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Computational Methods

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Homework Nbre 4

Exercise 1

Consider the following integral: $\int_1^3 \left(x^3 + x^2 + \frac{1}{x^2} \right) dx$

We give the exact value of the integral: 88/3.

(a) Compute an approximate value of this integral using the trapezoidal rule of integration with n=3 and give the true relative error.

(b) Compute an approximate value of this integral using the trapezoidal rule of integration with n=4 and give the true relative error and the absolute relative approximate error.

(c) Compute an approximate value of this integral using the Simpson's 1/3rd Rule of integration with n=4 and give the true relative error and the absolute relative approximate error.

Exercise 2

Consider the following integral: $\int_0^4 e^{-\frac{x^2}{2}} dx$

(a) Compute an approximate value of this integral using the trapezoidal rule of integration with n=3.

(b) Compute an approximate value of this integral using the trapezoidal rule of integration with n=4 and give the absolute relative approximate error.

(c) Compute an approximate value of this integral using the Simpson's 1/3rd Rule of integration with n=4 and give the absolute relative approximate error.

Solution

Exercise 1

a) $h = \frac{b-a}{n} = \frac{3-1}{3} = \frac{2}{3}$

$f(x) = x^3 + x^2 + \frac{1}{x^2}$

x	f(x)	α	α f(x)
1	3	**1**	3
$1+\frac{2}{3}=\frac{5}{3}$	7.767	**2**	15.535
$\frac{7}{3}$	18.331	**2**	36.664
3	36.111	**1**	36.111
Σ			91.310

$f(1) = 1+1+1 = 3$

$f(\frac{5}{3}) = \frac{125}{27} + \frac{25}{9} + \frac{9}{25} = \frac{125*25+25*3*25+9*27}{27*25} = \frac{5243}{675} = 7.767$

$f(\frac{7}{3}) = \frac{343}{27} + \frac{49}{9} + \frac{9}{49} = \frac{343*49+49*3*49+9*27}{27*49} = \frac{24253}{1323} = 18.331$

$f(3) = 27+9+\frac{1}{9} = \frac{27*9+9*9+1}{9} = \frac{325}{9} = 36.111$

$I \approx \frac{h}{2}\Sigma = \frac{\frac{2}{3}}{2} * 91.310 \approx 30.437$

$e_t = \frac{\frac{88}{3} - 30.437}{\frac{88}{3}} \approx -3.76\%$

b) $h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = 0.5$

$f(x) = x^3 + x^2 + \frac{1}{x^2}$

x	*f(x)*	α	α *f(x)*
1	**3**	**1**	**3**
1.5	**6.069**	**2**	**12.14**
2	**12.25**	**2**	**24.5**
2.5	**22.035**	**2**	**44.07**
3	**36.111**	**1**	**36.111**
Σ			119.821

$I \approx \frac{h}{2}\Sigma = \frac{0.5}{2} * 119.821 = 29.96$

$\in_t = \frac{\frac{88}{3} - 29.96}{\frac{88}{3}} * 100 = -2.12\%$

$|\in_a| = \frac{|29.96 - 30.44|}{29.96} * 100 = 1.60\%$

c) $h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = 0.5$

$f(x) = x^3 + x^2 + \frac{1}{x^2}$

x	*f(x)*	α	α *f(x)*
1	**3**	**1**	**3**
1.5	**6.069**	**4**	**24.28**
2	**12.25**	**2**	**24.5**
2.5	**22.035**	**4**	**88.14**
3	**36.111**	**1**	**36.111**
Σ			176.031

$I \approx \frac{h}{3}\Sigma = \frac{0.5}{3} * 176.031 = 29.34$

$$\epsilon_t = \frac{\frac{88}{3} - 29.34}{\frac{88}{3}} * 100 = -0.023\%$$

$$|\epsilon_a| = \frac{|29.34 - 29.96|}{29.34} * 100 = 2.11\%$$

Exercise 2

a) $h = \frac{b-a}{n} = \frac{4-0}{3} = \frac{4}{3} = 1.33$

$f(x) = e^{\frac{x^2}{2}}$

x	*f(x)*	α	α *f(x)*
0	1.0000	**1**	1.0000
$0 + \frac{4}{3} = \frac{4}{3}$	0.4111	**2**	0.8222
$\frac{8}{3}$	0.0286	**2**	0.0571
4	0.0003	**1**	0.0003
Σ			1.8797

$I \approx \frac{h}{2}\Sigma = \frac{\frac{4}{3}}{2} * 1.8797 = 1.2531$

b) $h = \frac{b-a}{n} = \frac{4-0}{4} = \frac{4}{4} = 1$

x	*f(x)*	α	α *f(x)*
0	1.0000	**1**	1.0000
1	0.6065	**2**	1.2131
2	0.1353	**2**	0.2707
3	0.0111	**2**	0.0222
4	0.0003	**1**	0.0003
Σ			2.5063

$$I \approx \frac{h}{2}\Sigma = \frac{1}{2} * 2.5063 = 1.2531$$

$$|\in_a| = \frac{|1.2531 - 1.2531|}{1.2531} * 100 = 0.0\%$$

c) $h = \frac{b-a}{n} = \frac{4-0}{4} = \frac{4}{4} = 1$

$$f(x) = e^{-\frac{x^2}{2}}$$

x	***f(x)***	**α**	**α *f(x)***
0	1.0000	**1**	1.0000
1	0.6065	**4**	2.4261
2	0.1353	**2**	0.2707
3	0.0111	**4**	0.0444
4	0.0003	**1**	0.0003
Σ			4.9888

$$I \approx \frac{h}{3}\Sigma = \frac{1}{3} * 4.9888 = 1.2472$$

$$|\in_a| = \frac{|1.2472 - 1.2531|}{1.2472} * 100 = 0.48\%$$