r^4(1+f(r))^6} \left(\frac{dr}{d\phi} \right)^2 + \frac{(1-f(r))^2}{r^2(1+f(r))^6}$$

ここで $u = \frac{1}{r}$ 、 $b = \frac{L}{E}$ とおくと

$$1 = r^4 \left(\frac{du}{dr} \right)^2$$

より

$$\begin{aligned}\left(\frac{1}{b}\right)^2 &= \frac{(1-f(1/u))^2}{(1+f(1/u))^6} \left(\frac{du}{d\phi}\right)^2 + \frac{u^2(1-f(1/u))^2}{(1+f(1/u))^6} \\ \therefore \left(\frac{du}{d\phi}\right)^2 &= G(u) := \left(\frac{1}{b}\right)^2 \frac{(1+f(1/u))^6}{(1-f(1/u))^2} - u^2\end{aligned}$$

4.2 パラメータの制約

$f(r)$ のとりうる範囲を確認しておく。

$$0 \leq f(r) \leq \frac{a}{2}$$

いくつかの条件から、パラメータの制約を考える

$$\begin{aligned}\rho(r) &= \frac{24\pi k f(r)^5}{\pi a^4 (1+f(r))^5} \\ p(r) &= \frac{8k f(r)^6}{\pi a^4 (1-f(r)^2)(1+f(r))^4}\end{aligned}$$

1) weak energy condition (WEC)

$$\begin{aligned}0 &\leq \rho(r) \\ 0 &\leq \rho(r) + p(r)\end{aligned}$$

解析すると

$$\begin{aligned}0 &\leq 1 + \frac{p(r)}{\rho(r)} \\ &= 1 + \frac{8k f(r)^6}{\pi a^4 (1-f(r)^2)(1+f(r))^4} \times \frac{\pi a^4 (1-f(r)^2)(1+f(r))^4}{8k f(r)^6} \\ &= 1 + \frac{f(r)}{3(1-f(r))} \\ \therefore f(r) &\leq 1, \frac{3}{2} \leq f(r)\end{aligned}$$

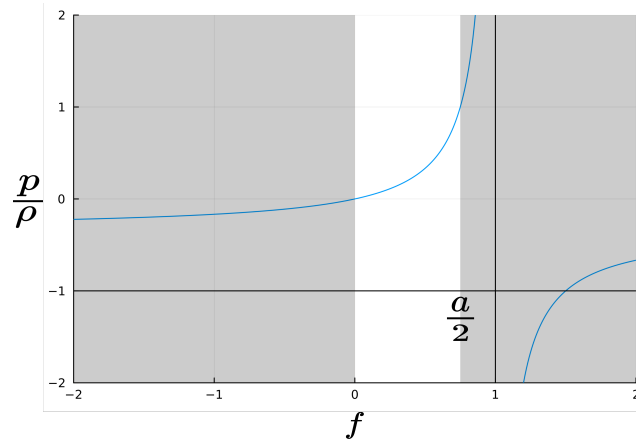


Figure 4.1: WEC

ここでの a の制約は

$$\frac{a}{2} \leq 1$$

2) strong energy condition (SEC)

$$0 \leq \rho(r)$$

$$0 \leq \rho(r) + 3p(r)$$

解析すると

$$0 \leq 1 + \frac{3p(r)}{\rho(r)}$$

$$= 1 + \frac{f(r)}{1 - f(r)}$$

$$\therefore f(r) \leq 1$$

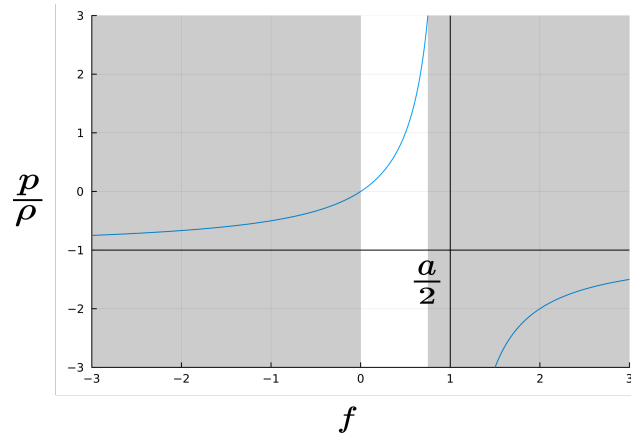


Figure 4.2: SEC

ここでの a の制約は

$$\frac{a}{2} \leq 1$$

3) dominant energy condition (DEC)

$$0 \leq \rho(r)$$

$$0 \leq \rho(r) + |p(r)|$$

解析すると

$$-1 \leq \frac{p(r)}{\rho(r)} \leq 1$$

$$-1 \leq \frac{f(r)}{3(1 - f(r))} \leq 1$$

$$\therefore f(r) \leq \frac{3}{4}, \frac{3}{2} \leq f(r)$$

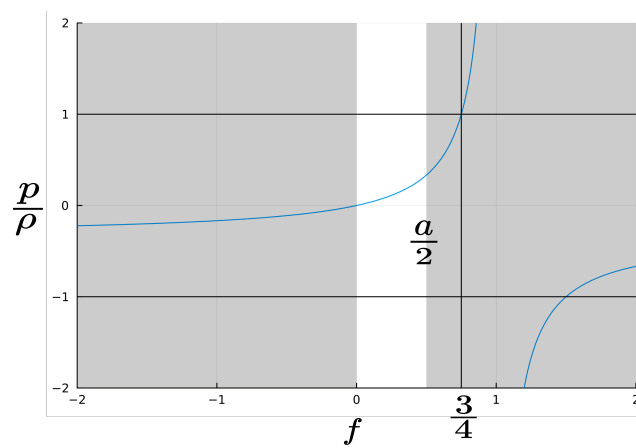


Figure 4.3: DEC

ここでの a の制約は

$$\frac{a}{2} \leq \frac{3}{4}$$

以上より、パラメータの制約は

$$0 \leq a \leq \frac{3}{2}$$

Chapter 5

16/12/2024

5.1 Buchdahl 時空の曲率

$$g_{ab} = \begin{pmatrix} -\frac{(1-f(r))^2}{(1+f(r))^2} & & \\ & (1+f(r))^4 & \\ & & r^2(1+f(r))^4 \end{pmatrix}$$

$$f(r) = \frac{a}{2\sqrt{1+kr^2}}$$

$$\frac{df}{dr} = -\frac{ar}{2(1+kr^2)^{\frac{3}{2}}} = -\frac{af(r)}{1+kr^2}$$

$$\Gamma^a_{bc} = \frac{1}{2}g^{ad} \left(\frac{\partial g_{db}}{\partial x^c} + \frac{\partial g_{dc}}{\partial x^b} - \frac{\partial g_{bc}}{\partial x^d} \right)$$

$$R^a_{bcd} = \frac{\partial \Gamma^a_{bd}}{\partial x^c} - \frac{\partial \Gamma^a_{bc}}{\partial x^d} + \Gamma^a_{ec} \Gamma^e_{bd} - \Gamma^a_{ed} \Gamma^e_{bc}$$

$$\frac{g_{tt}}{\partial x^r} = -\frac{af(r)}{1+kr^2} \cdot \frac{4(1-f(r))}{(1+f(r))^3}$$

$$\frac{g_{rr}}{\partial x^r} = -\frac{af(r)}{1+kr^2} \cdot 4(1+f(r))^3$$

$$\frac{g_{\phi\phi}}{\partial x^r} = 2r(1+f(r)) - \frac{af(r)}{1+kr^2} \cdot 4r^2(1+f(r))$$



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\bar{w}_j = () \int$$

メモ用

1 = 0

$$\begin{cases} 1 = 0 \\ 1 = 0 \end{cases} \tag{5.1}$$

(1)