# Problem Set 6

**Name:** Your Name

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**Problem 6-1.** —

**(a)** **DAG Relaxation**: [(a, b), (a, d), (d, b), (d, e), (b, e), (b, c), (b, f), (e, f), (f, c)]

**Dijkstra**: [(a,b), (a, d), (d, b), (b, c), (b, e), (b, f), (f, c), (e, f), (c, e)]

**(b)**

| v | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| $\delta(s, v)$ | 0 | 2 | 3 | 6 | 5 | 4 |

**Problem 6-2.** General shortest path problem in which we can use Bellman-ford, knowing that there is always some path between every two vertices with at most $k$ edges, implies that $|V|$ is upper bounded by $|E| \rightarrow |v| = O(|E|)$, and we can perform **DAG Relaxation** for $k$ levels.

- **If G has no negative cycles:** Let $v \in V$ be any vertex. Consider path $p = <v_0, v_1, ..., v_k>$ from $v_0 = s$ to $v_k = v$ that is a shortest path with minimum number of edges. No negative weight cycles $\rightarrow$ p is simple $\rightarrow k \leq |V| - 1$.
  After 1 pass through $E$, we have $d[v_1] = \delta(s, v_1)$, because we will relax the edge $(v_0, v_1)$ in this pass and we can't find a shorter path than this shortest path. After 2 passes through $E$, we have $d[v_2] = \delta(s, v_2)$, because in the second pass we will relax the edge $(v_1, v_2)$. After $i$ passes through $E$, we have $d[v_i] = \delta(s, v_i)$. After $k \leq |V| - 1$ passes through $E$, we have $d[v_k] = d[v] = \delta(s, v)$. this takes $O(k|E|)$.
- **If G has negative cycles:** we can perform the previous algorithm and then do **Bellman-ford** check After $k$ passes, if we find an edge that can be relaxed, it means that the current shortest path from $s$ to some vertex is not simple and vertices are repeated. Since this cyclic path has less weight than any simple path the cycle has to be a negative-weight cycle.

**Bellman-ford** takes $O(|V|(|V| + |E|))$ as in this case we just relax on $k$ levels and $|v| = O(|E|)$ so $O(K|E|)$.

**Problem 6-3.**

**Problem 6-4.**

**Problem 6-5.**

**Problem 6-6.**