Problem Set 7

Name: Your Name

Collaborators: Name1, Name2

Problem 7-1.
Subproblem: $x(i)$ is the maximum number of delegations from day i to day n. (suffixes), $i \in 1, \ldots, n+1$
Relate:

- we choose whether to campaign on day i or not.
- take the maximum from z and d within days $i, i+1, i+2$.
- $x(i) = max(d_i + z_{i-1)} + z_{i-2} + x(i+3)), (z_i + x(i+1))$

Topological Ordering: i is decreasing, and $x(i)$ requires $x(i+1)$ and $x(i+3)$.
Base:

- $x(n+1) = 0$
- $x(n) = d_n$
- $x(n-1) = max(d_{n-1} + z_n), (d_n + z_{n-1})$

Original problem:

- check whether $x(0)$ is greater than $\lfloor D/2 \rfloor + 1$ and determine if she win or not.

Time:

- sum D of all delegations from day i to day n is $O(n)$.
- n Subproblems each takes $O(1)$ time.
- total time is $O(n)$.

Problem 7-2.

- sort tigers by their age in decreasing order. T
- sort cages by their capacity in increasing order. C
- using dynamic programming, match the tigers T with subsequance of C with minimum discomfort $s_i - c_j$.

Subproblem:

- $x|(i, j)$: min total discomfort, matching $T[i :]$ with $C[j :]$.
- $i \in 0, 1, \ldots, n$
- $j \in 0, 1, \ldots, n^2$

Relate:

- $d(i, j) = s_i - c_j \geq 0$
- $x(i, j) = mind(i, j) + x(i + 1, j + 1), x(i, j + 1)$

Topological Ordering: i and j are decreasing, and $x(i, j)$ requires $x(i + 1, j + 1)$ and $x(i, j + 1)$.

Base:

- $x(n, j) = 0$
- $x(i, n^2) = \infty$

Original problem:

- $x(0, 0)$ is the minimum total discomfort.
- store parent of each node in a matrix P to reconstruct the solution.

Time:

- $O(n^3)$ Subproblems each takes $O(1)$ time.
- $O(n^3)$ time

Problem 7-3. Subproblem:
$x(u,t)$: contains weights of all paths from u to t for $u \in V$.
$L(u,n)$: number of odd paths from u to n
Relate:

- $x(u,t) = \sum_{j \in Adj^{+}(u)} x(u+1,t,n) + w(u,j)$
- $L(u,n) = L(u,n) + 1$ for every odd path from u to n in $x(u,t)$.

Topological Ordering: $x(u,t)$ requires $x(u+1,t,n)$ which is calculated before $x(u,t)$.

Base:

- $x(t,t) = 0$
- $L(i,j) = 0$

Original problem:

- $L(s,t)$ is the number of odd paths from s to t.

Time:

- $|V|$ Subproblem each takes $2\sum_{u \in v} deg^{+}(v) = O(|V| + |E|)$

Problem 7-4. same as **Coins** problem in lecture 16 but choose a subarray every time rather than a single coin.

Problem 7-5.

(a)

(b) Submit your implementation to `alg.mit.edu`.