

4장 연습문제

9.

$$E[X] = (1)\left(\frac{1}{10}\right) + (2)\left(\frac{1}{10}\right) + \dots + (10)\left(\frac{1}{10}\right) = 5.5$$

$$E[X^2] = (1^2)\left(\frac{1}{10}\right) + (2^2)\left(\frac{1}{10}\right) + \dots + (10^2)\left(\frac{1}{10}\right) = 38.5$$

$$\text{Var}(X) = E[X^2] - (E[X])^2 = 38.5 - 5.5^2 = 8.25$$

10.

$$(1) \quad \sum_{x=1}^{\infty} p(x) = \sum_{x=1}^{\infty} c\left(\frac{2}{3}\right)^x = c \frac{\frac{2}{3}}{1 - \frac{2}{3}} = 1 \quad c = \frac{1}{2}$$

$$(2) \quad P\left(\frac{1}{2} \leq X \leq \frac{7}{3}\right) = P(X=1) + P(X=2) = \frac{1}{2}\left(\frac{2}{3} + \left(\frac{2}{3}\right)^2\right) = \frac{1}{3} + \frac{2}{9} = \frac{5}{9}$$

16.

$$E[(X+Y)(2X-Y)]$$

$$= 2E[X^2] - E[Y^2] + E[XY]$$

$$= 2(\text{Var}(X) + (E[X])^2) - E[Y^2] + E[X][Y]$$

$$= 2(2 + 100) - 2 - 50 = 152$$

17.

$$E[X] = \frac{2+2}{3} = \frac{4}{3} \quad E[X^2] = \frac{2+4}{3} = 2 \quad \text{Var}(X) = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$$

$$E[Y] = \frac{2+2}{3} = \frac{4}{3} \quad E[Y^2] = \frac{2+4}{3} = 2 \quad \text{Var}(Y) = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$$

$$E[XY] = \frac{(1)(1) + (1)(2) + (2)(1)}{3} = \frac{5}{3} \quad \text{Cov}(X, Y) = \frac{5}{3} - \frac{16}{9} = -\frac{1}{9} \quad \rho = \frac{-\frac{1}{9}}{\sqrt{\left(\frac{2}{9}\right)\left(\frac{2}{9}\right)}} = -\frac{1}{2}$$