Vibrations & Waves

Circular Motion:

using polar coordinates is simpler for circular motion so the following are some important kinematic definitions in polar coordinates and periodic motion.

Angular Displacement $\bar{\theta} = \bar{\theta}_f - \bar{\theta}_i$, measured in rad.

Angular Velocity $\bar{\omega} = \frac{d\bar{\theta}}{dt}$, measured in rad/s.

Angular Acceleration $\bar{\alpha} = \frac{d\bar{\omega}}{dt}$, measured in rad/s^2.

Frequency $(f) = \frac{no.\, of\, turns}{\Delta t}$

Periodic time $(T) = \frac{2\pi}{\omega} = \frac{1}{f}$

Forced Occillations:

a spring system that is affected by an external force $F = F_0 cos(\Omega t)$ and a damping force $F_D = -rv$ will be described by the following equation of motion.

$$m\ddot{x} + k_s x + r\dot{x} = F_0 cos(\Omega t)$$

solving this differential equation we get

$x = Acos(\Omega t - \phi)$, where $A(\Omega) = \frac{F_0}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (2\alpha\Omega)^2}}$ and $\phi = tan^{-1}(\frac{2\alpha\Omega}{\omega_0^2 - \Omega^2})$.

the frequency of applied force that will result in the greatest amplitude is $\Omega_A = \sqrt{\omega_0^2 - 2\alpha^2}$ and that maximum amplitude is $A = \frac{F_0}{r\sqrt{\omega_0^2 - \alpha^2}}$.

if we want the spring to oscillate at maximum velocity however, then $\Omega = \omega_0$, the amplitude $A = \frac{F_0}{r\omega_0}$, and $v_{max} = \frac{F_0}{r}$.

Waves:

Waves can be classified based on three criteria, firstly:

Mechanical Waves: need a medium to propagate.

Electromagnetic Waves: doesn't need a medium.

secondly:

Transverse: the vibration is perpendicular to wave propagation.

Longitudinal: the vibration is parallel to wave propagation.

and lastly, **Travelling and Standing Waves**.

Wave equation

The function of a wave moving in a line is $y = f(kx - \omega t)$, from this we can find the differential equation for wave motion.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

However, we're only concerned with sinusoidal waves which behave according to this equation.

$$y(x,t) = Acos(kx - \omega t) = Acos(x - vt)$$

where $\omega = \frac{2\pi}{T}$ and $k = \frac{2\pi}{\lambda}$