Ultrasound imaging : Systems, signals and image processing

D. KOUAMÉ

University Paul Sabatier Toulouse III
IRIT, UMR CNRS 5505, Toulouse France

General Outline

- Introduction : Generalities on Medical imaging
- Part I : Physics of Ultrasound
- Part II : Ultrasound imaging
- Part III : Open challenges

- **Introduction : Generalities on Medical imaging**

❑ Aim

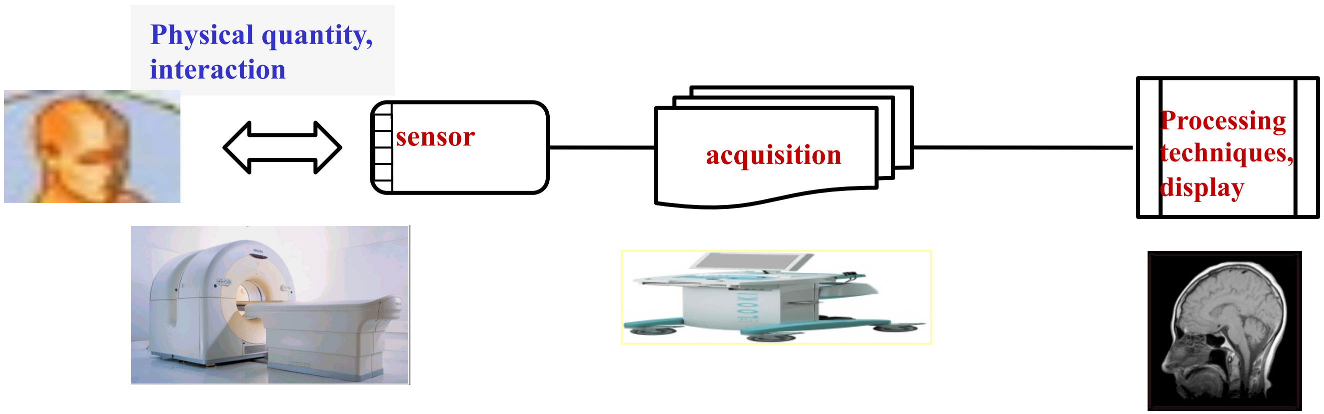
- Medical Imaging (MI) aims to collect data, process it in order to provide a set of qualitative and / or quantitative information relative to a body, a biological function.
- This information helps the physician in his work (diagnosis or therapeutic)
- In any MI system, there is a physical quantity that interacts either with a body directly or with an organ pre-conditioned (by an injectable product for example).
- MI involves, physics, chemistry, or biochemistry, instrumentation and signal and image processing.
- Medical imaging has its own particularities it is important to know before processing the medical images
- The fields covered by MI are 1D, 2D, 3D, 4D and their variants

❑ Brief Historical review

- End of the 19th century: M.I appeared with:
  - Discovery of X-rays by Roentgen in 1895 or
  -The piezoelectric effect by P. and M Curie in 1880
- Mid-20th century: 1st developement of MI systems
  - Ultrasound, X-ray scanner, PET, MRI
- Late 20th century and early twenty-first:
  - Development of complex systems MI
  - Multimodality systems (mixing different imaging systems),

❑ Imaging systems

- ***Each MI system uses a physical phenomenon (quantity). The image results from the interaction between the quantity and the organ to be considered***

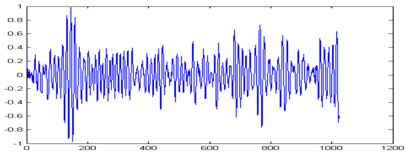


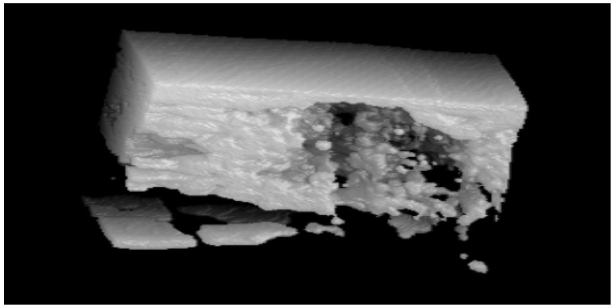
- ***The characteristics of the image come from the properties of the relevant physical quantity or law.***

- **This defines the imaging modality.**

❑ Characteristics

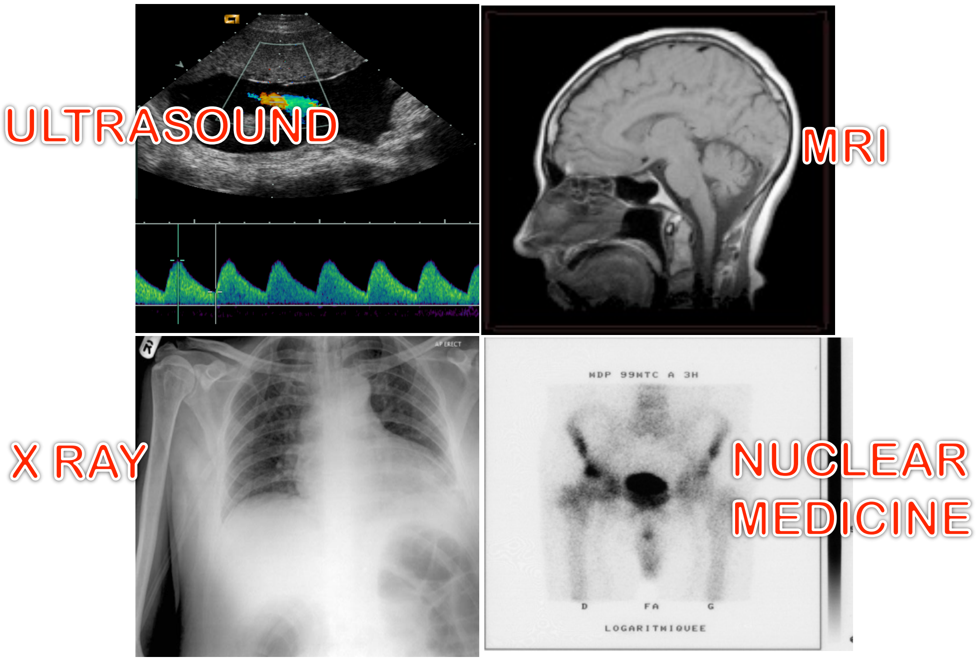
- **The physical laws**
  - **Attenuation**
  - **Penetration**
  - **scattering**
  - **…**
- **Through acquisition and image reconstruction**
  - **The slice/scan**
- **Result in the image types**



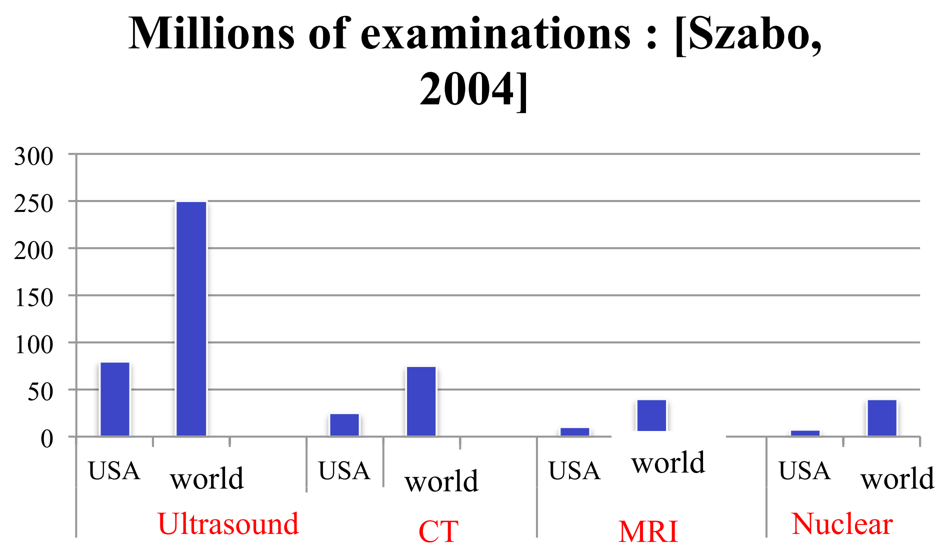


❑ Imaging Modality

- the 4 main MI modalities are : RX, MRI, US, NM



Medical imaging in numbers of examinations



All the big standard electronic companies have Medical Imaging departements: GE, Philips, Siemens, Toshiba, ...

Issues in terms of public health make this industrial area very dynamical

- **Physical laws of medical imaging modalities**

❑Waves

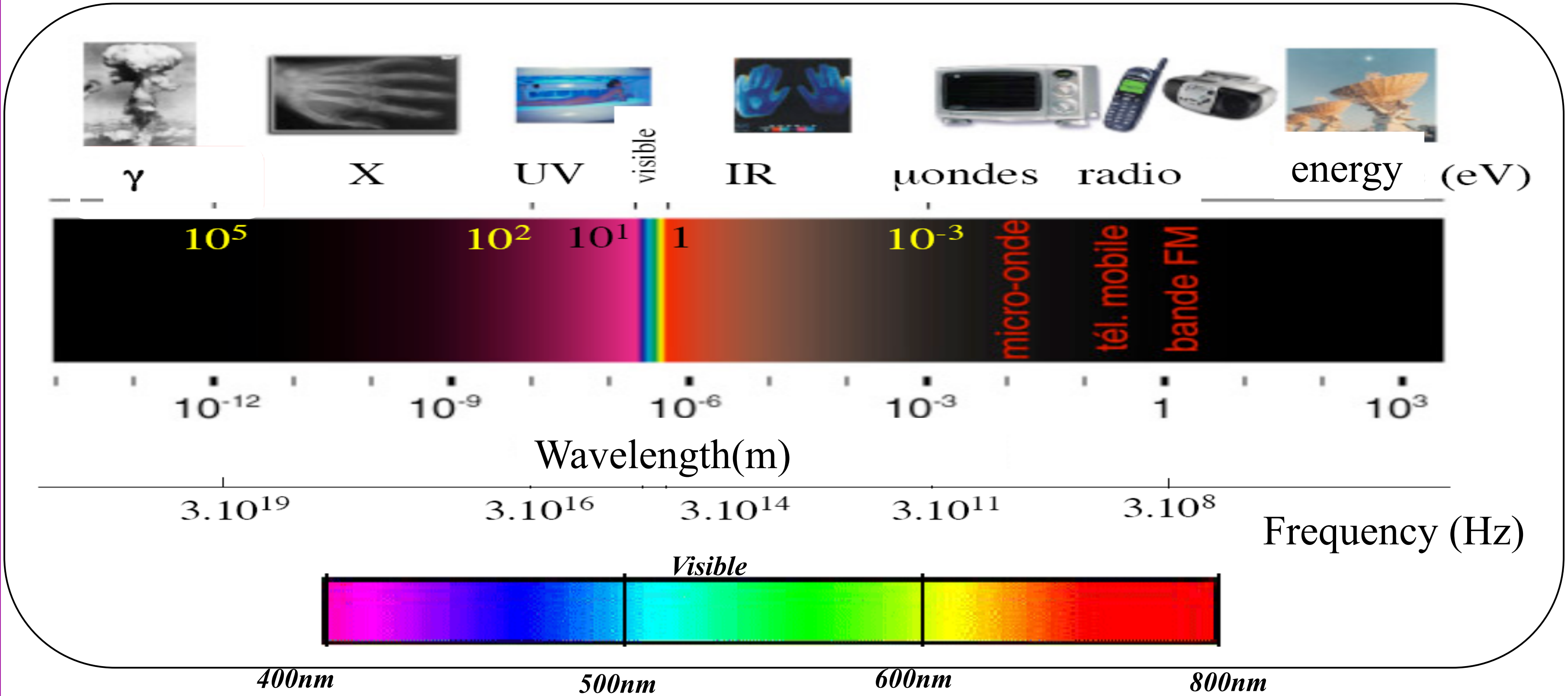
Medical imaging is based on either elementary particles or on waves coming from two environments

$E = h\nu$

$\lambda = \frac{c}{\nu}$ $c=3.10^8 m/s$

$1\ eV = 1{,}6 \cdot 10^{-19}\ J$ $h \approx 6{,}6 \times 10^{-34}\ J.s$

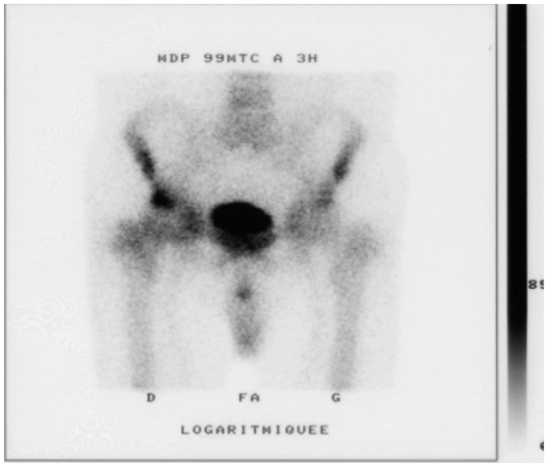
- **Electromagnetic environment**



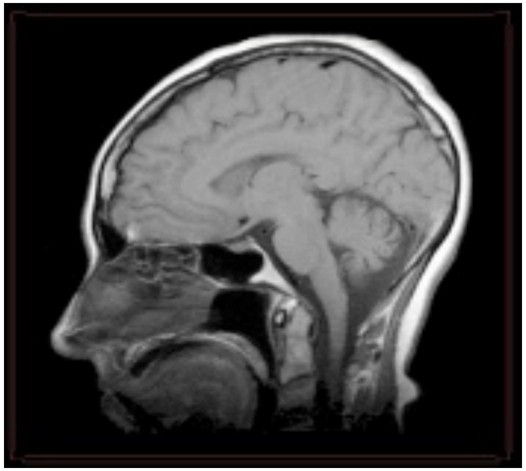
- **Acoustical environment (to be presented below in ultrasound part)**

Main Electromagnetic environment modalities

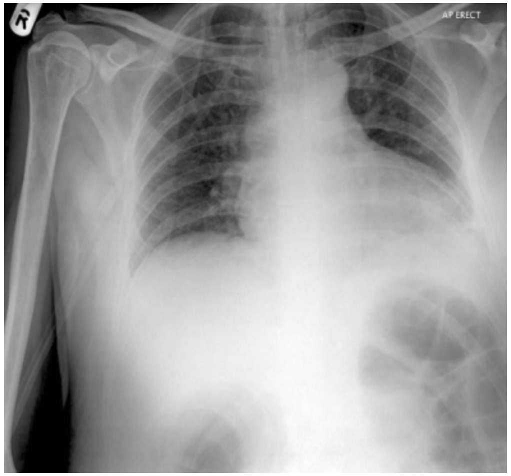
Nuclear medicine (PET, SPECT,...)



MRI



X-Ray



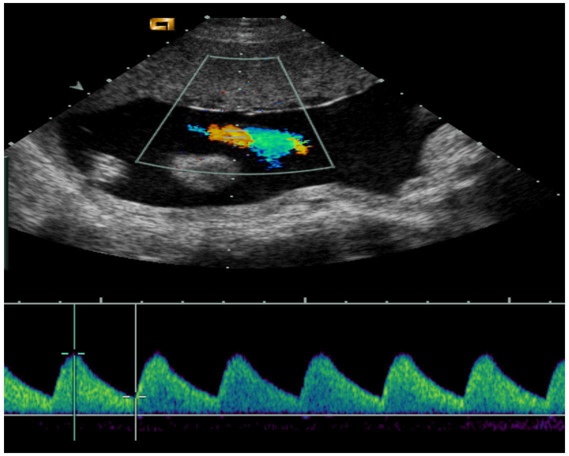
- **X-rays use attenuation of X-ray in the body. Excellent for seeing bones structure**
- **PET or SPECT study molecular distribution, prior tagged with radioactive atom**
- **MRI analyses the environment of hydrogen atoms in the body**

Open Challenges

- **Clinical : New methods**
  - **Diagnosis**
  - **Therapy**
- **And…**
- **Scientific**
  - **To follow in the case of ultrasound imaging**

….

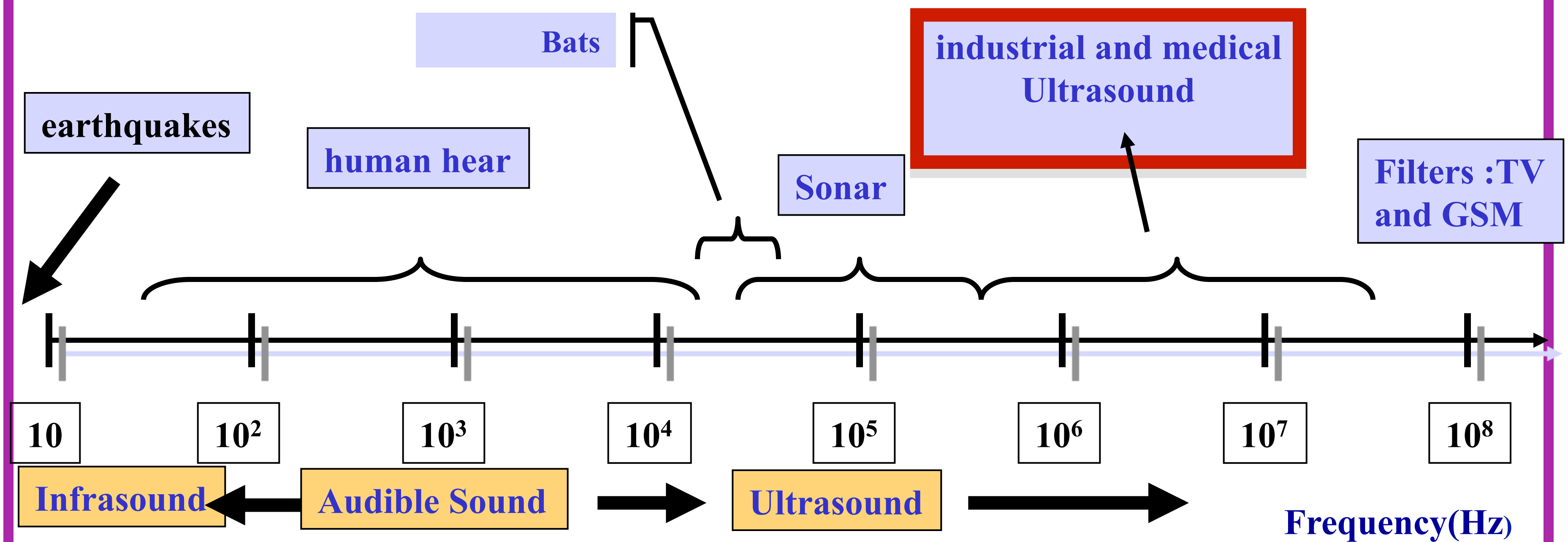
- **Part I : Physics of Ultrasound**



Outline

- Part I : Physics of Ultrasound
  - The Piezoelectrical transducer
  - Wave propagation & equations
  - Reflection/Transmission at Interfaces
  - Attenuation
  - Scattering
  - Doppler effect

❑Waves in acoustical environment



$\lambda = c/\nu$

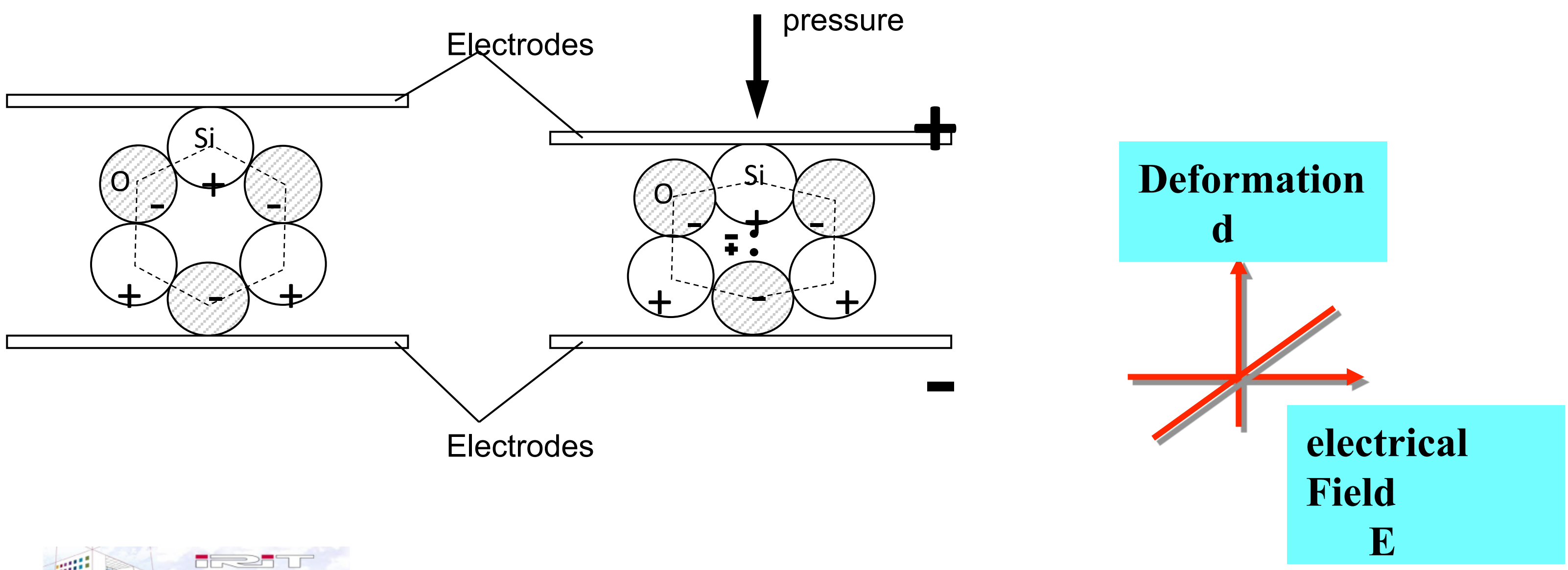
***c* depends on materials (approx 300m/s in air and 1500m/s in water and most biological tissues**

***ν* is frequency**

❑The Piezoelectrical transducer

❑ How can Ultrasound be generated ?

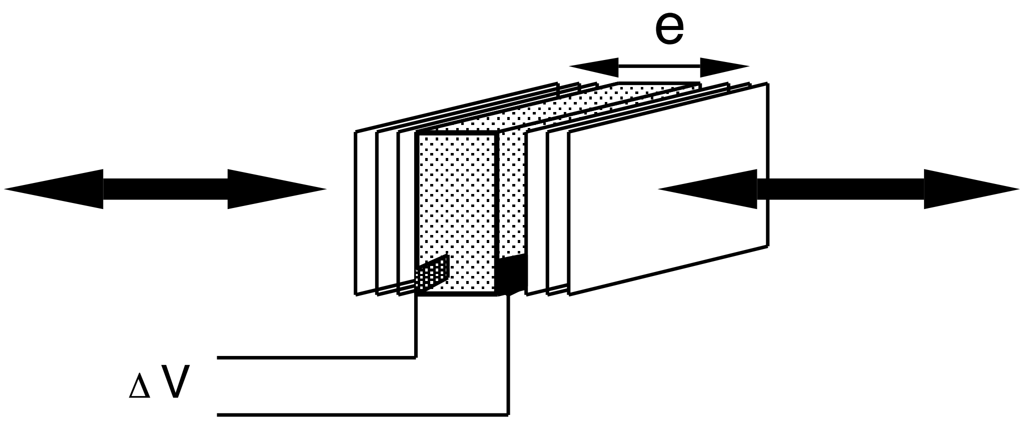
↳ Piezo-electric effect



❑ The transducer

electrical voltage ⇒ deformation

deformation ⇒ electrical voltage



❑ The Piezoelectrical transducer

Frequency characteristics of a transducer

- ***Frequency of resonance : ν_0***

  $\nu_0 = c/2e$ *or* $e = \lambda_0/2$ *e : thickness* λ_0 *:wavelength*

Example : material with speed of sound c = 3900 ms^{-1}

ν_0 (MHz)	1	2.25	3.75	7.5	10	30
e (mm)	1.95	0.86	0.52	0.26	0.195	0.06

- **Define the ratio : $Q = B/\nu_0$ B : Bandwidth at - 3 dB ou - 6 dB**

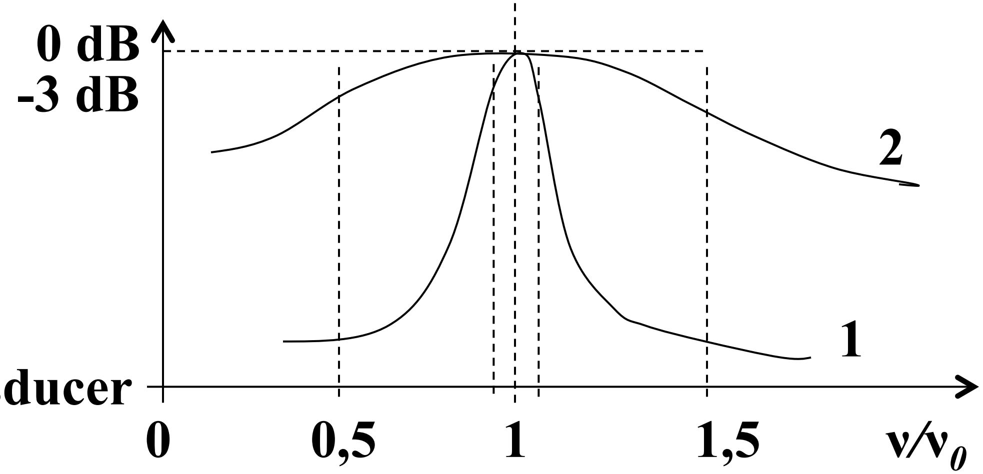
- **Frequency response**

  (2) large band transducer

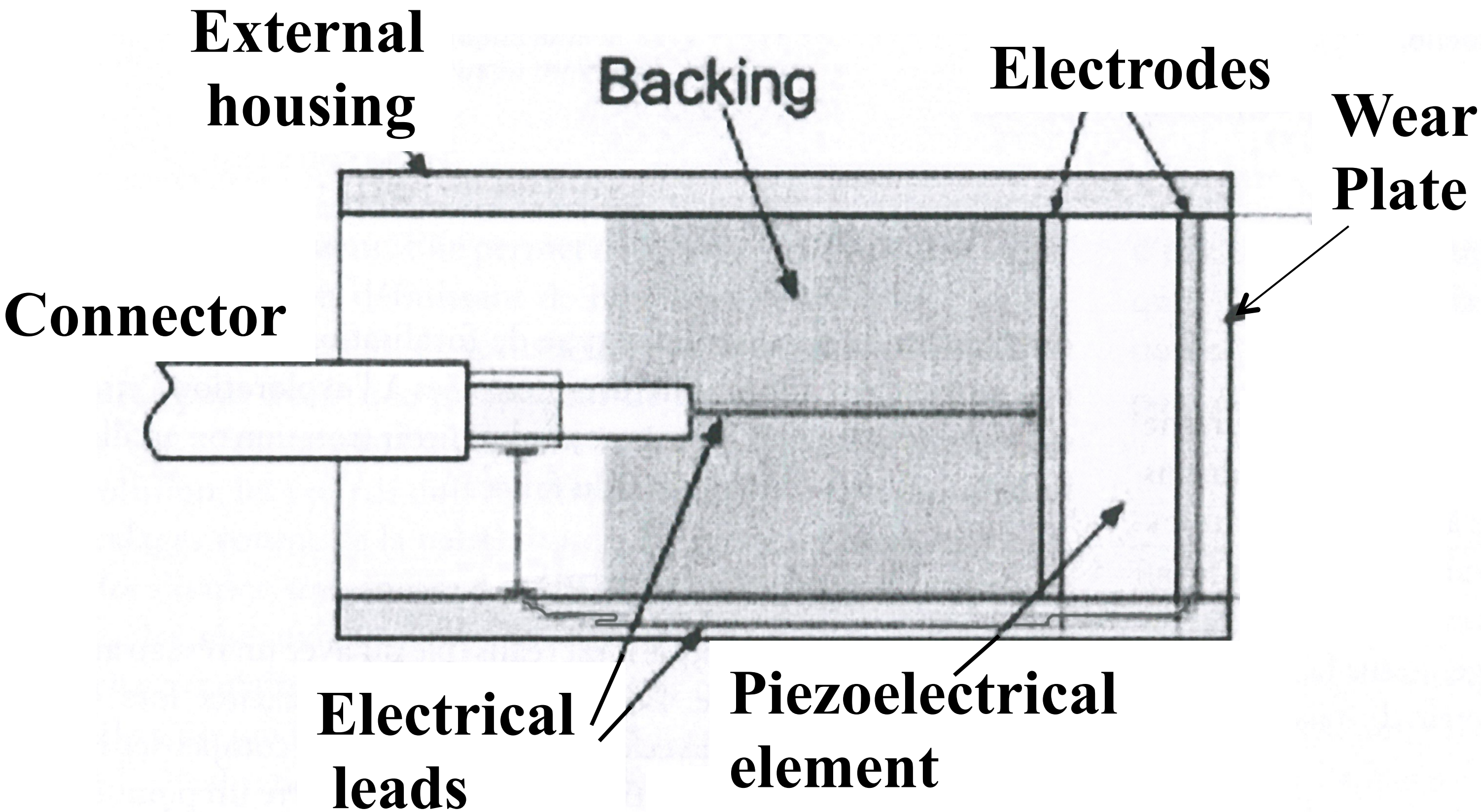
  Q = 10

  (1) Narrowband band transducer

  Q = 1



❑ UltrasonicTransducer (sensor)



Medical Ultrasound working frequency

MEDICAL DIAGNOSIS : 1 MHz < ν_0 < 30 MHz → 150 MHZ

1. 5 mm < λ < 0. 050 mm → 0. 010 mm

Choice of Transducer : function of

the organ of interest
its texture
its depth

To obtain the best possible RESOLUTION

Examples :

- **eye, skin, intravascular : ν_0 = 10 MHz to 50MHz**
- **liver, kidney… : ν_0 = 3, 5 – 7 MHz**
- **transcranian: ν_0=2 MHz**

❑Wave propagation & equations

- Acoustical waves are pressure. They propagate through matter by compression and expansion of the matter

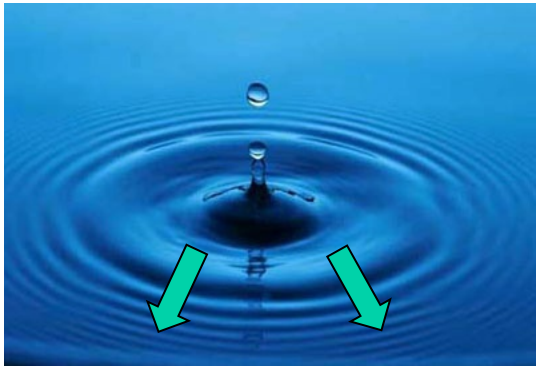
- A wave can be generated by compressing a small volume (particles) of material and then releasing it.

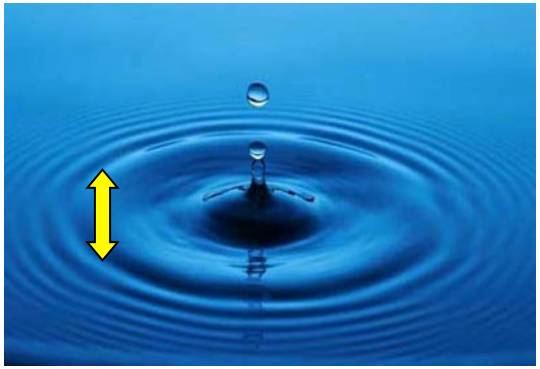
The elastic properties of materials cause the compressed volume to expand, making the neighborhood compressed and so on … and a wave is generated

Do not confuse:

Wave front propagation speed … and

Particles displacement velocity : They are in general different





Propagation of wave front at a certain speed

particles movement at another velocity

❑ Wave propagation

- In medical ultrasound, waves propagate, in most of time, in soft tissue. The particles move forth and back in the same direction that the acoustic wave is travelling. When this happens the acoustic wave is called *a longitudinal wave*

- Some harder material also support shear waves where particle move at right angle to the direction of propagation of acoustical wave

❑ 3D Acoustic Wave equation

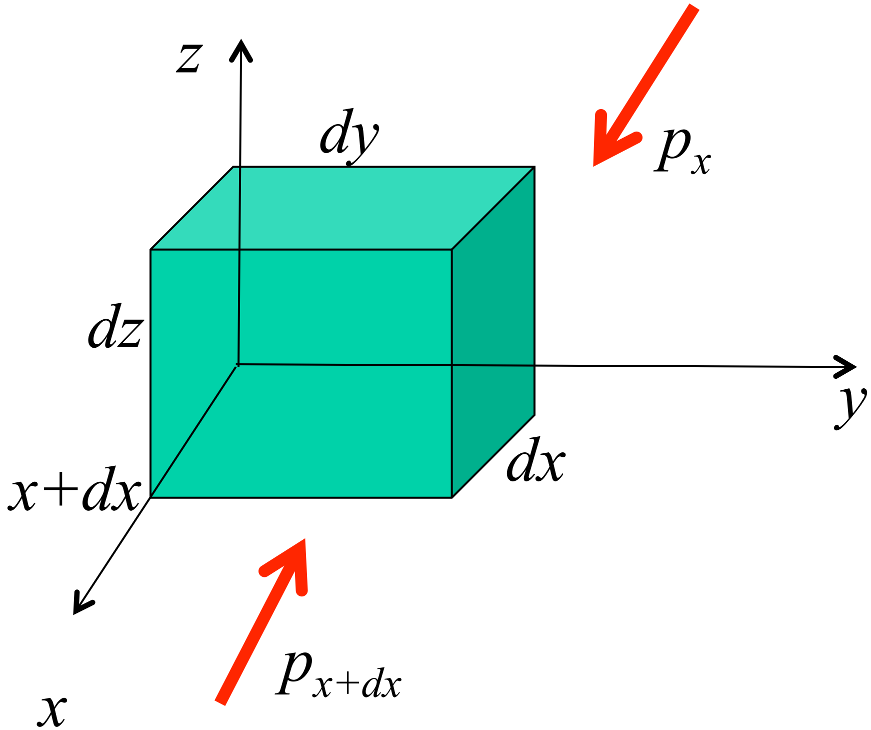
- **Important :**
  - **Acoustic wave is naturally 3D phenomenon i.e. it has spatial dependencies. For example, think about ordinary speech which spread out in all directions any time.**
  - **So whatever the physical quantity used to describe wave, it must depend on three spatial variables *x,y,z*, and time variable *t***

❑ 3D Acoustic Wave equation

- **Definitions:**
  - Particle displacement $\vec{u}(x,y,z,t);$ is associated with the compression and expansion of acoustic wave
  - Particle velocity *v(x,y,z,t) :* $\vec{v}(x,y,z,t) = \frac{d}{dt}\vec{u}(x,y,z,t);$
  - Acoustical pressure : Compression and expansion of a small volume is related to a local change in material's local pressure or acoustical pressure $p(x,y,z,t)$

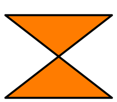
❑ 3D Acoustic Wave equation

Define

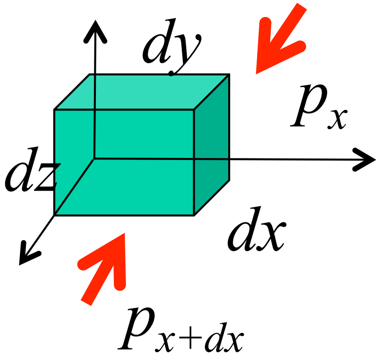


$$\vec{v}(x,y,z,t) \left| \begin{array}{l} v_x \\ v_y \\ v_z \end{array} \right.$$

In the following, we only consider 1st order approximaation



❑ 3D Acoustic Wave equation



Balance of forces in the face perpendicular to x

$$(p_x - p_{x+dx})dydz = m\gamma = \rho dx.dy.dz.\frac{\partial v_x}{\partial t}$$

$$-(p_{x+dx} - p_x)S = m\gamma = \rho S.\frac{\partial v_x}{\partial t}dx$$

$$-\frac{\partial p}{\partial x} = \rho_0 \frac{\partial v_x}{\partial t}; \qquad \rho = \rho_0 + d\rho \ \ with \ \rho \approx \rho_0$$

Balance of forces in the face perpendicular to x

❑ 3D Acoustic Wave equation

Doing so for the other faces, we obtain

$$\begin{cases} \dfrac{\partial p}{\partial x} + \rho_0 \dfrac{\partial v_x}{\partial t} = 0; \\ \dfrac{\partial p}{\partial y} + \rho_0 \dfrac{\partial v_y}{\partial t} = 0; \\ \dfrac{\partial p}{\partial z} + \rho_0 \dfrac{\partial v_z}{\partial t} = 0; \end{cases} \Longleftrightarrow \boxed{\rho_0 \frac{\partial \vec{v}}{\partial t} + \overrightarrow{grad}\ p = 0}$$

(Euler's equation)

❑ 3D Acoustic Wave equation

$$\rho_0 \frac{\partial \vec{v}}{\partial t} + \overrightarrow{grad}\ p = 0$$

means that the derivative of $\vec{v}$ comes from a potential (gradient).
It is the same situation for $\vec{v}$ which also comes from a ptential Φ

$$\begin{cases} \dfrac{\partial \Phi}{\partial x} + v_x = 0; \\ \dfrac{\partial \Phi}{\partial y} + v_y = 0; \\ \dfrac{\Phi}{\partial z} + v_z = 0; \end{cases} \Leftrightarrow \quad \vec{v} = -\overrightarrow{grad}\ \Phi$$

$$\Rightarrow \quad \rho_0 \overrightarrow{grad} \frac{\partial \Phi}{\partial t} = \overrightarrow{grad}\ p$$

$$\Rightarrow \quad p = \rho_0 \frac{\partial \Phi}{\partial t}$$

❑ 3D Acoustic Wave equation

Equation of continuity or equation of mass conservation:

The mass variation of the parallelepiped is equal to $\dfrac{\partial(\rho dxdydz)}{\partial t}dt$

is due to the difference of the fowrate between two opposite faces.

Thus for x, we have :

$$(\rho v_x)_x dydzdt - (\rho v_x)_{x+dx} dydzdt = -\frac{\partial(\rho v_x)_x}{\partial x} dxdydzdt$$

And for all the faces it comes :

$$\frac{\partial(\rho)}{\partial t} dxdydzdt = -\left[\frac{\partial(\rho v_x)_x}{\partial x} + \frac{\partial(\rho v_y)_y}{\partial y} + \frac{\partial(\rho v_z)_z}{\partial z}\right] dxdydzdt$$

$$\rho \, div(\vec{v})$$

❑ 3D Acoustic Wave equation

Finally, $$\frac{\partial(\rho)}{\partial t} = -\rho_0\, div(\vec{v})$$ *Mass conservation equation*

Using

$$\vec{v} = -\overrightarrow{grad}\,\Phi = -\nabla\,\Phi \quad \textbf{\textit{we have}} \quad \frac{\partial(\rho)}{\partial t} = \rho_0\left[\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}\right] = \rho_0\nabla^2\Phi$$

$$\nabla^2 = Laplacian; \quad \nabla = gradient$$

Define a volume V_0 and its varation dV, we have : $$\frac{dV}{V_0} = -\frac{d\rho}{\rho_0}$$

Define the coefficient of adiabatic compressibility : $$\chi = -\frac{1}{V_0}\frac{dV}{dp}$$

Thus $$\chi = -\frac{1}{V_0}\frac{dV}{dp} = \frac{1}{\rho_0}\frac{d\rho}{dp} \quad \textbf{\textit{or}} \quad \frac{1}{\chi\rho_0} = \frac{dp}{d\rho}$$

❑ 3D Acoustic Wave equation

Thus $$\frac{1}{\chi\rho_0} = \frac{dp}{d\rho} = \frac{\frac{\partial p}{\partial t}dt}{\frac{\partial \rho}{\partial t}dt}$$, **or** $$\frac{\partial p}{\partial t} = \frac{1}{\chi\rho_0}\frac{\partial \rho}{\partial t}$$

Since $p = \rho_0 \dfrac{\partial \Phi}{\partial t}$ **and** $\dfrac{\partial \rho}{\partial t} = \rho_0 \nabla^2 \Phi$

we have $$\rho_0 \frac{\partial^2 \Phi}{\partial t^2} = \frac{1}{\chi\rho_0}\frac{\partial \rho}{\partial t} = \frac{1}{\chi}\nabla^2\Phi$$

Or finally $$\boxed{\frac{\partial^2 \Phi}{\partial t^2} - \frac{1}{\rho_0 \chi}\nabla^2\Phi = 0}$$ **:** *acoustical wave equation 1 (AWE1)*

❑ 3D Acoustic Wave equation

Definition : ***The quantity*** $c = \dfrac{1}{\sqrt{\chi\rho_0}}$ ***is the called speed of sound (in m/s).*** *We will call it sometimes SOS*

The waves travels at this speed

Using the speed of sound the AWE1 comes to $$\frac{\partial^2\Phi}{\partial t^2} - c^2\nabla^2\Phi = 0$$

By use of Euler's equation it can be shown that we have a similar equation with *p, i.e.*

$$\boxed{\frac{\partial^2 p}{\partial t^2} - c^2\nabla^2 p = 0}$$

acoustical wave equation 2 (AWE2)

❑ 3D Acoustic Wave equation

- *In Part II, we will see the general actual solution of the wave equation.*

- *some basic considerations will allow to get a quite good and enough insight about the solution of this wave equation for the time being*

❑ 3D Acoustic Wave equation

- **Summary** $\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$; $\nabla^2 = Laplacian = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

- **Définitions** $Z = \rho c = Acoustic\ impedance$

$$p = Zv$$

- **Acoustic Intensity** $I = pv = \frac{p^2}{Z}$

- **Analogy to electrical circuits p is analogous to voltage and v to current**

- **Remember c and v are not the same in general.**

- **Z is an importante characteristics of ultrasound waves :**

Unit of Z : kgm^{-2}s^{-1}; 1kgm^{-2}s^{-1}=1Rayl

❑ Acoustical characteristics for some materials

Medium	Density kg/m^3 ρ	Speed of sound m/s c	Characteristic acoustic impedance $kg/[m^2 \cdot s]$ Z
Air	1.2	333	0.4×10^3
Blood	1.06×10^3	1566	1.66×10^6
Bone	$1.38 - 1.81 \times 10^3$	2070 – 5350	$3.75 - 7.38 \times 10^6$
Brain	1.03×10^3	1505 – 1612	$1.55 - 1.66 \times 10^6$
Fat	0.92×10^3	1446	1.33×10^6
Kidney	1.04×10^3	1567	1.62×10^6
Lung	0.40×10^3	650	0.26×10^6
Liver	1.06×10^3	1566	1.66×10^6
Muscle	1.07×10^3	1542 – 1626	$1.65 - 1.74 \times 10^6$
Spleen	1.06×10^3	1566	1.66×10^6
Distilled water	1.00×10^3	1480	1.48×10^6

We can then consider that **c** is approximatively 1500m/s in most of biological tissue .

❑ Plane Wave

Definition : ***If an acoustical wave varies in only one spatial direction and time, it is called a plane wave***

For example, if $p(x,y,z,t)$ were constant for any x and y for a given z and then $p(x,y,z,t) = p(z,t)$ propagate along z-direction and (AWE2) becomes one dimensional wave equation

$$\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial^2 p}{\partial z^2} = 0 \qquad \textit{(PWE)}$$

In the following** z **will considered to the direction of plane wave propagation.

❑ Plane Wave

The general solution of (PWE) can be written as

$$p(z,t) = a_v(t - c^{-1}z) + a_r(t + c^{-1}z); \qquad (SG1)$$

It can be easily verified by direction substitution into (PWE)

- $a_v(t - c^{-1}z)$ *and* $a_r(t + c^{-1}z)$ ***are interpreted resp. as the*** *forward travelling wave* ***and the*** *backward travelling wave* .

- *The only requirement is that they must be twice differentiable.*

- *They satisfy indendently the wave equation. So one of then might be zero. So only the forward wave can be considered in a given a given medium*

❑ Plane Wave

Two important examples that satisfy (PWE) which are used in classical ultrasound imaging

▪ ***Ex1: Sinusoidal function*** $p(z,t)=\cos(z-ct)$

Its parameters are the frequency f, the wave number k and the wavelength λ $f=\dfrac{kc}{2\pi}=\dfrac{\omega}{2\pi};\quad \lambda=\dfrac{2\pi}{k}=\dfrac{c}{f}$

▪ ***Assume that a medical ultrasound system operates at 3.5MHz. What is its wavelength?***

❑ Plane Wave

▪***Assume that a medical ultrasound system operates at 3.5MHz. What is its wavelength?***

$$p(z,t) = \cos(z - ct)$$

Considering the SOS c=1500m/s; λ=1500/3.5$\times 10^6$=429μm

Although, it may not be obvious at this point, the resolution of the ultrasound system is of order λ. This means that ultrasound system can achieve submillimetric resolution

❑ Plane Wave

- ***Ex2: Impulse (Dirac) function***

$$p(z,t) = \delta(z - ct)$$

Generally, it uses for large band system

- ***Assume that a transducer is pointing at +z direction. At time t=0 it generates an acoustic wave*** $a(t) = e^{-t/\tau}$ ***Assume the SOS is c=1500m/s.***

a)What is the form of the forward wave in +z direction

b) At what time the leading edge hits an interface which is at 20 cm of the transducer?

❑ Spherical Waves

- *In an isotropic material, a spherical wave can be generated by a small, local disturbance in the pressure.*

- *A spherical wave depends only on time t and the radius* $r=(x^2+y^2+z^2)^{1/2}$ *from the source of the disturbance assumed to be at (0,0,0).*

In this case, using (AWE2), the pressure travelling in the radial direction can be shown to verify

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rp)-\frac{1}{c}\frac{\partial^2 p}{\partial t^2}=0 \qquad \text{(SWE)}$$

This is called spherical wave equation

❑ Spherical Waves

The general solution of (SWE) can be written as

$$p(r,t) = \frac{1}{r} a_e(t - c^{-1}r) + \frac{1}{r} a_i(t + c^{-1}r); \quad (SG2)$$

It can be easily verified by direction substitution into (SWE)

- $a_e(t - c^{-1}r)$ *and* $a_i(t + c^{-1}r)$ ***are interpreted resp. as the*** *outward travelling wave* ***and the*** *inward travelling wave* .

- *The only requirement is that they must be twice differentiable.*

- *Generally there is no inward travelling wave so*

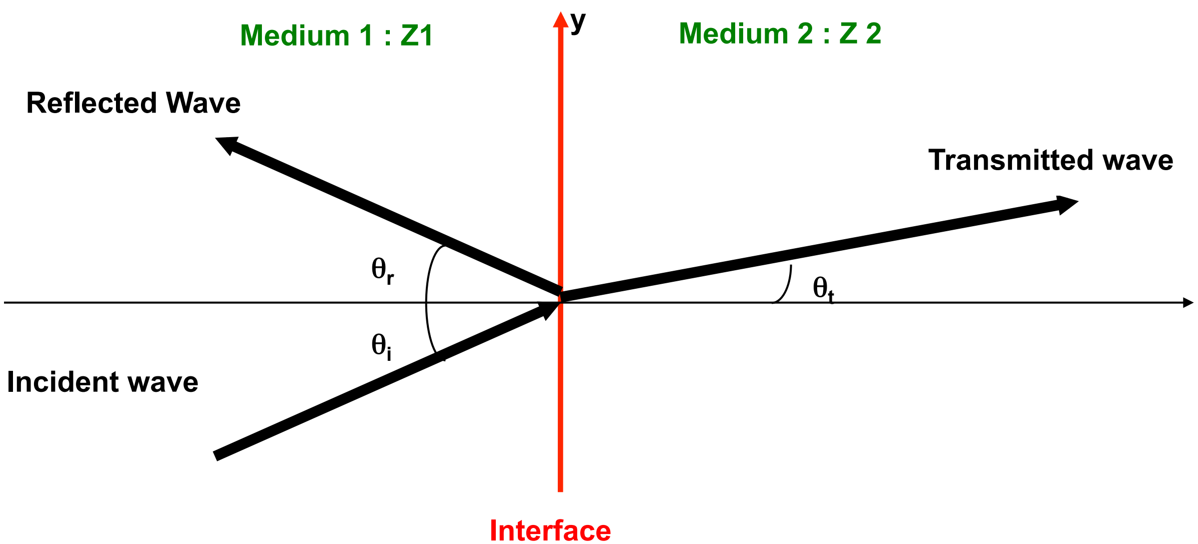
$$\boxed{p(r,t) = \frac{1}{r} a_e(t - c^{-1}r)}$$

❑ Spherical Waves

$$p(r,t) = \frac{1}{r} a_e(t - c^{-1} r)$$

❖ ***This looks like plane wave equation except for the factor 1/r which cause spherical wave to lose amplitude as it propagate radially outward***

❑Reflection/Transmission at Interfaces



$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{c_2}{c_1} \qquad Snell's\ law$$

E_i : incident energy; E_r reflected energy; E_t Transmitted energy

$$R = \frac{E_r}{E_i}; \qquad T = \frac{E_t}{E_i} \qquad and \quad we\ have: \quad T + R = 1$$

❑ Reflection/Transmission

•It can be easily shown that :

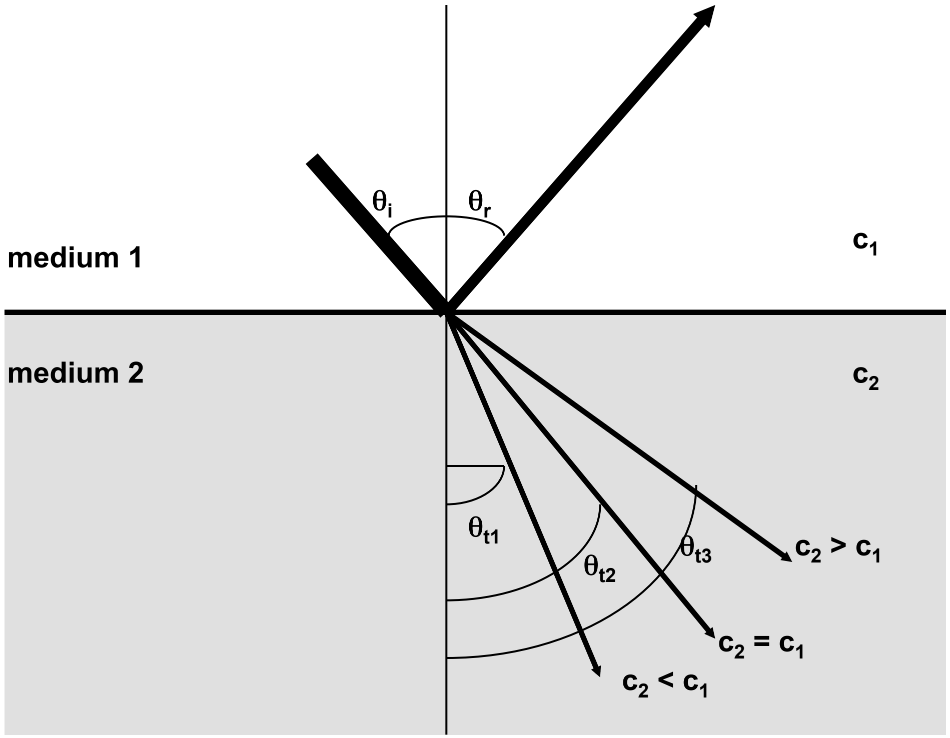
$$R = \left(\frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t} \right)^2 \qquad T = \frac{4 Z_1 Z_2 \cos\theta_i \cos\theta_t}{\left(Z_2 \cos\theta_i + Z_1 \cos\theta_t \right)^2}$$

Normal Incidence

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2 \qquad T = \frac{4 Z_1 Z_2}{\left(Z_2 + Z_1 \right)^2}$$

➢ Differents types of reflection

- REFLECTION ON A PLANE SURFACE : SPECULAR REFLECTION (MIRROR EFFECT)



$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{c_2}{c_1}$$

We have a specular effect and when c2 > c1 there is a total reflexion if θ_i is greater than critical angle

$$\theta_c = \arcsin\left(\frac{c_1}{c_2}\right)$$

➢ Differents types of reflection

- **REFLECTION ON A PLANE SURFACE : SPECULAR REFLEXION (MIRROR EFFECT)**

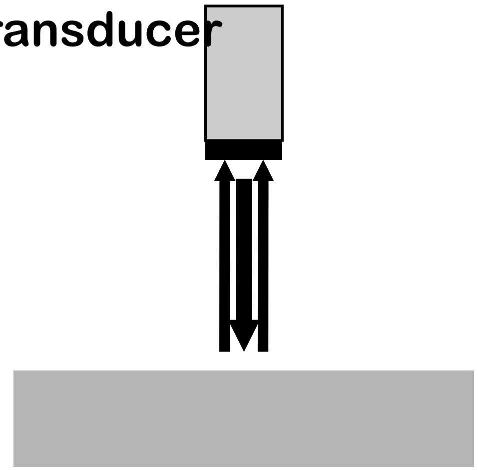
Assume that the medium 1 is fat and the medium 2 is muscle.
a)For plane wave with incident $\theta_i = 45°$
find the reflected and the transmitted angles

b) Find the critical angle

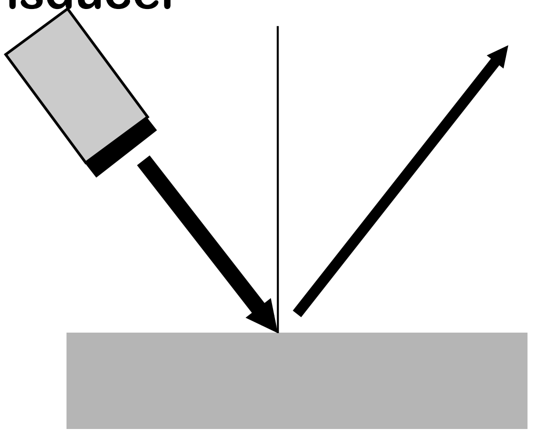
➢ Differents types of reflection

- **REFLECTION ON A PLANE SURFACE : SPECULAR REFLEXION (MIRROR EFFECT)**

- **Transducer**



Transducer



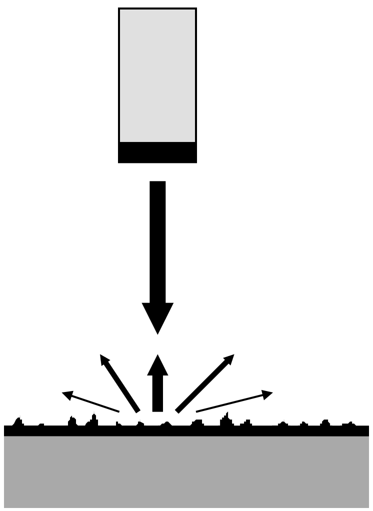
If $\theta_i \neq$ 0 no received wave

If θ_i = 0
The received wave is maximum

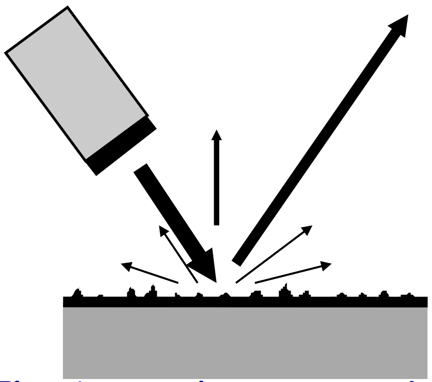
➢ Differents types of reflection

▪ **REFLCTION ON A SURFACE ROUGH (USUALLY TISSUES) OR ON VERY SMALL TARGET**

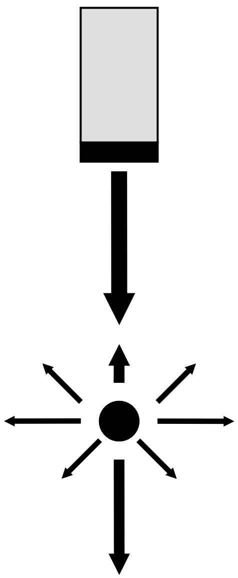
- **Backscattered wave = wave reflected in all the directions (see Scattering part)**



The transducer receives the specular, normal wave and the backscattered waves



The transducer receives the the backscattered waves



When dimension of the target is smal (compared to λ), the wave is scattered in all the direction and the target is called scatterer

➢ Examples

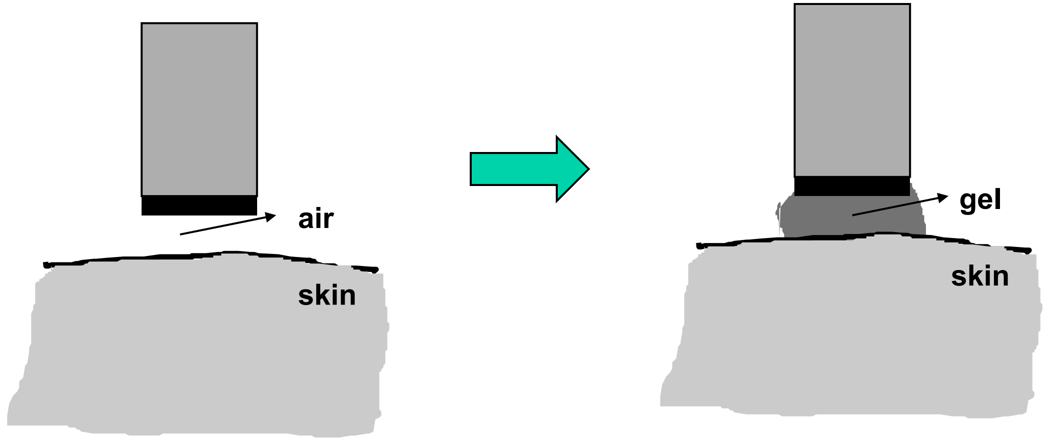
1) air → water or tissues

air: ρ = 1,293 kg/m3 , c = 340 m/s , Z1 = $0{,}00044.10^6$ kg/m2.s

water: ρ = 10^3 kg/m3 , c = 1480 m/s , Z2 = $1{,}48.10^6$ kg/m2.s

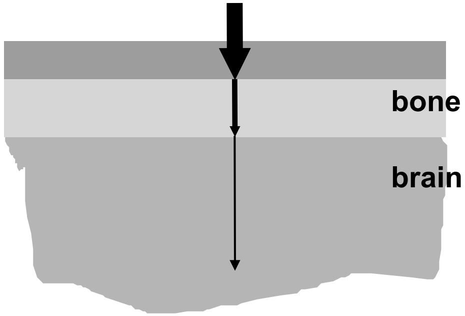
$$T = \frac{4Z_1Z_2}{(Z_1 + Z_2)^2} = 1{,}2.10^{-3} \qquad TdB = 10 \log T = -29 \text{ dB}$$

Very few energy is transmitted and thus use of gel is very important



➢ Example

2) Water or tissues →bone



Z1 water = 1,48 MRayl , Z2 *bone* = 6 Mrayl

T water→ bone = 0,64; tissue → bone → brain

loss 36% loss 36%

Very high lost in the transmission

+ additional attenuation (see the following)

❑Attenuation

Actually, while propagation, the amplitude of an acoustic wave experiences a decrease. This lost in the wave amplitude (signal) is called Attenuation.

It is due to several mechanisms (absorption, scattering,mode conversion)

❑ Attenuation

Suppose a forward plane wave $p(z,t)$ *is travelling in +z direction,*
where $p(0,t) = A_0 s(t)$

In the absence of attenuation(ideal case), $p(z,t) = A_0 s(t - z / c)$

Due to attenuation we have actually $p(z,t) = A(z) s(t - z / c)$

❑ Attenuation

$$p(z,t) = A(z)s(t - z/c) \quad \textbf{(ATW1)}$$

A(z) is the actual amplitude of the wave and is dependent on the z-position of the wave

$$A(z) = A_0 e^{-\alpha_L z} \quad \textbf{(ATW2)}$$

The amplitude decay is modeled as :

α_L is called amplitude attenuation factor and has unit m^{-1}

This model is phenomenological i.e. it agrees well in practice but not easily supported by theory. In particular (ATW1) no longer satisfy the wave equation(PWE)

Attenuation

From $A(z) = A_0 e^{-\alpha_L z}$

$$\alpha_L = \frac{1}{z}\ln\left(\frac{A(z)}{A_0}\right) \qquad \textit{(ATW3)}$$

The unit is called sometimes :Nepers.cm^{-1} (instead of cm^{-1})

Since generally the gain in amplitude is expressed in dB,
The amplitude attenuation factor defined in dB.cm^{-1} is defined by

$$\alpha = 20\log_{10}(e)\alpha_L \approx 8.69\alpha_L$$

α called attenuation coefficient

Don't confuse them: the one is in Nepers.cm^{-1}r and the other in dB.cm^{-1}.

❑ Attenuation

Actually the attenuation coefficient depends on the frequency f

A good model is $$\alpha(f) = \beta f^{m}$$

m is slightly greater than 1 for most biological tissues

For example b=0.5 and m=1.2 in homogeneous liver

SUMMARY

$$\boxed{A(z) = A_0 \times e^{-\alpha_L z}}$$

Attenuation Coeffficient (dB/cm)

$$\alpha(f) = \beta f^m$$

Medium	Attenuation à 1 MHz en dB/cm	Attenuation Dependence de law : $\alpha(f)/\beta$
Air	12	f^2
Water à 20°C	0,0022	f^2
Blood	0,18	$f^{1.3}$
Brain	0,85	f
Fat tissue	0,63	≈f
Soft tissue	0,81	≈f
Muscle	1,3 – 3,3	≈f
Skin	1,3 - 3	≈f
Lung	41	1/f

Source S.T. WELLS 77

The higher the frequency ⇨ The higher the attenuation

❑ Exo

Suppose a 5 Mhz acoustic pulse travels from a transducer through 2cm of fat, then encounters an interface with the liver at normale incidence.
a)At what time interval after the transmitted pulse will the reflected pulse (i.e. the echo) arrive back at the transducer ?

b) Taking both attenuation and reflexion losses into account, what will be the amplitude loss in decibels of the returning echo?

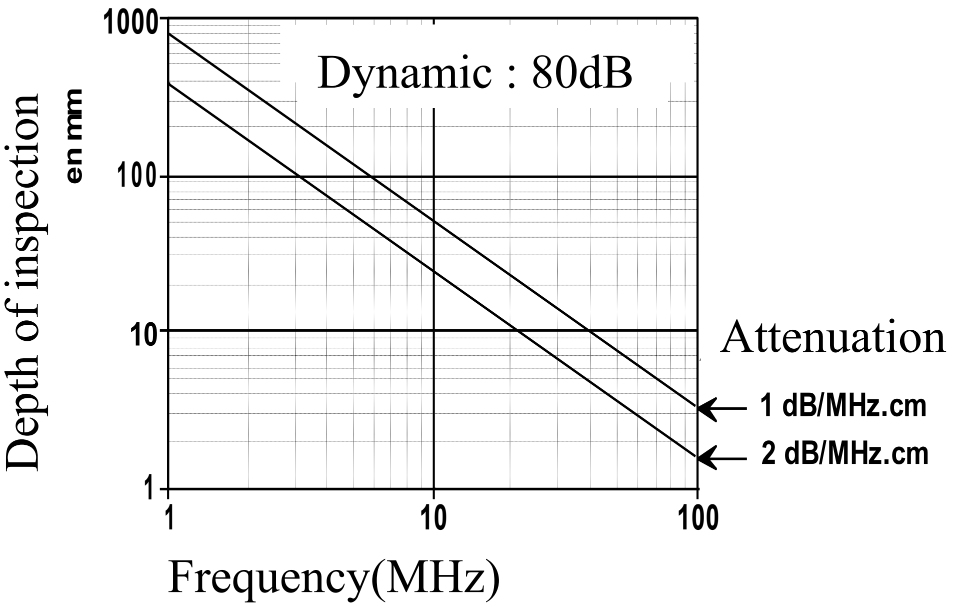
The attenuation of the medium limits the depth of inspection

➲ behaves like a low pass filter

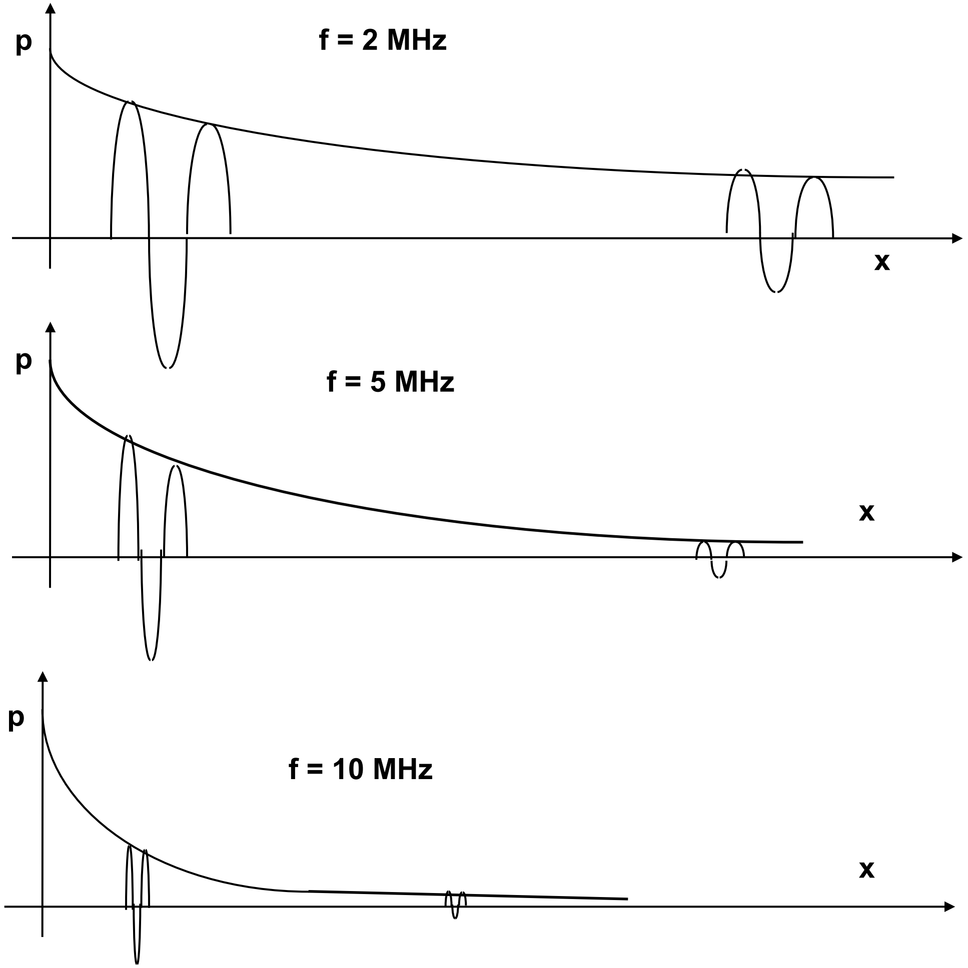
➲ The deph of imaging is limited

$$P\max = \frac{Dy}{2 \times \beta \times f^{m}}$$

- Pmax : Maximum depth of inspection (imaging) cm
- Dy : dynamic of the imaging system in dB
- β : attenuation of the propagating medium in dB/cm/MHz
- f : ultrasounf frequency in MHz
- m : frequency dependence law of attenuation of the medium



Exemple



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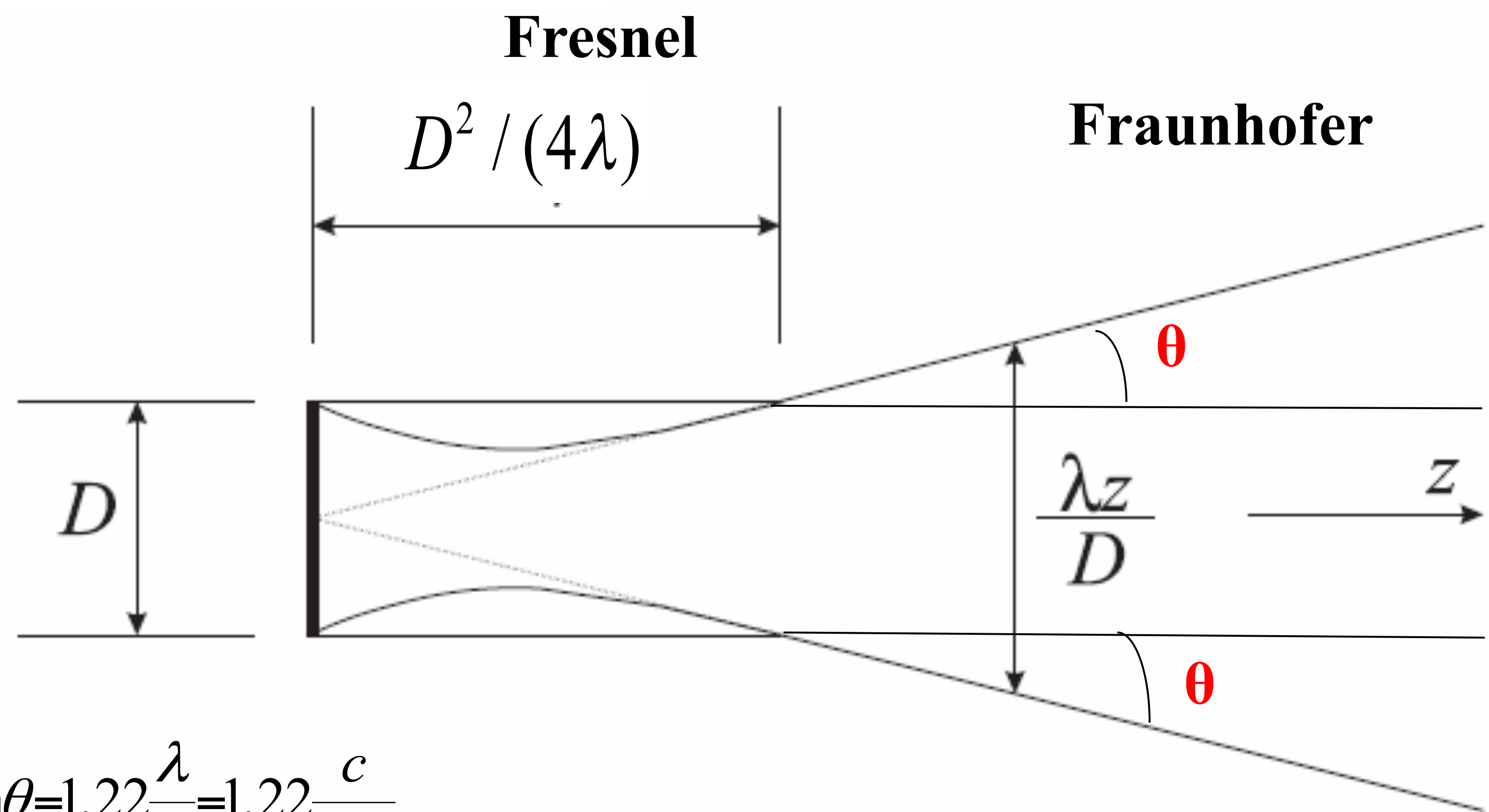
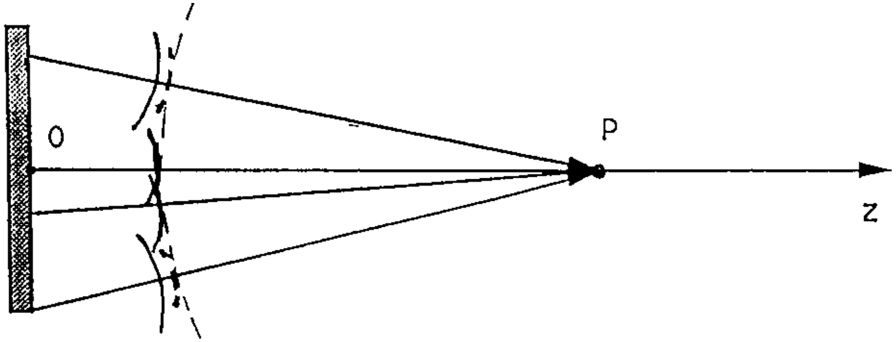
Non focussed beam : acoustic beal of a circular plan transducer of diameter D



$$\sin\theta = 1{,}22\frac{\lambda}{D} = 1{,}22\frac{c}{fD}$$

❑ Focussing and Apodization

- **Beam focussing** : **Aim = Increase the acoustic energy on propagation axis and reduce it otherwhere in order to narrow the beam**



Imperfect Summation in P because the wave arrive at different time

FOCUSSING Put them in phase by delaying them so that they arrive at the same moment in P.

APODIZATION Performs weighted sum of the waves arriving in P.

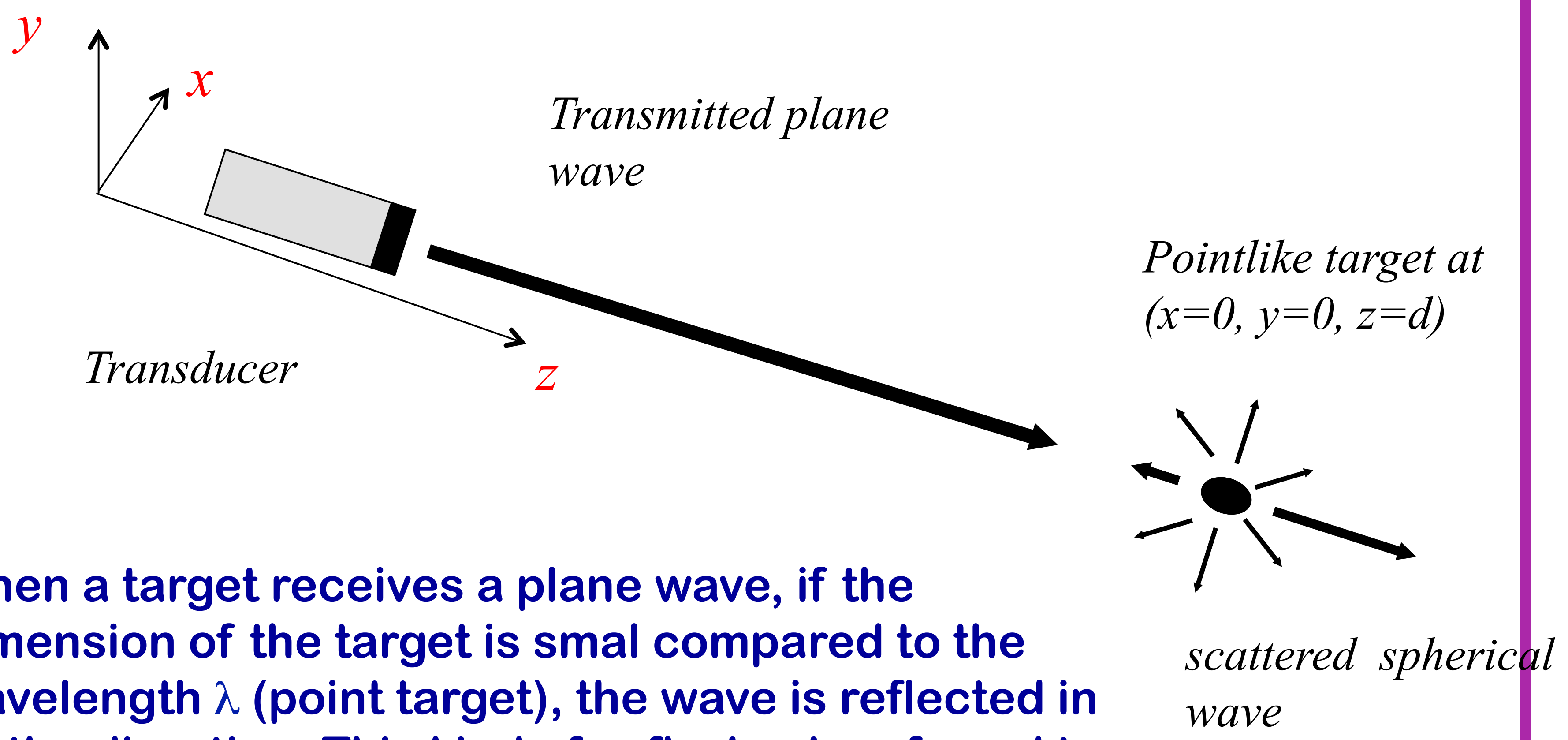
❑Scattering

Many targets in the body are much smaller than the wave length when excited by an incident acoustic wave

These targets do not reflect but they vibrate as small spherical particles giving rise to spherical wave(in all the direction). This is called SCATTERING. This small particles are called SCATTERERS.

The amplitude of this spherical wave (called scattered waves) is a fraction of the incident wave

❑ Scattering



When a target receives a plane wave, if the dimension of the target is smal compared to the wavelength λ (point target), the wave is reflected in all the direction. This kind of reflexion is referred to as scattering. The scattered wave is spherical

The target is called scatterer

❑ Scattering

Consider the plane wave, attenuated

$$p(z,t) = A_0 e^{-\alpha_L z} s(t - c^{-1} z)$$

Incident upon a small pointlike target (satterer) located at (0,0,d) in the +z direction.

This scatterer acts as spherical source, converting a fraction R of the incident wave into a spherical wave centered at (0,0,d).

R is called reflection coefficent and is a property of individual target and the medium.

❑ Scattering

Treating (0,0,d) as the origin, i.e. r is the radius from (0,0,d), the resulting scattered wave is :

$$p(z,t) = \frac{Re^{-\alpha_L r}}{r} A_0 e^{-\alpha_L d} s(t - c^{-1}z - c^{-1}r)$$

This equation at the core of ultrasound imaging equation, we will see later

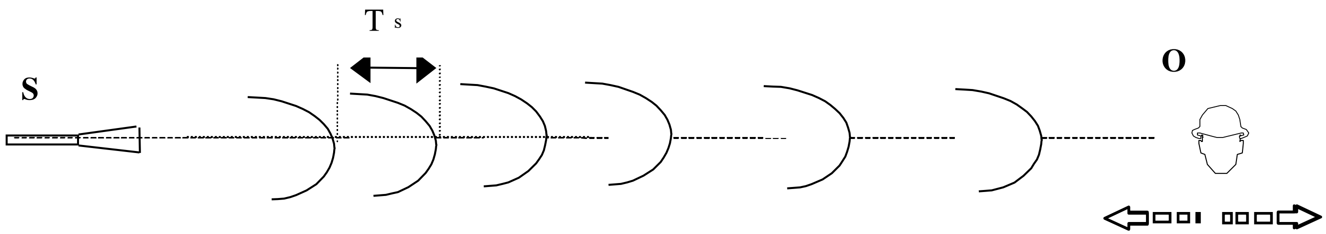
The attenuation can be reduced by using an time varying amplification referred to as Time Gain Compensation (TGC)

❑Doppler effect

The Doppler effect or Doppler shift is change in frequency due to the relative movement between the source and the receiver (of waves).

Now consider the following

A source S emetting a sound with a time period Ts and a receiver O having a relative movement



fixed source- mobile receiver

If the receiver, located at x_0, moves away from the source with a constant velocity v in the same direction as the tops, its location is given by :

$$x(t) = x_0 + vt$$

The first sound hits him in O, the second in O1, the following in Oi, i=2,... Due to this movement, the sounds hit him with apparent frequency

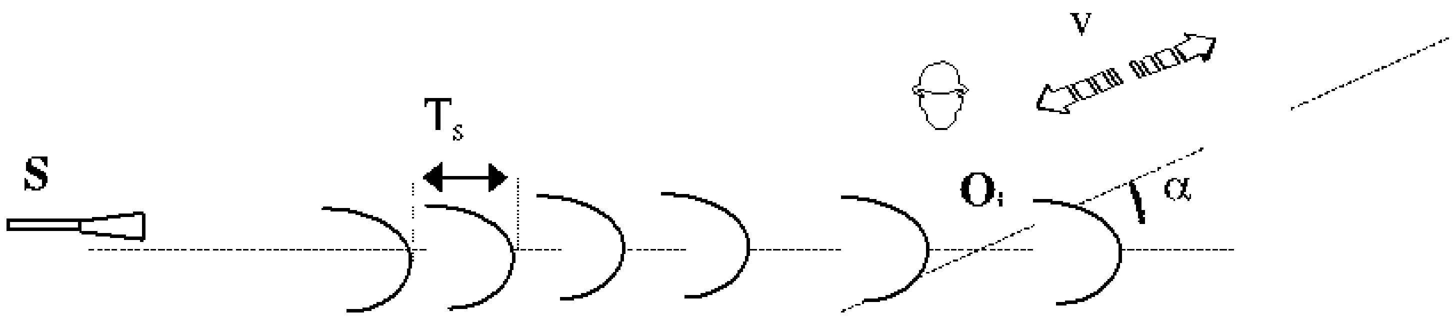
$$F_a = 1/T_a.$$

Between two tops, the receiver travels a distance $d = v\,T_a$.
The same distance, is travelled by the sound with the speed c, can also be written :

$$d = c\,(T_a - Ts)$$

By substitution,

$$T_a = T_s \frac{c}{c - v} \quad ou \quad F_a = F_s \frac{c - v}{c}$$



If the receiver moves in direction α

$$F_a = F_s \frac{c - v\cos\alpha}{c}$$

In the case of a mobile Source moving in a direction β with a velocity v ' and a fixed receiver, we have similarly

$$F_a = F_s \frac{c}{c - v'\cos\beta}$$

In the general case where the Source is mobile in a direction β with a velocity v ' and the receiver is also mobile in a direction α with a velocity v we have

$$F_a = F_s \frac{c + v\cos\alpha}{c - v'\cos\beta}$$

In ultrasound Doppler, a one tranducer-probe (or 2-transducer-probe) is used for transmitting (a wave with frequency f_o) and receiving the wave. A scatterer moving in a medium is then first, a moving receiver and secondly, a moving source; so

$$\alpha = \beta = \theta; \quad et\ v = v'$$

Thus

$$F_a = f_0 + f_d = f_0 \frac{c + v\cos\theta}{c - v\cos\theta}$$

We have thus **Emitting frequency** **Doppler frequency**

$$F_a = f_0 + f_d = f_0 \frac{c + v\cos\theta}{c - v\cos\theta}$$

Moreover: $v << c$

Doing a Taylor development

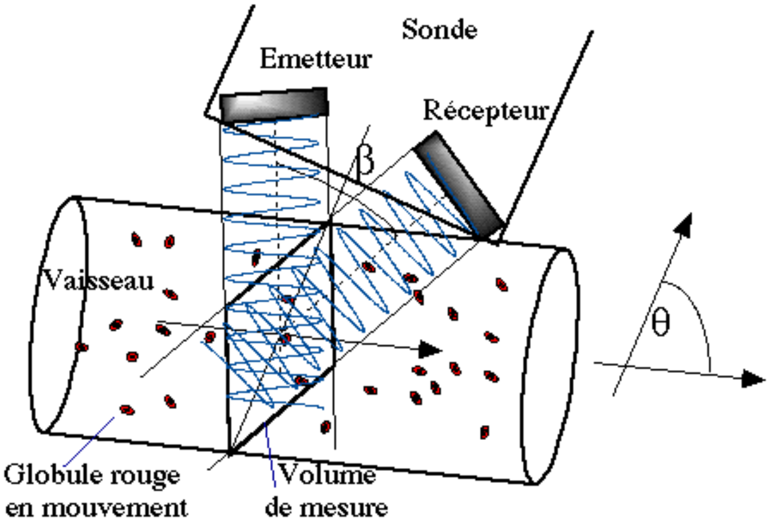
$$f_d \approx f_0 \frac{2v\cos\theta}{c}$$

In practice, two principles are used to obtain Doppler frequency:

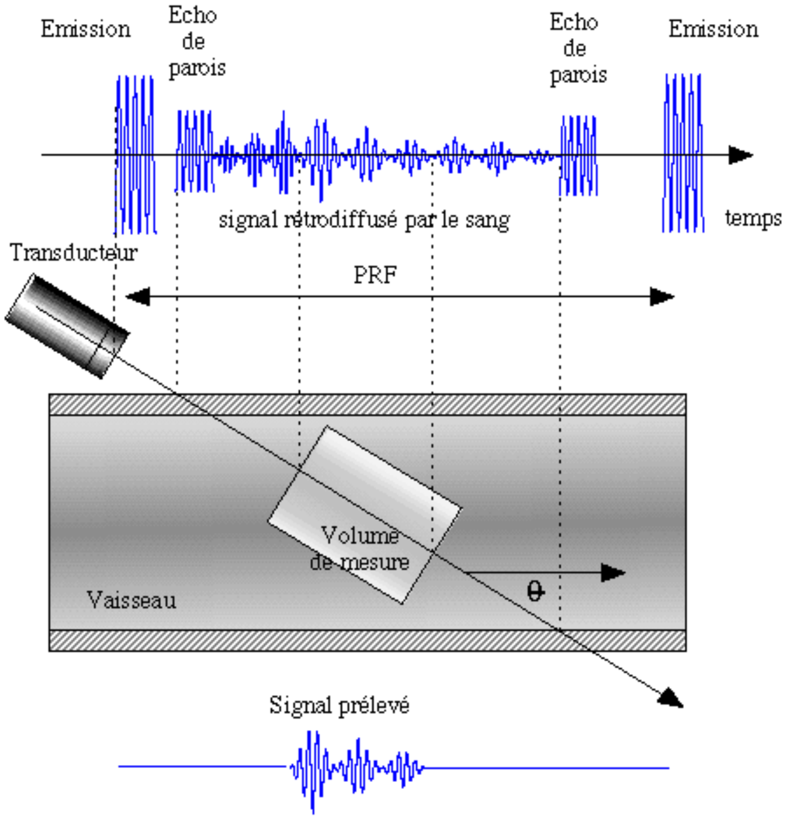
the Continuous wave Doppler and thet pulse Wave Doppler

Doppler systems

Continuous waveDoppler system



Pulse wave Doppler system

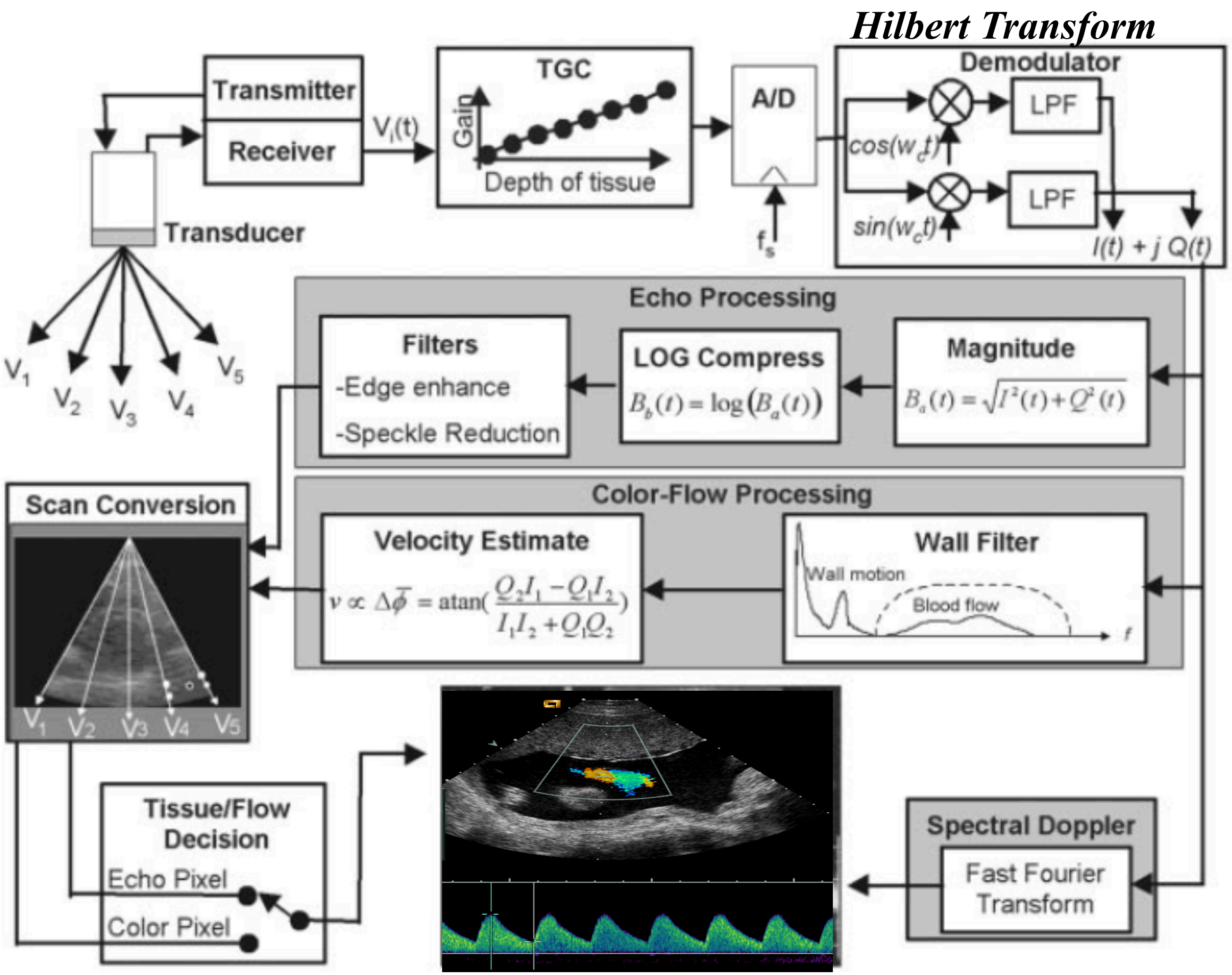


- **Part II : Ultrasound imaging, signals and images**

Outline

- **Part II : Ultrasound imaging : signal and image**
  - Overview typical diagnostic ultrasound machine
  - Components of an imaging system
  - Ultrasound imaging: elementary principles
  - Ultrasound images modes : A, TM, B
  - Type of Ultrasound images modes, and acquisition
  - Image formation : pulse echo equation and scattering
  - 2D and 3D imaging
  - Beamforming : Synthetic aperture
  - Signal and Image Processing Model

Overview of typical diagnostic ultrasound machine



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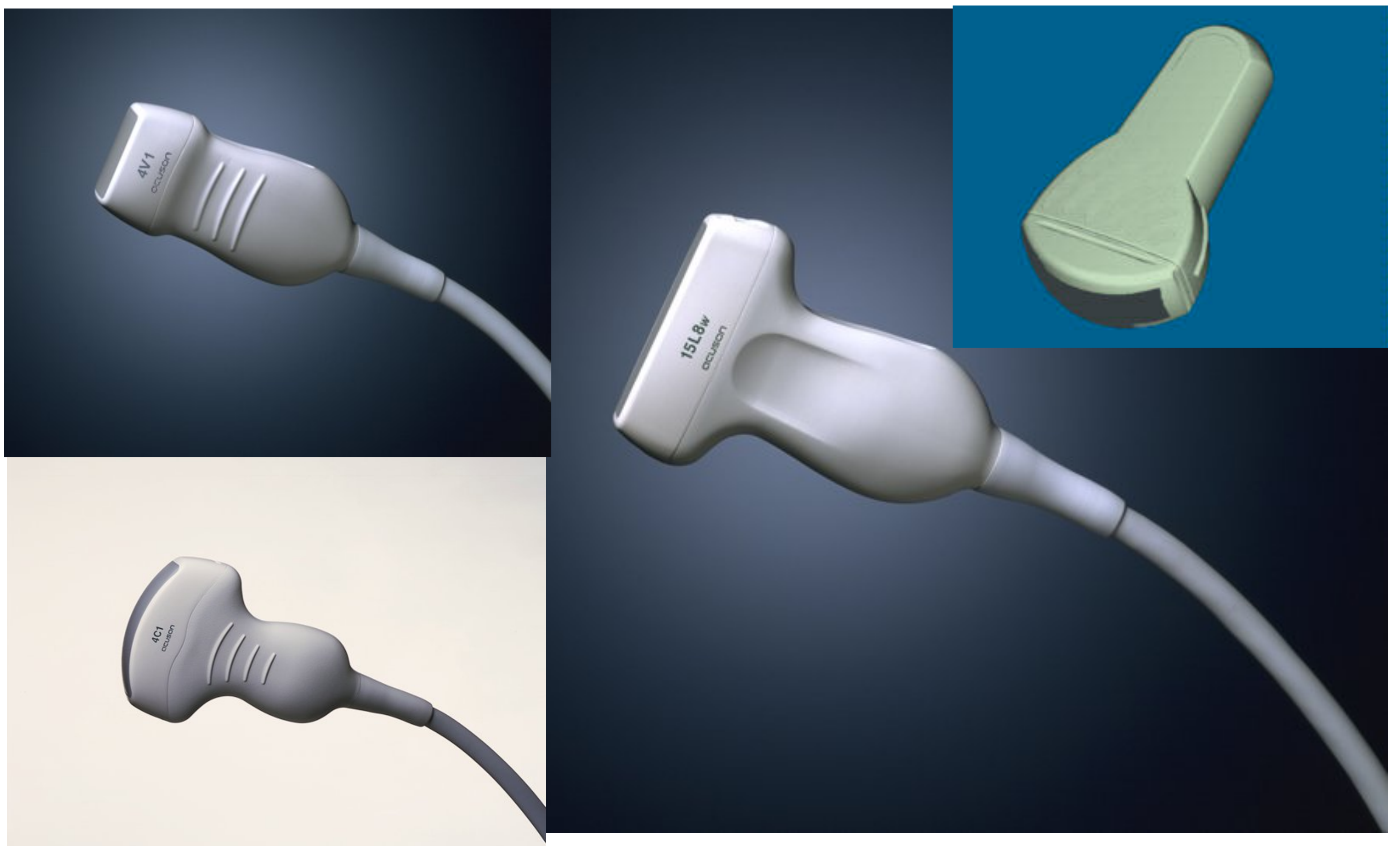
George York and Yongmin Kim, Ultrasound Processing and Computing: Review and Future Directions, Annu. Rev. Biomed. Eng. 1999. 01:559–588

The components of an Imaging System

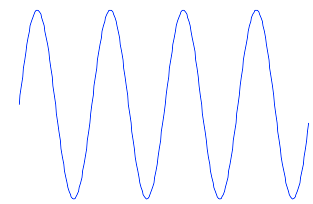
Ultrasound probes

One or multiple piezo-electrical element(s) are embedded in the so called probes

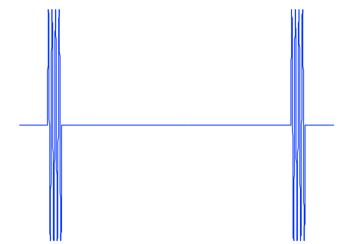


❑ Electrical transmission signal

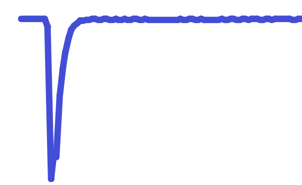
•**Continuous sinusoide**



• **Pulse wave**



• **High voltage Impulse wave**



Exemple

PANAMETRICS
221 Crescent St Waltham MA [illegible]
Tel: 800-225-8330, 617-[illegible]
Fax: 617-899-1552

TRANSDUCER DESCRIPTION

PART NO.: V324
SERIAL NO.: 203543
DESIGNATION: 50 MM PTF

FREQUENCY: 25.00 MHz
ELEMENT SIZE: .25 in. DIA.

TEST INSTRUMENTATION

PULSER/RECEIVER: 5601
DIGITAL OSCILLOSCOPE: LECROY 9450
TEST PROGRAM: TP103 VER. 2.1
CABLE: RG-58A/U 4FT

TEST CONDITIONS

PULSER SETTING: ENERGY:1; DAMPING:50
RECEIVER SETTING: ATTN:14dB; GAIN:46dB
TARGET: .125 IN SILICA; WATER PATH: 49.7586mm
JOB CODE: T/200

MEASUREMENTS PER ASTM E1065

FOCAL LENGTH ----- 49.76mm

WAVEFORM DURATION:
-14DB LEVEL -- .104US
-20DB LEVEL -- .125US
-40DB LEVEL -- .224US

SPECTRUM PARAMETERS:
CENTER FREQ. ---- 25.5MHz
PEAK FREQUENCY -- 26.3MHz
-6DB BANDWIDTH -- 49.01 %

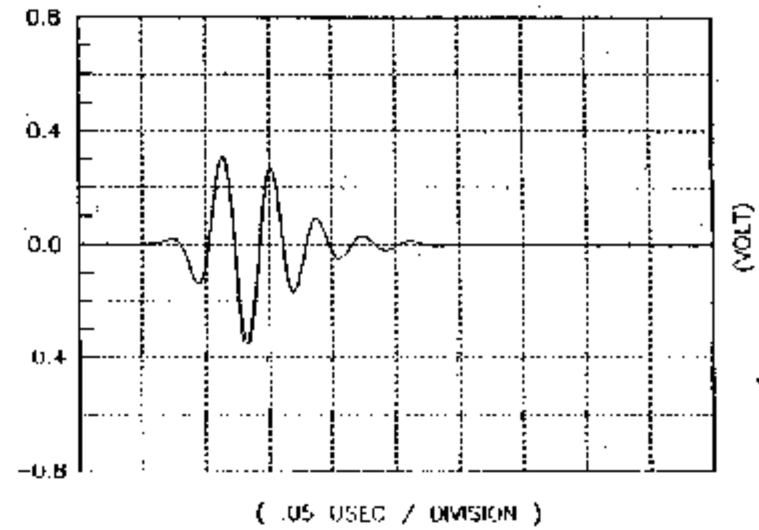
COMMENTS:

SOFRANEL
SOCIETE FRANÇAISE D'ELECTROPHYSIQUE
59, rue Parmentier
78500 SARTROUVILLE

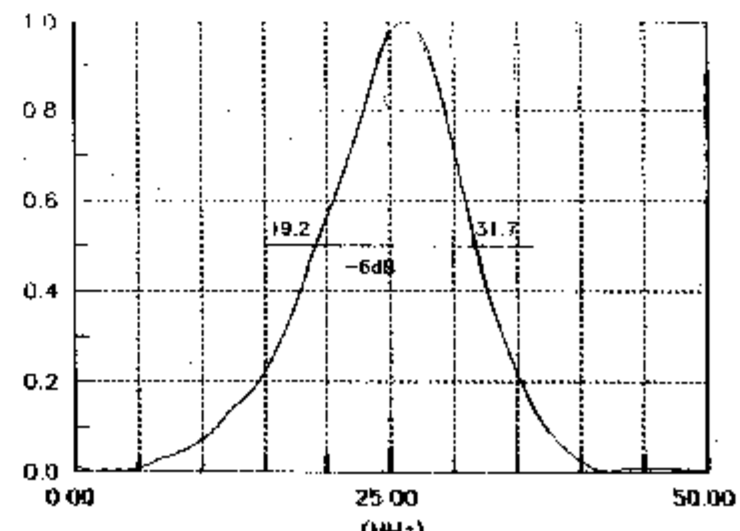
** ACCEPTED.

TECHNICIAN. (3) DATE: 03-02-1995

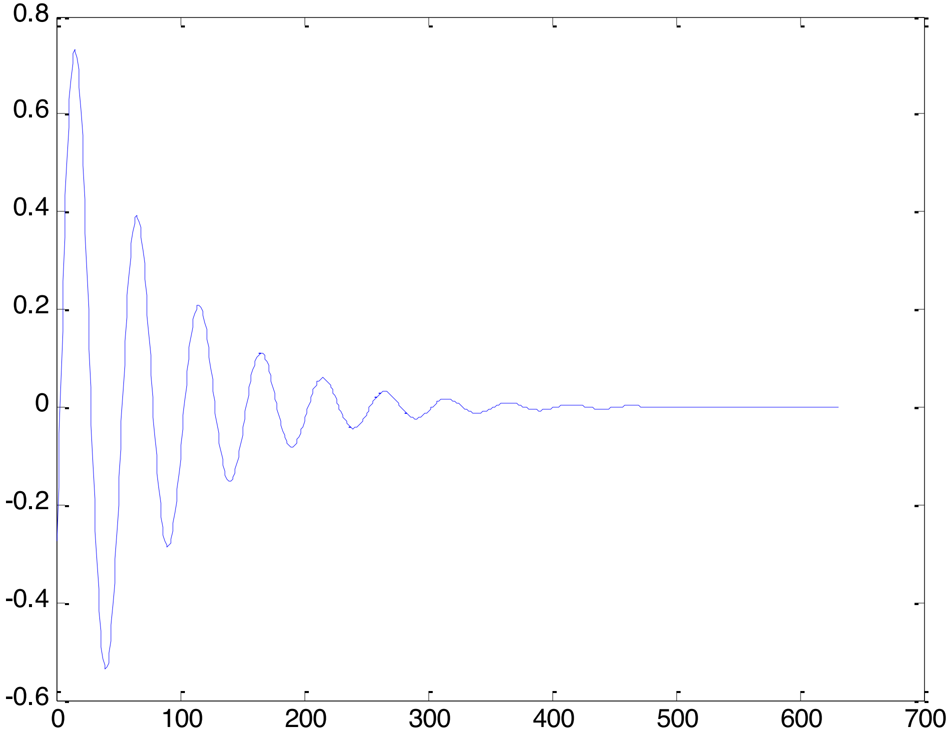
SIGNAL WAVEFORM



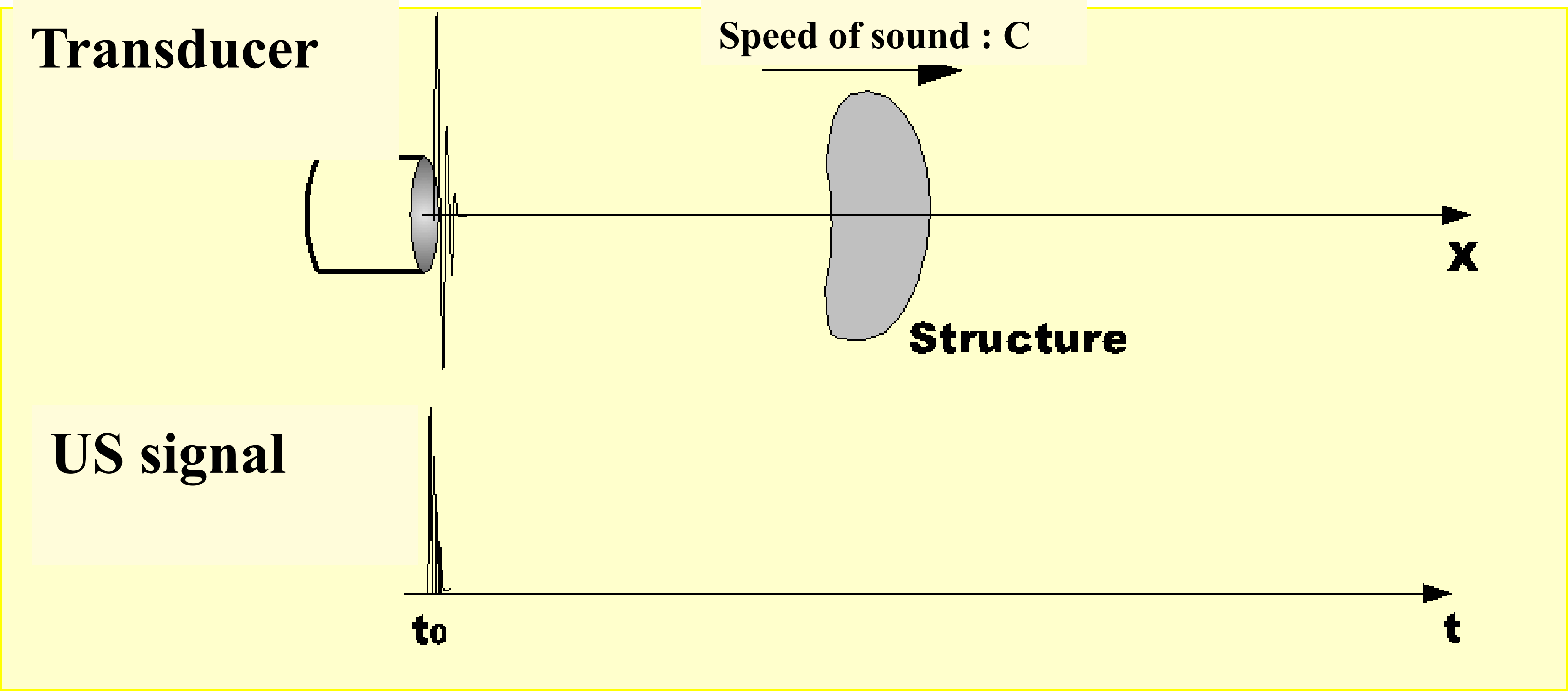
FREQUENCY SPECTRUM



Exemple of signal

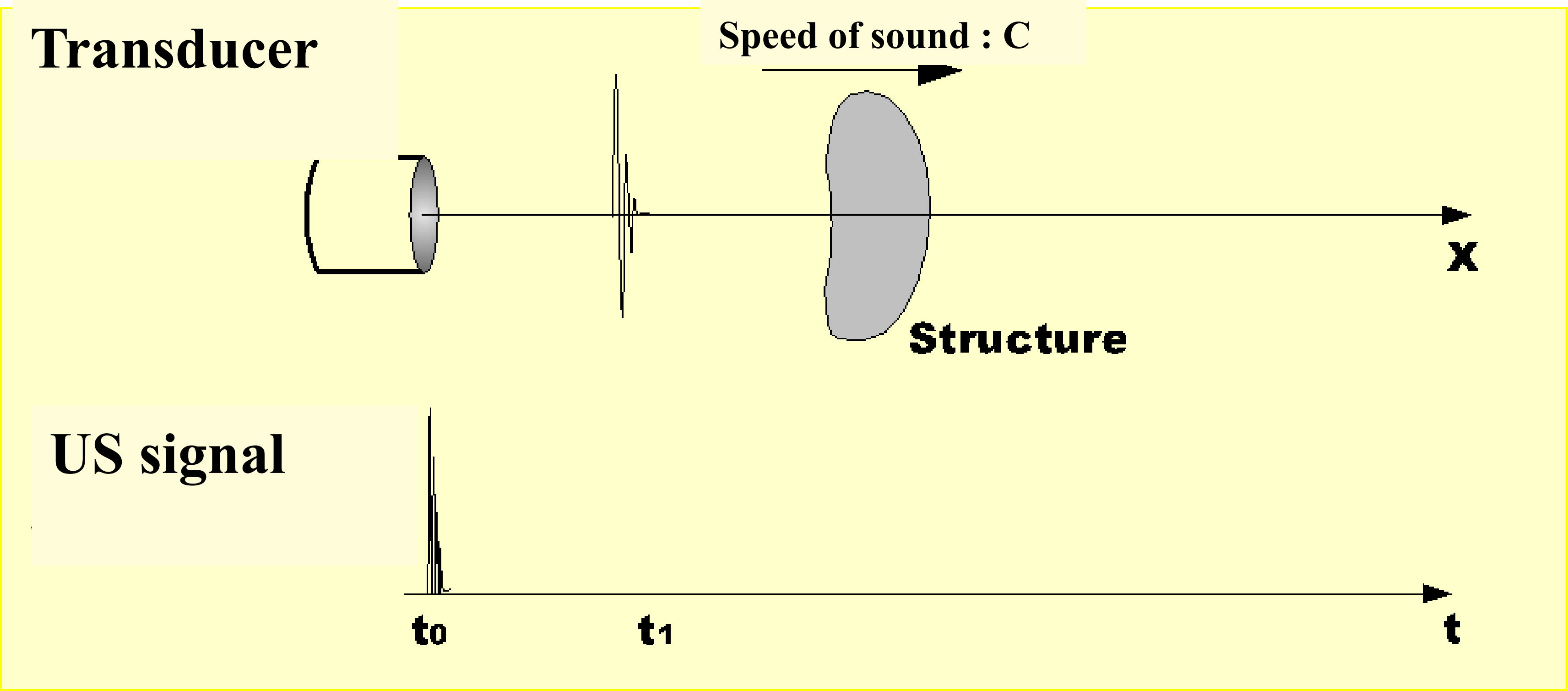


❑Ultrasound imaging: elementary principles



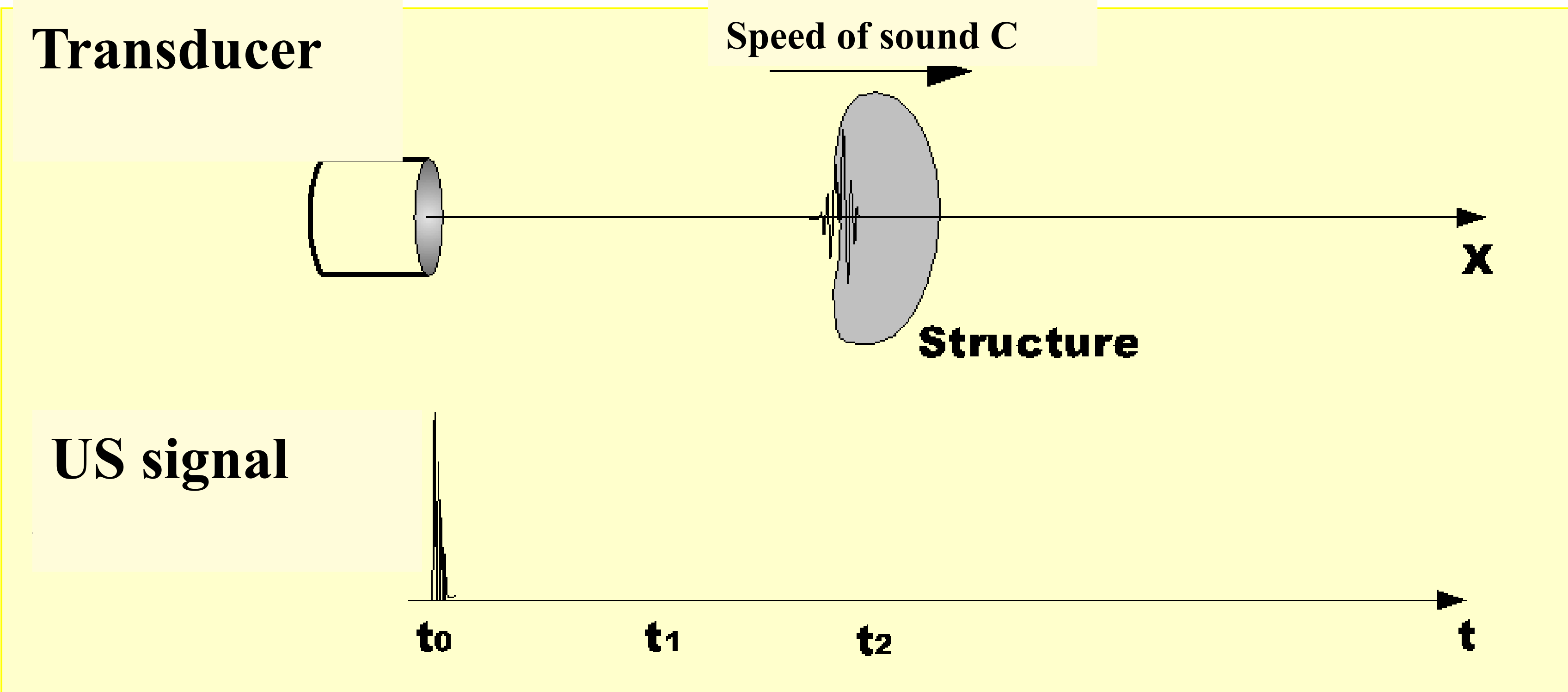
At t_0 ⇨ transmission of a short duration ultrasound wave.

❑ Ultrasound imaging: elementary principles



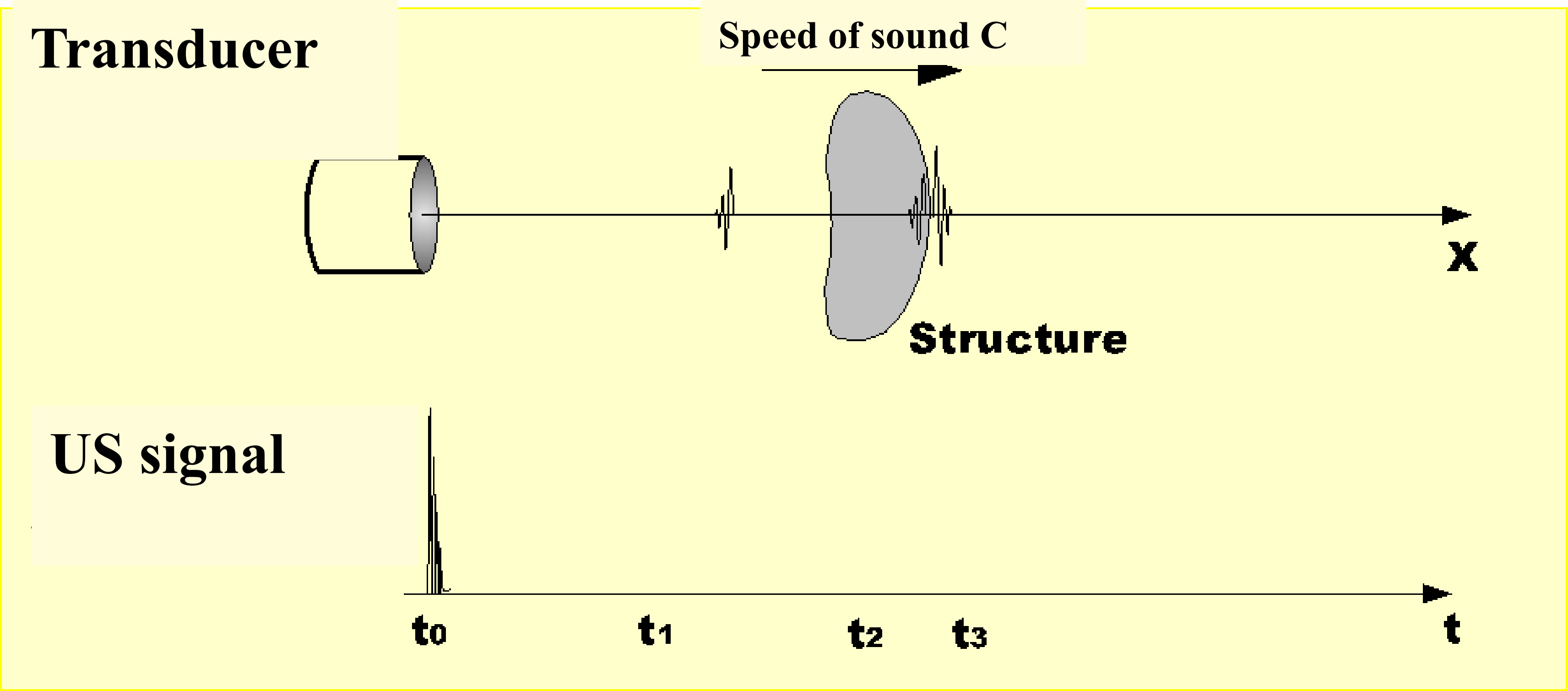
À t_1 ⇨ propagation of the wave in the medium at speed of sound *c*.

❑ Ultrasound imaging: elementary principles



À t_2 ⇨ generation of an echoe at the first interface (1^{st} echoe).

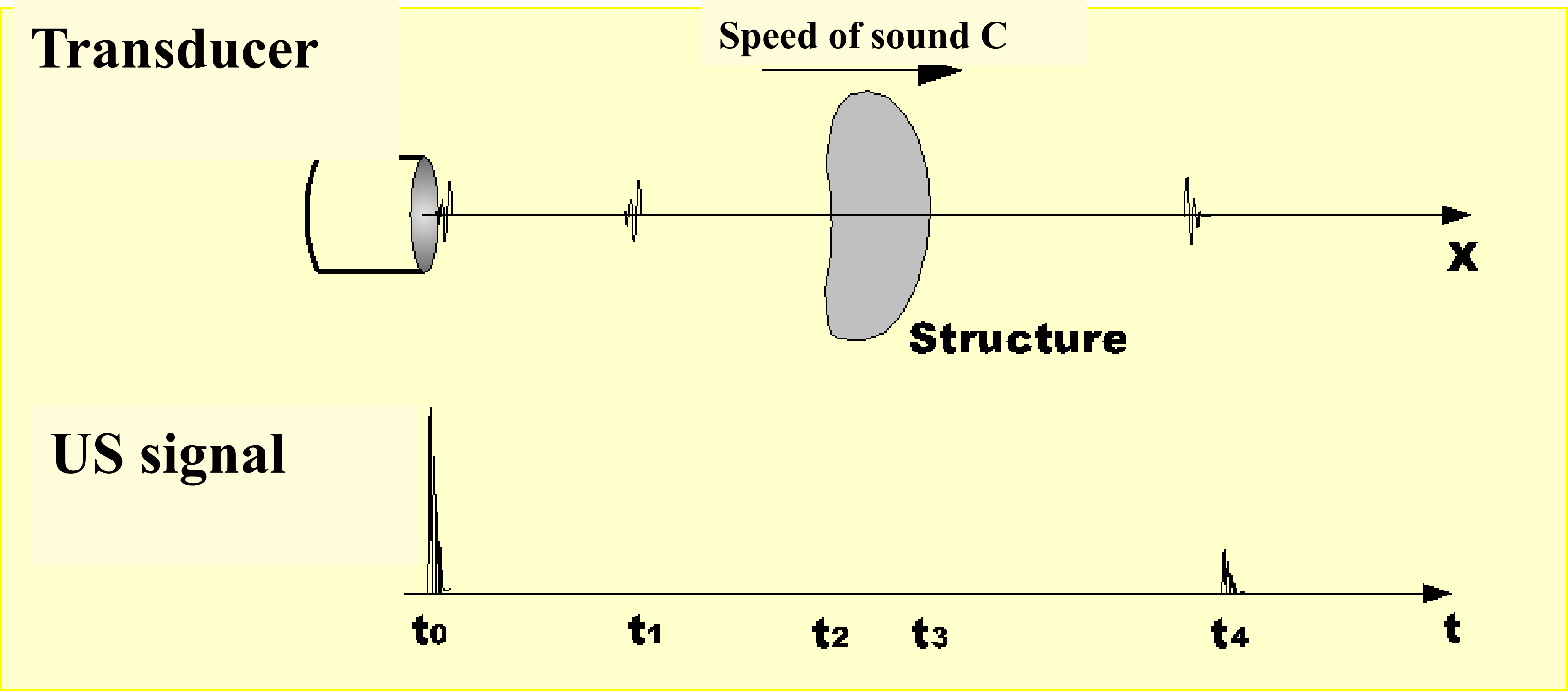
❑ Ultrasound imaging: elementary principles



À t_2 ⇨ generation of an echoe at the first interface.

À t_3 ⇨ generation of an echoe at the second interface(2nd echoe).

❑ Ultrasons: principes de l'échographie

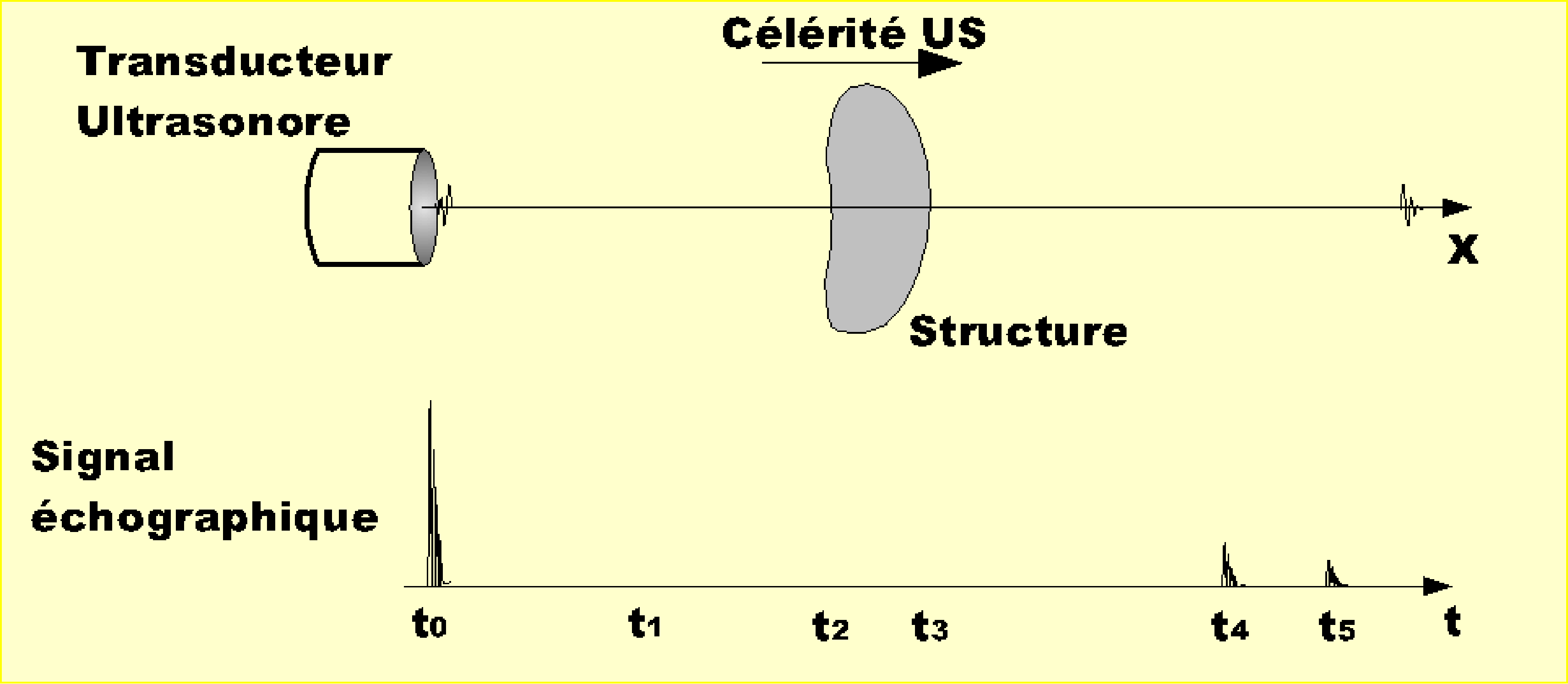


À t_2 ⇨ generation of an echoe at the first interface(first echoe).

À t_3 ⇨ generation of an echoe at the second interface(second echoe).

À t_4 ⇨ The first echoe is back to the transducer.

❑ Ultrasons: principes de l'échographie



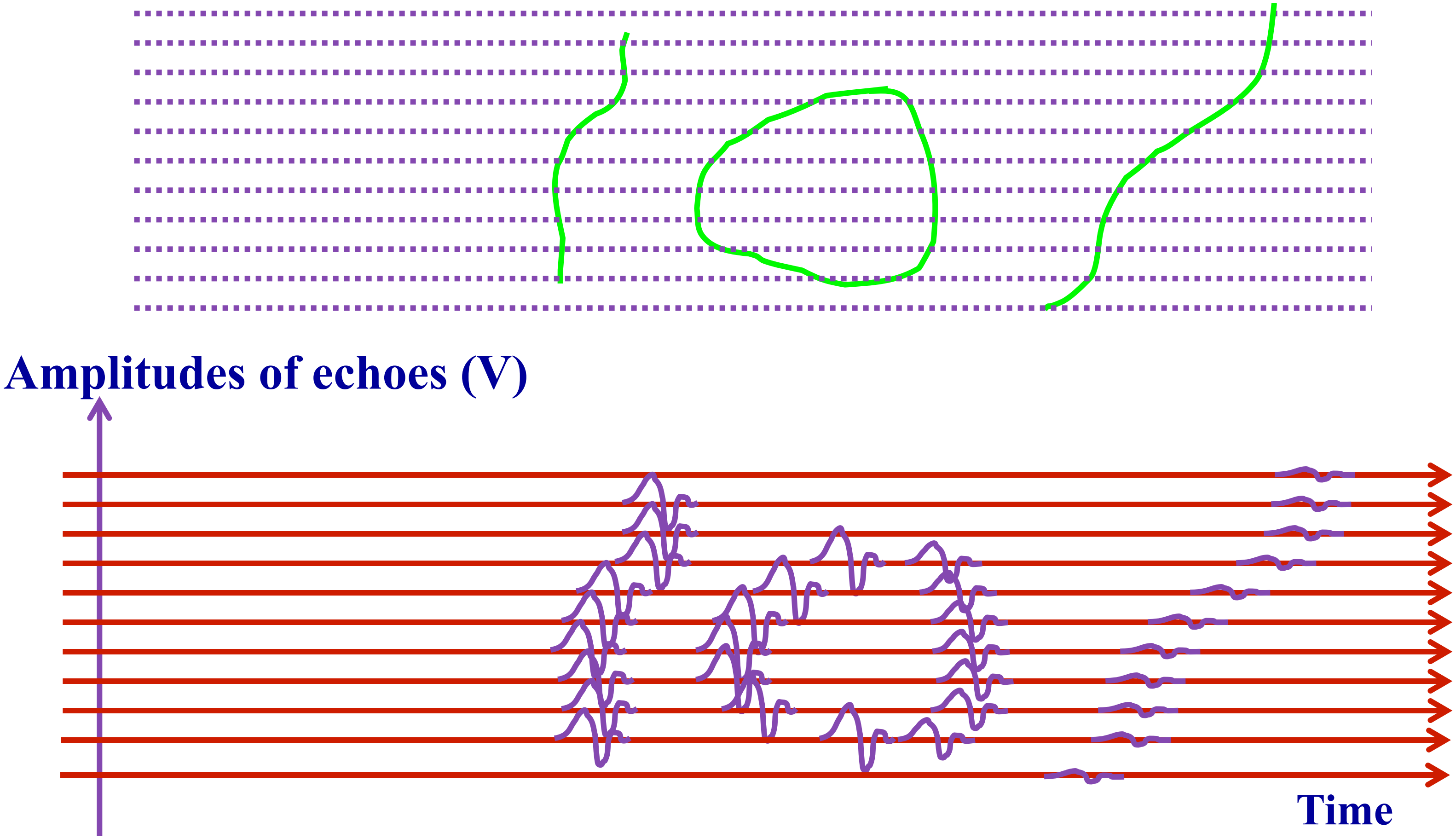
À t_2 ⇨ generation of an echoe at the first interface(first echoe).

À t_3 ⇨ generation of an echoe at the second interface(second echoe).

À t_4 ⇨ The first echoe is back to the transducer

À t_5 ⇨ The second echoe is back to the transducer and display.

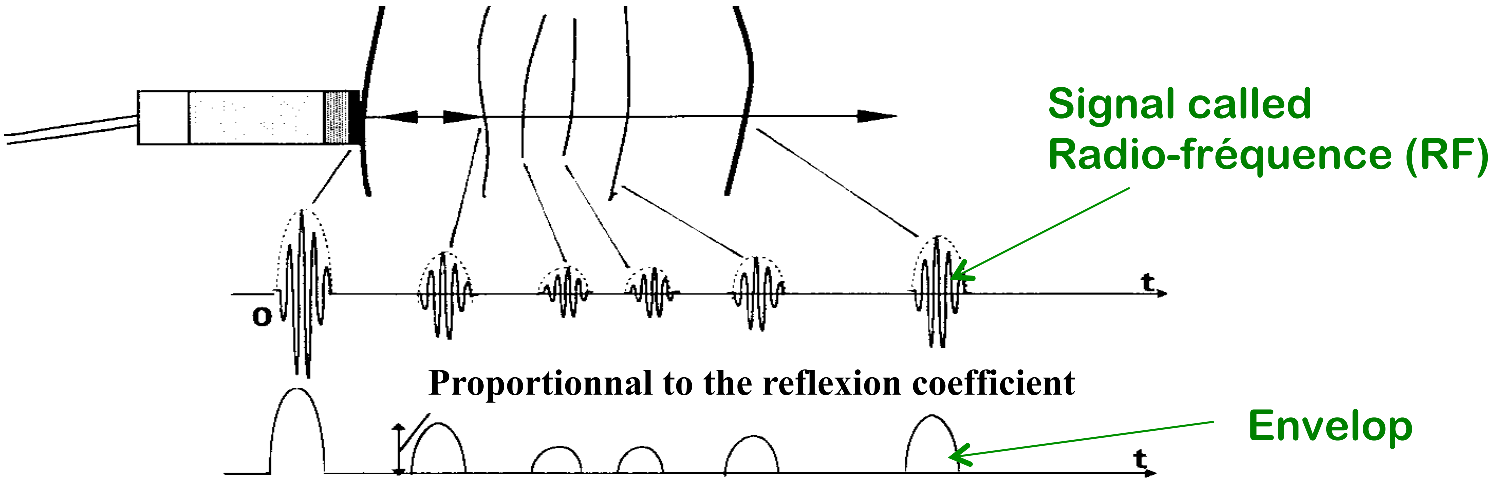
❑ Ultrasound imaging: elementary principles



There exists different image display modes

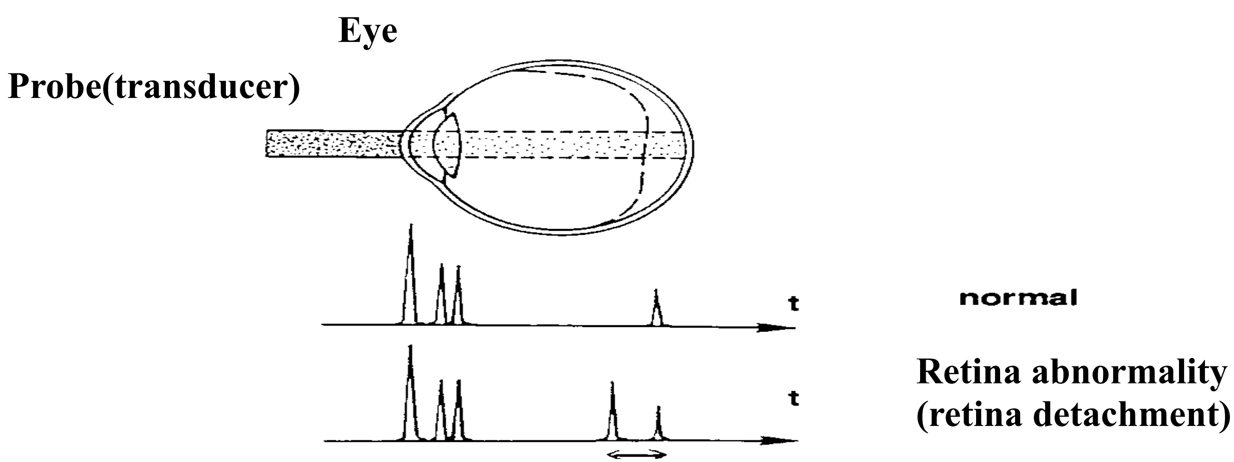
- **Ultrasound images modes : A, TM, B**

➢ Amplitude Mode or A mode

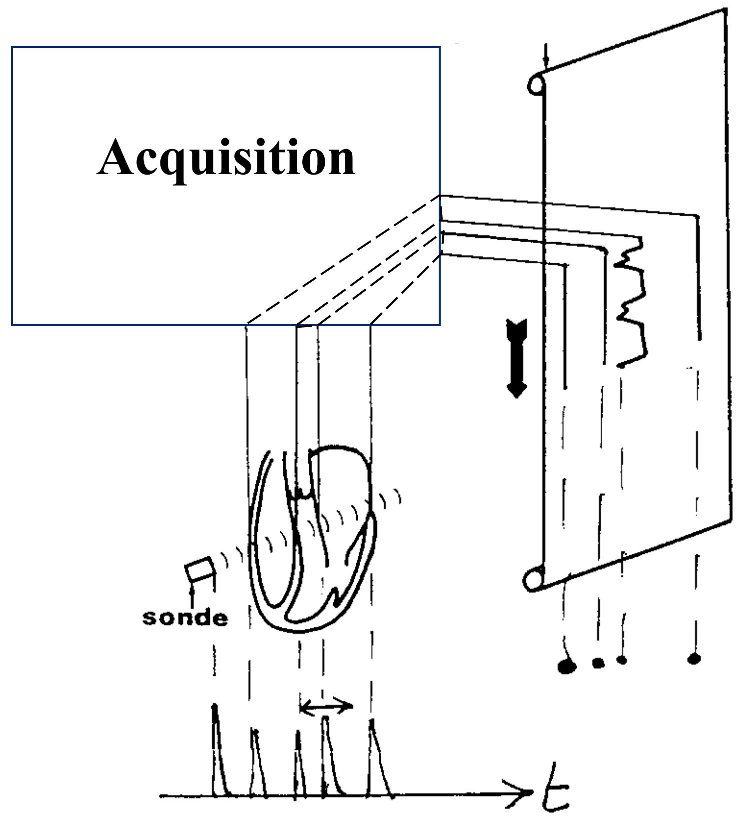


One dimension exploration for : - distance

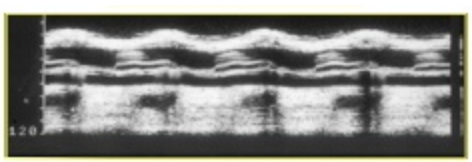
Application : distance and thickness measurement



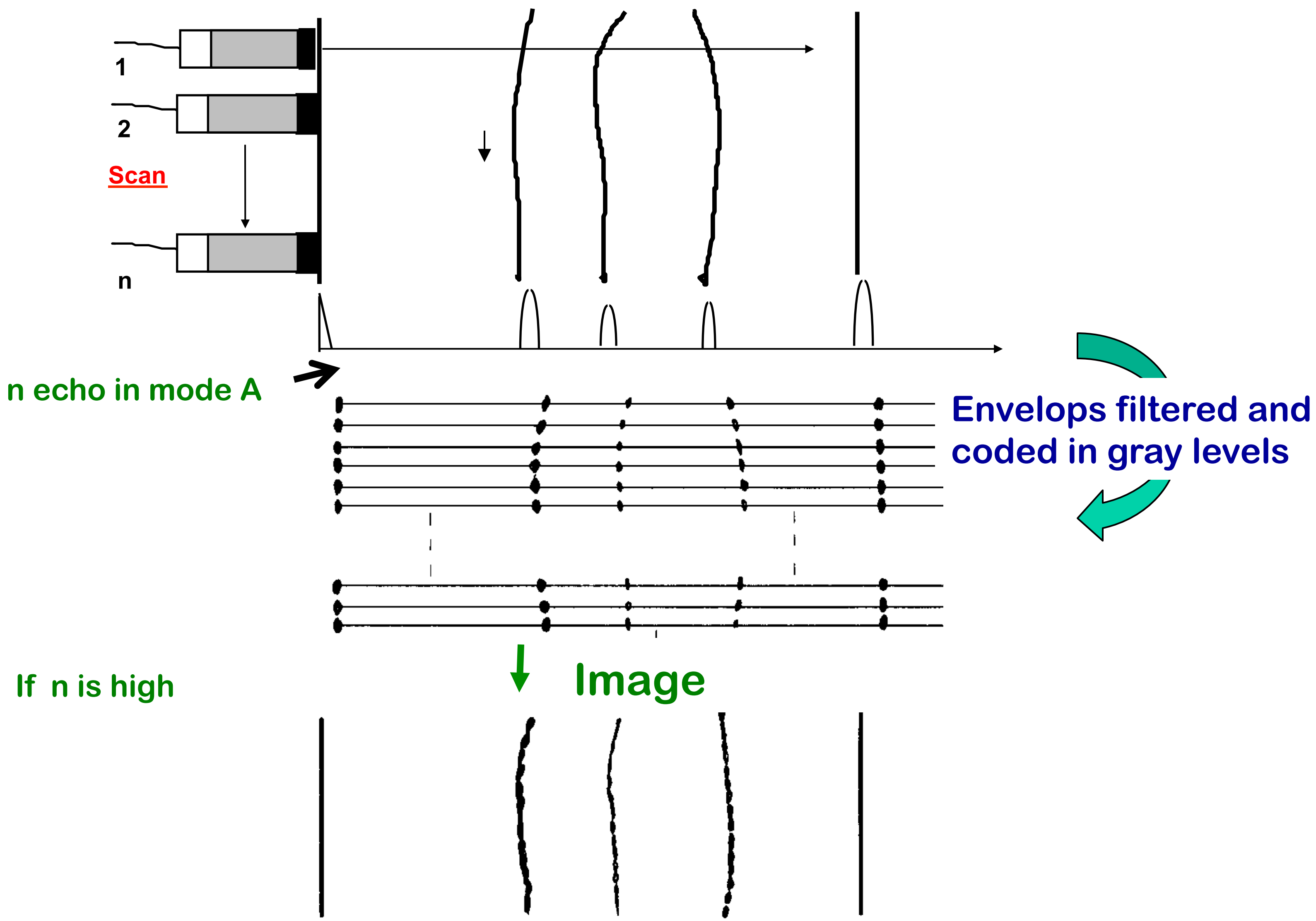
➢ Time Movement or TM mode



Display as a matrix of mode A columns



➢ Brightness Mode or B mode



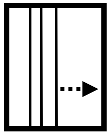
❑Type of ultrasound images and acquisition

Ultrasound probe: scans a slice → échographic slice

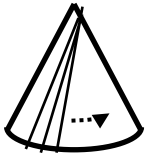
Image = superposition of lines corresponding to the different directions of ultrasound faisceau

3 types of images

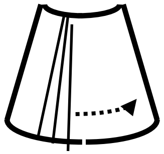
Rectangle scan = parallel lines



Sector scan = divergent lines

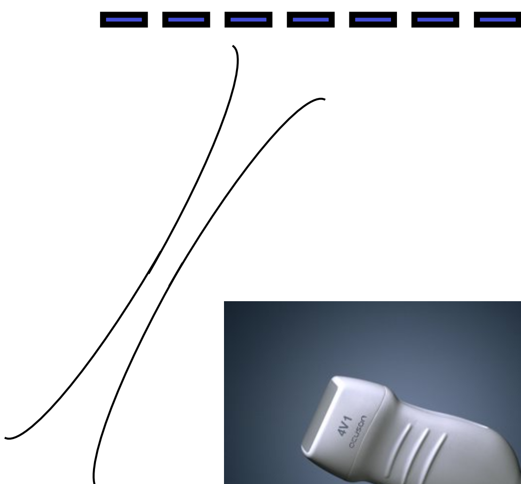


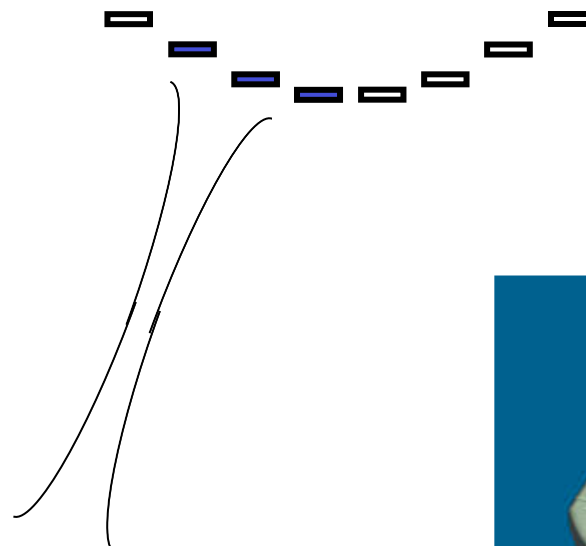
Truncated sector= divergent lines



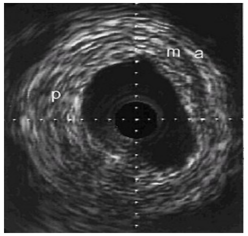
- ***Linear scan***

- ***sector scan***

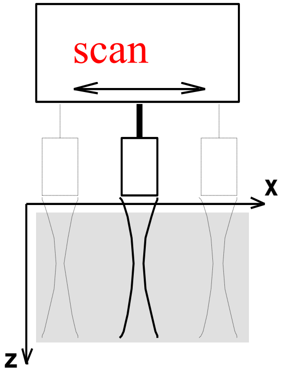




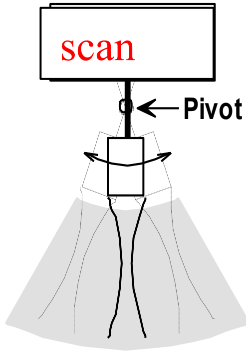
ACQUISITION OF AN IMAGE



2D B mode scan

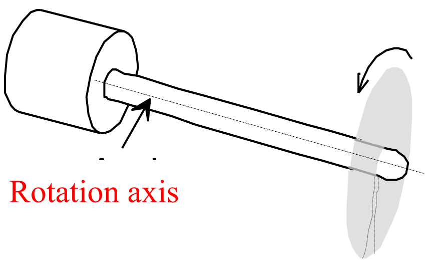


Rectangular image



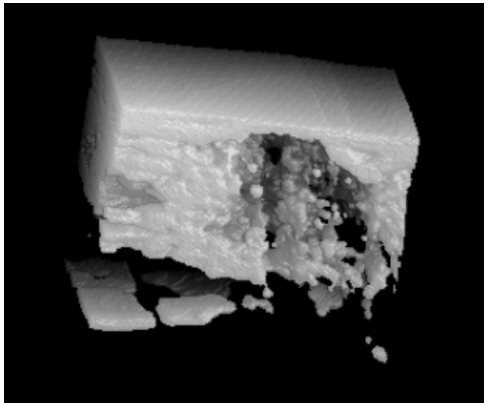
Sector image

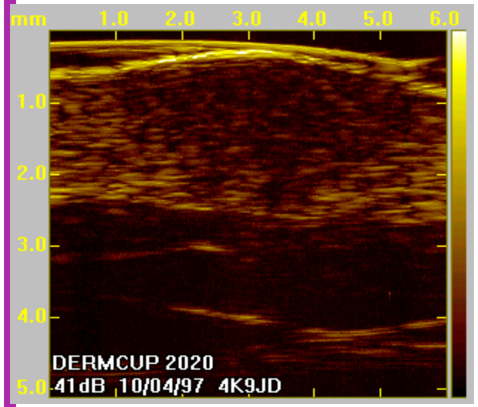
b

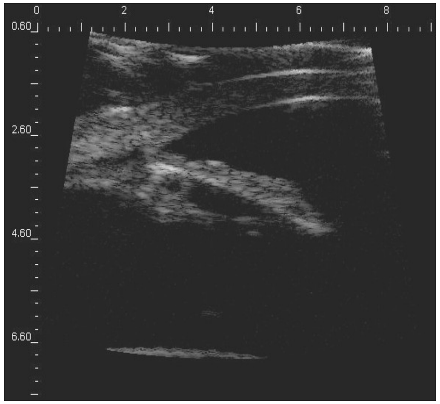


Circle image

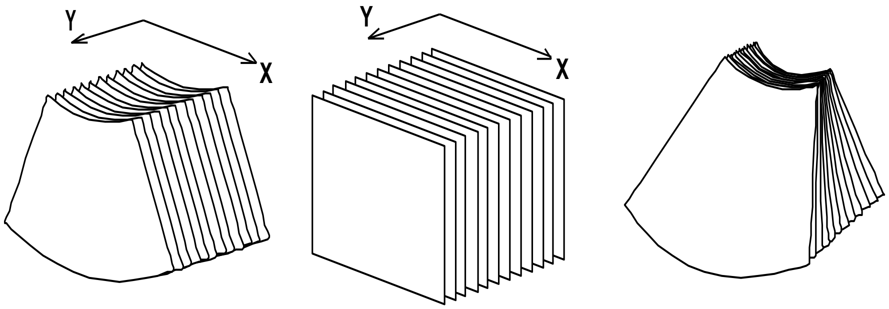
c







3D scan



a b c

❑Image formation : pulse echo equation and scattering

We consider here linear propagation and imaging.

❑ 3D Acoustic Wave equation, in the presence of scatterers

Remember : *We consider in the first part the case of homogeneous medium. In this case the wave propagation equation is (AWE2)*

$$\frac{\partial^2 p}{\partial t^2} - c^2 \nabla^2 p = 0$$

Now : *consider the small variation of the density* $\Delta\rho(\mathbf{r})$ *and the compressibility* $\Delta\chi(\mathbf{r})$ *in the medium, due the presence of scatterers i.e, the density, the compressibility vary from their mean value*

$$\rho(\mathbf{r}) = \rho_0 + \Delta\rho(\mathbf{r}); \quad \chi(\mathbf{r}) = \chi_0 + \Delta\chi(\mathbf{r})$$

❑ 3D Acoustic Wave equation, in the presence of scatterers

Then it can be shown after some derivations, e.g. [Morse 1968, Jensen 1991] that

$$\boxed{\nabla^2 p(\mathbf{r},t) - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}(\mathbf{r},t) = \frac{1}{c_0^2}\frac{\Delta\chi(\mathbf{r})}{\chi_0}\frac{\partial^2 p}{\partial t^2}(\mathbf{r},t) + \nabla\cdot\left[\frac{\Delta\rho(\mathbf{r})}{\rho_0 + \Delta\rho(\mathbf{r})}\nabla p(\mathbf{r},t)\right]}$$

(NWE1)

- *Different versions of this equation exist since* χ, ρ *and c are related*
- *The two terms on the right side account for the scattering. They vanish for homogeneous medium, i.e :* $\Delta\chi(\mathbf{r}) = 0$ *and* $\Delta\rho(\mathbf{r}) = 0$

❑ 3D Acoustic Wave equation, in the presence of scatterers

- *Solving this equation is not obvious in general. Doing some approximations this equation can be solved either in spatial(e.g. [Jensen 1991]) or frequency (e.g. [Ng 2006], Insana1993]) domain*

- *We present the resolution in frequency domain by adopting the assumptions and methodology followed by Ng 2006 because it is simple and quite pretty. First, we briefly recall it and use the results as basis to explain ultrasound signals image formation.*

Define

$$\boxed{H_t(\mathbf{r},\omega) \equiv \int_A a_t(\mathbf{r}_a)\exp[-j\omega\tau_t(\mathbf{r}_a)]\frac{\exp(-j\frac{\omega}{c_0}\left|\mathbf{r}-\mathbf{r}_a\right|)}{2\pi\left|\mathbf{r}-\mathbf{r}_a\right|}d^2\mathbf{r}_a}$$

(TRPSF)

$H_t(\mathbf{r},\omega)$ *is the Spatial transmit transfert function.*

Its temporal inverse Fourier transform is the transmission spatial point spread function (PSF)

Thus

$$\boxed{P_i(\mathbf{r},\mathbf{r}_0,\omega) = j\omega\rho_0 V_n(\omega) H_t(\mathbf{r}-\mathbf{r}_0,\omega)}$$

Here again define :

$$H_r(\mathbf{r},\omega) \equiv \int_A a_r(\mathbf{r}_a)\exp[-j\omega\tau_r(\mathbf{r}_a)]\frac{\exp\left(-j\frac{\omega}{c_0}\left|\mathbf{r}-\mathbf{r}_a\right|\right)}{2\pi\left|\mathbf{r}-\mathbf{r}_a\right|}d^2\mathbf{r}_a$$

(TRPSF)

$H_r(\mathbf{r},\omega)$ *is the Spatial received Transfert function.*

Its temporal inverse Fourier transform is the received spatial point spread function (PSF).

After some derivations[Ng 2006] it comes:

$$\boxed{R(\mathbf{r}_0,\omega) \approx \int_{R^3} H_{pe}(\mathbf{r}_1 - \mathbf{r}_0,\omega) f(\mathbf{r}_1) d^3\mathbf{r}_1}$$

where

$H_{pe}(\mathbf{r},\omega)$ *is the pulse echo kernel and is defined by :*

$$\boxed{H_{pe}(\mathbf{r},\omega) \equiv E_m(\omega) H_t(\mathbf{r},\omega) H_r(\mathbf{r},\omega) \text{ with } E_m(\omega) = j\omega^3 V_n(\omega) W_n(\omega)}$$

$f(\mathbf{r})$ *is the reflectivity function. It characterizes acoustic property of the medium:*

Attenuation effect

Not including dispersive attenuation is, not a serious drawback of the theory, as this change of the pulse can be embedded into the already spatially varying $H_{pe}(\mathbf{r},\omega)$.

Here again A model for the attenuation is

$$\boxed{Att(f,\mathbf{r}) \equiv \exp\left(-\beta f^{m}\left|\mathbf{r}\right|\right)}$$

Finally TGC also apply

Thus by taking the inverse Fourier transform over ω *the time domain RF signal is*

$$\boxed{r(\mathbf{r}_0,t) \approx \int_{R^3} h_{pe}(\mathbf{r}_1-\mathbf{r}_0,t)\, f(\mathbf{r}_1)\, d^3\mathbf{r}_1}$$

where:

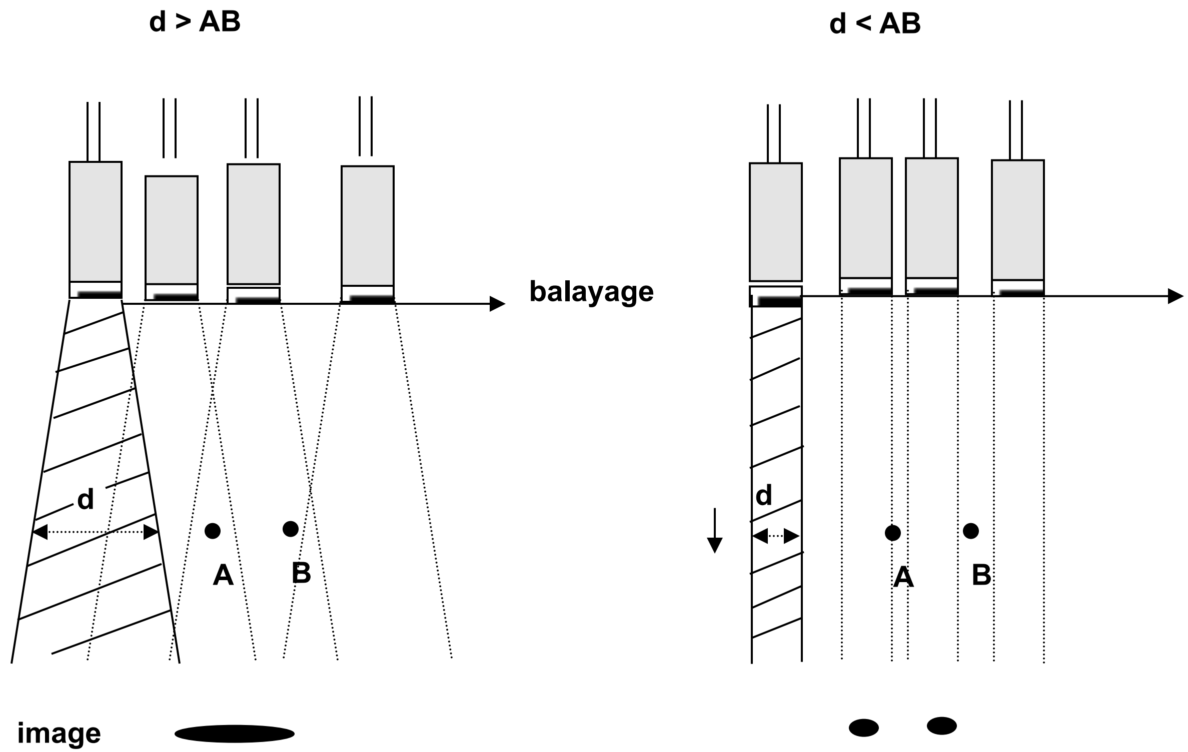
$$\boxed{h_{pe}(\mathbf{r},t) \equiv e_m(t) \underset{t}{*} h_t(\mathbf{r},t) \underset{t}{*} h_r(\mathbf{r},t) \text{ with } e_m(t) = FT^{-1}[\,j\omega^3 V_n(\omega) W_n(\omega)]}$$

and:

$$\boxed{f(\mathbf{r}) \equiv \frac{\rho_0^2 \chi_0}{2}\left[\frac{\Delta\chi(\mathbf{r})}{\chi_0} - \frac{\Delta\rho_0(\mathbf{r})}{\rho_0 + \Delta\rho_0(\mathbf{r})}\right]}$$

FT^{-1} stands for is the inverse Fourier transform :

❑ Exemple lateral resolution



The form of the beam is very important for correctly imaging

❑ Exemple lateral resolution

$$r_{ax} = \frac{2c}{B}$$

$$r_{lat} = \lambda \frac{L_f}{D} = \lambda \frac{Focal\ Distance}{Diameter}$$

✧ The IQ signal and speckle

Defining (x,y,z) coordinates, respectively as lateral azimuthal and axial, and assuming that the active aperture is at z=0; and $\mathbf{r}_1=(x,y,z)$

$$r(x_0,y_0,t)\equiv r(x_0,y_0,0,t)\approx\iiint h_{pe}(x-x_0,y-y_0,z,t)f(x,y,z)\,dx\,dy\,dz$$

(RFEQ)

This equation is RF signal equation

The complex-value demodulated IQ signal is defined by

$$\tilde{r}(x_0,y_0,t)\equiv\exp(-j\omega_0 t)\left[r(x_0,y_0,t)+jHT[r(x_0,y_0,t)]\right]$$

HT denotes Hilbert Transform.

ω_0 is the transducer center frequency (working frequency)

✧ The IQ signal and speckle

Thus the complex IQ signal is

$$\boxed{\tilde{r}(x_0, y_0, t) \approx \iiint \tilde{h}_{pe}(x - x_0, y - y_0, z, t)\,\tilde{f}(x, y, z)\,dx\,dy\,dz}$$

with the complex analytical kernel $\tilde{h}_{pe}$ *defined by*

$$\boxed{\tilde{h}_{pe}(x_0, y_0, z, t) \equiv \exp[-j(\omega_0 t - 2k_0 z)]\left[h_{pe}(x_0, y_0, z, t) + jHT[h_{pe}(x_0, y_0, z, t)]\right]}$$

$$k_0 = \frac{\omega_0}{c_0}$$

And the complex value reflectivity $\tilde{f}$ *is defined by*

$$\boxed{\tilde{f}(x, y, z) \approx \exp(-j2k_0 z)\,f(x, y, z)}$$

✧ The IQ signal and speckle

- *In some situations finding from IQ signal the complex value kernel or complex reflectivity may be easier*

- *Thus these derivations establish a model the soft tissue as a homogeneous fluid with pointlike scattering targets.*

- *This target behave as complex phasors with amplitudes given by $|f(x, y, z)|$ and phase determined by the axial depth z.*

- *These scattering targets are uncorrelated with each other and so their amplitudes are effectively random.*

- *This is speckle*

✧ The IQ signal and speckle

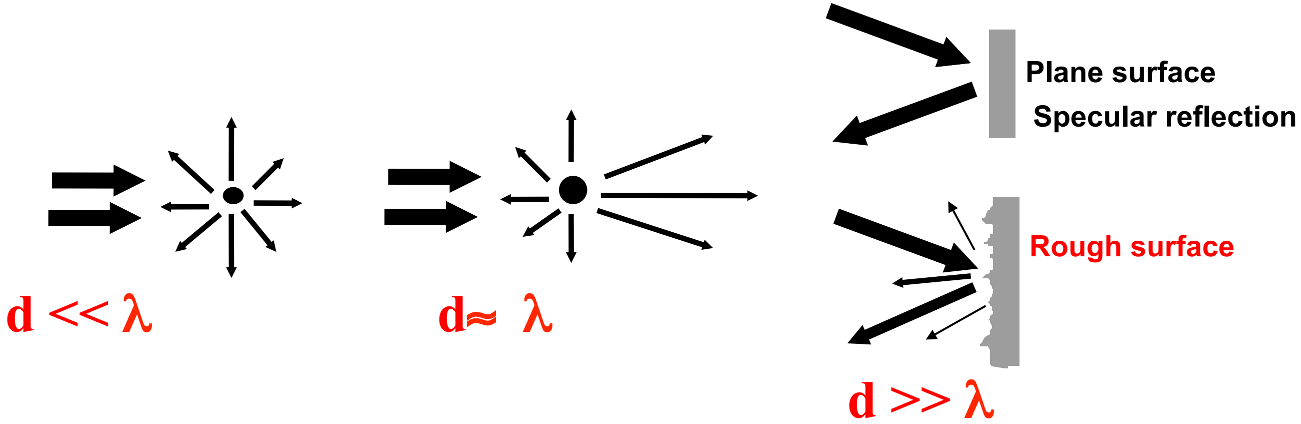
The integral expression

$$\boxed{\tilde{r}(x_0, y_0, t) \approx \iiint \tilde{h}_{pe}(x - x_0, y - y_0, z, t)\, \tilde{f}(x, y, z)\, dx\, dy\, dz}$$

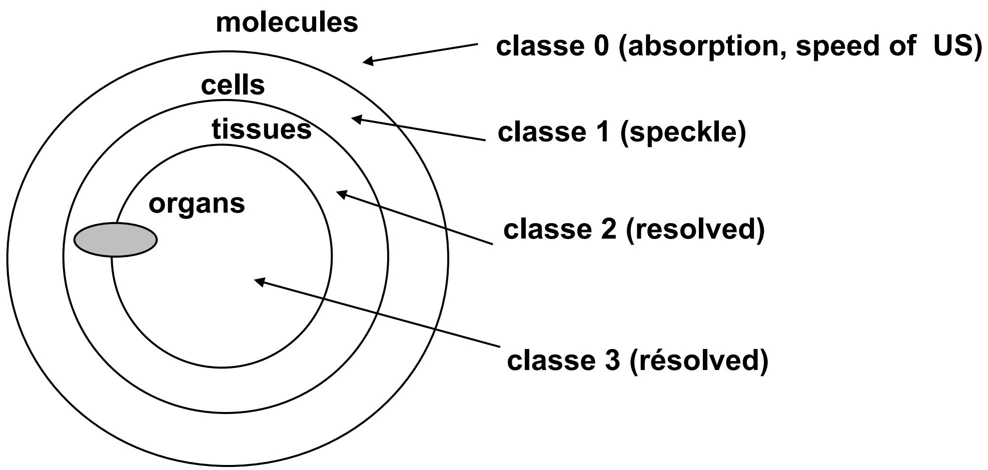
may therefore be considered to describe the weighted sum of complex phasors with random amplitude and random phase

Classically due to high concentration of scatterers the RF $r(x_0, y_0, t)$ *signal is assumed to be Gaussian distributed, and its magnitude which the envelop of the RF signal* $|\tilde{r}(x_0, y_0, t)|$ *is assumed to be Rayleigh distributed*

❑ the speckle

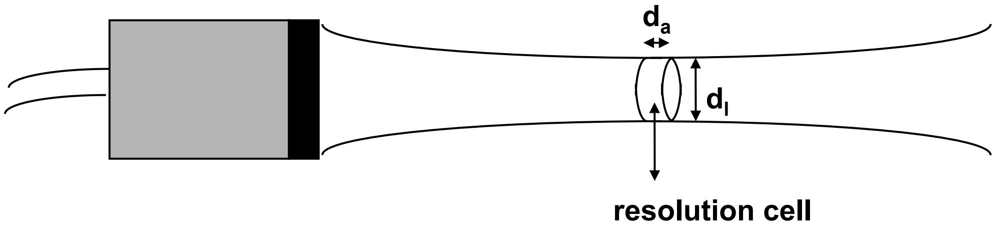


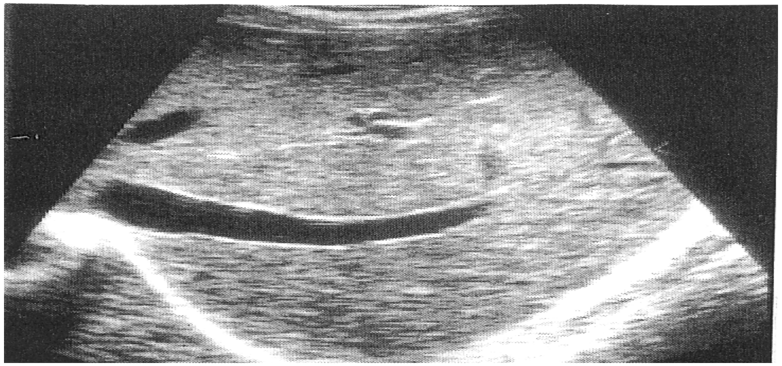
The target, depending on its dimension and its shape has a scattering function → signal et images with random behaviour.



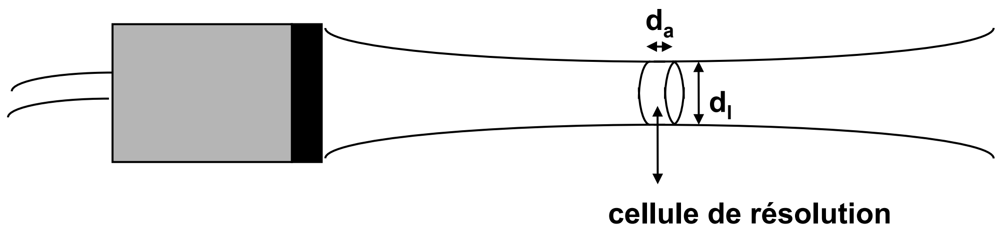
❑ the speckle

B mode image is a convolution of PSF with scattering function of the scaterrer
Only target of classes 2 et 3 are resolved with classical frequency (1-50Mhz).





❑ the speckle



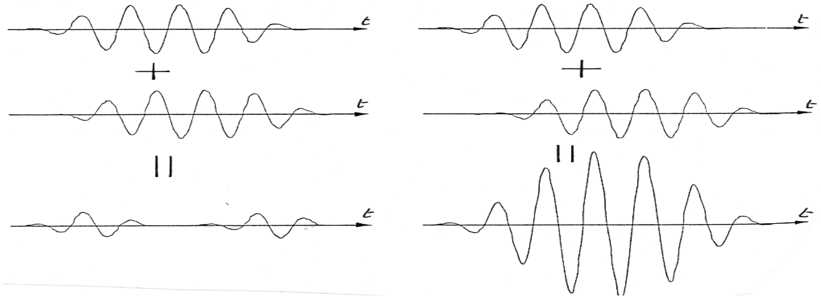
The speckle is the result of interference between the waves scaterred by the differents targets in the resolution cell

If the density of the targets or the scaterrers is high in the resolution cell →speckle

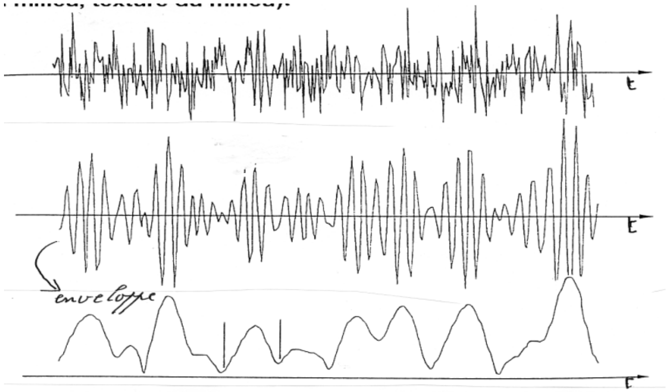
If the density tends to 1 → classe 2 (resolved)

❑ Scaterring : the speckle

Interferences as function of the distance between two targets:



The signal shape due to the speckle as function of the distance between all the targets (density, texture of the medium)



❑ the speckle

In Conclusion :The speckle is thus a random process

Due to high number of scaterers in the acquisition volume,
the Rf signal from the scaterers is assumed to have a Gaussian distribution (Law of large numbers)

Thus the pdf of the envelop (image en mode B), is assumed to have a Rayeigh pdf

$$p_A(a) = \frac{a}{\sigma^2} exp\left(-\frac{a^2}{2\sigma^2}\right), a \geq 0$$

-Or a Rice pdf , if there exists a cohorence component (deterministic)

$$p_A(a) = \frac{a}{\sigma^2} exp\left(-\frac{a^2+s^2}{2\sigma^2}\right) I_0\left(\frac{as}{\sigma^2}\right) a \geq 0$$

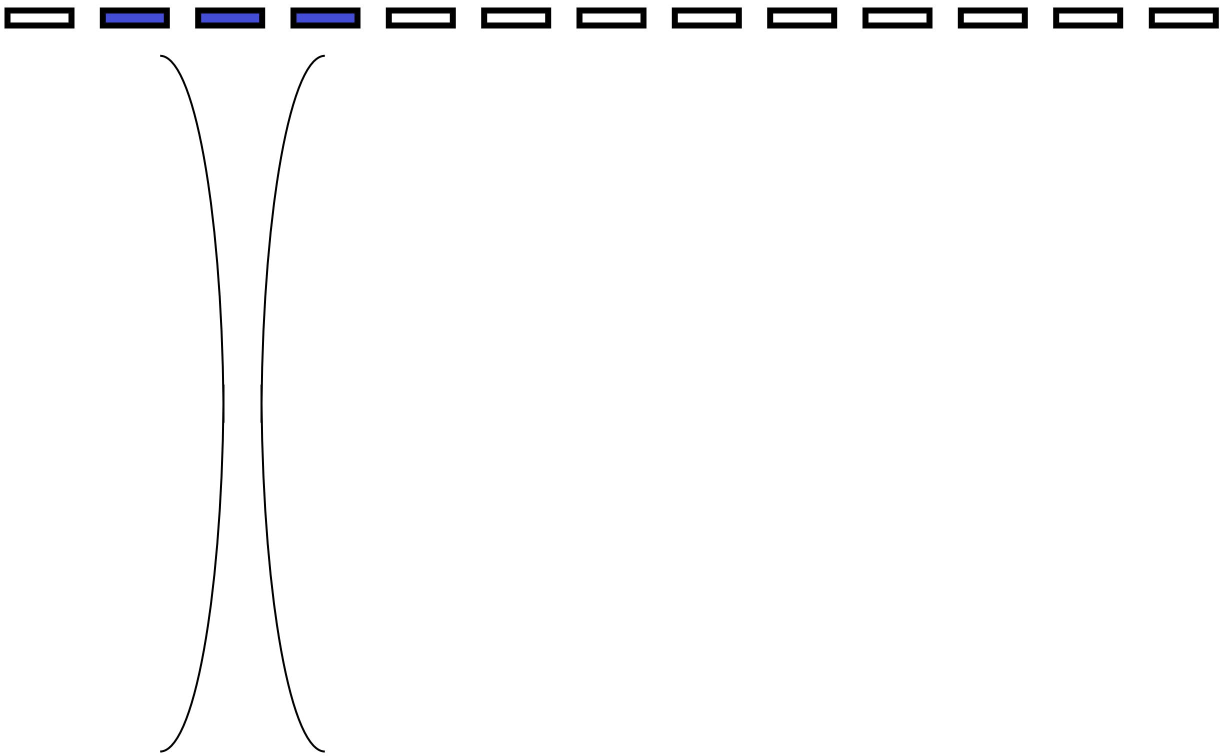
s/σ=parameter of the law; I_0 = Bessel function

❑2D and 3D Imaging

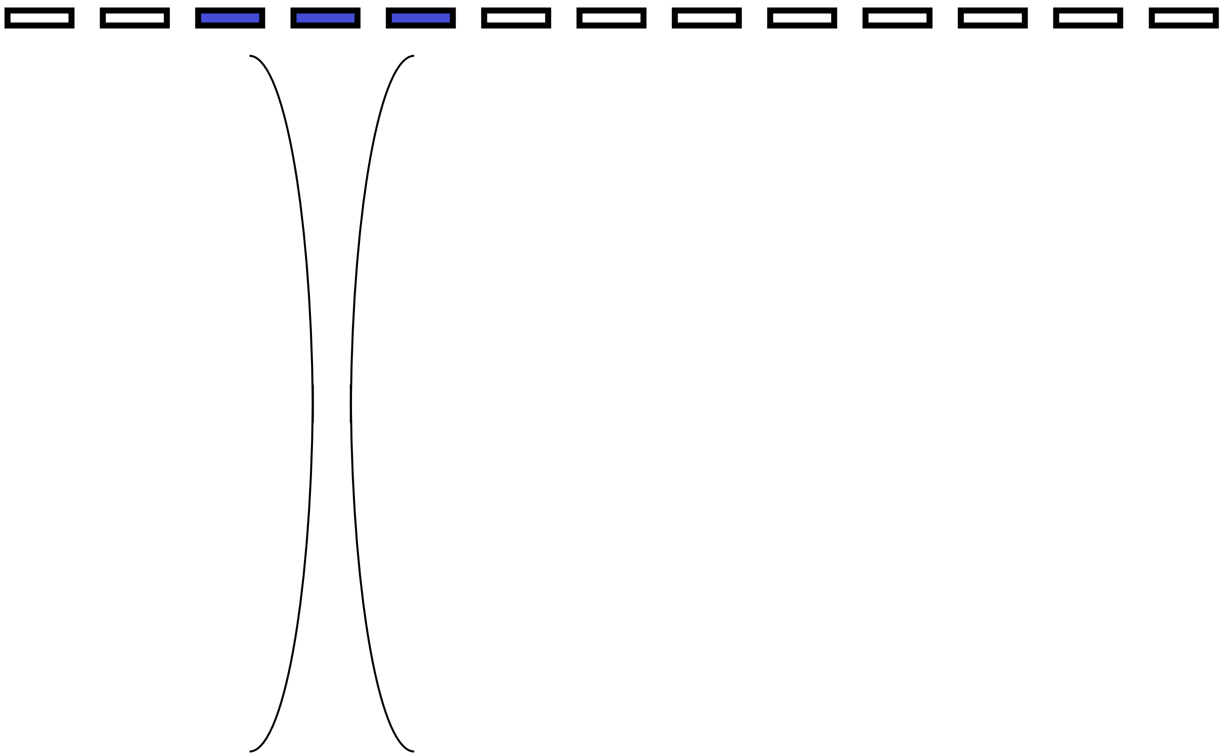
The derivations presented above to explain the RF signal apply in 2D imaging provided appropriate scans and beamforming are used

Linear Acquisition : principle

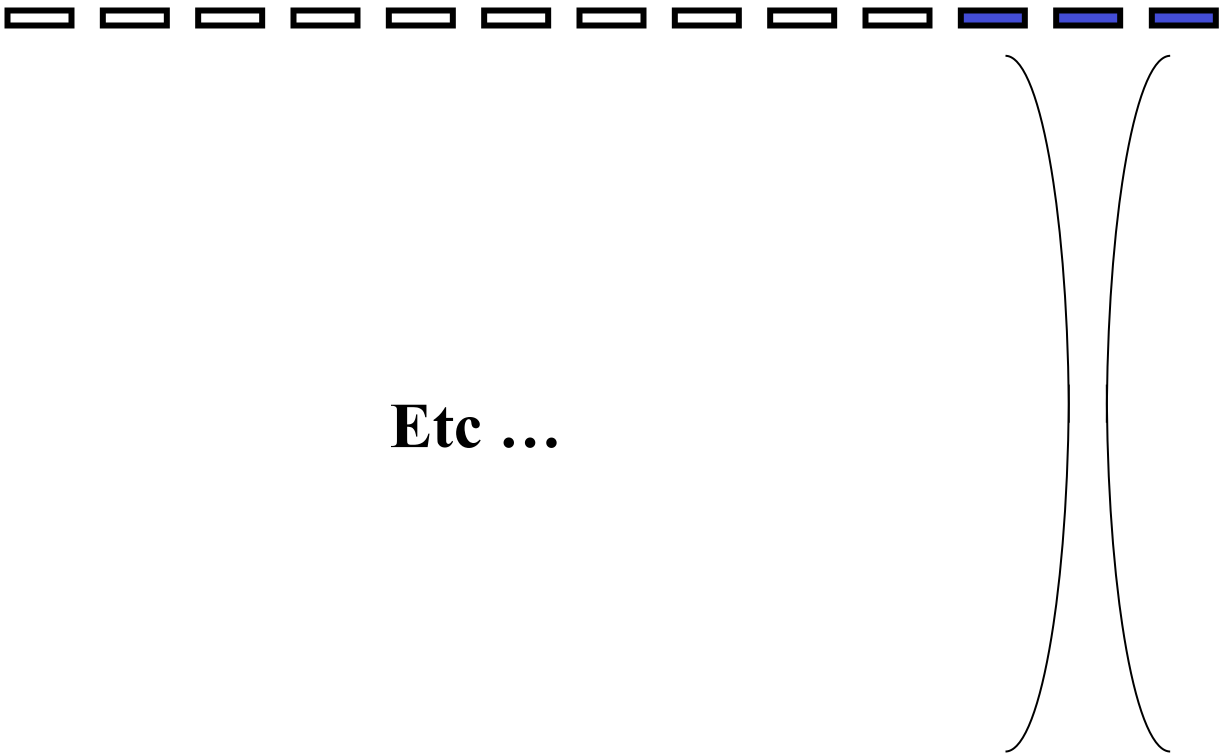
Linear Acquisition : principle



Linear Acquisition : principle

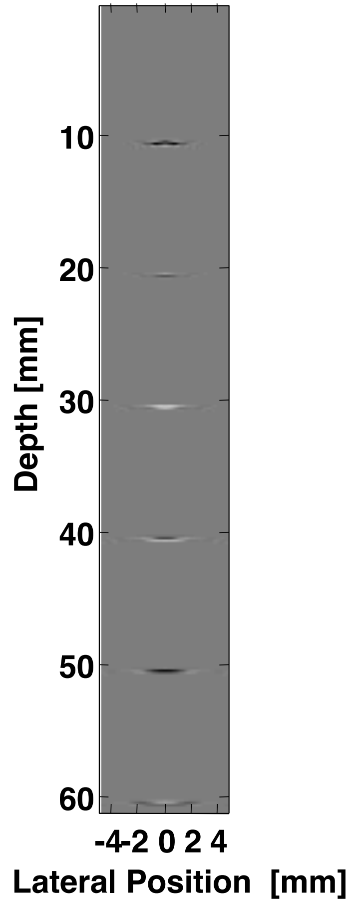


Linear Acquisition : principle

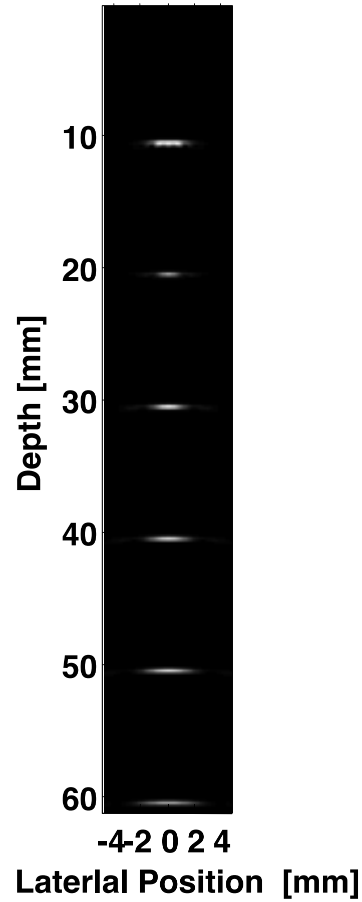


Etc …

RF image and envelop of many scaterrers

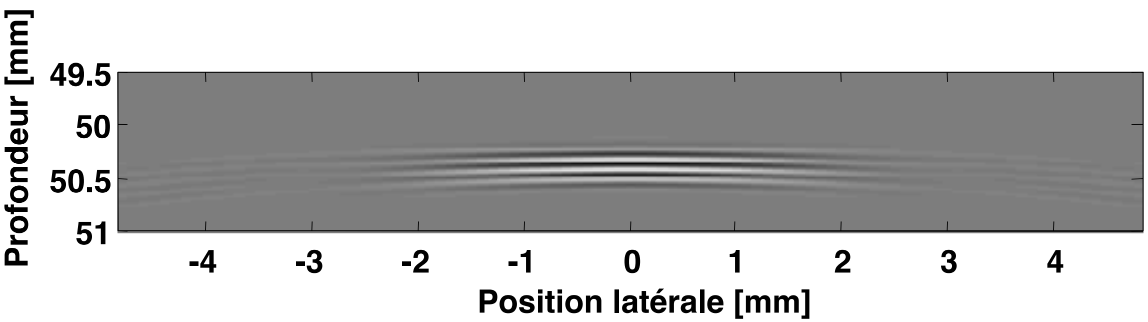


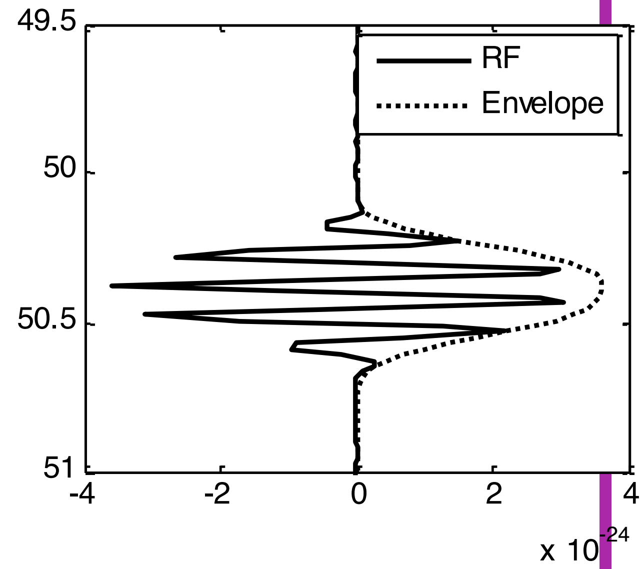
RF

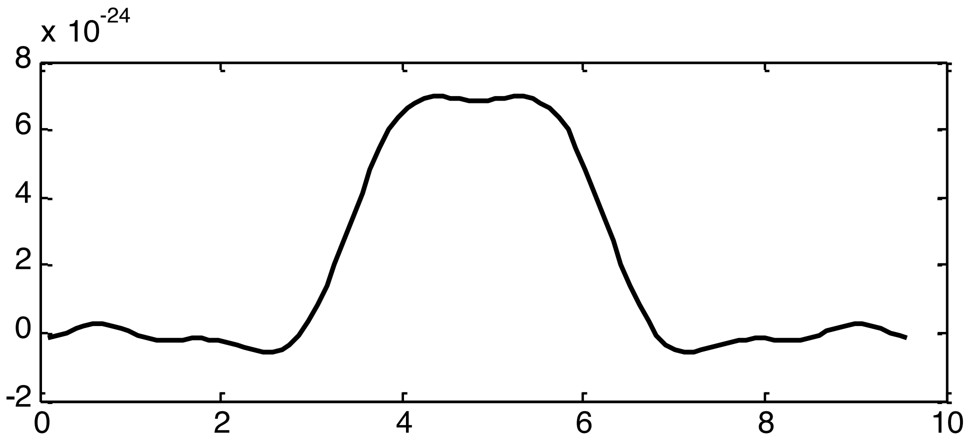


Envelop

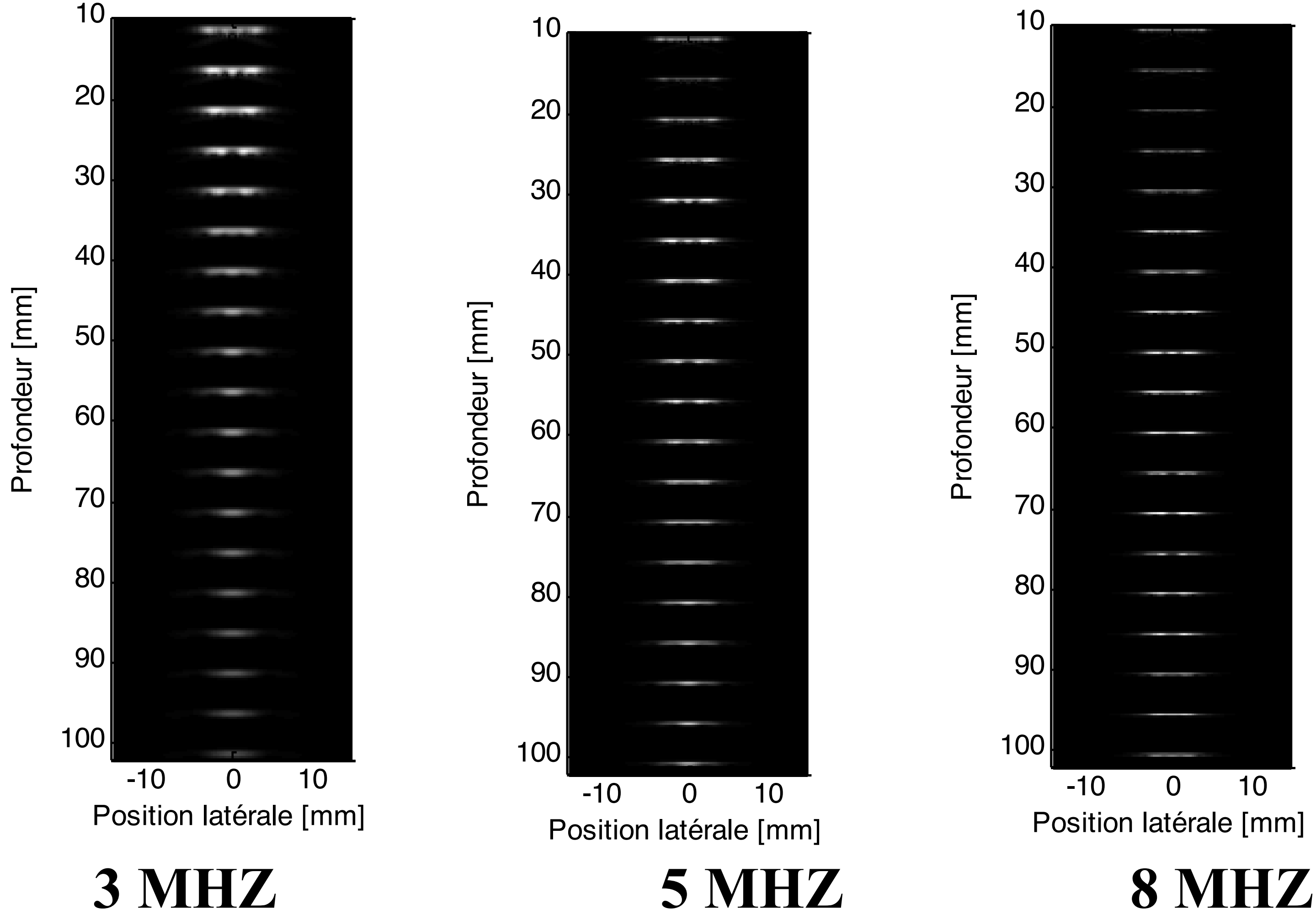
Zoom on the PSF at 50mm



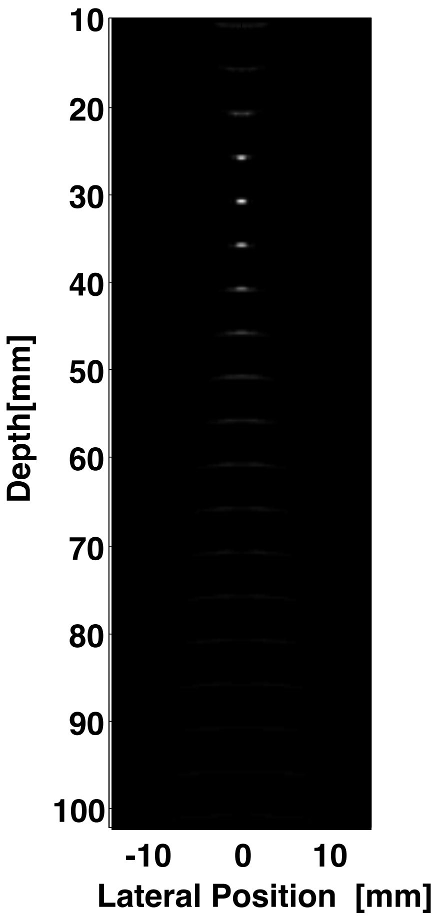




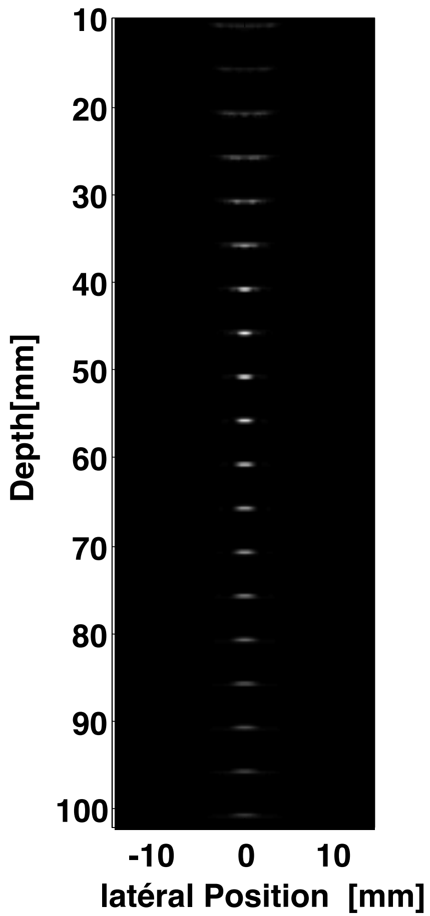
Effect of US frequency



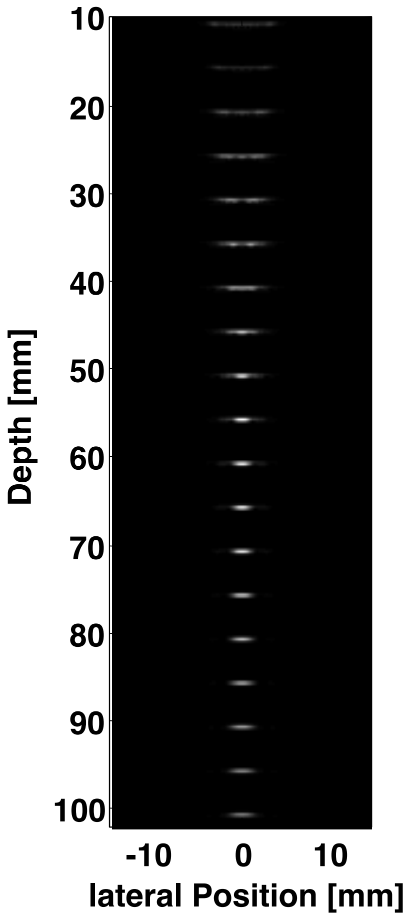
Effect of focussing



30 mm

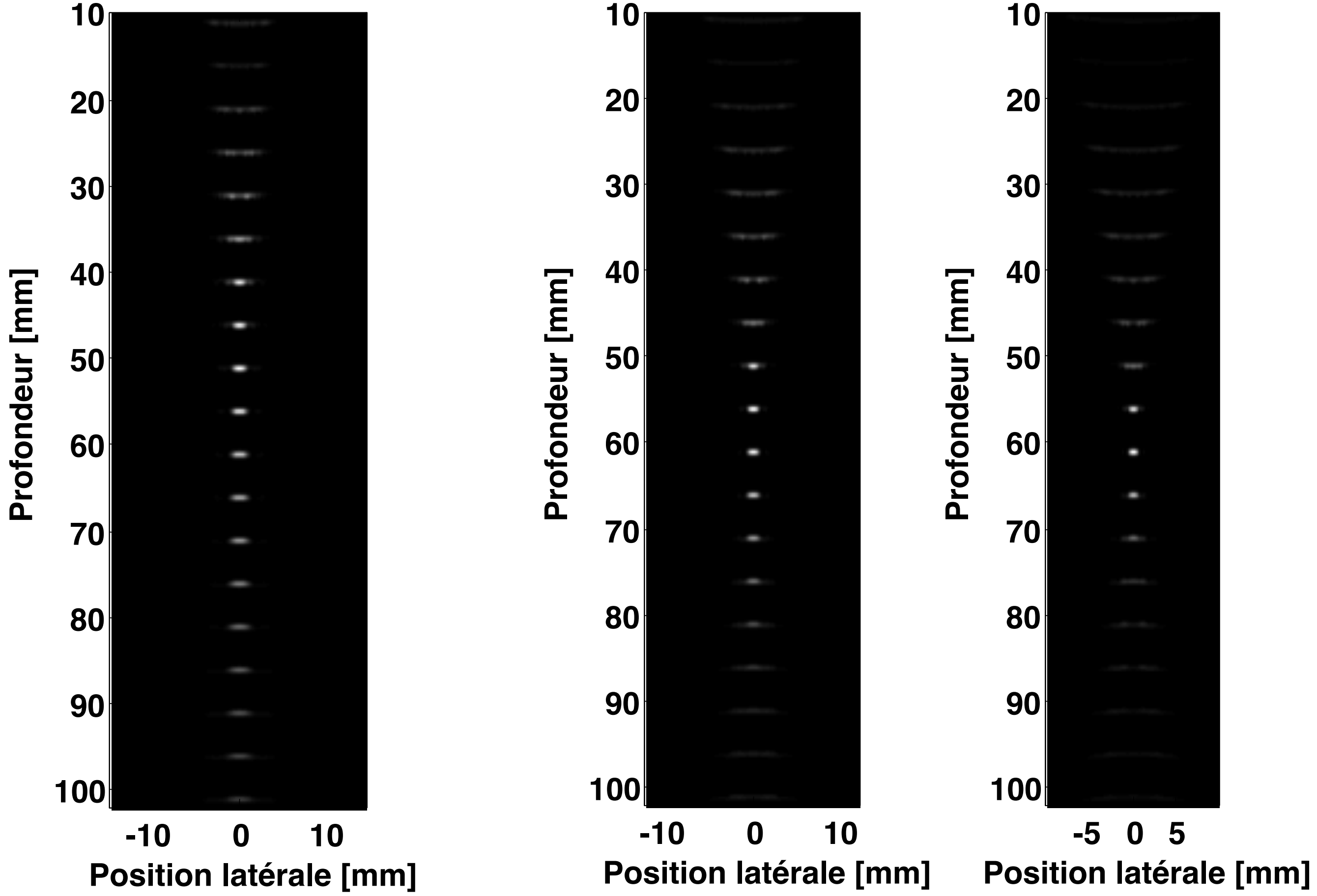


60 mm



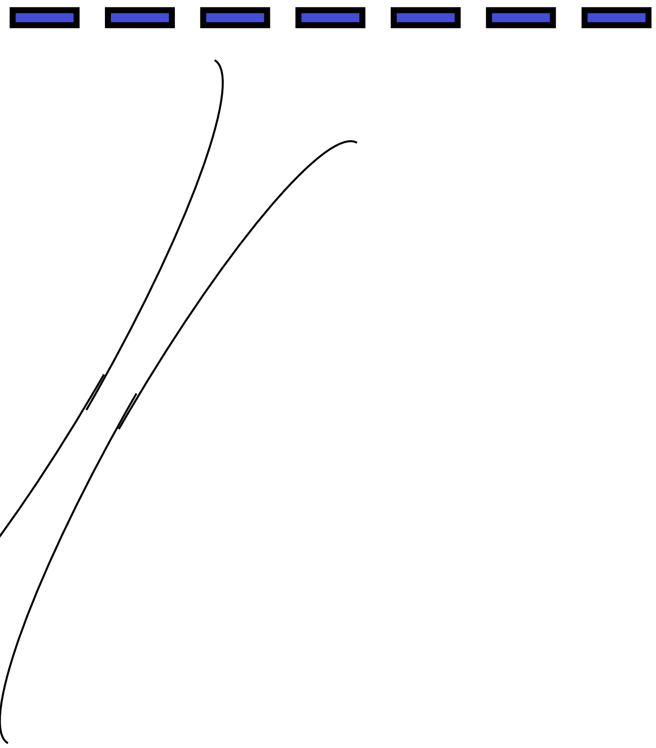
90 mm

Effects of number of elements

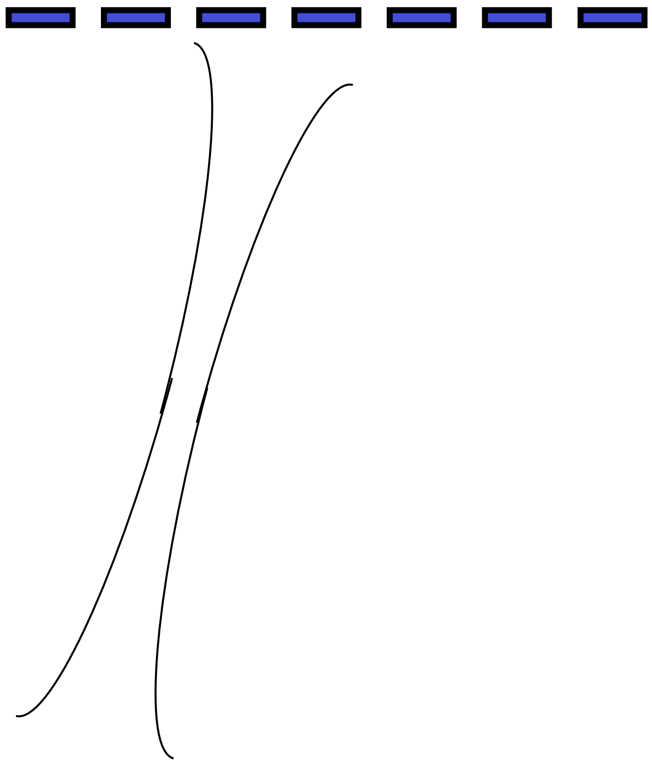


32 active elements **48** **64**

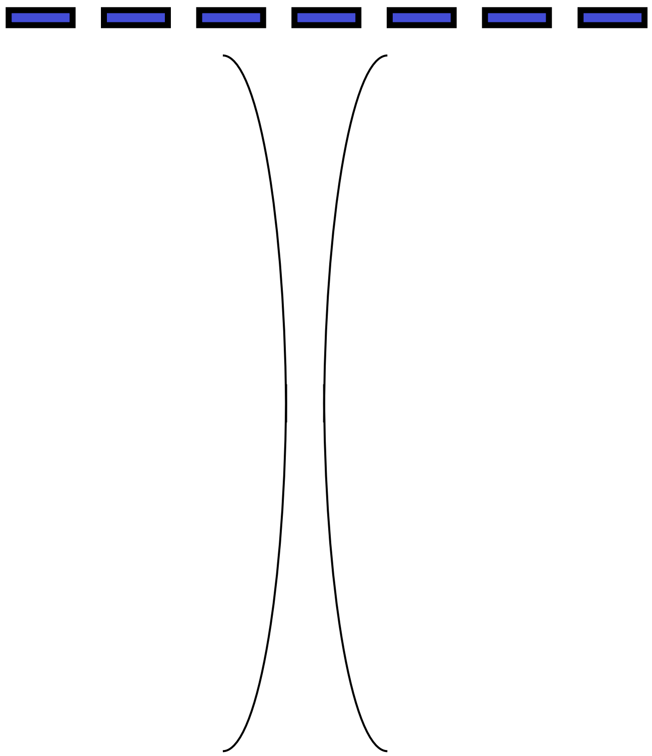
Sector scan: principle



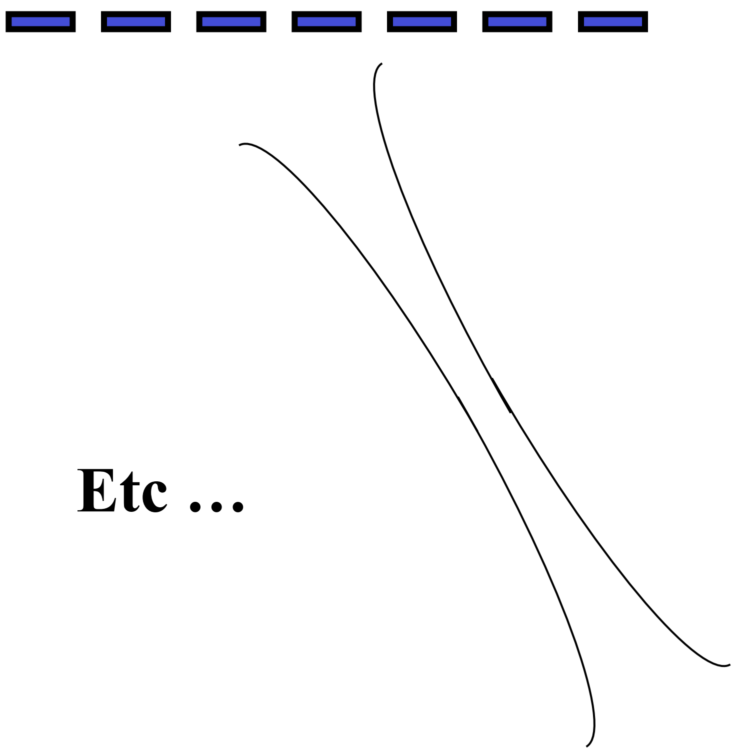
Sector scan: principle



Sector scan: principle

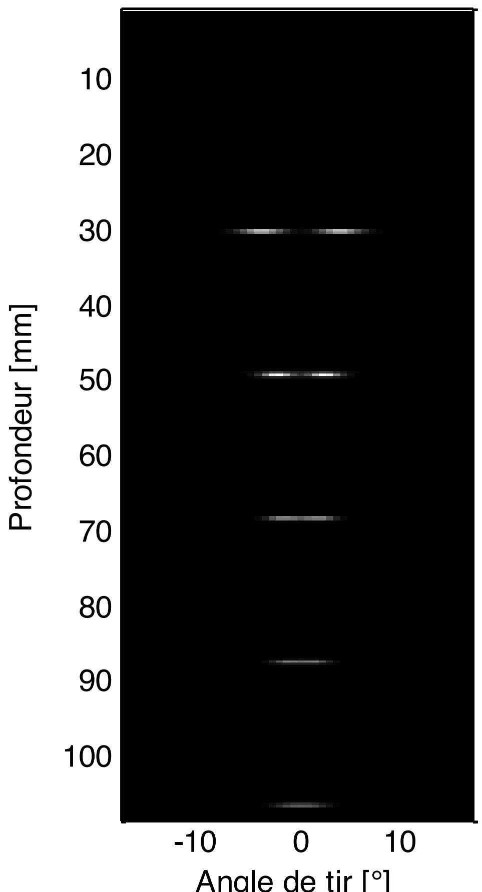


Sector scan: principle

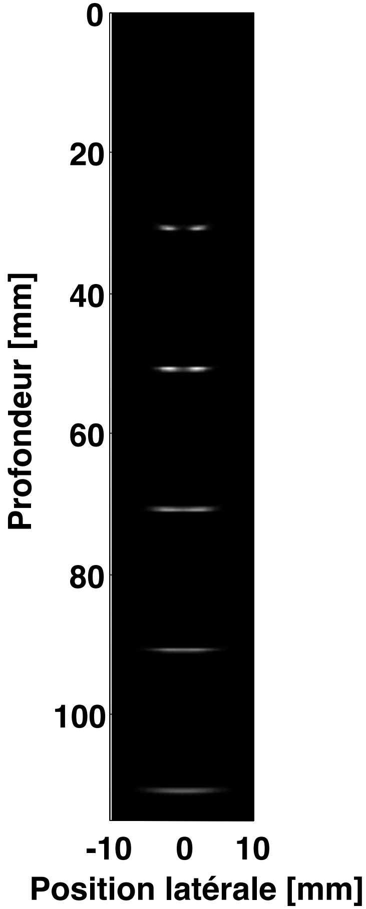


Etc …

Sector images



Polar coordinates

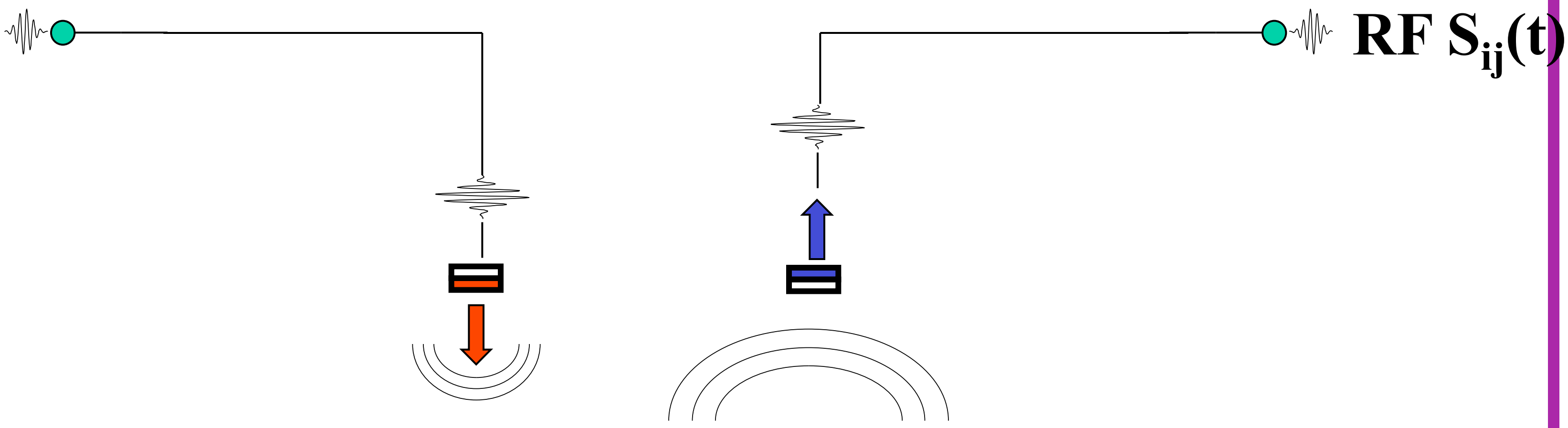


Cartesian coordinates

❑ Beamforming : Synthetic aperture

- Transmission on each element sparately
- Reception of the signal from the all the elements
- Dynamical recombination by post processing

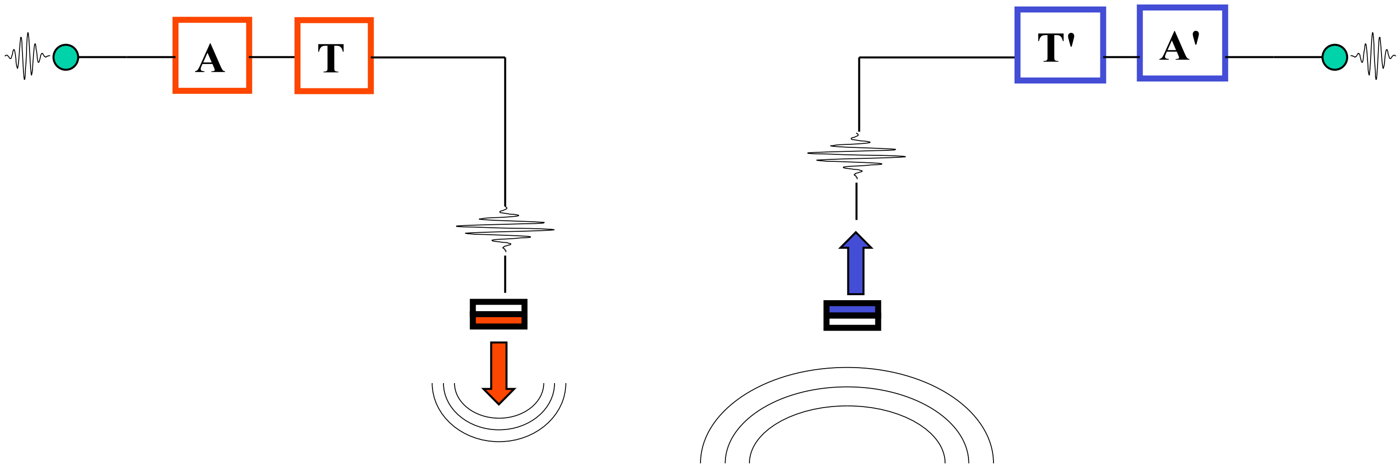
A single element



A single element transmits a wave, a single element receives:

The received signal = $S_{ij}(t)$

Delay at transmission and reception



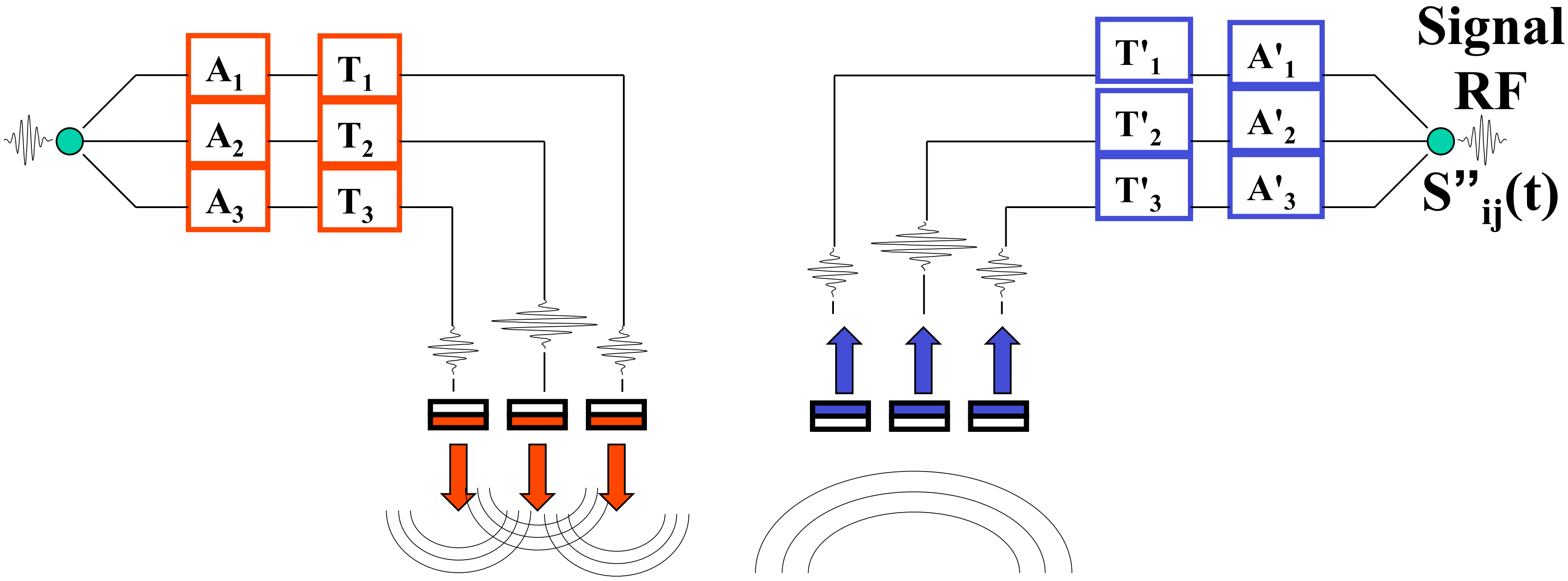
Signal RF

$S'_{ij}(t)$

We introduce a delay and a apodization at transmission and reception

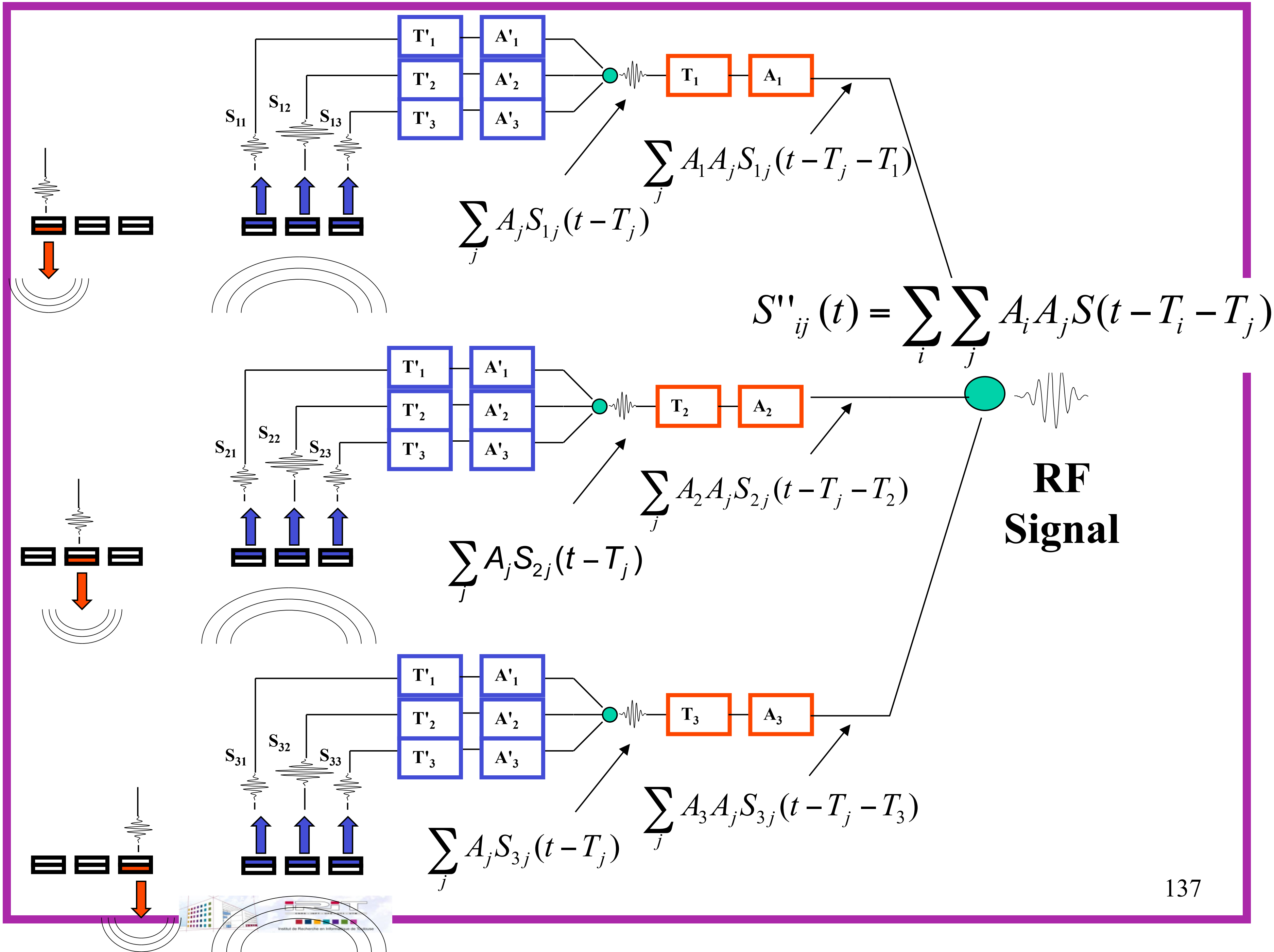
$$S'_{ij}(t)=A*A'S_{ij}(t-T-T')$$

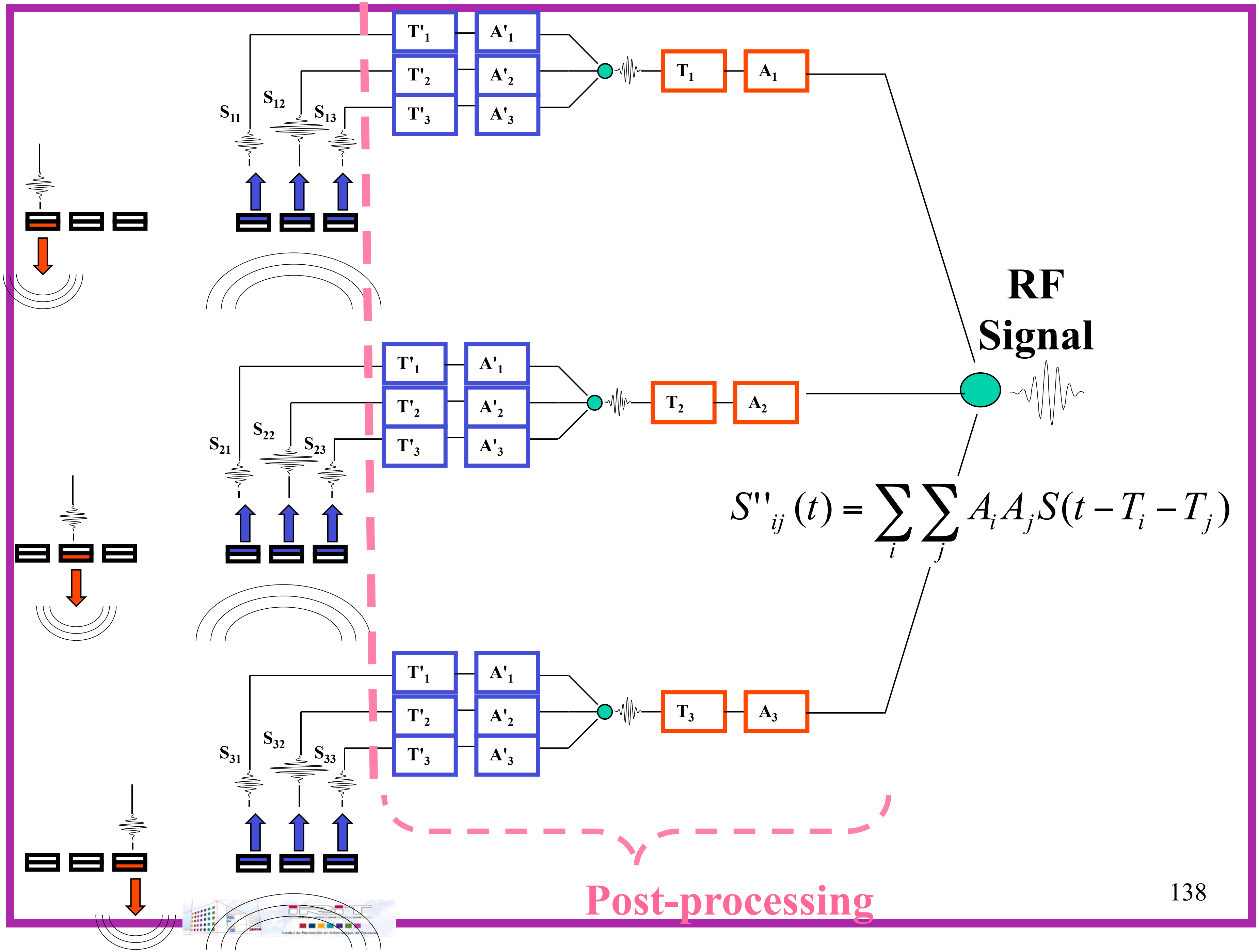
Conventional case



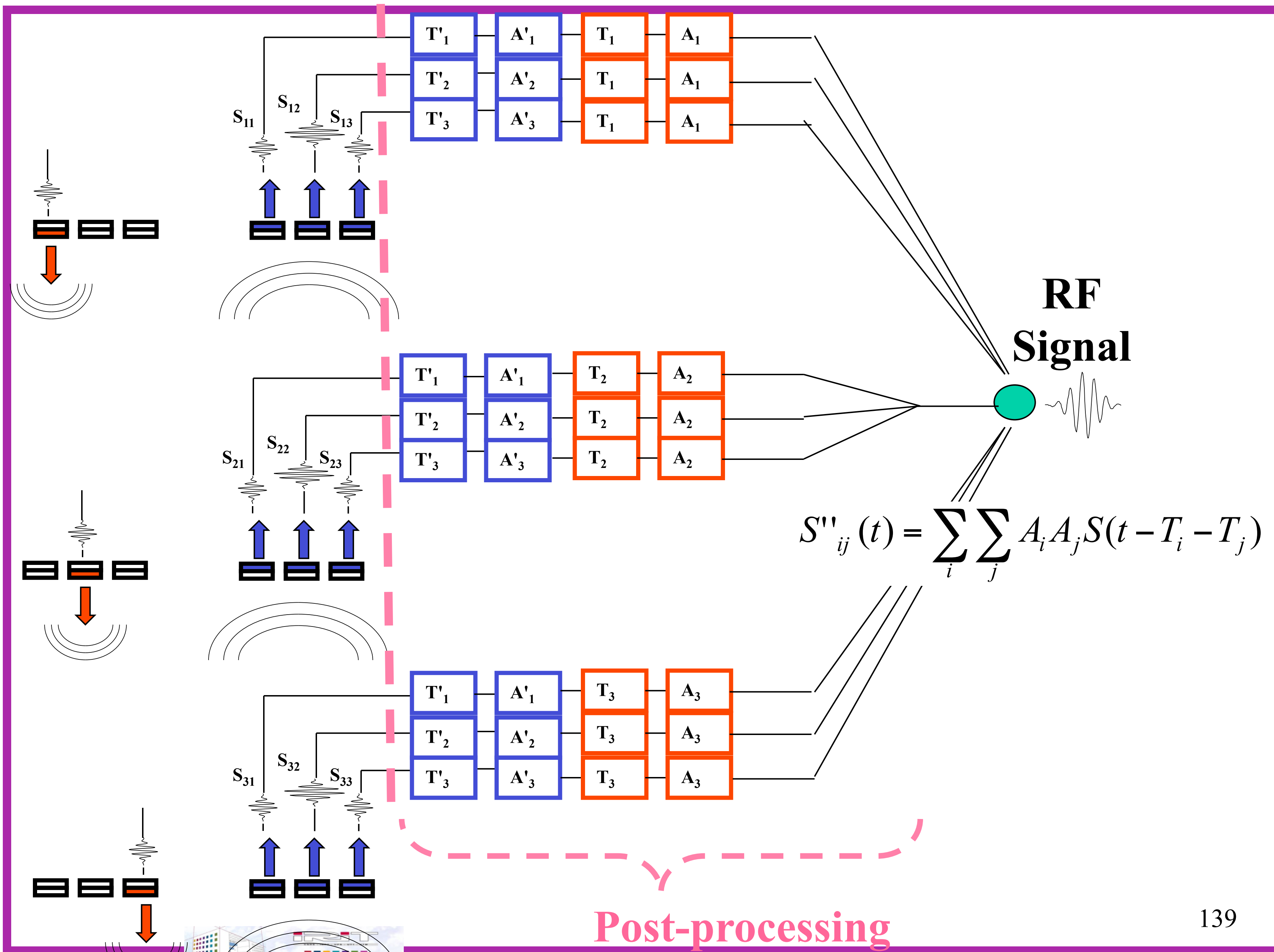
$$S''_{ij}(t) = \sum_i \sum_j A_i A_j S(t - T_i - T_j)$$

For convenience, the elements are excited simultaneously but with delay


T'_1
A'_1
T'_2
A'_2
T'_3
A'_3
T_1
A_1
T_2
A_2
T_3
A_3
S_{11}
S_{12}
S_{13}
S_{21}
S_{22}
S_{23}
S_{31}
S_{32}
S_{33}
$\sum_j A_1 A_j S_{1j}(t - T_j - T_1)$
$\sum_j A_j S_{1j}(t - T_j)$
$S''_{ij}(t) = \sum_i \sum_j A_i A_j S(t - T_i - T_j)$
$\sum_j A_2 A_j S_{2j}(t - T_j - T_2)$
$\sum_j A_j S_{2j}(t - T_j)$
RF
Signal
$\sum_j A_3 A_j S_{3j}(t - T_j - T_3)$
$\sum_j A_j S_{3j}(t - T_j)$

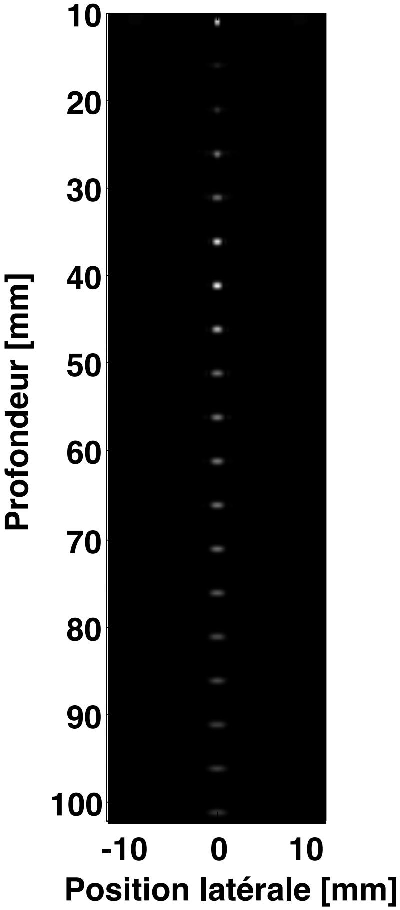


$$S''_{ij}(t) = \sum_i \sum_j A_i A_j S(t - T_i - T_j)$$

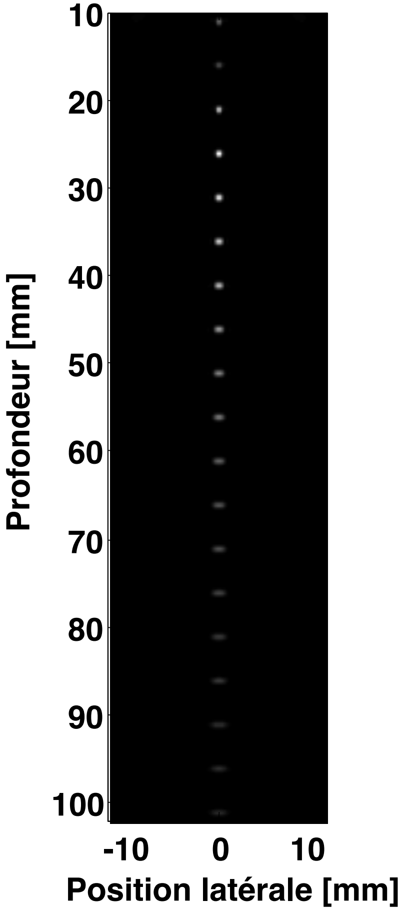


$$S''_{ij}(t) = \sum_i \sum_j A_i A_j S(t - T_i - T_j)$$

Some illustrations



Tx zones Rx dynamic



Tx & Rx Dynamic

❑ Image frame rate

Pulse repetition rate : While imaging, a pulse is sent and another pulse can be sent again only after all the echoes from the previous pulse vanish (remain under detection level)

Thus while imaging at maximum depth of penetration *dp* the pulse repetition interval T_R is :

$$T_R \geq \frac{2d_p}{c}$$

The pulse repetition rate f_R is

$$f_R = \frac{1}{T_R}$$

Suppose N pulses are required to generate an image, the image frame rate F is

$$F = \frac{1}{NT_R}$$

Exemple

If an ultrasound system operating in B mode requires 512 pulses to generate an image. Assume the transducer is sensitive to at most 80 dB

If the material being imaged has a SOS c= 1540 m/s and β=1dBcm^{-1}MHz^{-1} and m= 1 the working frequency to achieve a frame rate of 15 frame/s is approximatively 4MHz :

$$T_R = \frac{1}{NF} = \frac{1}{512 \times 15} = 0.13ms$$

$$T_R \geq \frac{2d_p}{c} = \frac{2}{c} \times \frac{D}{2\beta f^m} = \frac{1}{1540} \times \frac{80}{1 \times f} \Rightarrow$$

$$f \geq \frac{1}{154000} \times \frac{80}{1 \times 1.3 \times 10^{-4}} = 3.99MHz$$

Thus 2D case is directly obtained from 1D case. From this, 3D extension is straightforward

❑Signal and Image Processing Model

The RF signal equation (RFEQ) which can be extended to 2D or 3D can be formulated in discrete case by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

where

y *is sampled discrete 2D or 3D image*

H *is the PSF kernel including transmit and received PSF*

x *is the sampled unknown reflectivity*

n *is the model noise including electronic, acoustic noise and all other physical approximations that are made while developing the RF signal*

H *is spatial (or time)-varying*

y, H, x *and* **n** *are in the lexicographical form, i.e. are transformed in vectors. This equation is a convolution equation*

❑ SOME IMPORTANT REMARKS ON ULTRASOUND IMAGING

Ultrasound imaging is :

- ***characterized by speckle***

The speckle can be used to characterize structure in a US image. It is however in many cases an issue since it reduces readability of the images

- ***characterized by limited resolution***

The resolution is limited by the transducer working frequency, focussing features and electronic control or artifacts

- ***is fast in 2D imaging, quite fast in 3D***

Frame rate is limited by the imaging system (number of RF lines, working frequency, attenuation,…)

❑ CONCLUSIONS

All these features are severe limitations for ultrasound imaging in many clinical applications.

Overcoming them constitutes open challenges.

Most of these challenges are investigated using device-based techniques (Transducers optimization, device-based compound imaging,…)

Here we present Image-processing-based approaches

Outline

Part III Challenges in US imaging

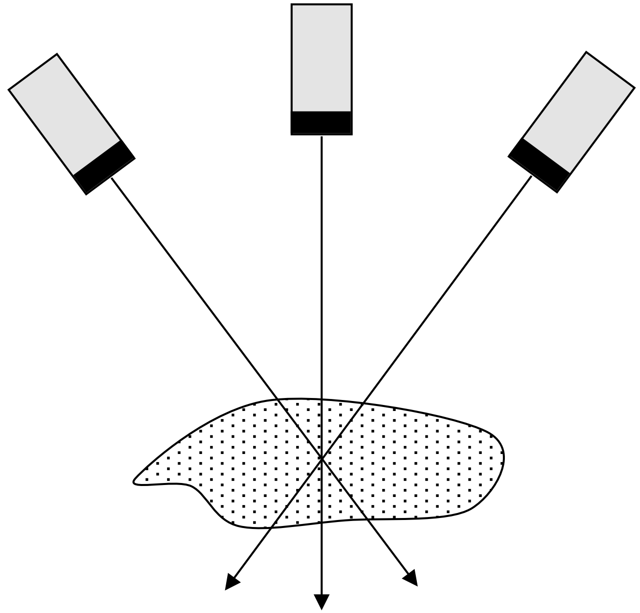
- Despeckling
- Tissue characterization using speckle
- Ultrasound Image restoration
- Ultrasound Image Segmentaion
- Fast ultrasound imaging and Compressive sampling
- Ultrasound Doppler Flow estimation

❑Despeckling

The aim here is to reduce or remove speckle.
One of the first approaches for ultrasound imaging despeckling is referred to as compound imaging.

It can also be performed using device-based averaging :

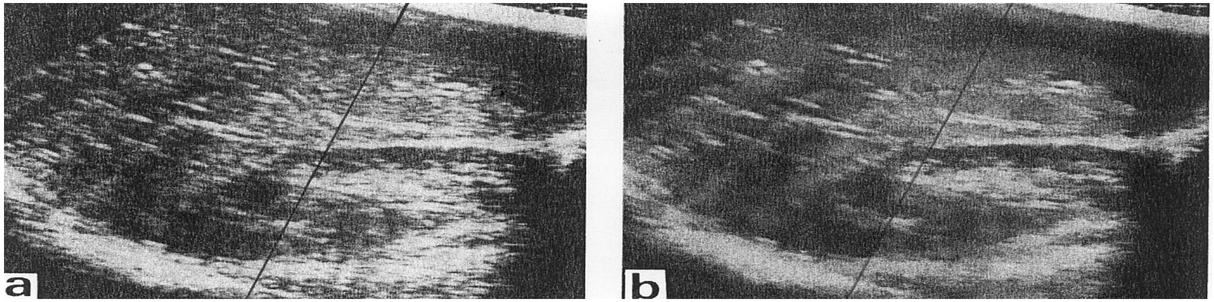
❑ Spatial compound imaging



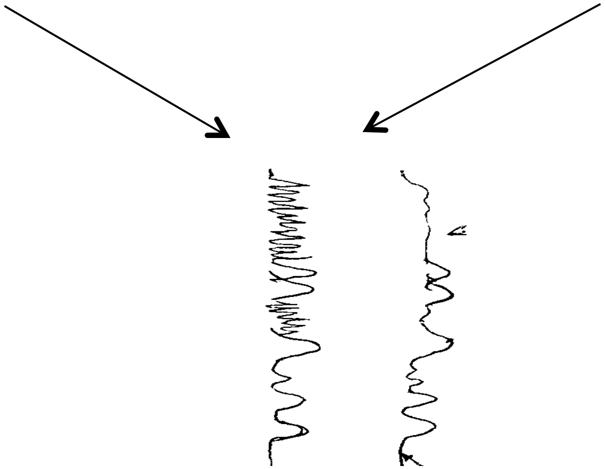
By compound scans → averaged speckle → reduction

⬊ **Spatial Compound imaging**

❑ Example of basic spatial speckle reduction technique



Adaptive Filtering



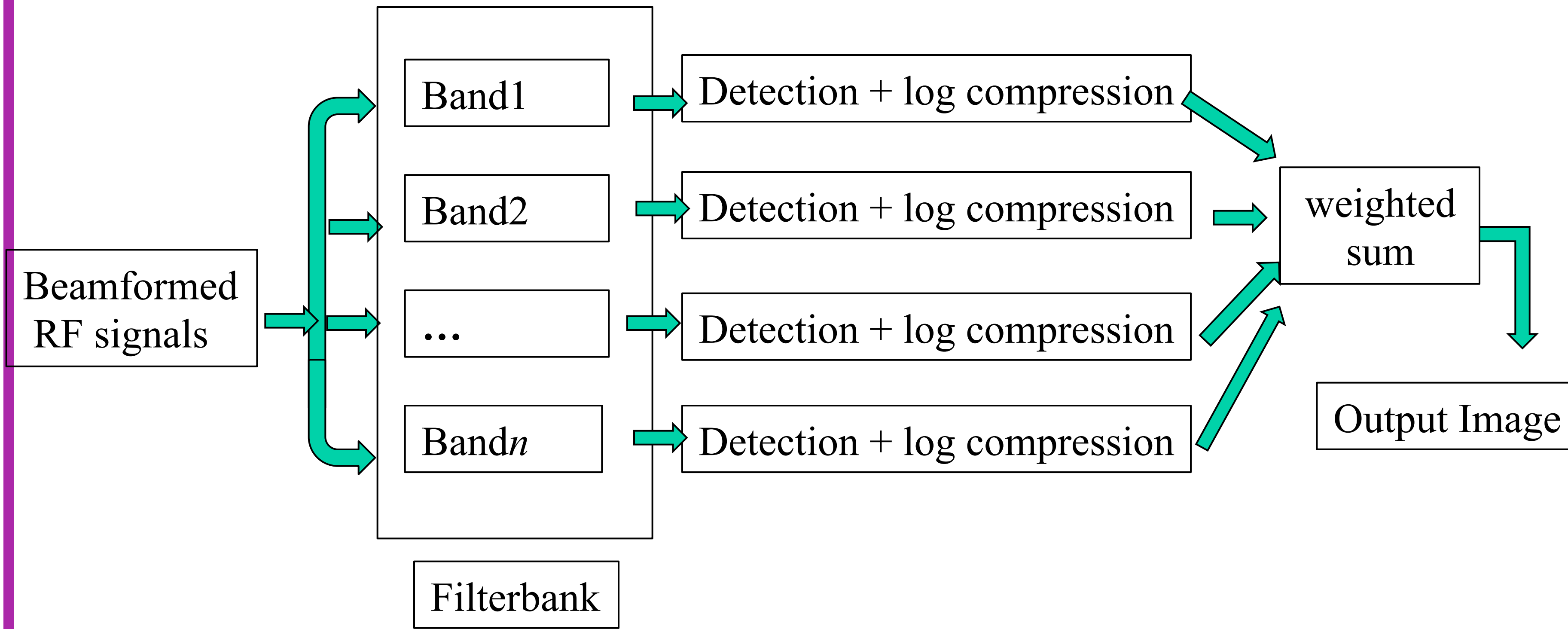
Before Filtering

After filtering

Be carefull with the filtering effects

There also exists frequency compound imaging

- *The basic principle consists in dividing the signal RF into overlapping frequencies using a multiple of band-pass filtering or filterbank.*
- *The output of each of the filters is then individually detected and log compressed. Finally, they are weighted and added .*



The choice of the filters have given birth a various works

- *Beside, the despeckling problem may be seen as a general denoising*
- *One of first ideas consist in modeling the speckle as multiplicative noise*
- *Thus envelop of the RF image (signals) say s(n,m) is model as the product of the "true" envelop f(n,m) and a noise g(n,m) i.e.*

$$\boxed{s(n,m) = f(n,m)g(n,m) + e(n,m)}$$

where e(n,m) is an addtitive noise

- *Although this multiplicative noise has some theoretical limitations [Tur 1982] , it has been successfully used in ultrasound imaging.*

- *Thus neglecting the additive noise e(n,m),and taking the log of s(n,m), it comes*

$$\boxed{ls(n,m) = lf(n,m) + 1g(n,m);\ where\ lx = \log\left[x\right]}$$

- *And this is a problem of additive noise removing. And there exist in the Image processing literature many works by beginning by the Wiener filter to more sophisticated ones*

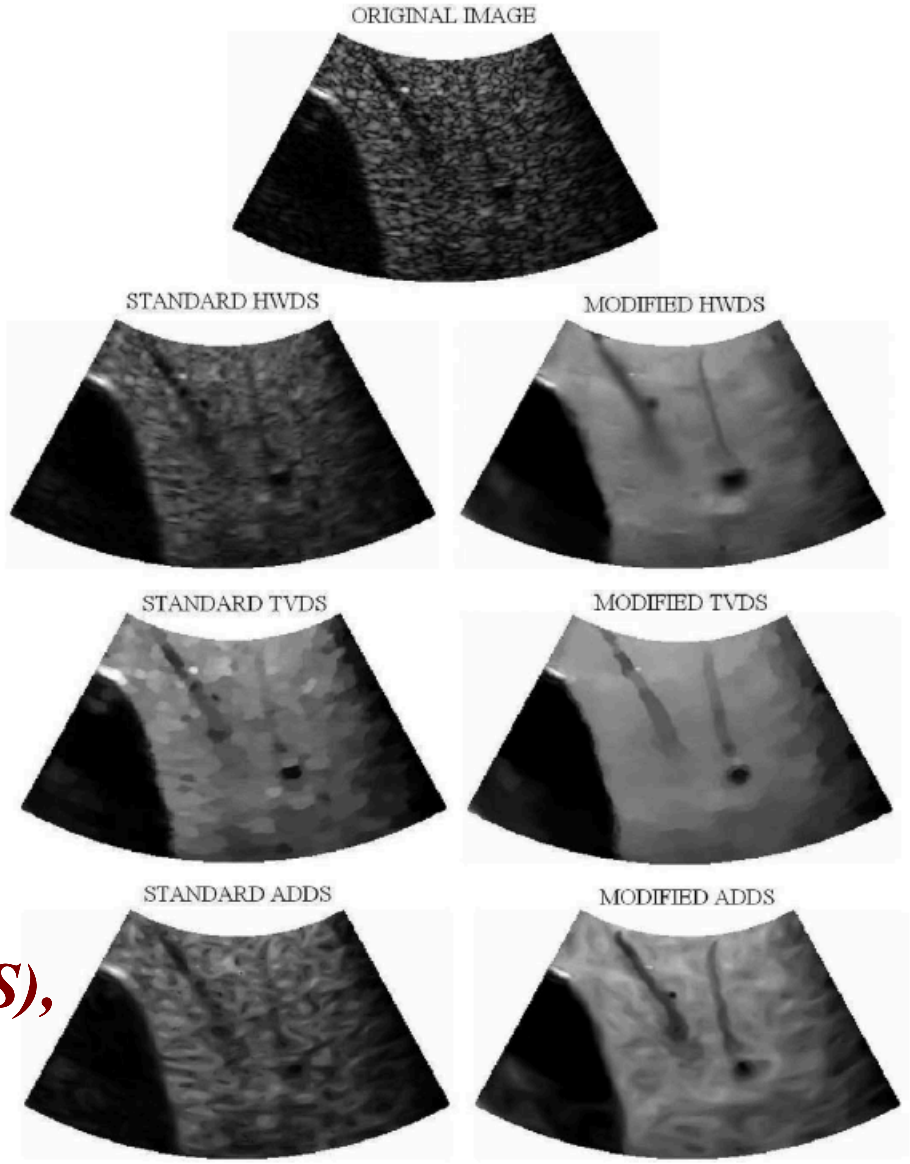
- *Classically lg is assume to be white Gaussian noise. This can yield difficulties in rejecting the spiky noise lg,*
- *[Michailovtich 2006] proposed a more accurate method consisting in a preprocessing procedure (estimating and subsequently subtracting its spiky components via the computation of the residual of ls], before a filter is applied to ls(n,m) to reject lg(n,m).)*
- *Different filters were revisited with and without this pre-processing : wavelet denoising, total variation filtering, and anisotropic diffusion, and showed better performance*

Exemple of illustration

wavelet denoising (HWDS),

total variation filtering(TVDS),

and anisotropic diffusion(ADDS),



Results from [Michailovtich 2006]

❑Tissue characterization using speckle

- *If despeckling can be used to highlight some big anatomical structures, in many clinical situations it is desirable to keep a sufficient level of speckle to allow tissue characterization.*

- *To do so statistical distributions of speckle have been widely studied in the literature.*

- *The first works assumed that (see previous parts) that the statistical distribution of the RF signals is Gaussian and therefore, their envelops are Rayleigh distributed*

❑ Tissue characterization using speckle

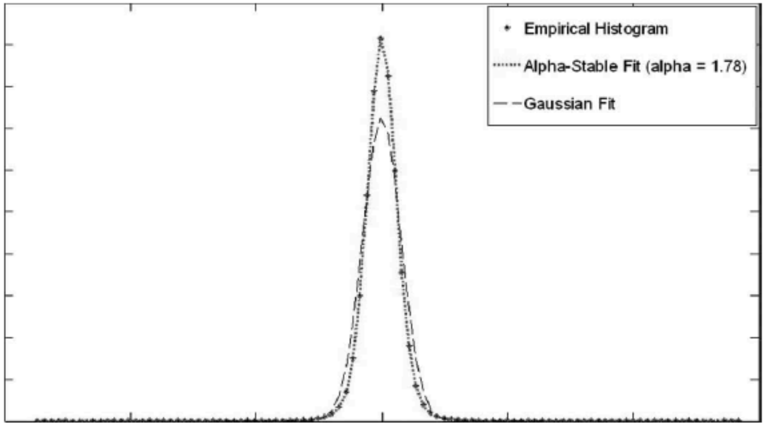
- *However many works and namely works from P. M. Shankar's group (e.g [Shankar 1993]) proposed other distributions for the ultrasound envelop and M-mode images : K, Gamma, Nakagami distributions*

- *Recently [Pereyra 2012] proposed non-Gaussian α-stable statistics for RF signal of skin tissues*

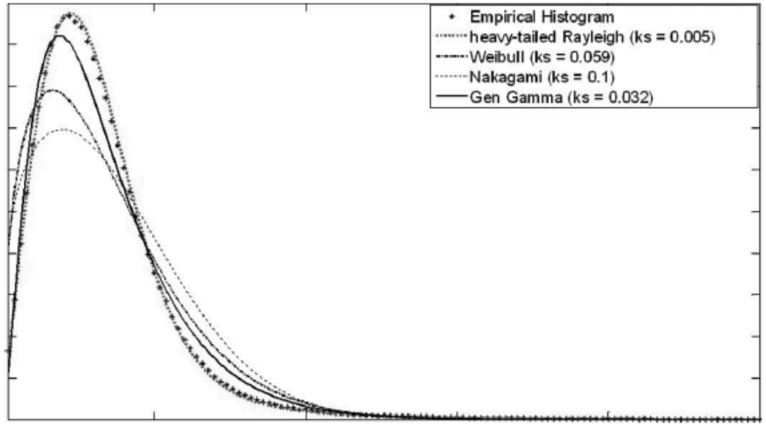
- *and also a generalized (heavy-tailed) Rayleigh distribution for the envelop signal of skin tissues*

- *and finally showed that these are more accurate and can be used to identify cancer tissue in ultrasound images*

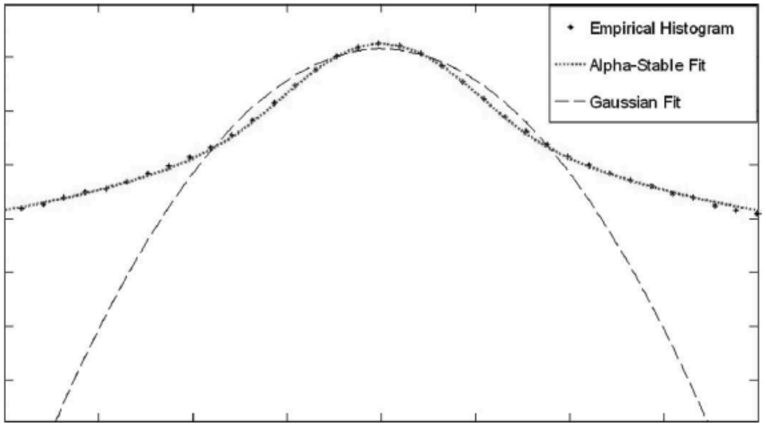
Exemple of Results



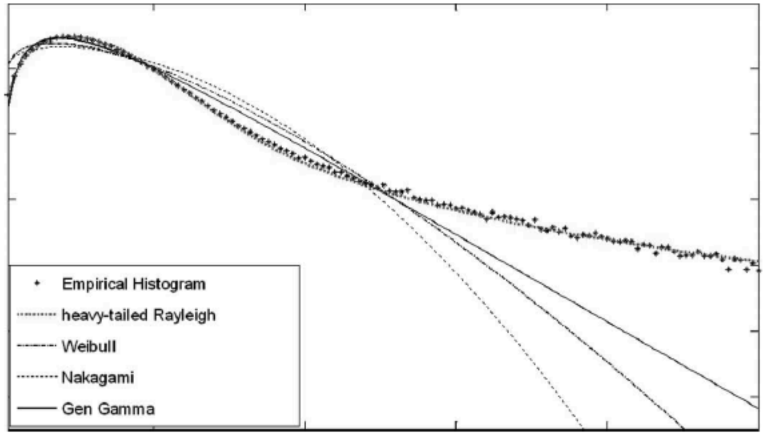
Comparison of the empirical probability density functions obtained from forearm dermis, and the corresponding estimations using the SαS and Gaussian distributions.



Comparison of the empirical envelope probability density functions obtained from forearm dermis, and the corresponding estimations using the heavy-tailed Rayleigh, generalized gamma, Weibull, and Nakagami distributions.



Comparison of the tails by means of a logarithmic plot of the probability density functions.



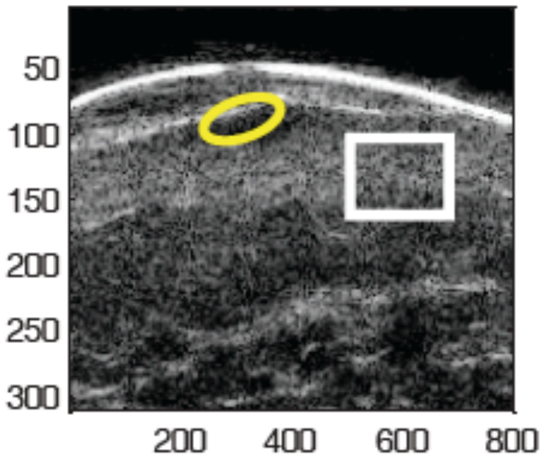
Comparison of distributions tails by means of a logarithmic plot of the probability density functions.

[Pereyra 2012]

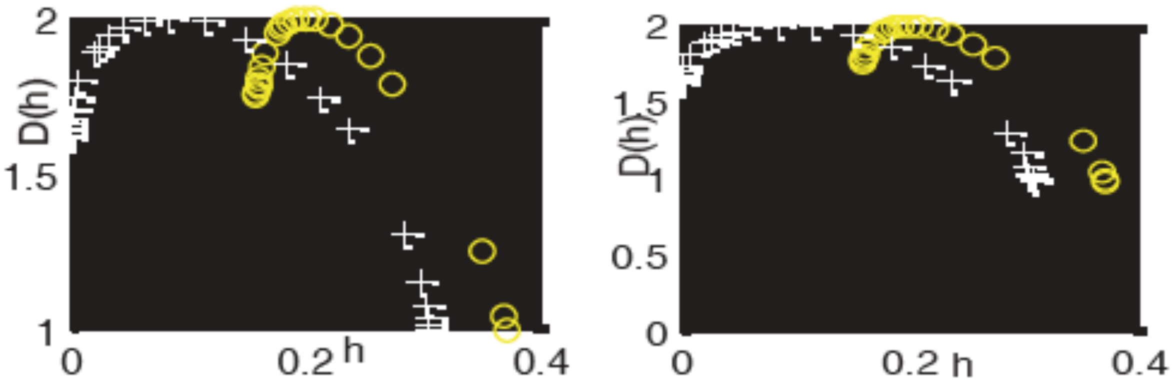
❑ Tissue characterization using speckle

- *The speckle can also be characterized in terms of fractals or multifractals ;*
- *e.g [Djeddi 2010] showed that the multifractal features, (namely the multifractal spectrum) change when the speckle contents change.*
- *This can be used to distinguish healthy tissue from pathological ones*

Exemple of illustration



Ultrasound skin images: the yellow ellipse delimits a melanoma and the white square surrounds normal dermis.



Estimation of the multifractal spectrum from the 4 samples above

❑ Tissue characterization using speckle

- *These statistical or fractal features can subsequently be used to perform ultrasound image segmentation and constitute an interesting alternative to classical segmentation approaches*

- *However the tissue characterization using speckle remains opened. And many works are still in progress*

❑Ultrasound image restoration

- *Image restoration is one of most important challenge in ultrasound imaging, due to the specific nature of ultrasound images*

- *The models for ultrasound image restoration need to take into account this specific nature*

❑Ultrasound image restoration

The model introduced previously can be used

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

From the previous ultrasound image presentation, one knows that

H *is unknown, although there are available knowledge about it. Moreover* **H** *is time (or spatial) varying*

x *is unknown reflectivity to be found, with available information on its statistics*

n *has known or assumed statistical information*

IRIT
Institut de Recherche en Informatique de Toulouse

❑ Ultrasound image restoration

Restoration becomes solving an inverse problem with either variational or/and Bayesian approaches . For instance

Some works investigate the problem

$$\arg\min_{H,X} \|\mathbf{y} - \mathbf{Hx}\|_2^2 + \lambda \|\mathbf{\Gamma x}\|_p^p + \mu \|\mathbf{H} - \mathbf{H}_0\|_2^2$$

$\mathbf{\Gamma}$ *is a special prior such TV, Fourier or Wavelet transform*
p= 1 is typical value

❑ Ultrasound image restoration

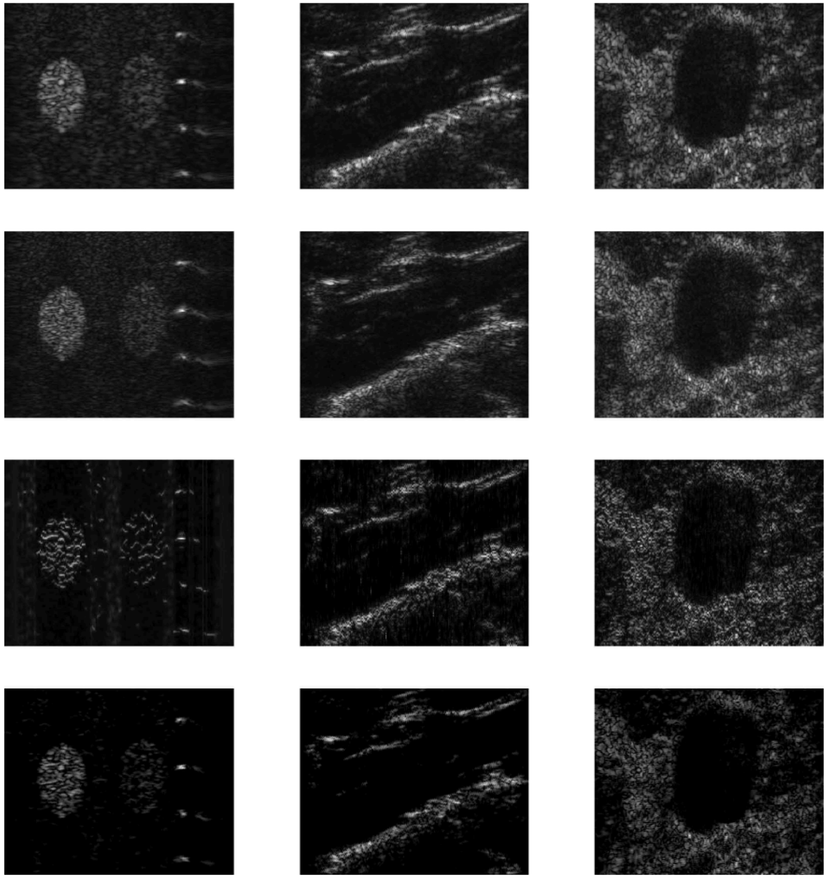
The estimation of **H** *is tricky in general*

In deconvolution framework,for instance it can be accessed by incorporating prior through parametric Inverse Filtering Approach : [Michailovtich 2007] or ADMM approach by dealing without [Yu 2012],

The problem can also be investigated through super-resolution approach by either performing device-based estimation of H [Ellis 2010]or via ADMM approach [Morin, ICIP 2013] Other alternatives consist in using some high resolution spectral models,e.g, [Ploquin, ICIP 2010]

Exemple of illustration

Visual performance of different deconvolution methods: (top row) original envelope images; (second row) restored images by the cepstrum based method; (third row) restored images by the hybrid parametric inverse filtering method; (bottom row) restored images by ADMM by Yu2012.



[Yu 2012]

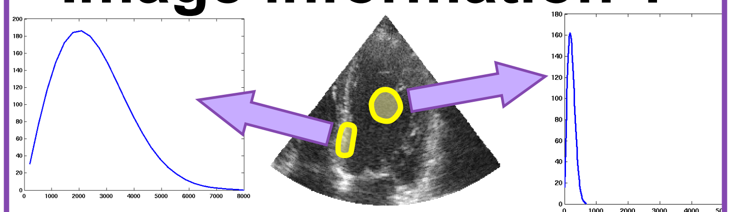
❑Ultrasound image Segmentation

Segmentation of medical ultrasound is a very active research field and, as other ultrasound signal and image processing tasks, needs a quite good knowledge about ultrasound imaging for succeeding

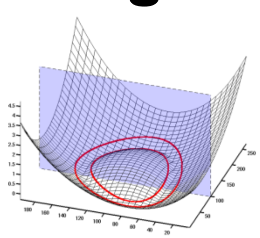
Ultrasound image Segmentation : overview

- Typologie of segmentation approaches : a typology

Image information ?

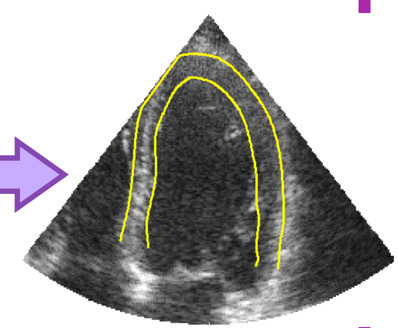


A priori knowledge ?



Active contours
Machine learning
Clustering
Active Shape models
Markov random fields
Watershed transform
Atlas based
…

Formalization / algorithm ?

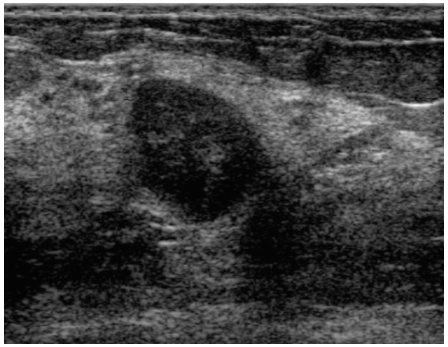


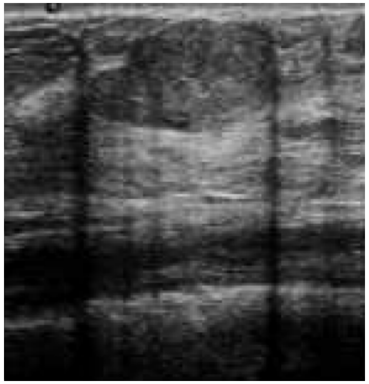
What makes the segmentation of medical ultrasound image specific ?

- ***US Image specificities***

– Speckle:

- *Contour-based approaches based on edge detection operators difficult to use*
  → Image enhancement via speckle reduction

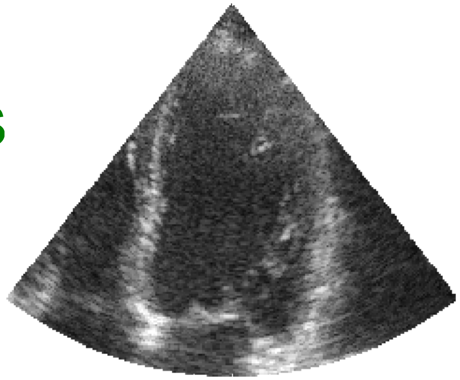




- *Speckle = information*
  → Use of region-based features: texture or statistical parameters

– Shadows, drop out, inhomogeneities

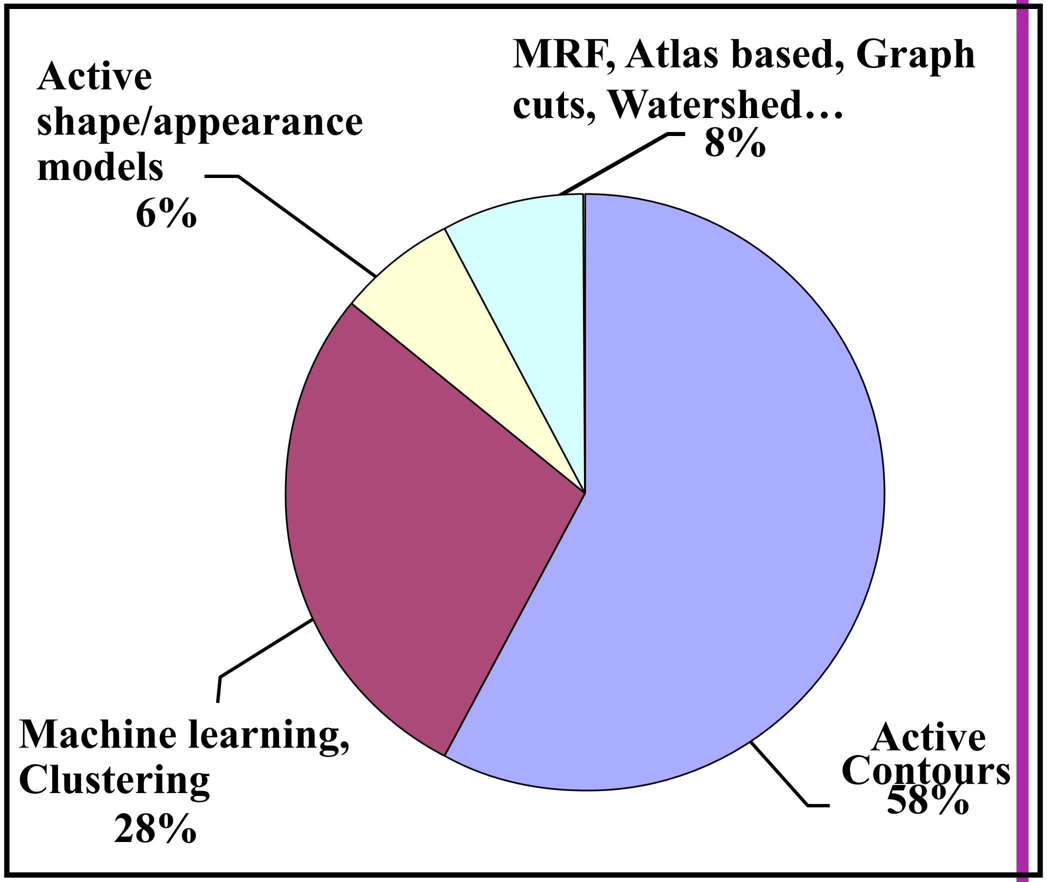
- *Missing boundaries*
  → a priori constraints are important



Main segmentation approaches

- Active contours
- Machine learning and clustering
- Active shape/ appearance models
- Markov random field, atlas based, graph cuts, watershed, ...

Percentage of papers for each main approach

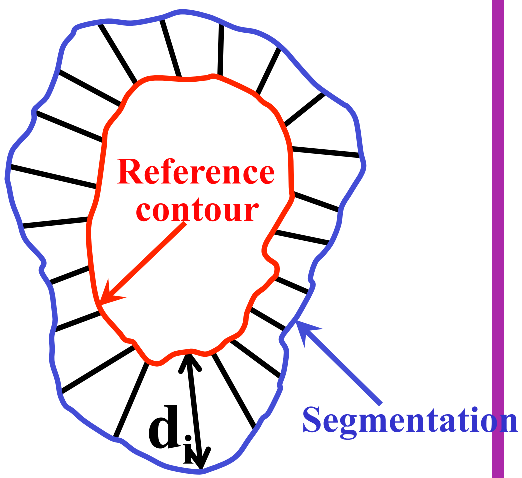


• *Evaluation of US Image segmentation*

1. Quantifying accuracy: basic approach

Define a ground truth ??

- ***Select a metric: Dice, Haussdorf distance, Mean absolute distance (MAD), etc.***
- ***Use the metric to compare the segmentation result to the reference***



$$\mathrm{MAD} = \frac{1}{N}\sum_{i=1}^{N} d_i$$

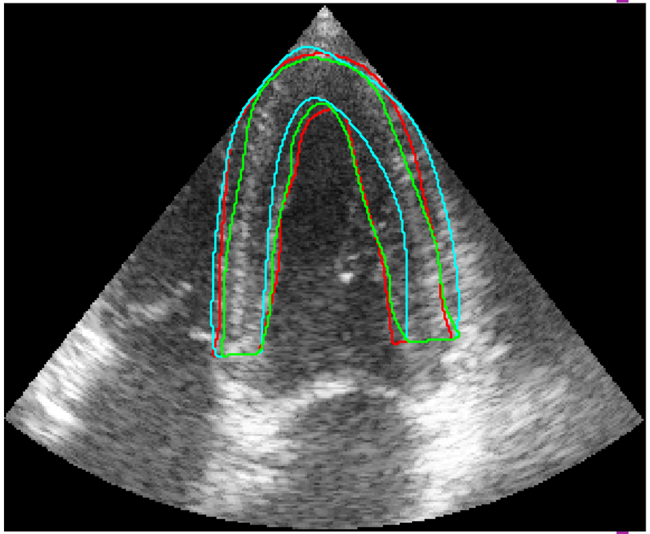
2. Comparison with other state-of-the art segmentation algorithms

● *Building a reference data*

Contours traced by a medical expert from in vivo images

● A heavy task:

- *Gather a representative image database*
- *Solve possible ethico-legal issues (anonymization, informed consent, etc.)*
- *Involve several physicians to perform manual delineation*



● *Consequences*

- *Evaluation performed often on a reduced set of data (typically 50 to a few hundreds)*
- *Sharing of the reference data is rare (ethico-legal issues)*

❑Fast Imaging and compressive sensing

Many fast acquisition schemes exist and need a compromise between frame rate and resolution (plane wave, parallel beamforming, simultaneous coded excitations…)

The limitations of these approaches are compromise between SNR, resolution, artifacts and the achieved frame rate.

Some recent studies have attempted to remediate this compromise, at the expense of increasing energy or nonlinear effects that reduce the efficiency of artifact suppression,

Recently some new attempts have been done with compressive sampling with promising results

Compressive sensing in medical ultrasound

Conventional sampling:

- *Shannon's theorem:*

$$f_s \geq 2 f_{\max}$$

- *What if unknown or infinite bandwidth?*

- *3D US imaging* → huge data volume + real time issues

Quick recall

- *CS = novel theory aiming to* ***reduce data volume***

[Candès et al. 2006]

- *CS applied in medical imaging (MRI and tomography)*

[Lustig et al. 2007, Provost et al. 2009]

➔*Aim =* ***combine US and CS***

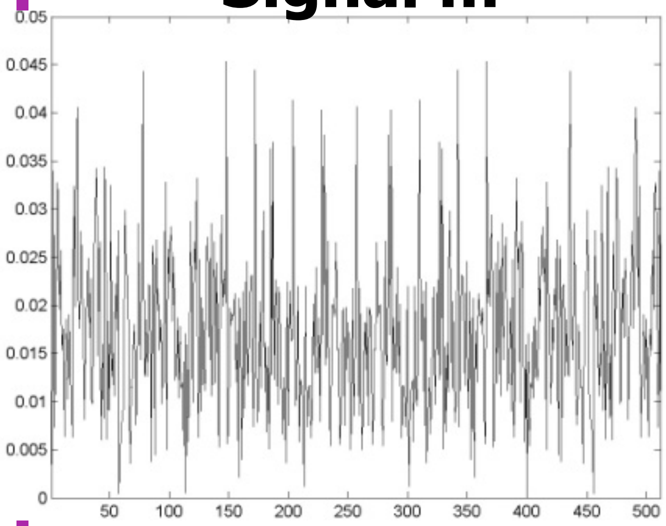
→*Reduce the amount of data*

→ *Increase the acquisition rate*

Quick recall

Theory of compressed sensing - Sparsity

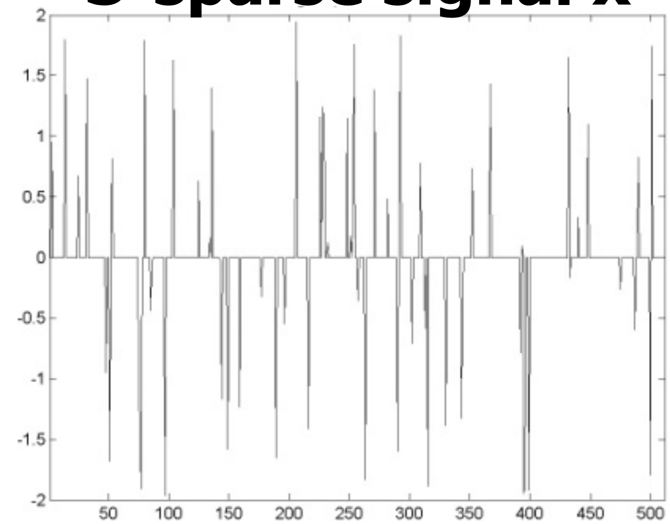
Signal m



Sparsifying transform Ψ

Fourier, wavelets, etc.

S-sparse signal x



Sparsity:

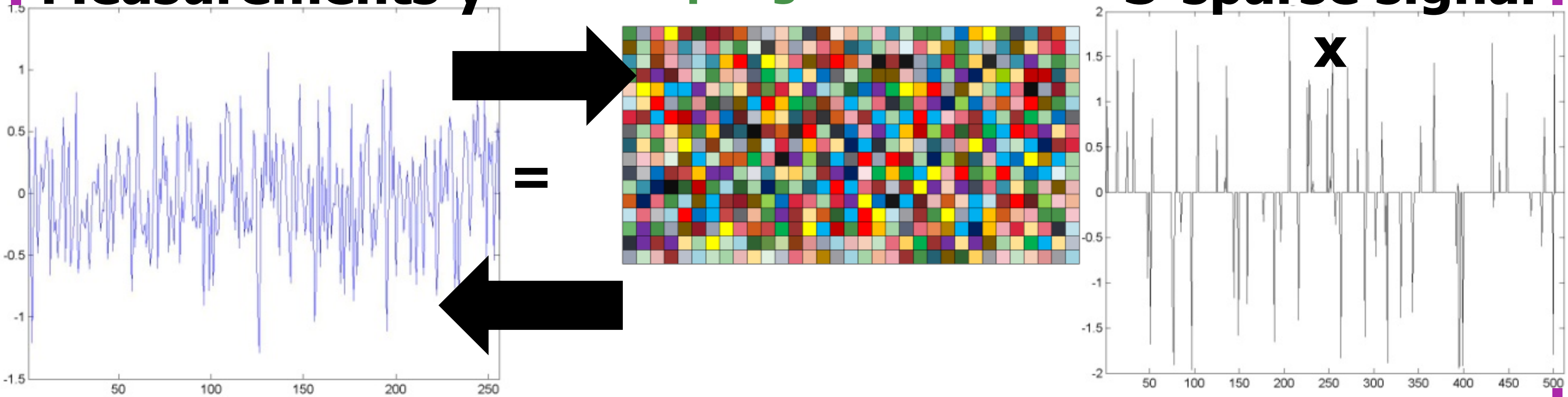
$$x = \Psi\, m$$

$$m = \Psi^{-1}\, x$$

Quick recall

Theory of compressed sensing - Sampling

Measurements y **=** **Sampling scheme Φ** **S-sparse signal x**



Sampling:

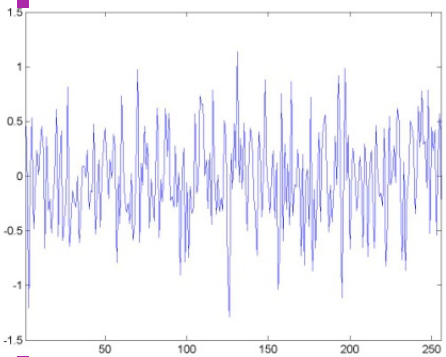
$$y = \Phi x$$

$$\{x_1, x_2, x_3, \ldots\} = \Phi^{-1} y$$

Quick recall

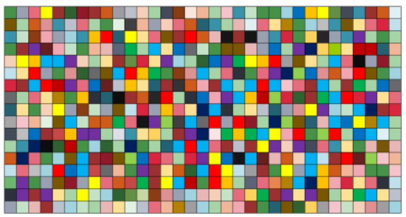
Theory of compressed sensing - Incoherence

Mesurements y

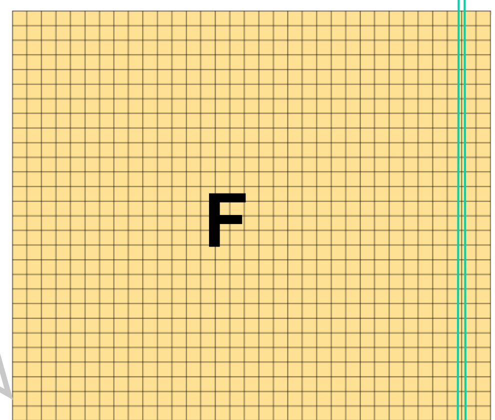


=

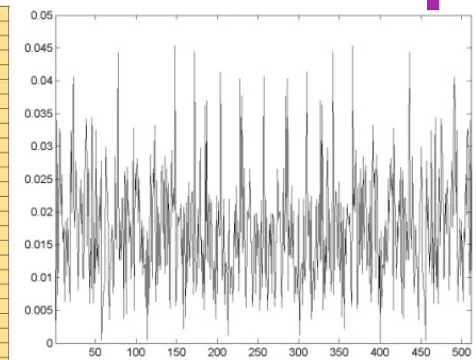
Sampling matrix Φ



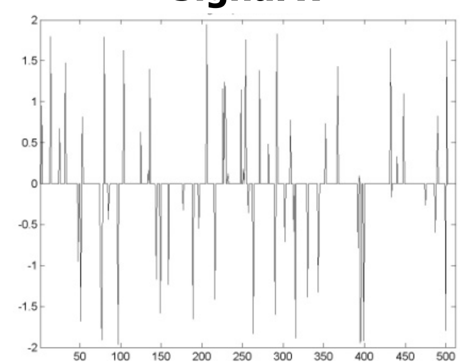
Sparsifying matrix Ψ



Signal m



Signal x



Incoherence:

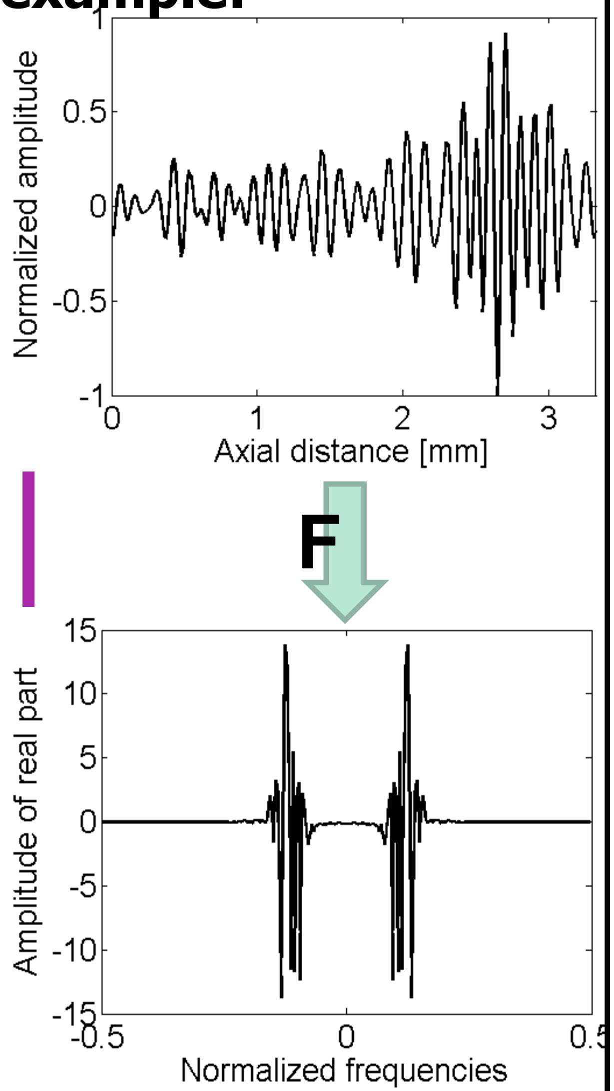
$$\mu(\Phi,\Psi)=\sqrt{n}\max_{1\le k,j\le n}\left|\left\langle \phi_k,\psi_j\right\rangle\right|$$

[Candès *et al.* 2008]

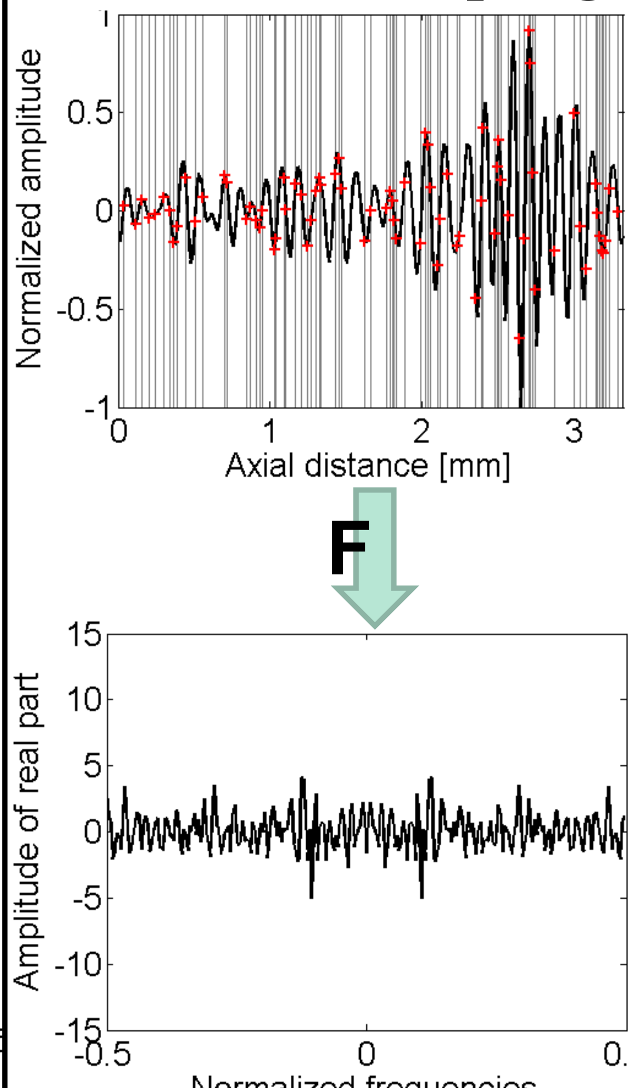
Quick recall

Theory of compressed sensing - Incoherence

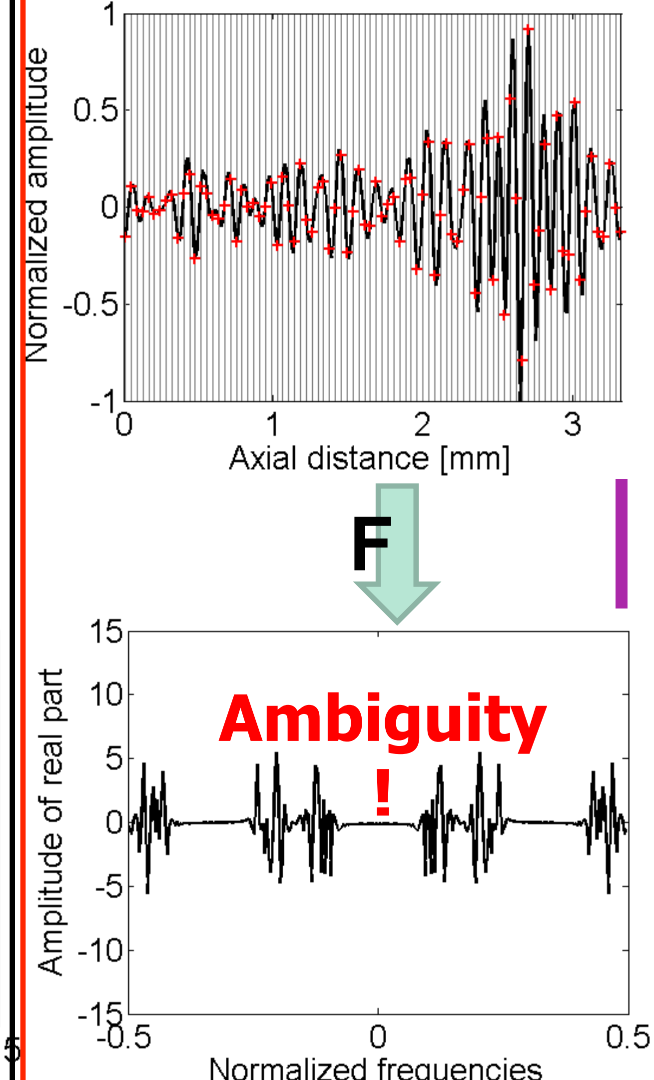
A simple example:



Incoherent sampling

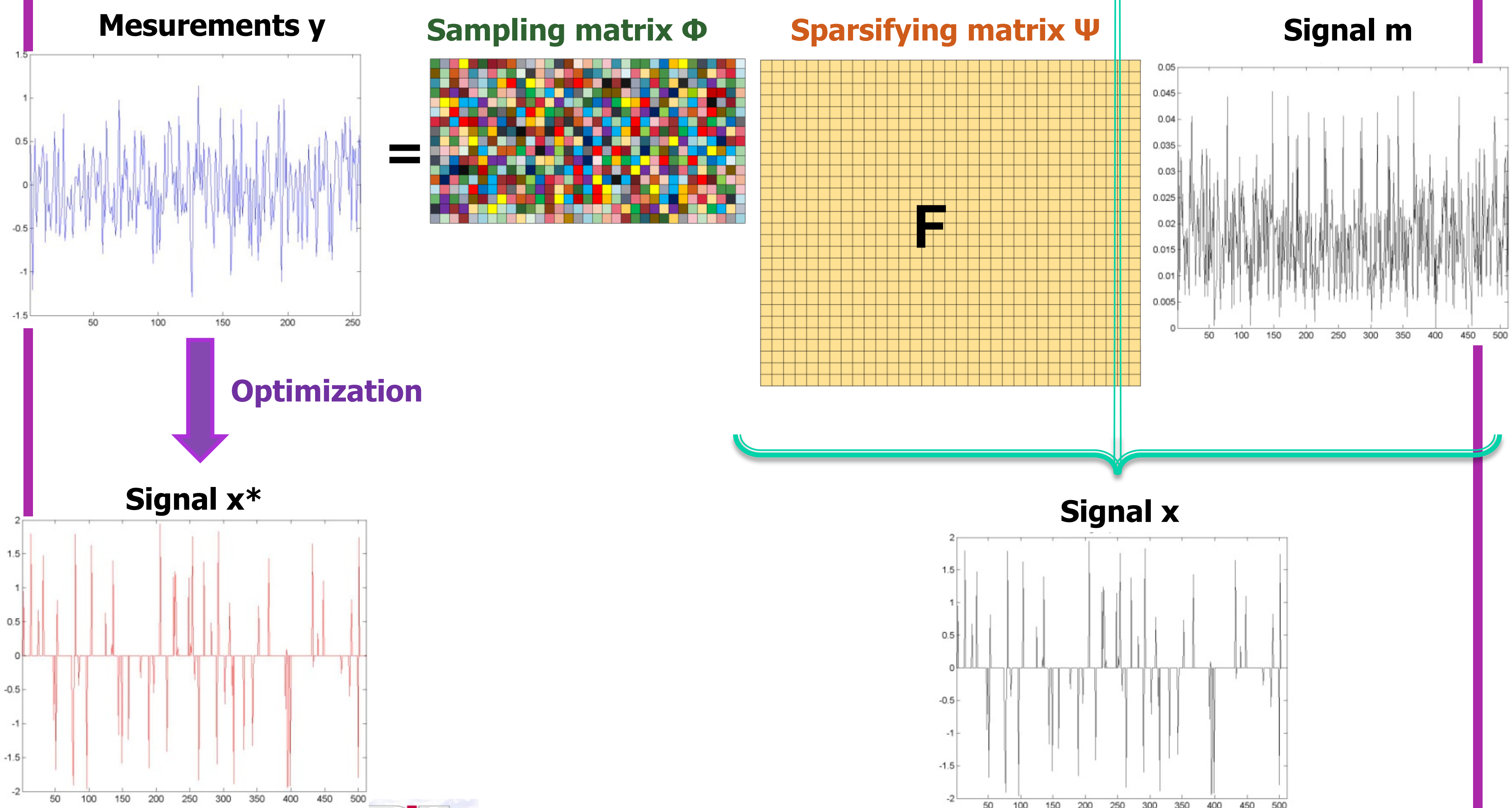


Coherent sampling



Quick recall

Theory of compressed sensing - Reconstruction



Quick recall

Theory of compressed sensing - Summary

- m = image or signal
- x = S-sparse signal
- y = mesurements

$\xrightarrow{\Psi}$ $\boxed{x = \Psi m}$ $\xrightarrow{\Phi}$ $\boxed{y = \Phi m}$

- Φ (K,N) = sampling matrix (K<<N)

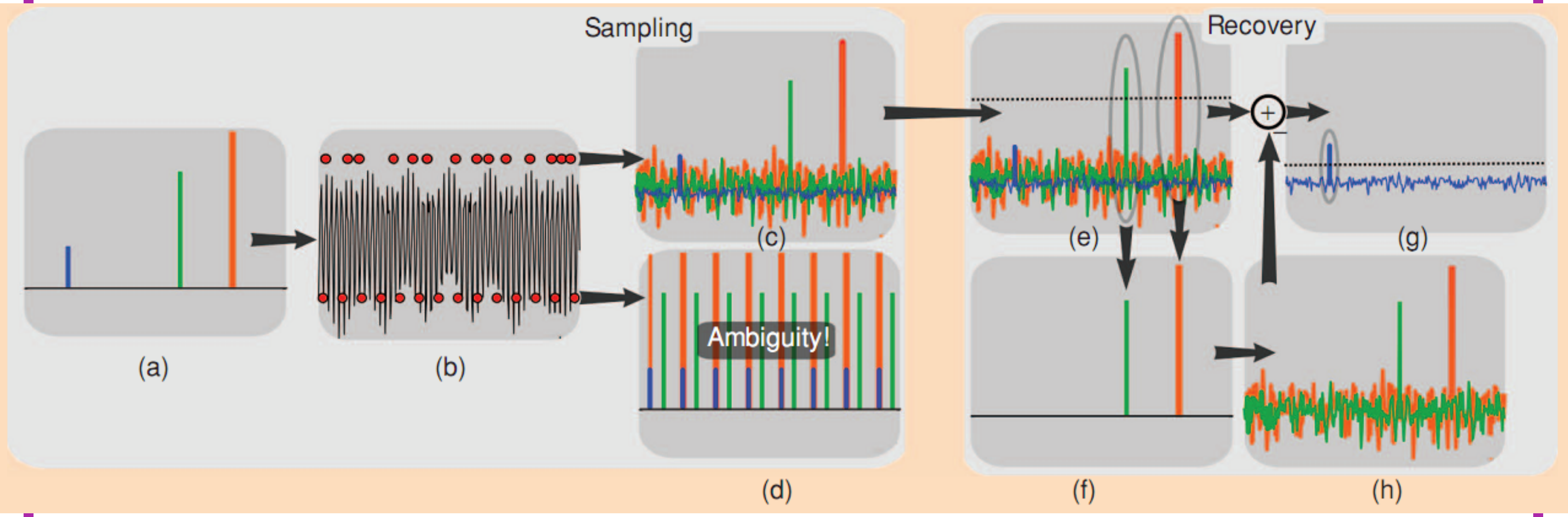
<u>Condition of success:</u> $K \geq C \,.\, \underbrace{S}_{\text{sparsity}} \,.\, \underbrace{\mu^2(\Psi, \Phi)}_{\text{coherence}}.\log N$

To recover x from y:

$$\min \|x\|_1 \quad \text{s.t.} \quad \boxed{\begin{aligned} y &= \Phi m \\ &= \Phi\Psi^{-1}x \end{aligned}}$$

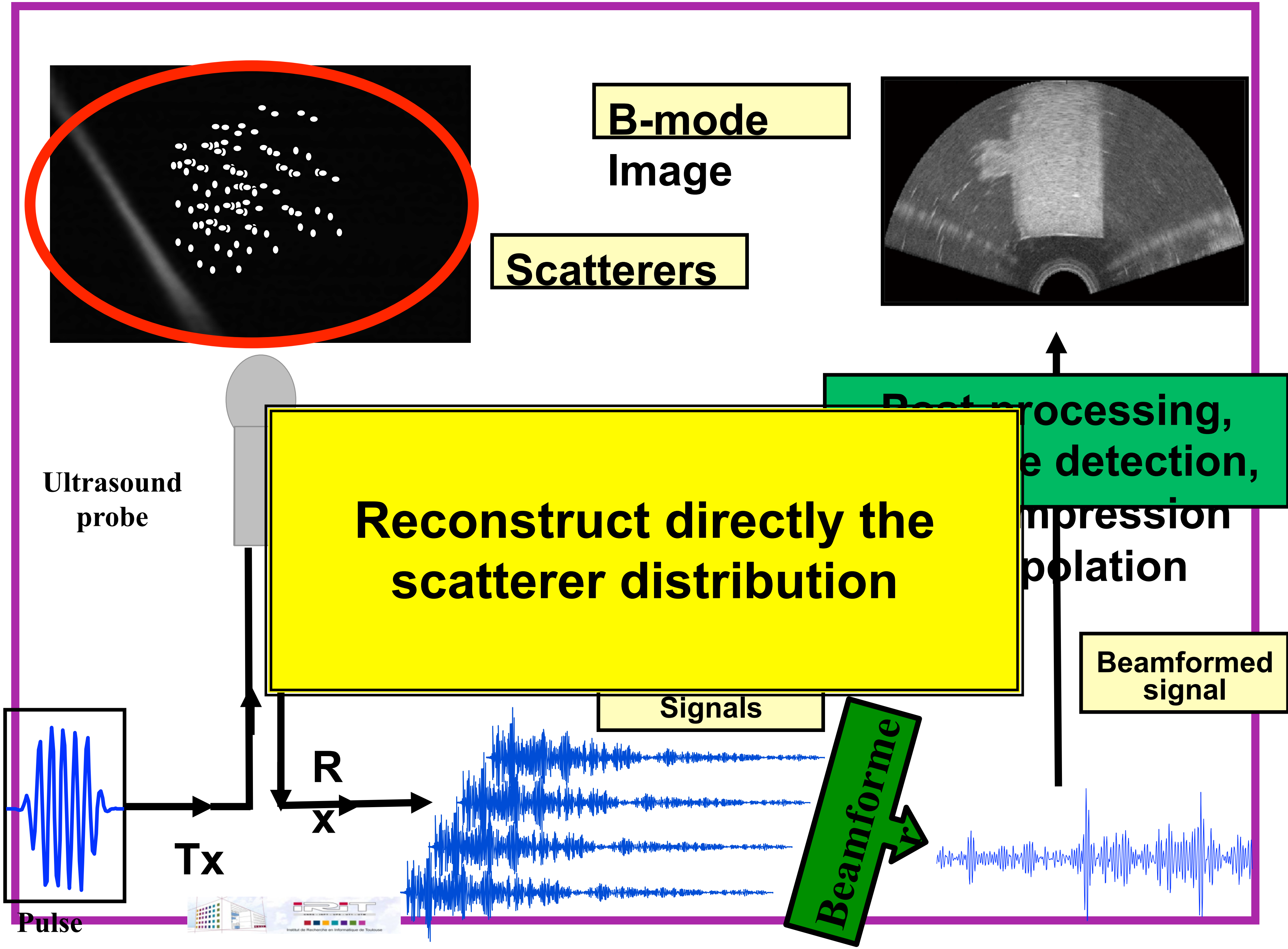
[Candès *et al.* 2007]

Quick recall
Theory of compressed sensing - Optimization



[Lustig *et al.* 2008]

CS methodology in Ultrasound imaging


B-mode
Image
Scatterers
Ultrasound probe
Reconstruct directly the scatterer distribution
Signals
Beamformed signal
R
X
Tx
Pulse
Beamformer

CS in US: sparse scatterer distribution

Assumption made by several authors and solved in different ways [Eldar, Schmitz, Shen, Zhuang]

Problem considered in [Shiffner et al., Biomed Tech, 2012]

- *Objective* ***fast imaging*** *without lost of spatial resolution*
- ***Plane wave*** *insonification at angle* θ
- *Modelling the RF signals received at each element writing* ***the direct scattering problem***

- *As seen previously*

$$p^{sc}(e_\theta) = G(e_\theta)\gamma_\kappa$$

$p^{sc}(e_\theta)$ **: signal received at each element**

$G(e_\nu)$ **: propagation and scattering**

γ_κ **:** N_x **x** N_z **scatterers on a regular lattice**

CS in US: sparse scatterer distribution

Solving the inverse scattering problem becomes

a CS-related problem if γ_κ is supposed sparse

Link with the CS problem

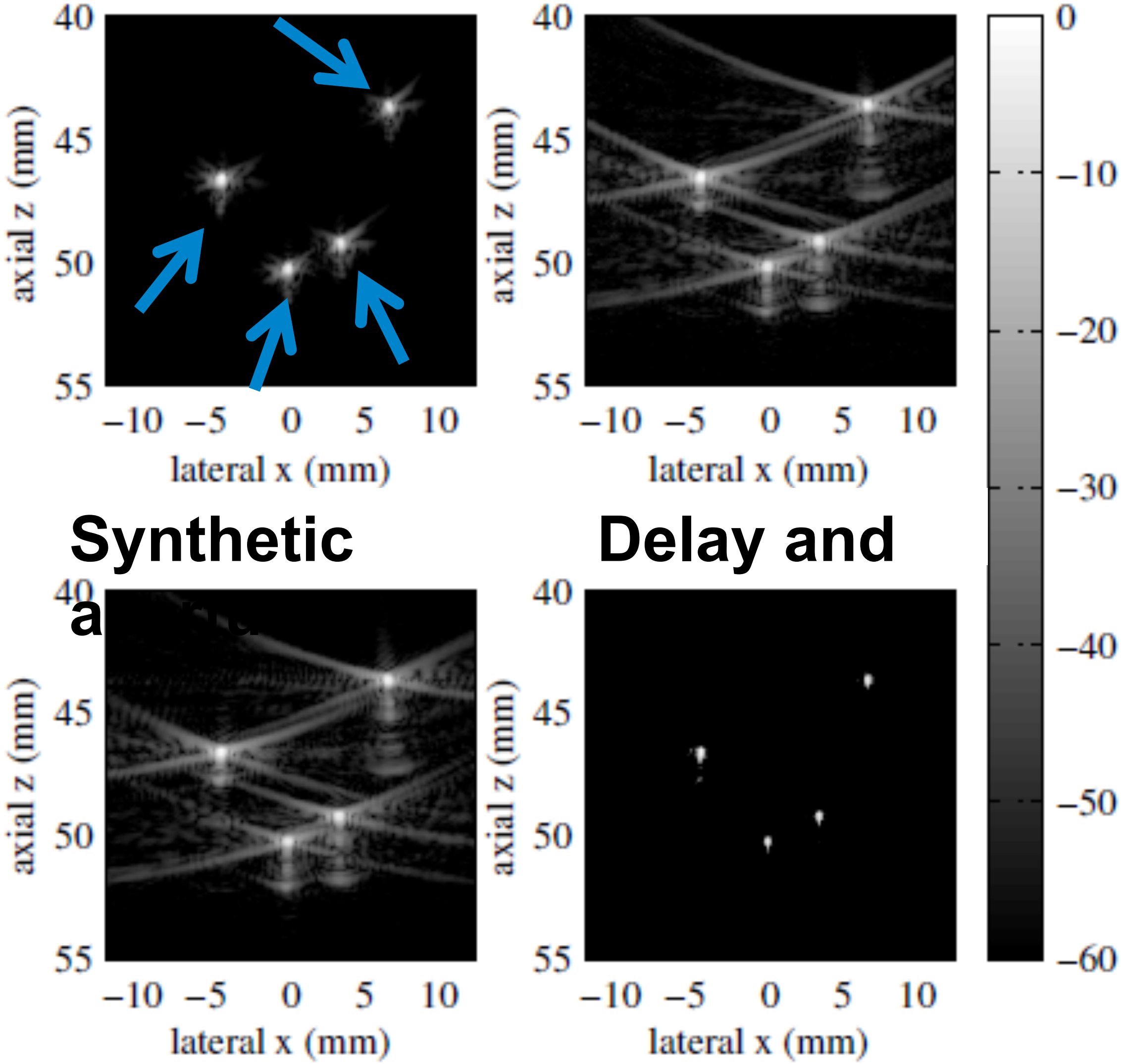
$$p^{sc}(e_\theta) = G(e_\theta)\gamma_\kappa \Longleftrightarrow y = Av$$

$$y = \boldsymbol{R\Phi\Psi} v \text{ with } \begin{cases} y = p^{sc}(e_\theta) & \textit{The measured signals} \\ \boldsymbol{R} = \boldsymbol{I} & \\ \boldsymbol{\Phi} = G(e_\theta) & \textit{Measurement matrix} \\ \boldsymbol{\Psi} = \boldsymbol{I} & \\ v = \gamma_\kappa & \textit{The sparse « representation »} \end{cases}$$

CS in US: sparse scatterer distribution

4 isolated scatterers

[Shiffner et al., Biomed Tech, 2012]



Synthetic a

Delay and

Fourier back-

CS

➔ Works well when the assumption is respected

CS in US: sparse scatterer distribution

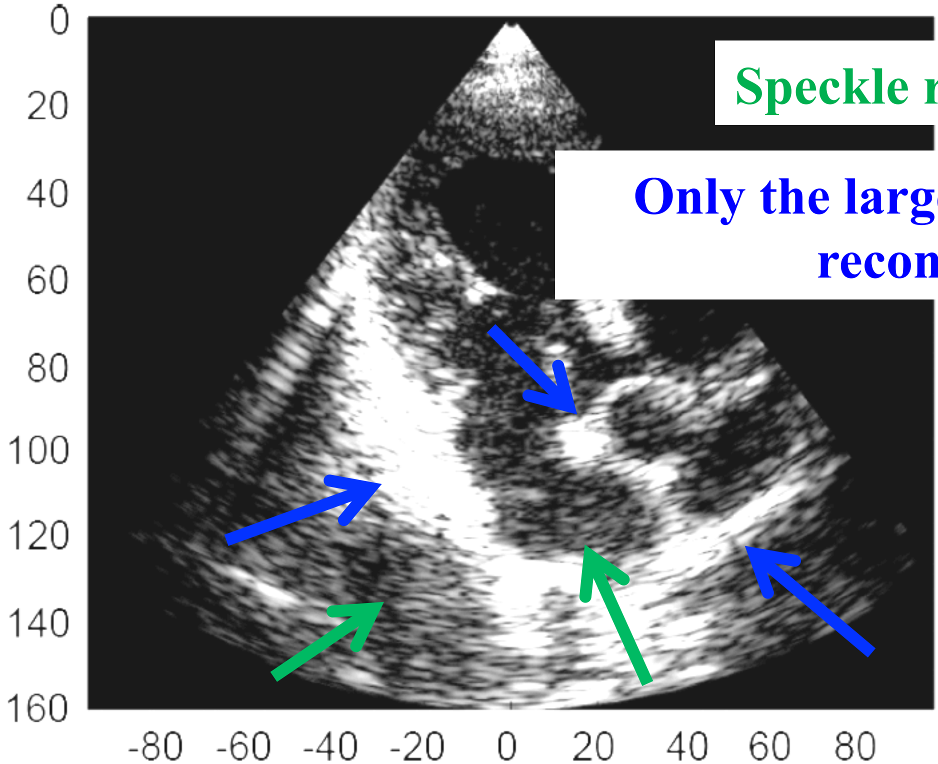
Same assumption: sparse distribution map

Other approach: Finite Rate of Innovation and Xampling

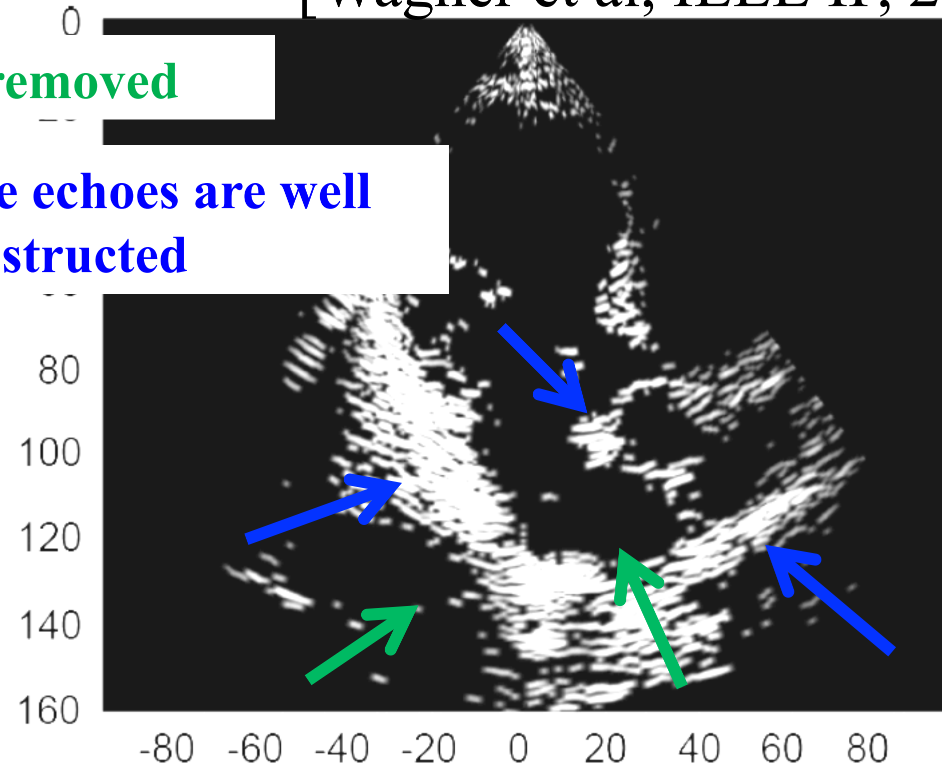
[Wagner et al, IEEE IP, 20

Speckle removed

Only the large echoes are well reconstructed

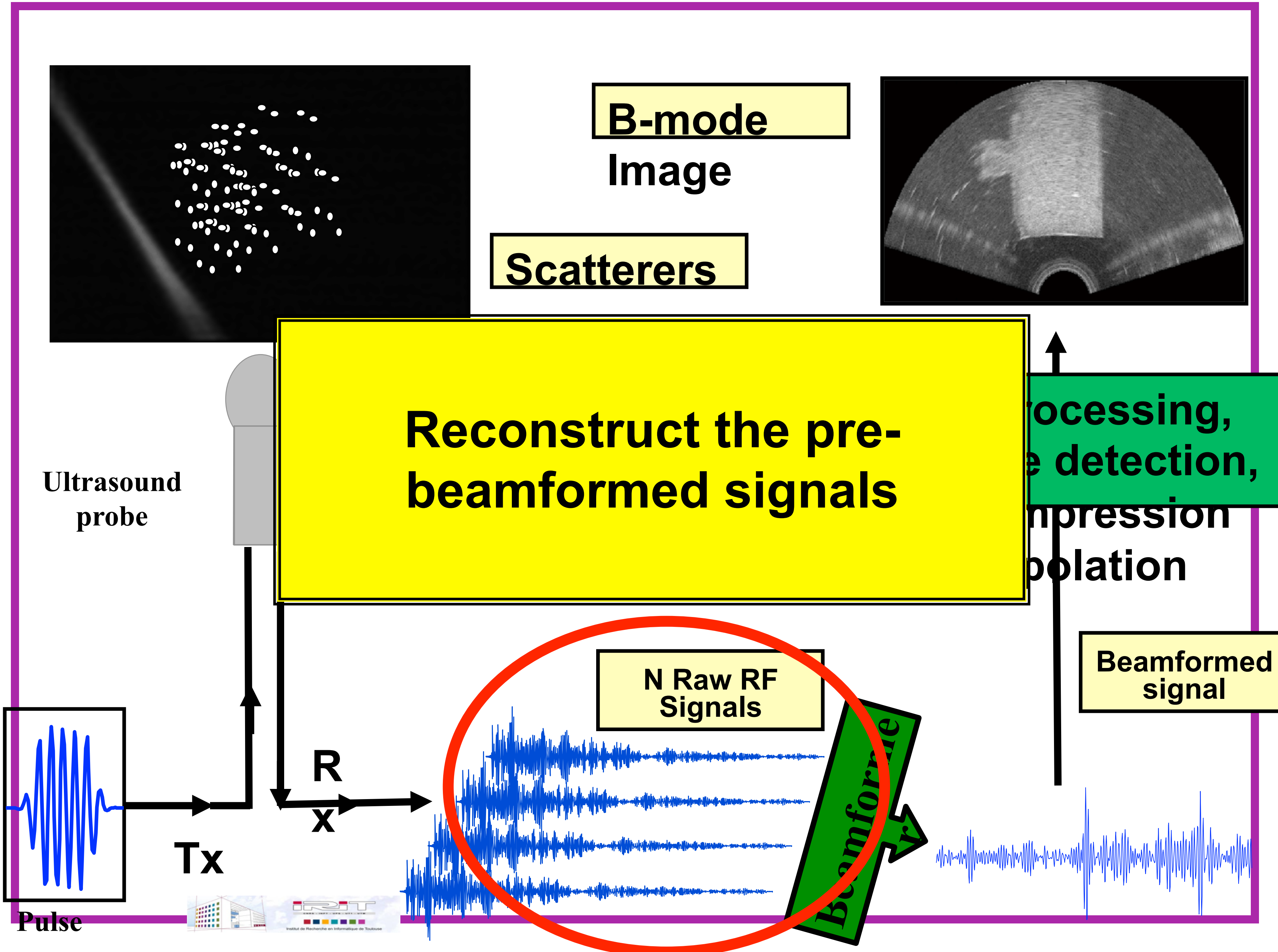


Original cardiac image



Reconstructed cardiac image

➔ **Scatterer distribution perhaps not so sparse…**


B-mode
Image
Scatterers
Reconstruct the pre-beamformed signals
rocessing,
e detection,
npression
polation
Ultrasound probe
N Raw RF Signals
Beamformed signal
R
x
Tx
Pulse

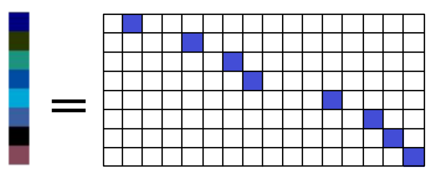
CS in US: sparse channel data

Problem considered

- ***Conventional imaging (focused Tx)***
- ***Objective: test if removing channel data is possible***
- ***Signals to reconstruct : raw channel data*** x

→ remove elements??

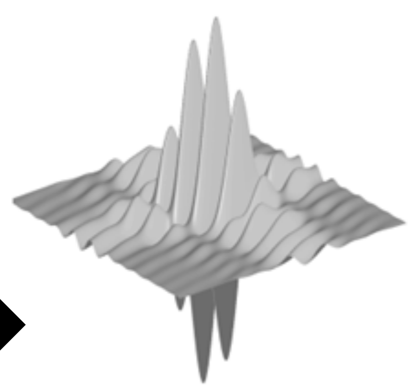
- ***Random acquisition***

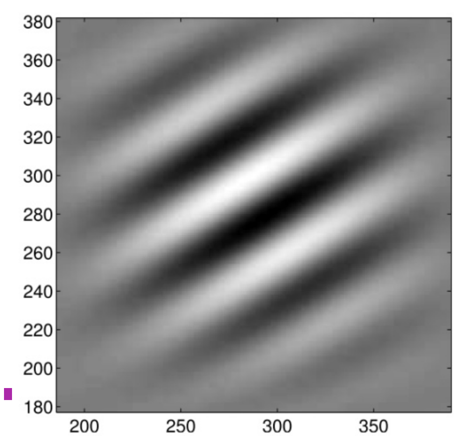


$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

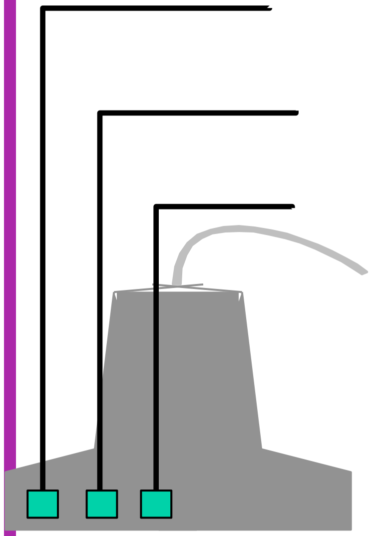
$$y = \quad \boldsymbol{R} \quad \phi = I \quad x$$

- ***Three transforms tested*** Ψ
  - ***Fourier***
  - ***Wavelets***
  - ***Wave Atoms***

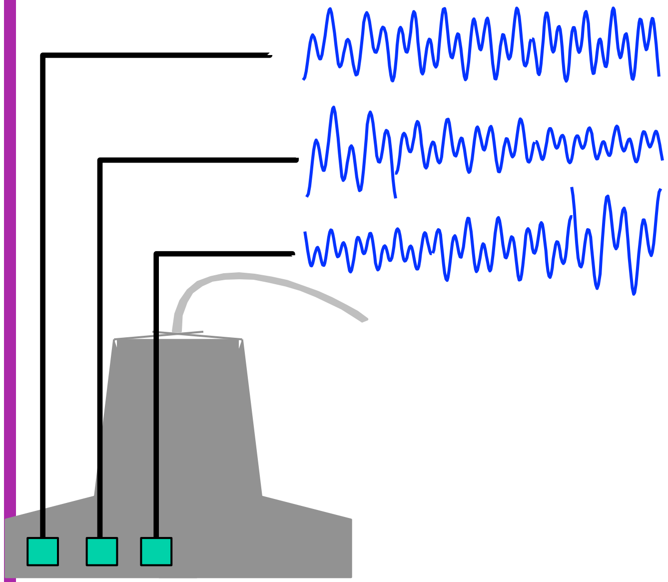




CS in US: sparse channel data



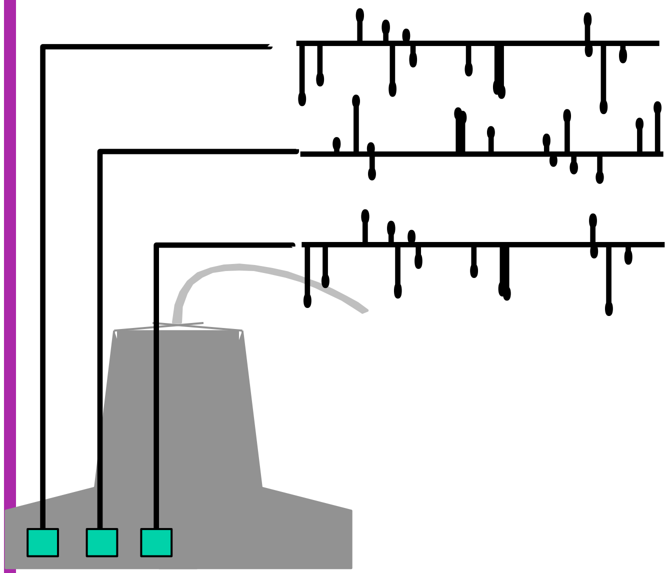
CS in US: sparse channel data



Complete chanel data set

$$x$$

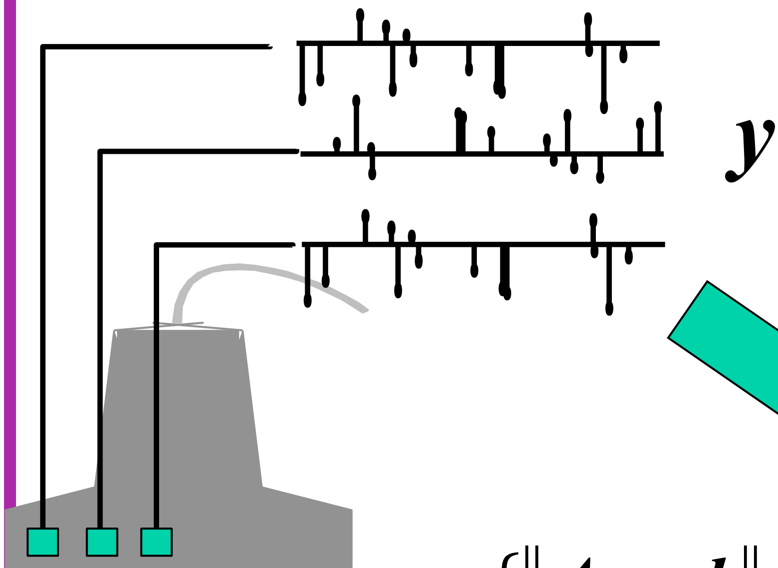
CS in US: sparse channel data



Random undersampling

$y = Rx$ **(Measurment matrix ϕ = Identity)**

CS in US: sparse channel data



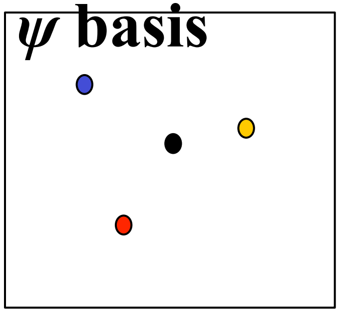
Reconstruction in the sparse basis ψ

$$\begin{cases} \|Ax - b\|_{l_2} < \varepsilon \\ \|x\|_{l_1} \ \text{minimum} \end{cases}$$

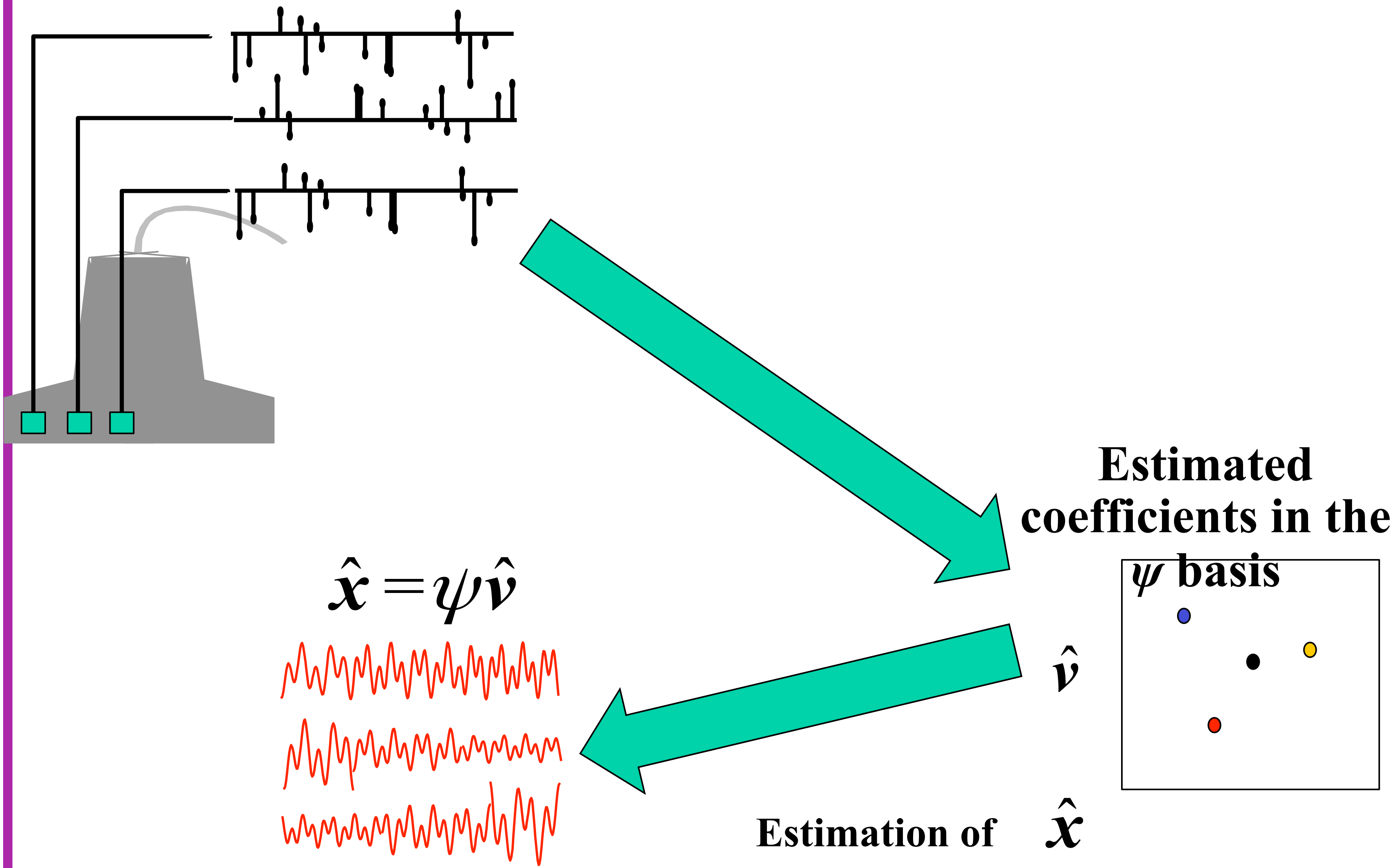
Basis pursuit using minimization

Estimated coefficients in the ψ basis

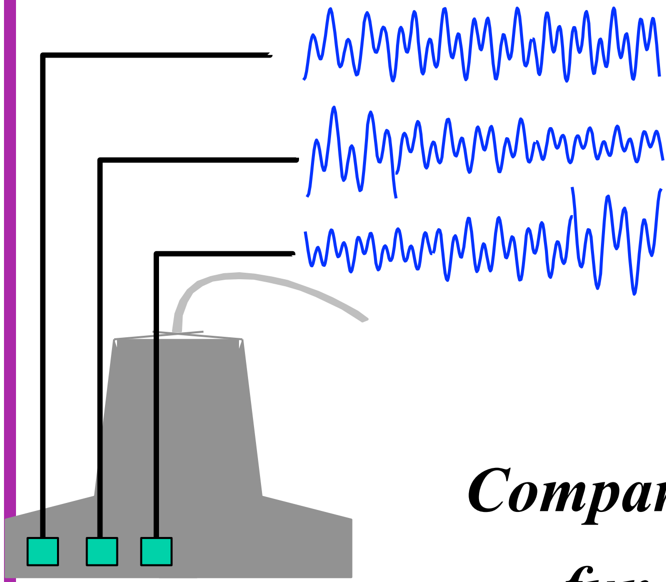
$\hat{v}$



CS in US: sparse channel data

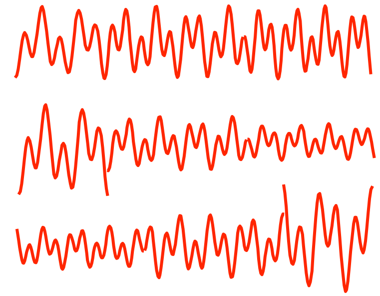


CS in US: sparse channel data



?????

$$x = \hat{x}$$



Comparaison original / reconstructed signals

function of undersampling

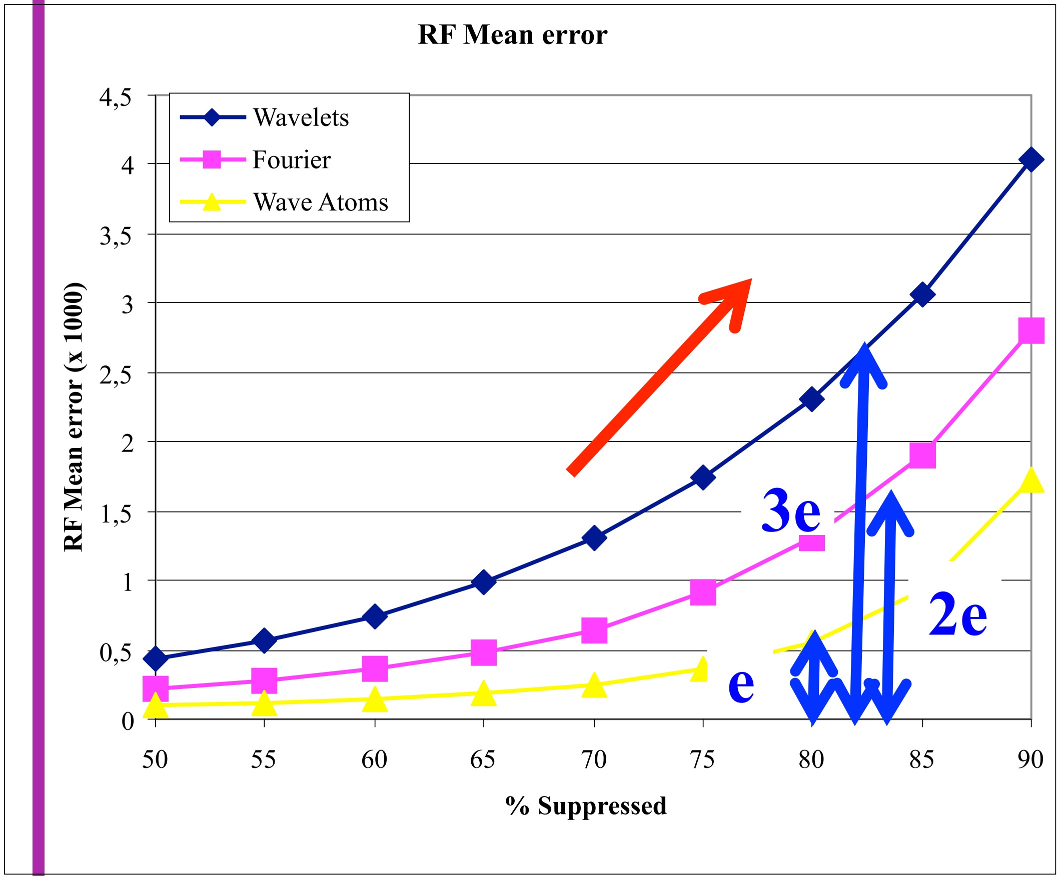
sparsifying basis

Field II simulations [Jensen et al. 1992 & 1996]

Measurements with Ultrasonix MDP + DAQ

CS in US: sparse channel data

Reconstruction error as a function of removed data and basis ψ

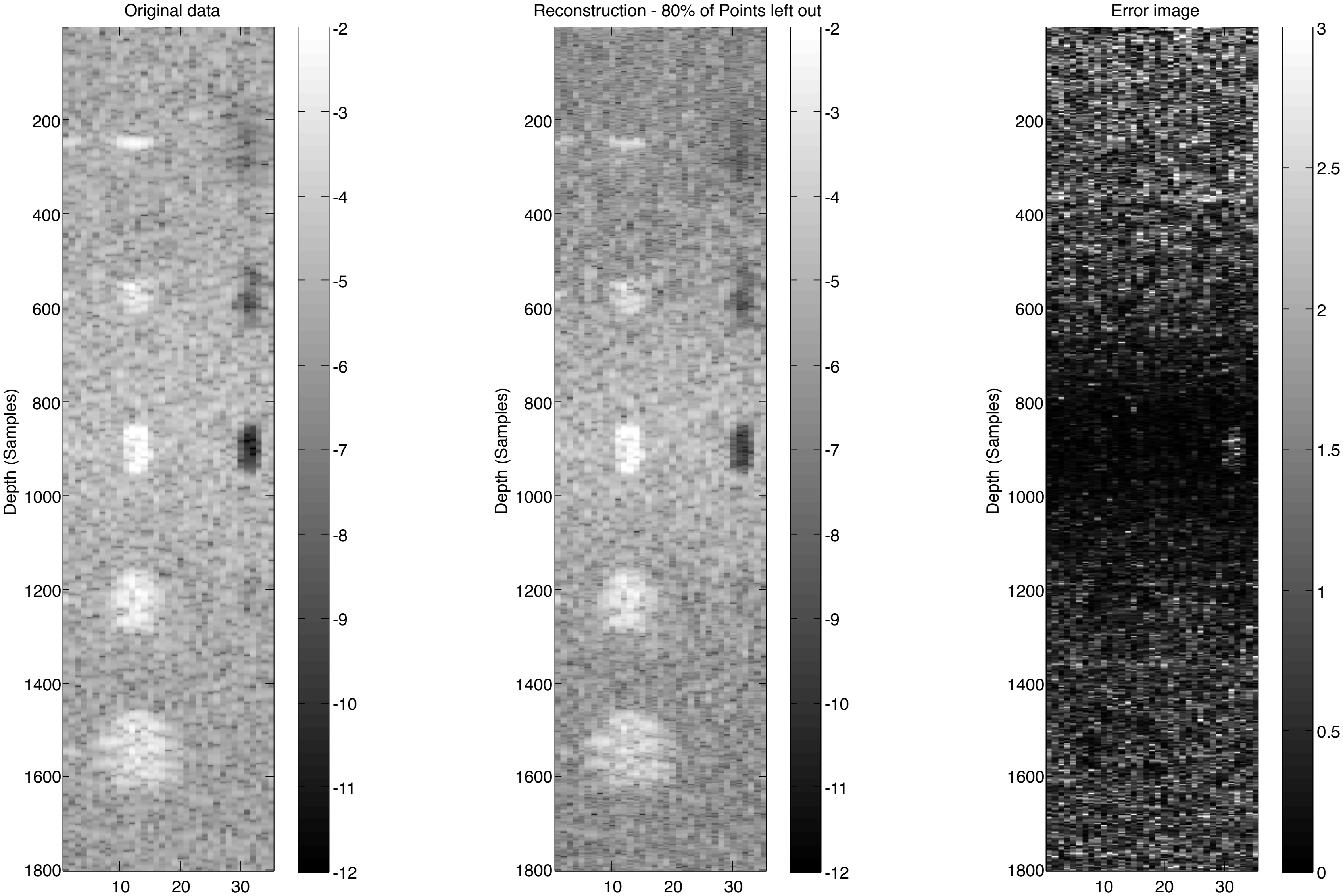


- **Error increase with decreasing quantity of data**

- **Error e with wave atoms**
  - **2x smaller than with Fourier**
  - **3x smaller than wavelets**

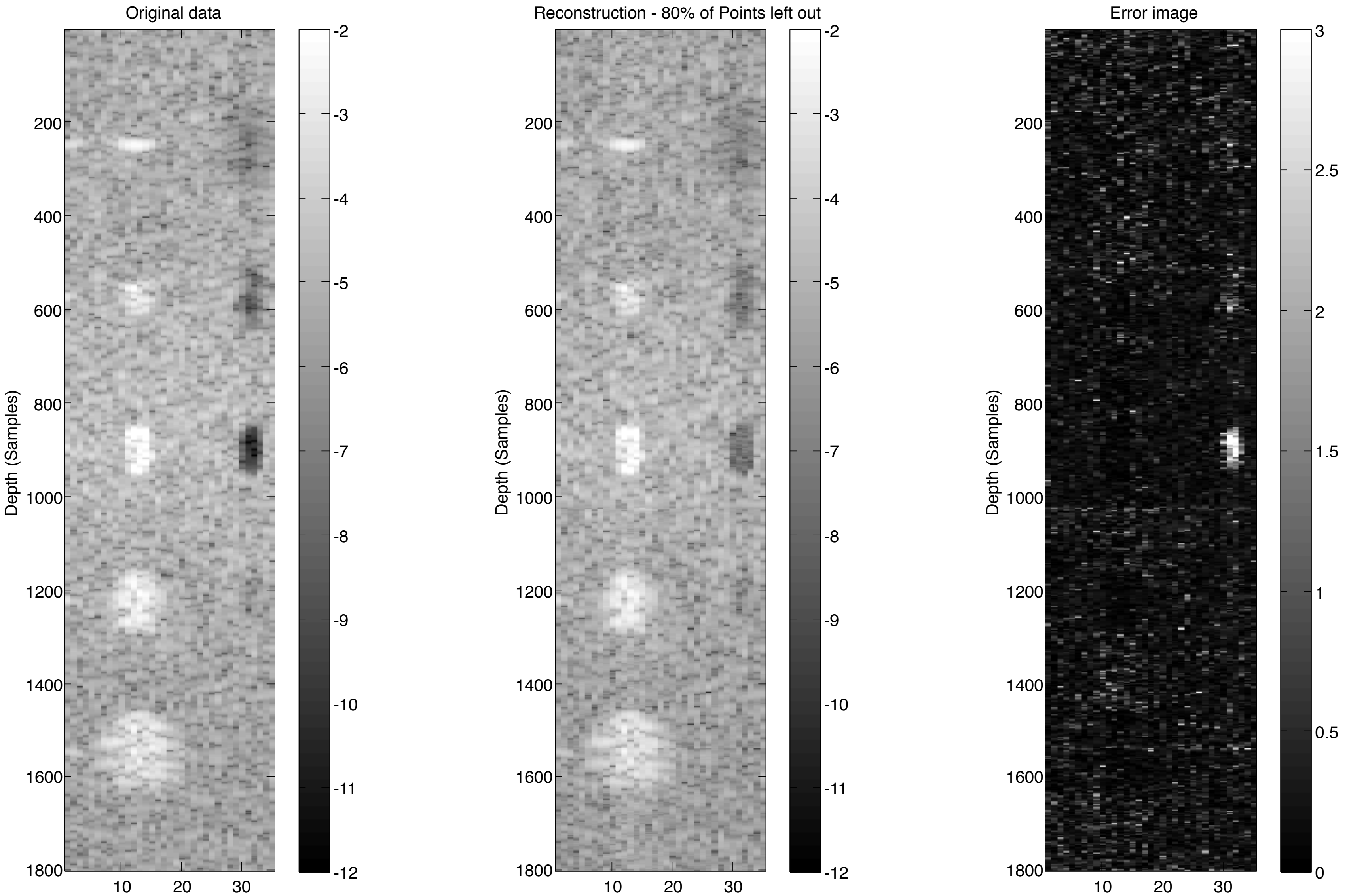
CS in US: sparse channel data

B-mode – **Wavelets** (80% undersampling)



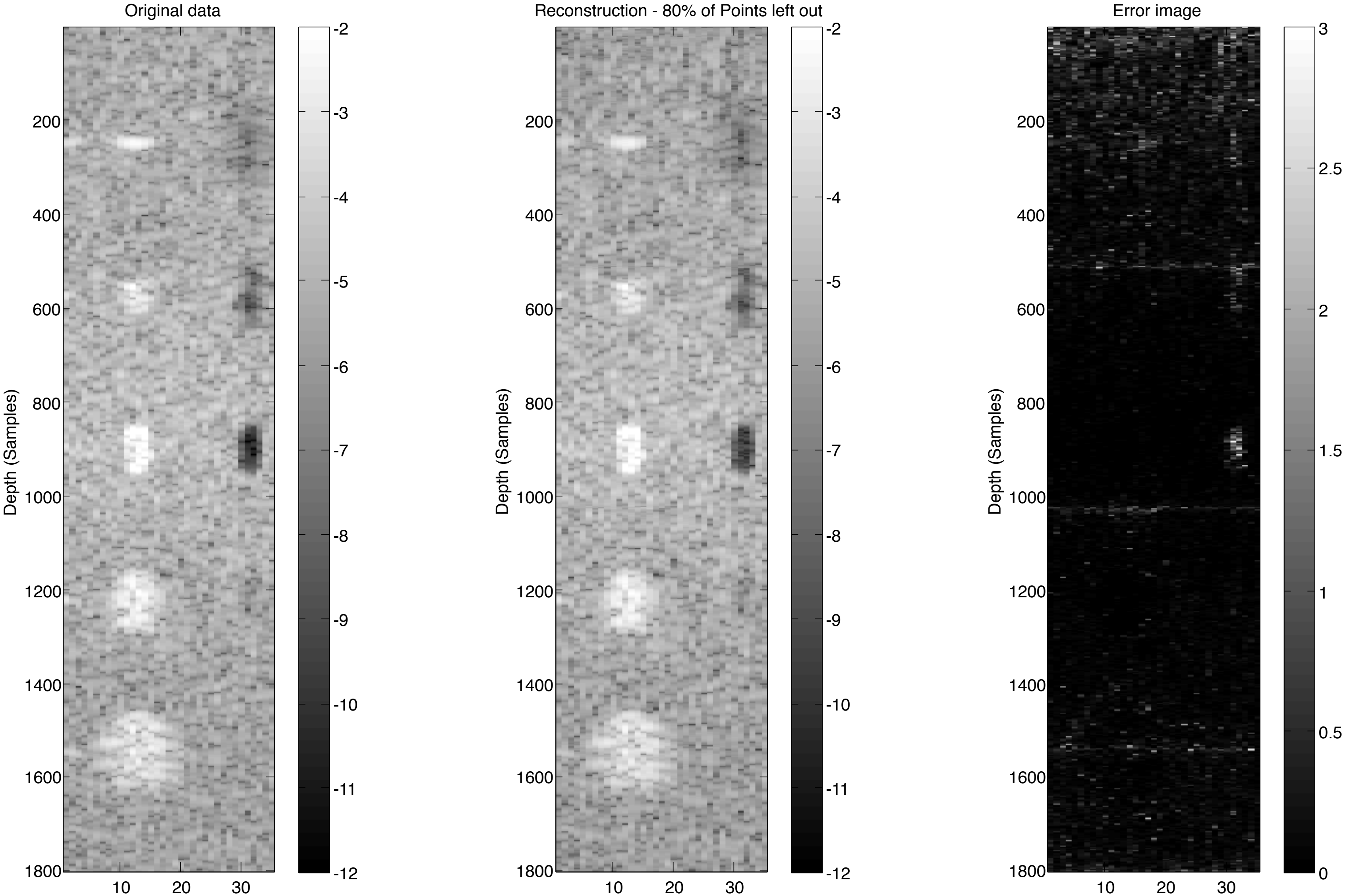
CS in US: sparse channel data

B-mode - **Fourier** (80% undersampling)



CS in US: sparse channel data

B-mode - **Wave atoms** (80% undersampling)

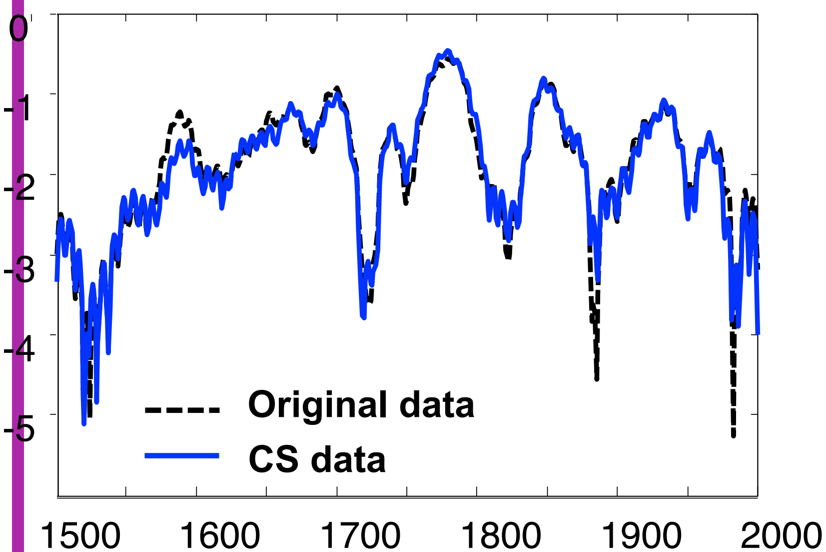


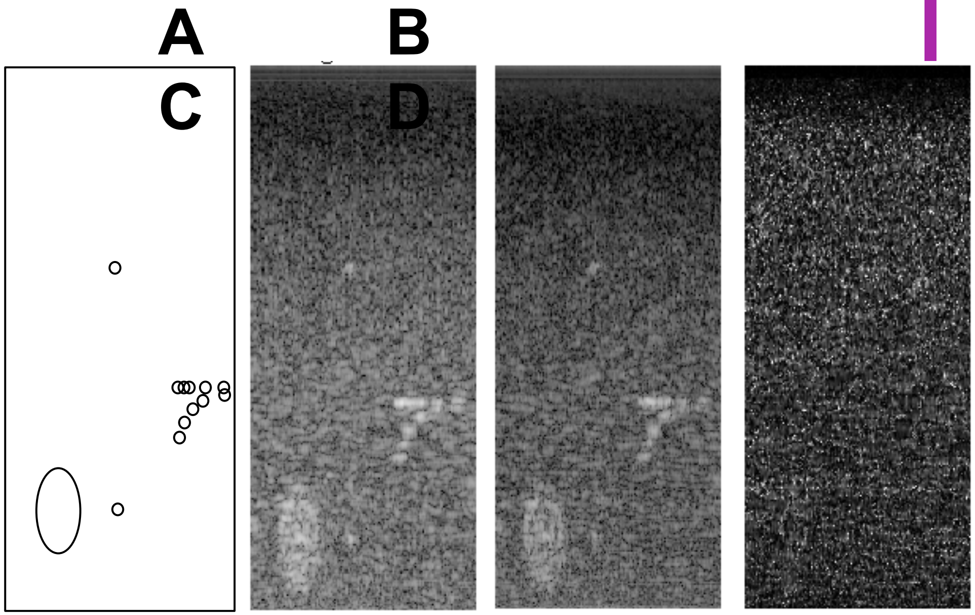
CS in US: sparse channel data

Ultrasonix MDP with Sonix DAQ

CIRS model 054GS

[Liebgott *et al.*, Ultrasonics 2012, in press]





Original and **CS B mode signals** obtained from from 80% dowsampled (wave-atoms).

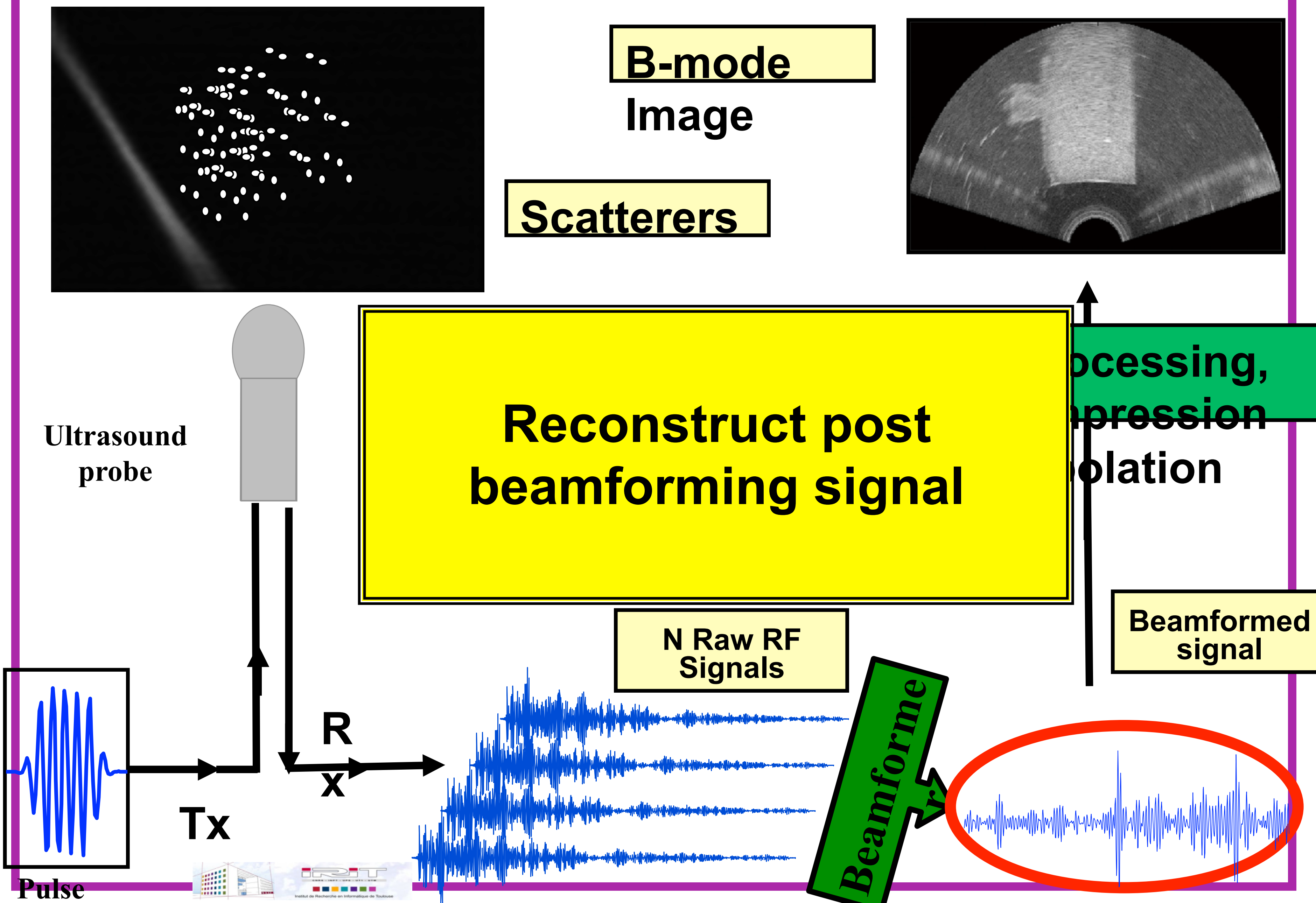
Corresponding B mode images

A : phantom structure

B : original image

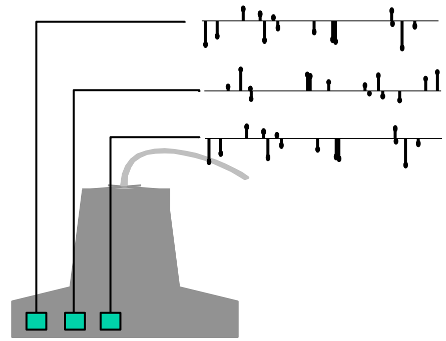
C : CS image

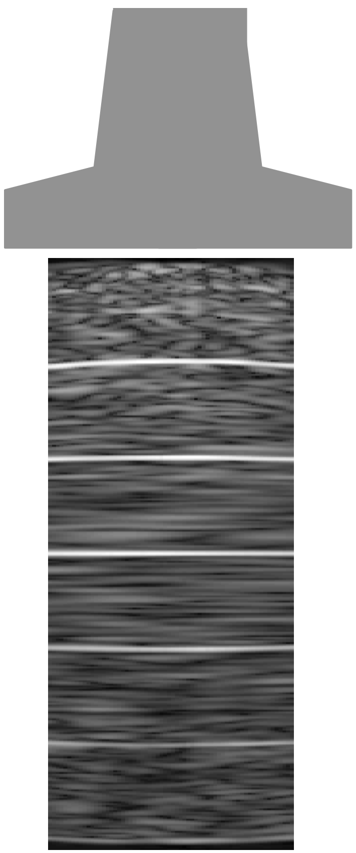
D : error


B-mode
Image
Scatterers
Reconstruct post
beamforming signal
Ultrasound
probe
N Raw RF
Signals
Beamformed
signal
Beamforme
r
R
x
Tx
Pulse

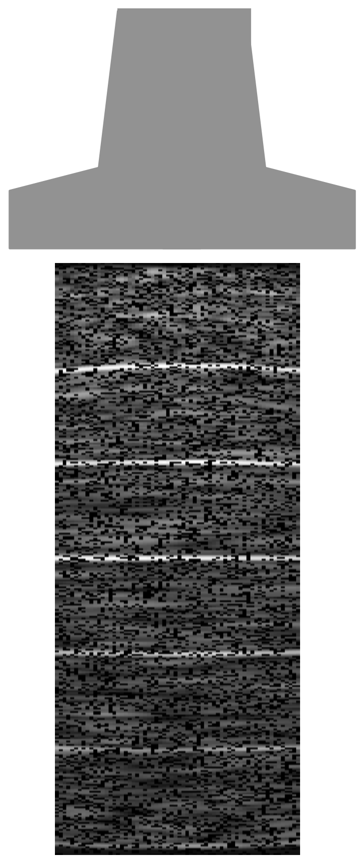
CS in US: sparse channel data

Uniform random sampling





Complete data

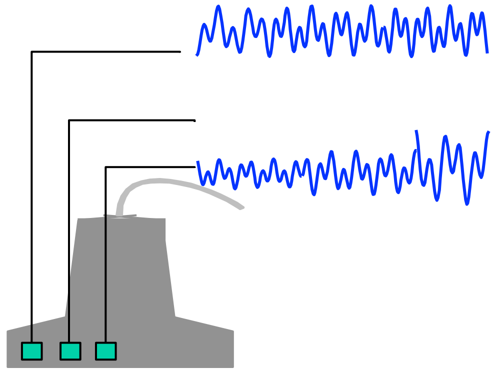


Downsampling scheme

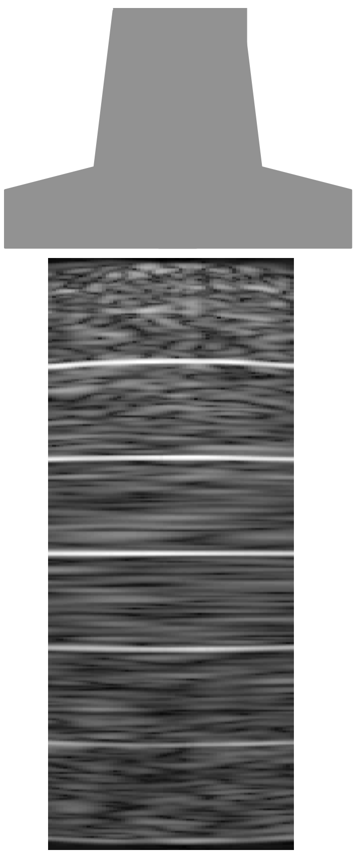
++ Reconstruction is possible

The acquisition sheme is probably just as complicated as processing the complete channel data

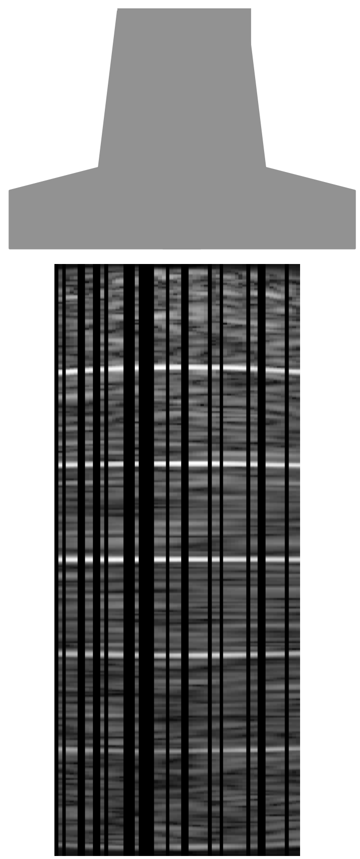
CS in US: sparse channel data



Channel-wise
undersampling



Complete data



Downsampling scheme

++ Technically easy = remove elements

-- does not work as good as uniform random

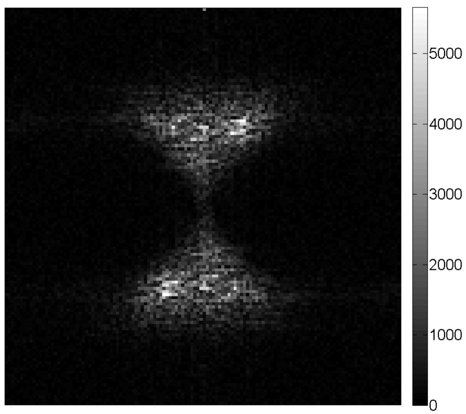
CS in US: sparse beamformed signal

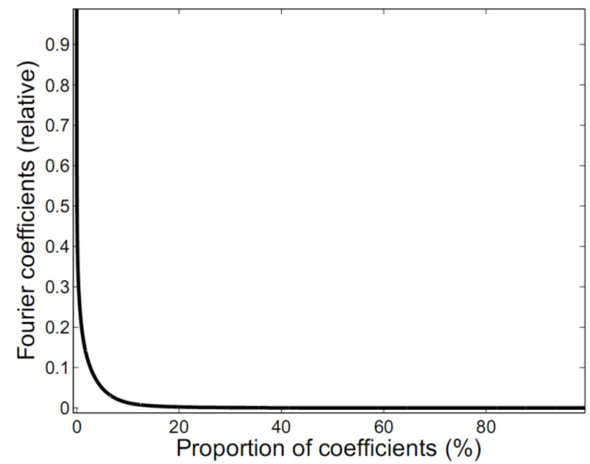
[Dobigeon *et al.*, 2011]
[Quinsac *et al.*, 2011]

Problem considered

- Reconstruction of bandlimited beamformed RF images x
- Objective save acquisition time and acquire less data
- Assumption of sparsity in the 2D Fourier domain $x = \psi v$

Fourier transform of US image



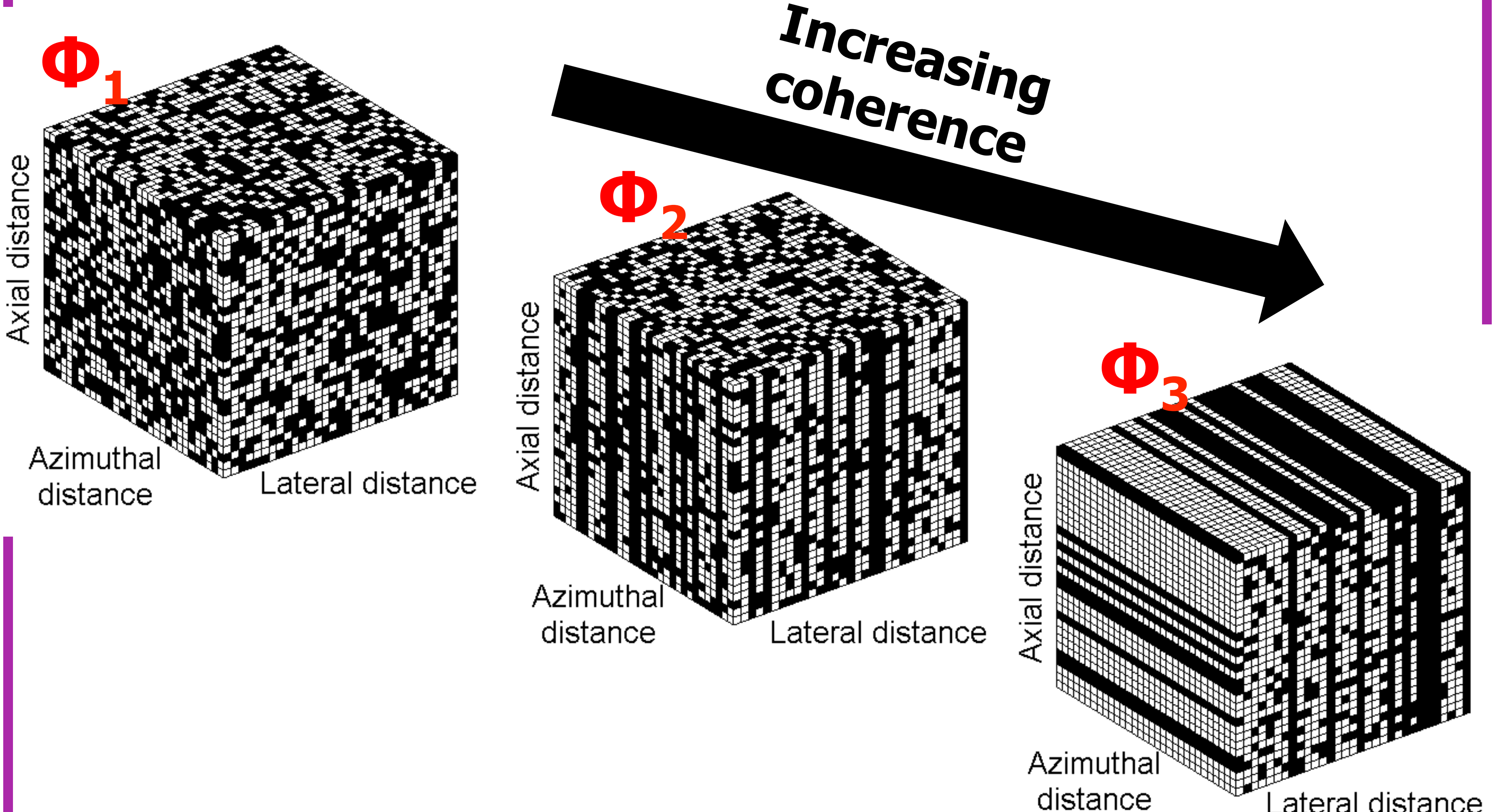


Most of coefficients close to zero

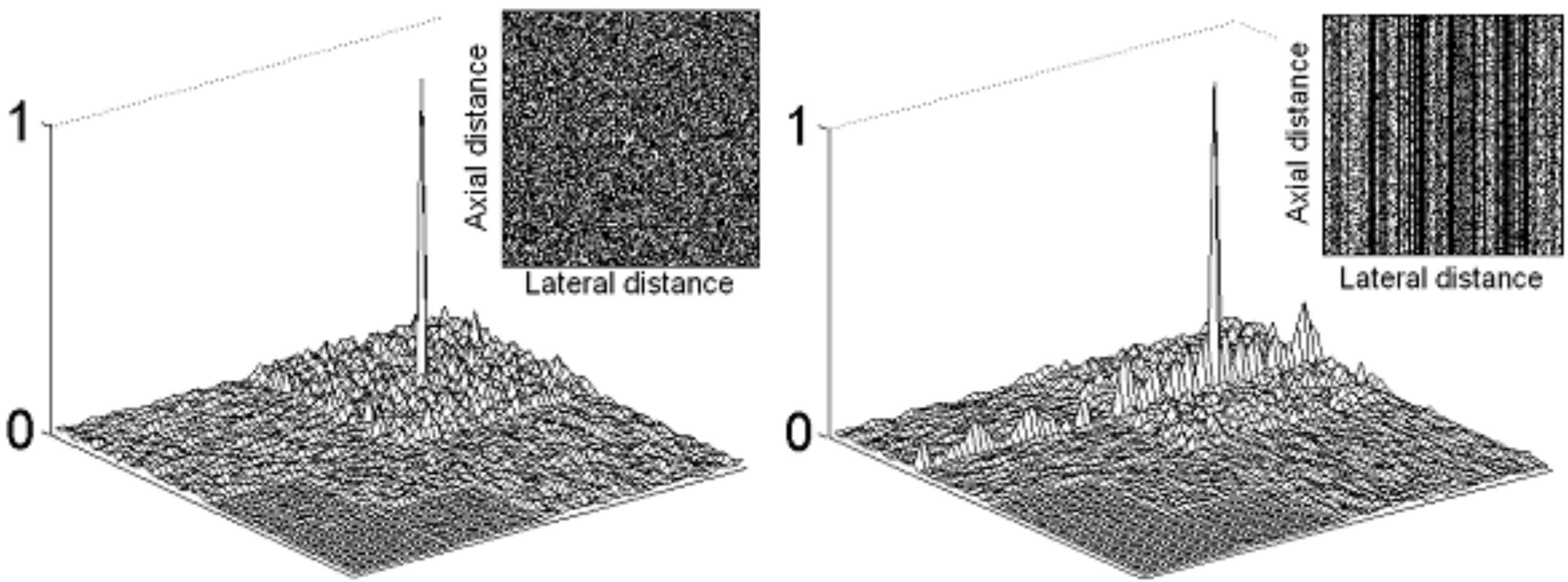
- Acquisition Φ
  - Randomly skip RF lines
  - Random sampling of the acquired RF lines
- Variational or Bayesian reconstruction

CS in US: sparse beamformed signal

Method for CS in US imaging - Sampling



Method for CS in US imaging - Incoherence



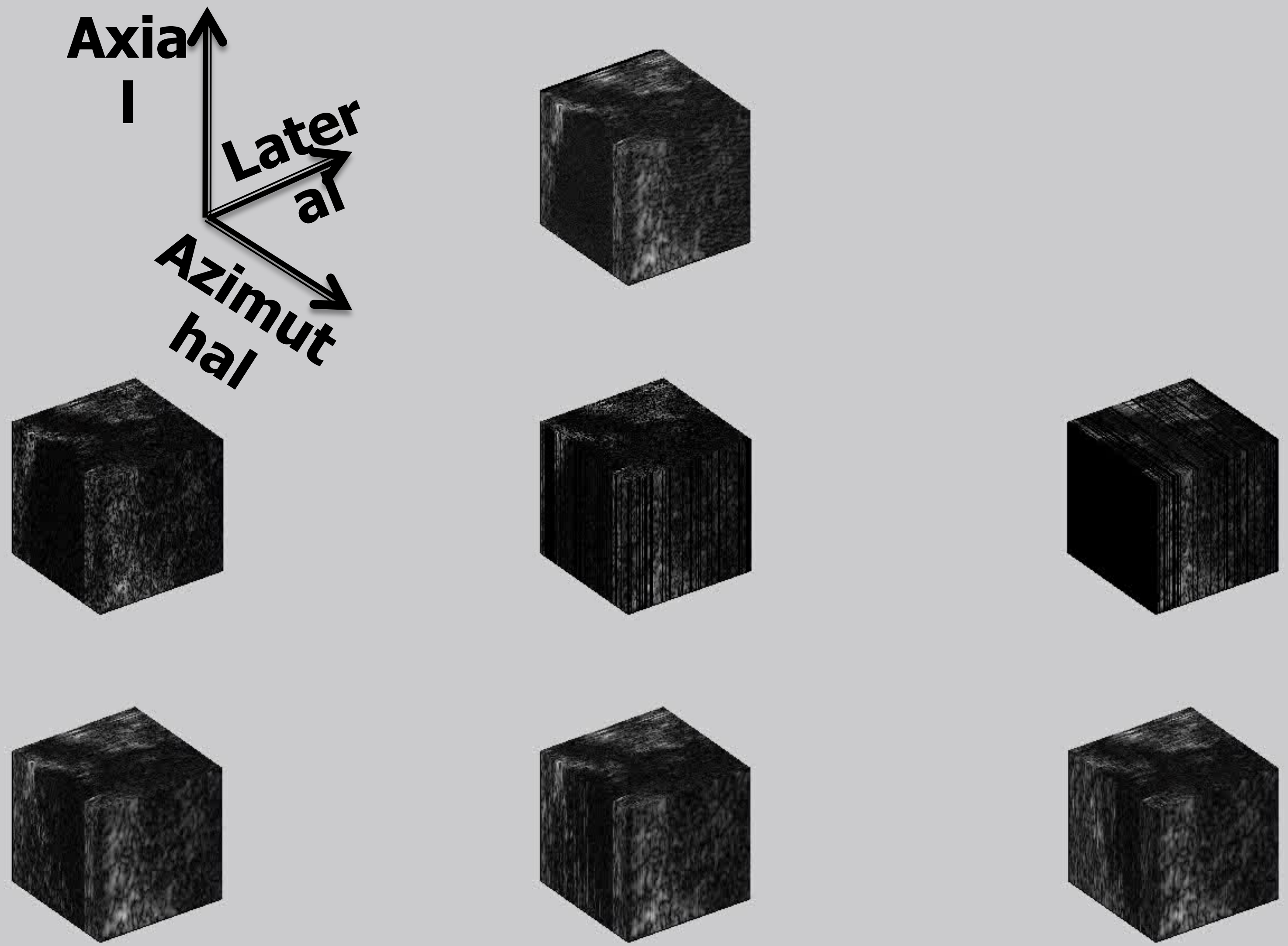
(a) TPSF for Φ_1 (b) TPSF for Φ_2

$$TPSF(i;j) = e_j^T F \Phi^{-1} \Phi F^{-1} e_i$$

Results on 3D US images

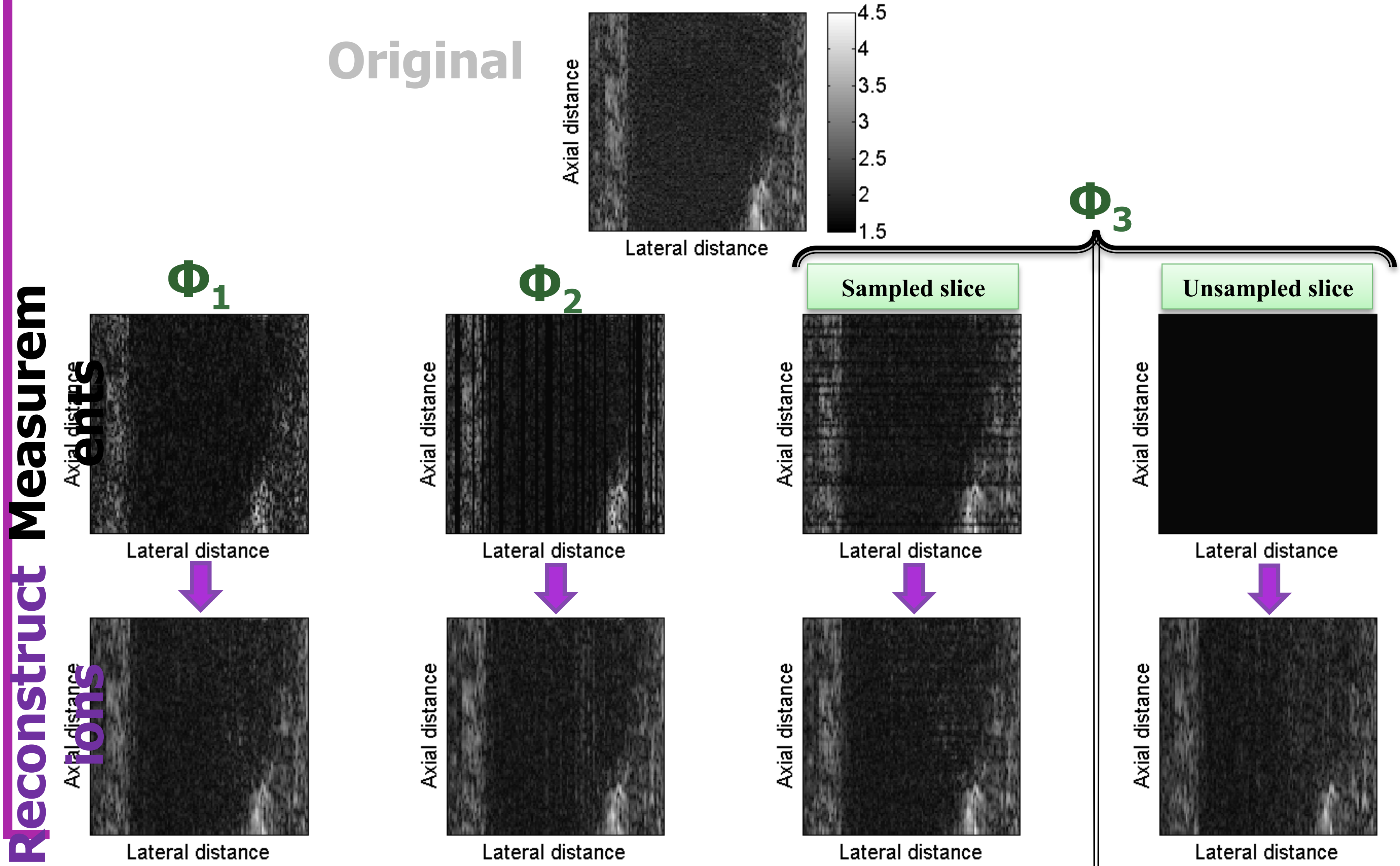
CS in US: sparse beamformed signal

Results on 3D US images - Volumes



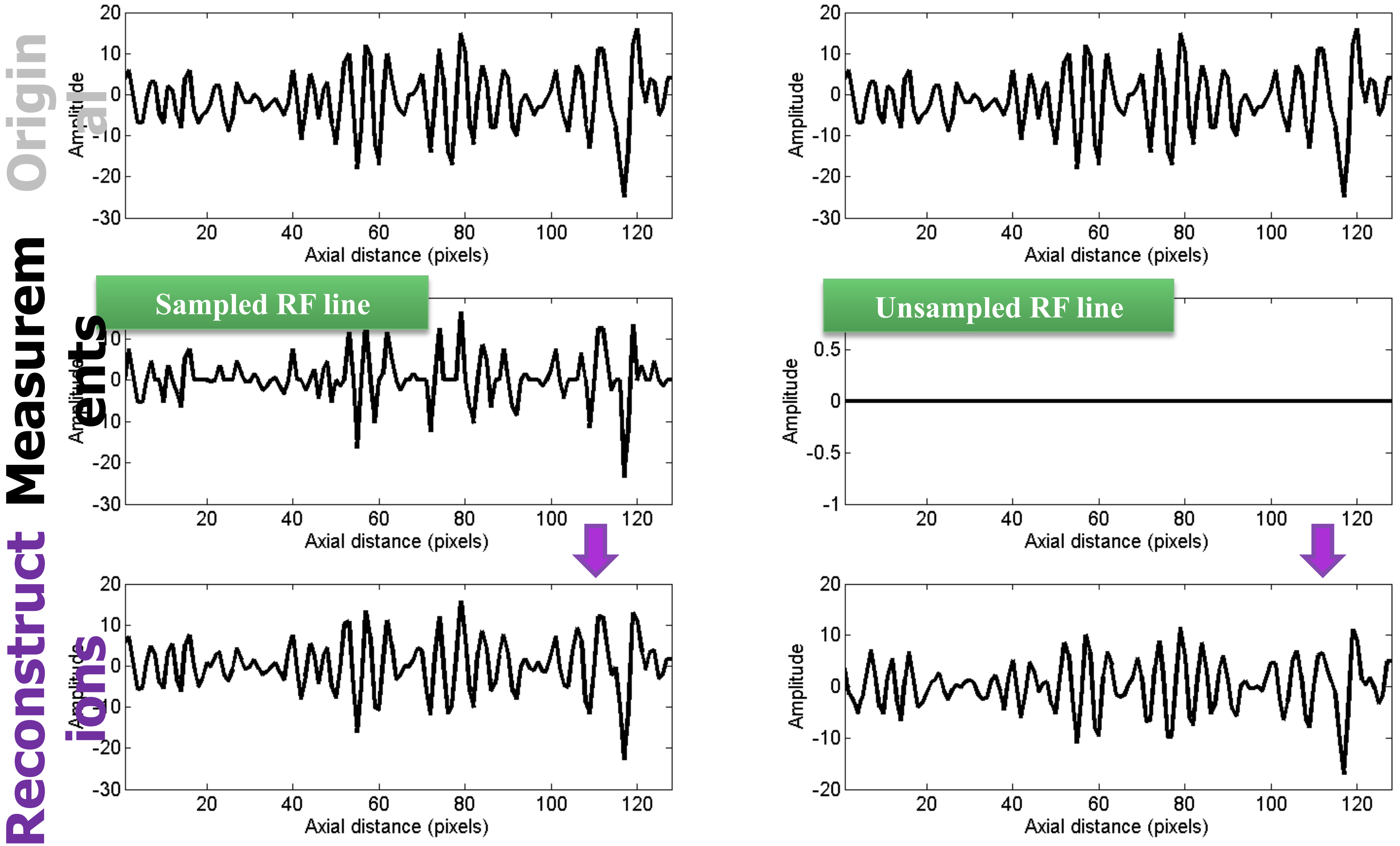
CS in US: sparse beamformed signal

Results on 3D US images - Slices



CS in US: sparse beamformed signal

Results on 3D US images – RF lines

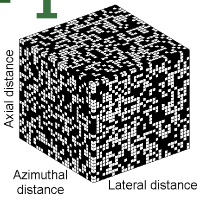


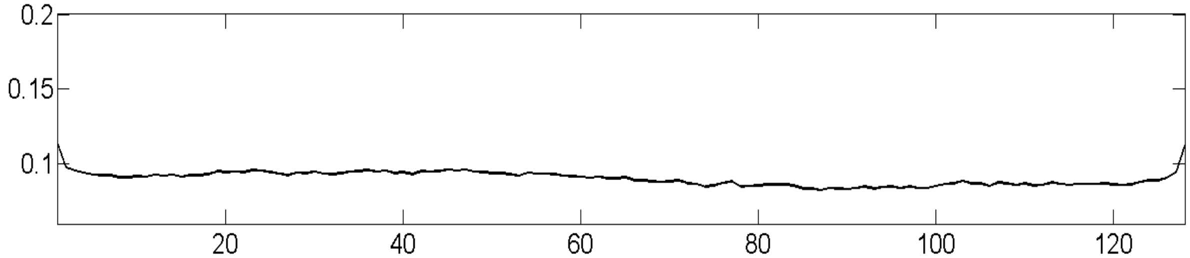
CS in US: sparse beamformed signal

Results on 3D US images - Errors

Mean NMRSE

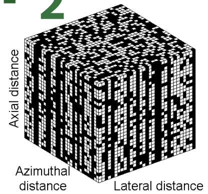
Φ_1

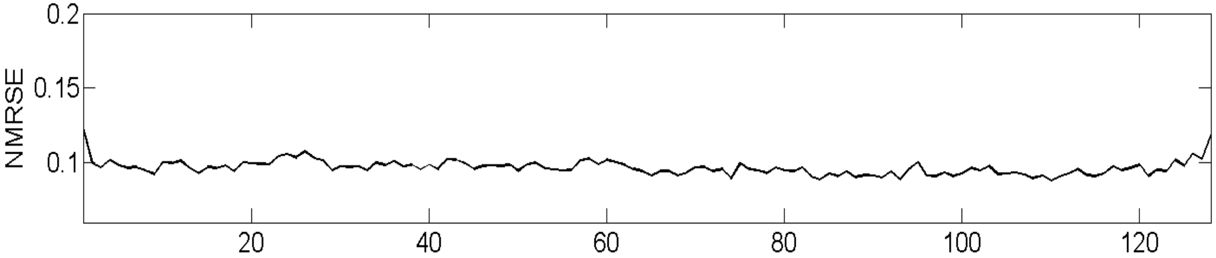




$0.090 \pm 4.4E^{-4}$

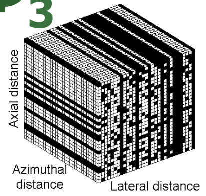
Φ_2

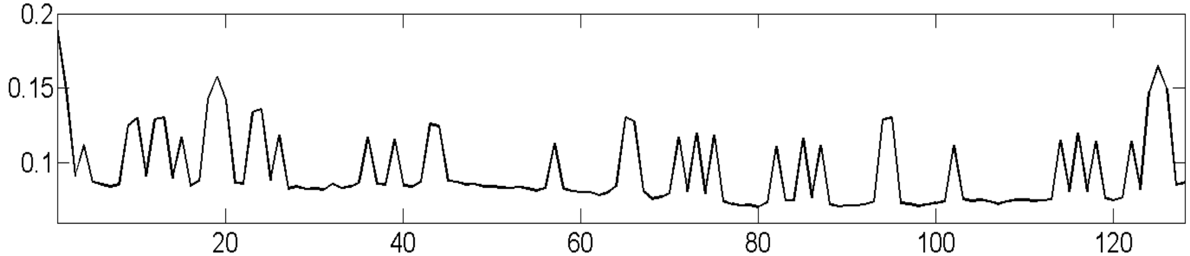




$0.097 \pm 4.4E^{-4}$

Φ_3





$0.094 \pm 20E^{-4}$

Slice number

An alternative of this appraoch is bayesian reconstruction, which allows more flexibility to take into account priors on the ultrasound images

CS in US: sparse beamformed signal

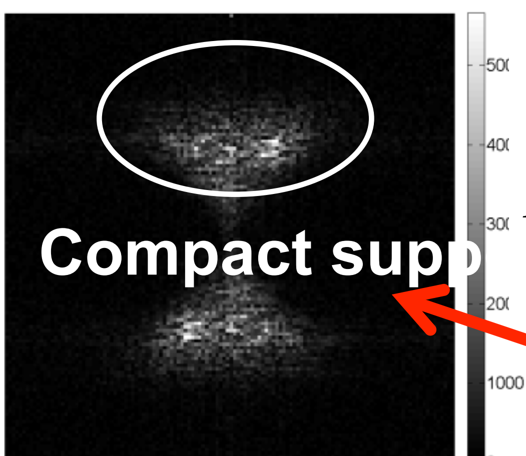
Bayesian approach : model completed by prior assumption on data statistics

$$p(v|y) \propto p(y|v)\,p(v)$$

- **Prior knowledge on v , the 2D Fourier transform of the RF image**
  - **sparse**
  - **Gaussian statistics of the RF images ➔ Gaussian statistics of the non-zero coefficients of v**

v

  - **++ compact support of the non-zero coefficients**



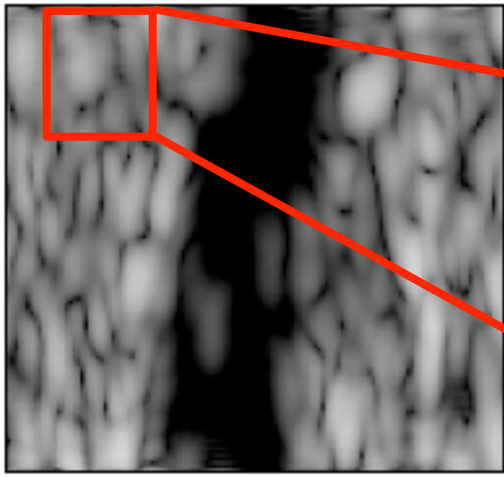
$$p\left(v_i \middle| \sigma_v^2, w\right) = (1-w)\delta\left(\left|v_i\right|\right) + \left(\frac{w}{\pi\sigma_v^2}\right)\exp\left(-\frac{\left|v_i\right|^2}{\sigma_v^2}\right)$$

Many zeros

Gaussian statistics

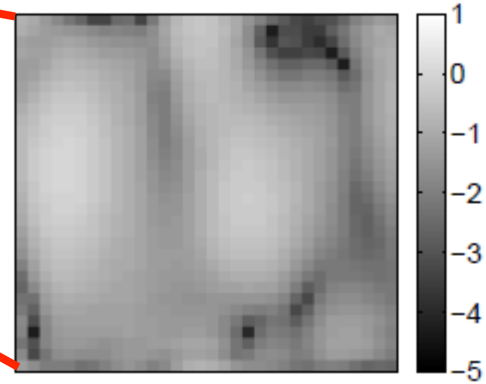
CS in US: sparse beamformed signal

Small block extracted from an RF image

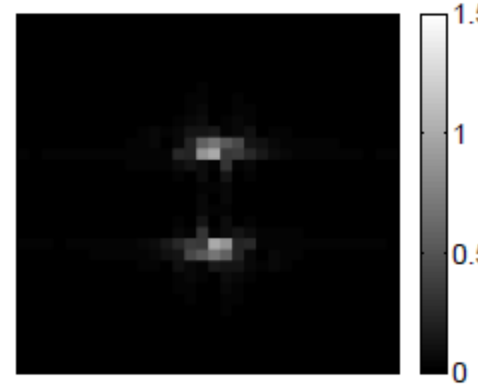


Original data

Spatial domain

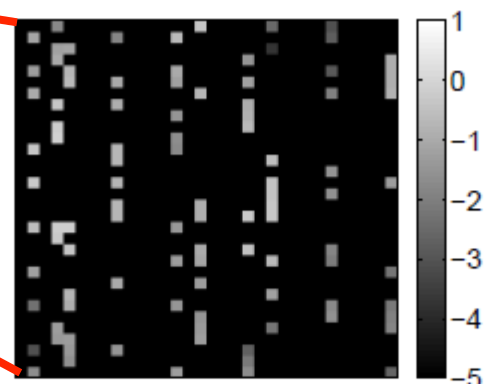


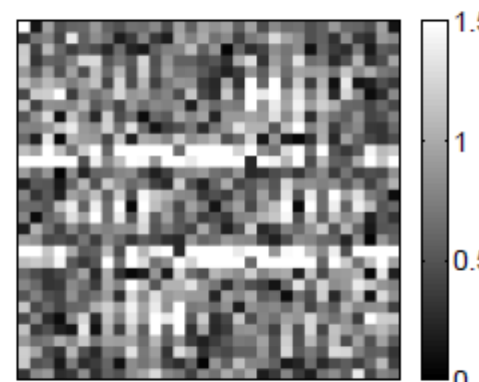
Fourier domain



Compressed acquisition

Measurement / Observation





CS in US: sparse beamformed signal

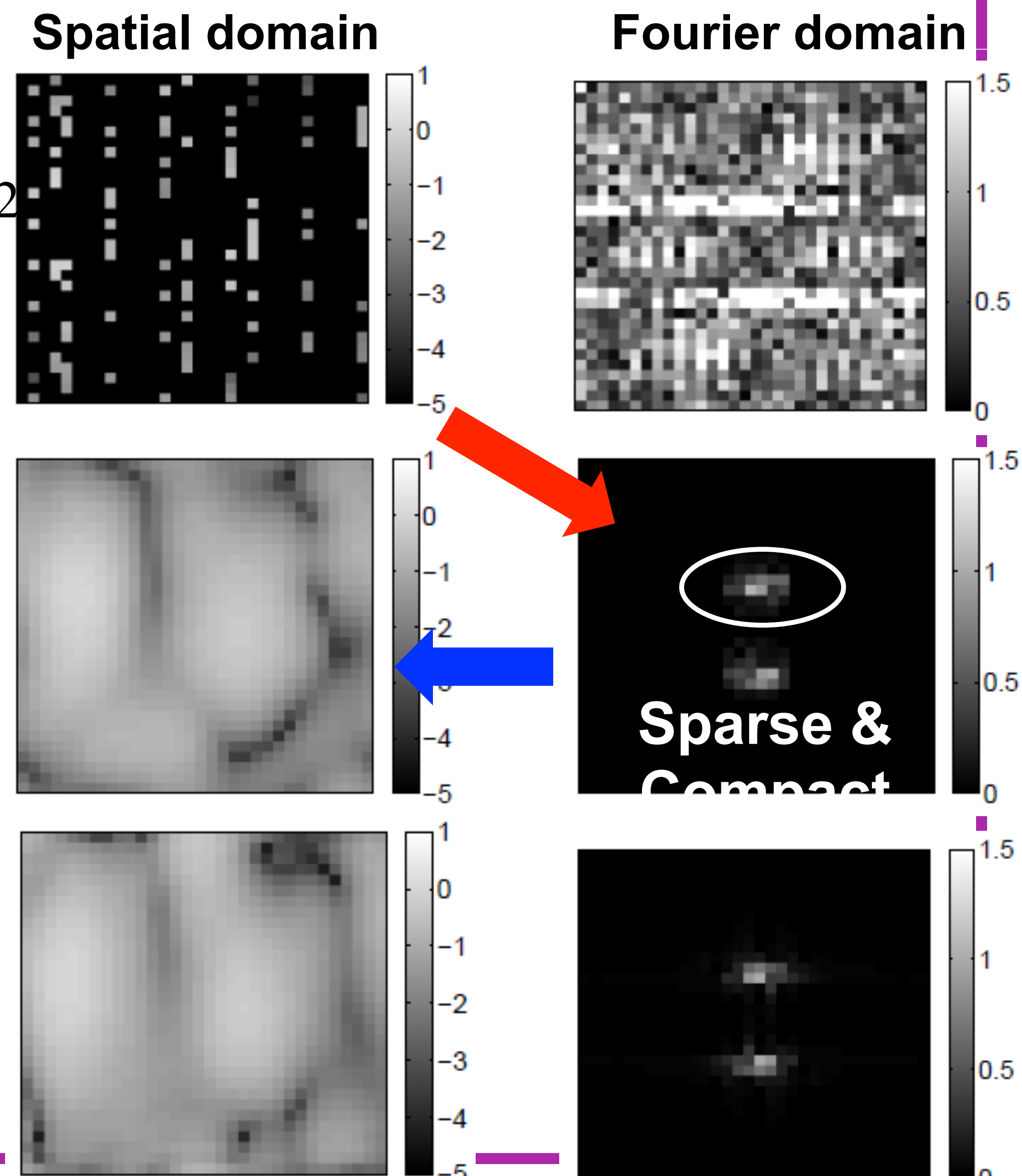
1) Compressed acquistion Φ_2

Corresponding 2D Fourier transform

2) L_1 minimisation in the Bayesian framework → v

3) Use of transform ψ to recover the x

Comparison with initial complete data



CS in US: sparse beamformed signal

1) Compressed acquistion Φ_2

Corresponding 2D Fourier transform

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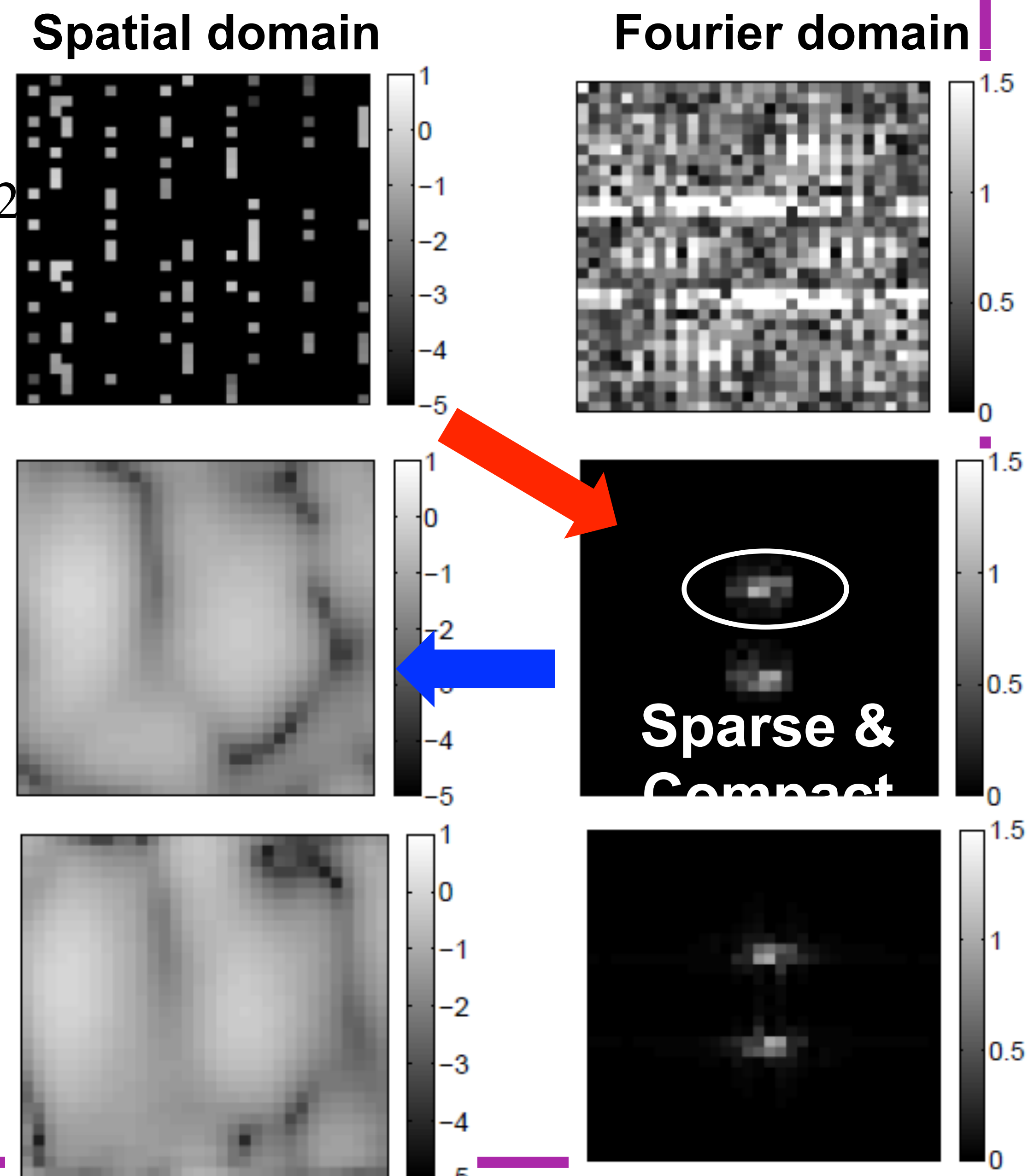
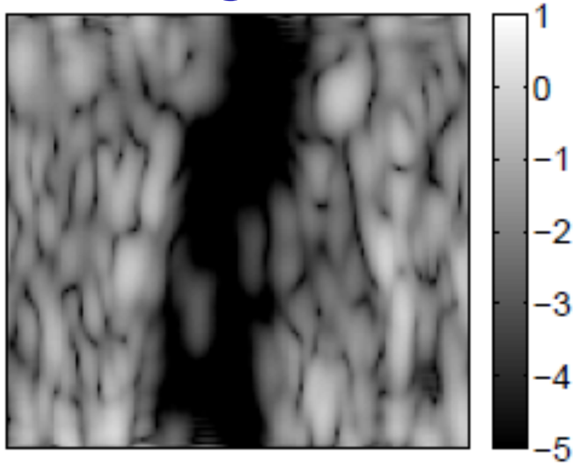
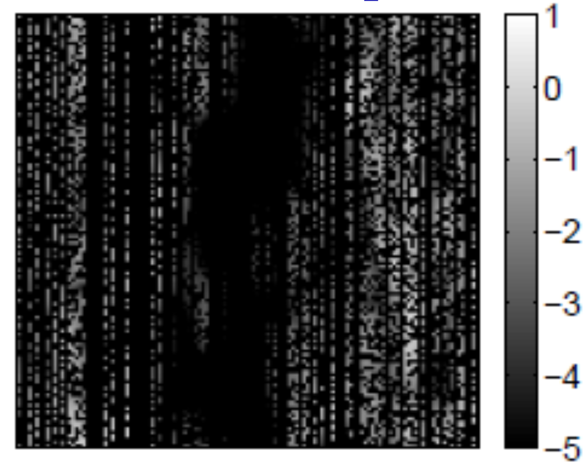


Comparison with convex relaxation

Original data

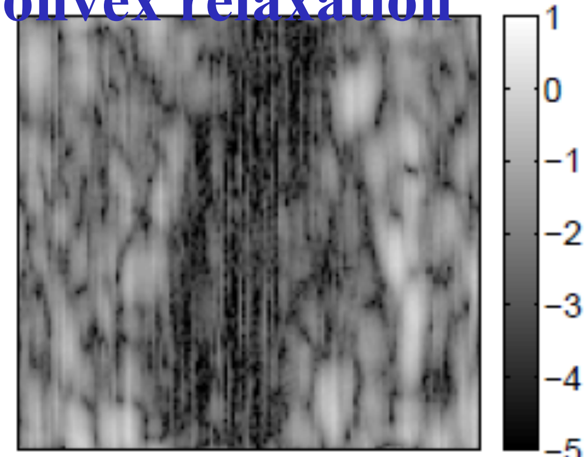


Measured samples

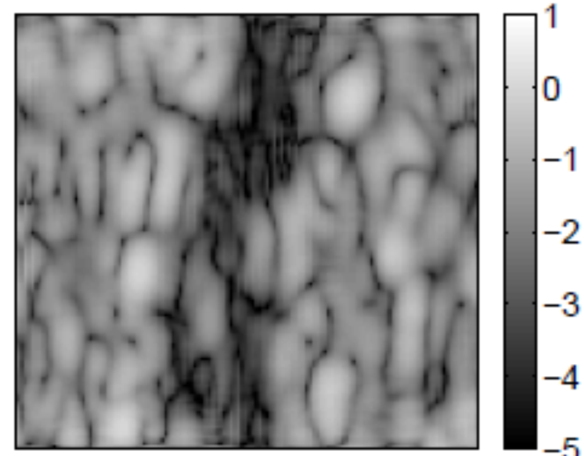


Basis pursuit

Convex relaxation



(Bayesian framework)



➔ Better reconstruction thanks to richer *prior* information

Discussions

Potential interest:

speed up acquisition while keeping spatial resolution

reduce needed data (remove elements ??)

Feasability at different stages of US data formation has been proven

Compressed Ultrasound imaging needs

- *A good sparsifying basis*
- *An incoherent acquisition strategy*
- *Dedicated acquisition material*
- *Efficient reconstruction algorithms*

Everything has to be done !!

❑Ultrasound Doppler Flow estimation

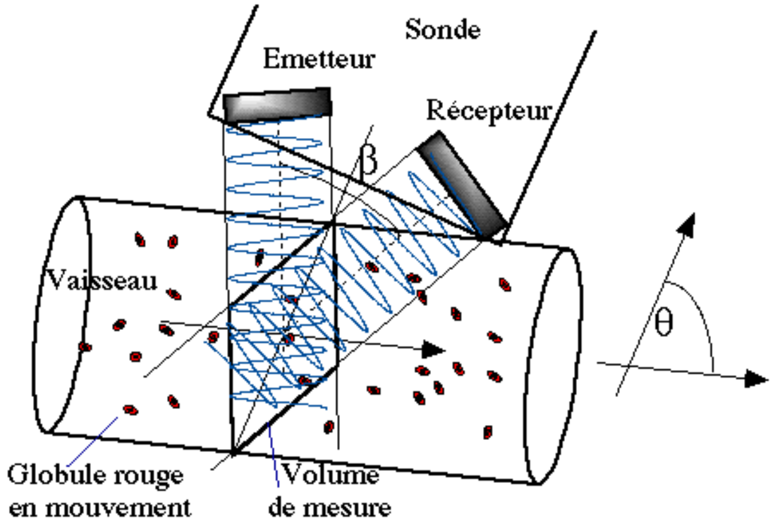
In Us imaging there are:

- echography imaging
-and Dopper imaging (for mainly flow and movement)

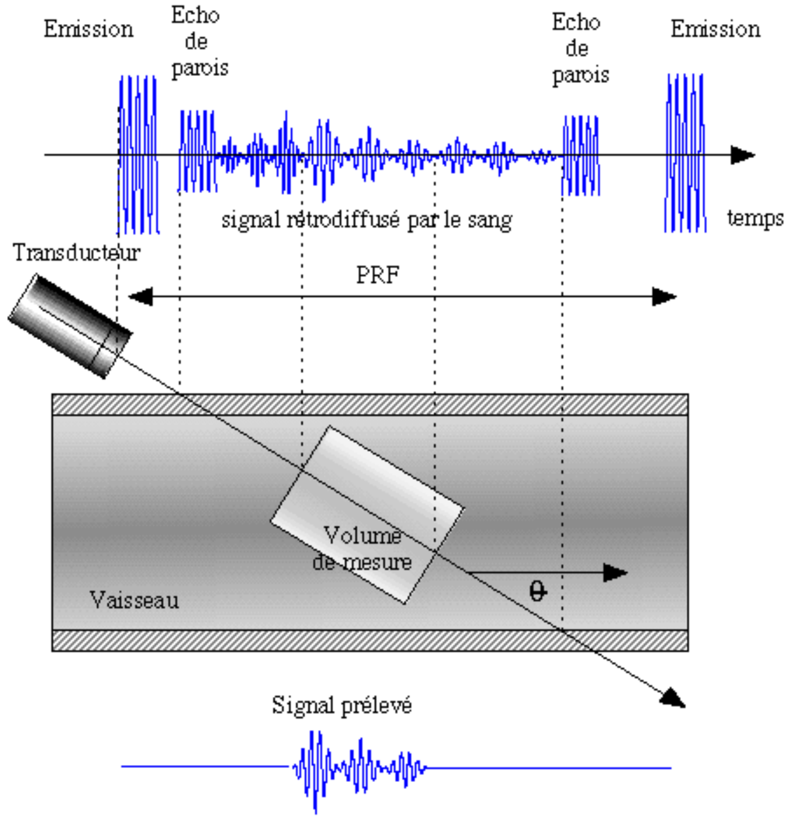
Introduction to Doppler signal

Doppler System

continuous wave Doppler



Pulse wave Doppler

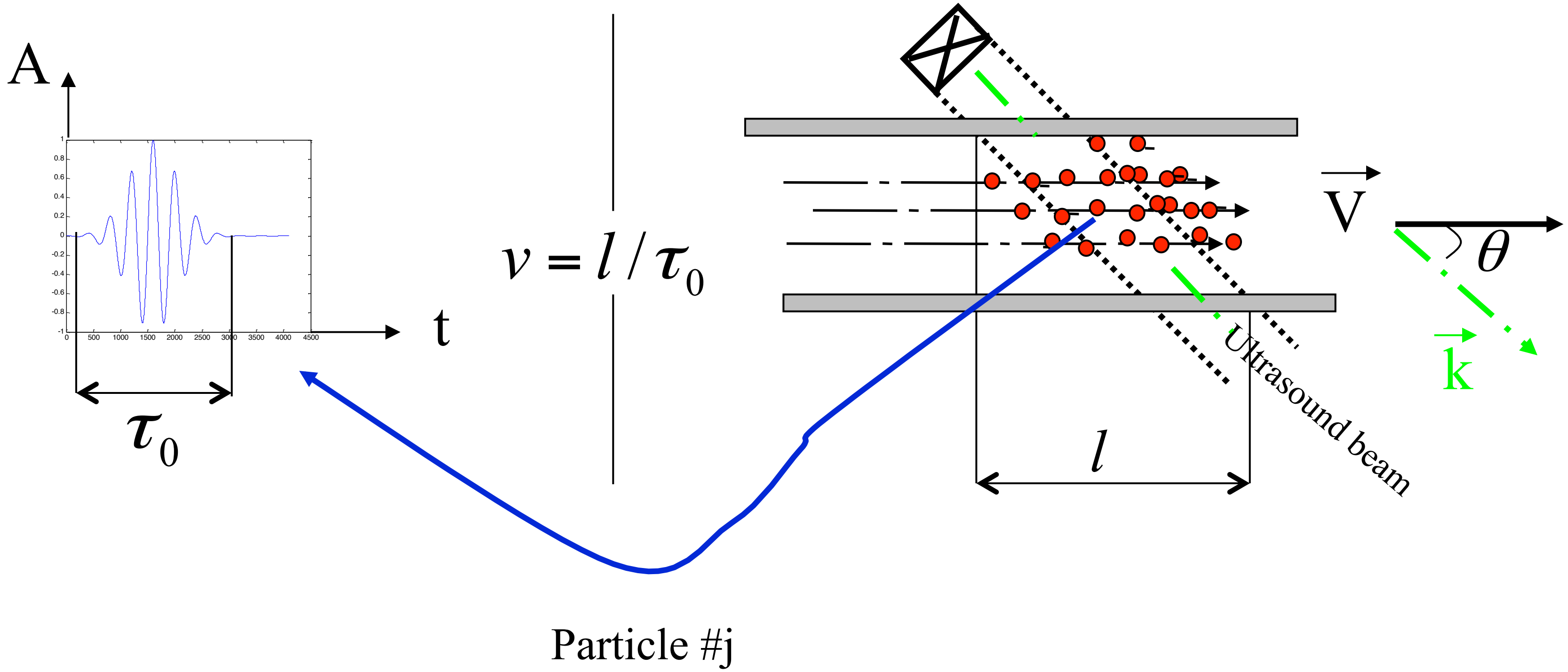


Doppler signal

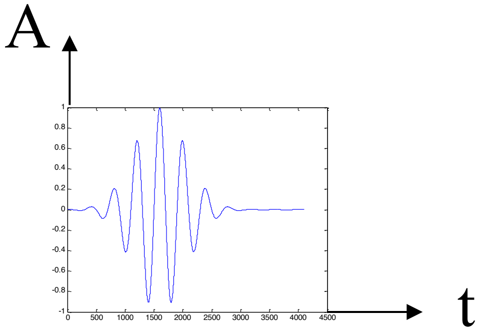
Introduction to Doppler signal

Hypothesis on the nature of the flow

- Stationnairy flow
- large number of targets (gaussian statistics)
- All the targets (particles) move with the same velocity



Introduction to Doppler signal



Signal backscaterred by a particle $\# j$

$$S_j(t) = C_j D_j \cos[(\omega_0 + \omega_d)t + \phi_j]$$

**angular Doppler frequency :*

$$\omega_d = \vec{k}\vec{V} = \frac{\omega_0 V \cos(\theta)}{c}$$

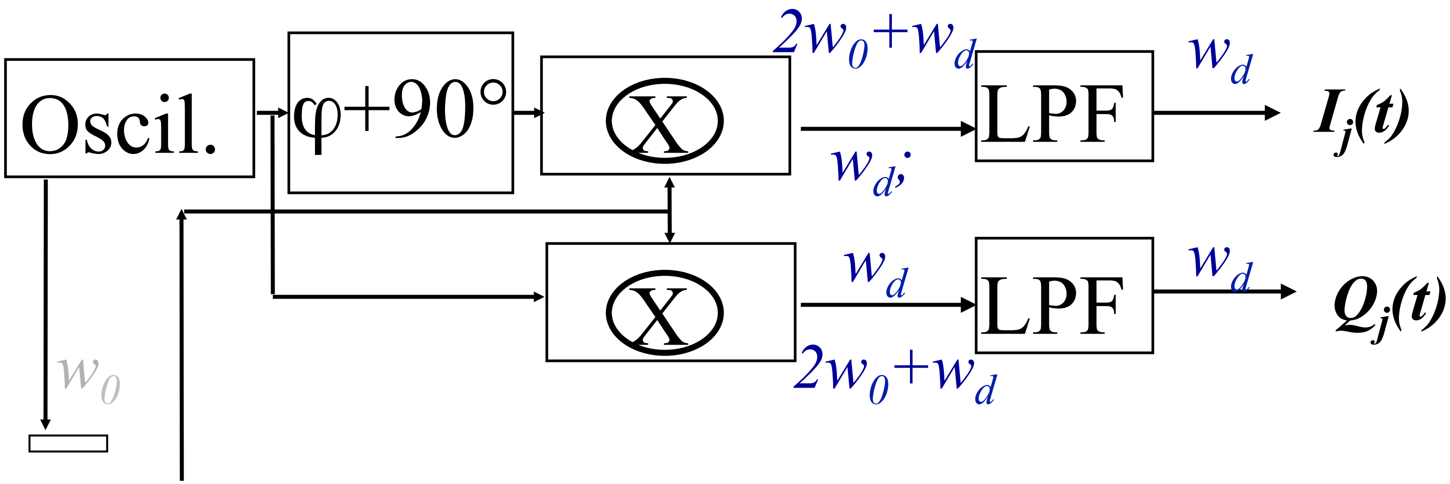
**random Phase due to: random location of the particles*

** Coefficient linked to properties of de reflectiviy of particles*

** Transmitted US intensity. Non uniform in the measurement volume (Transducer Directivity diagram)*

Introduction to Doppler signal

After demodulation and low pass filtering ...



$$S_j(t) = C_j D_j \cos[(\omega_0 + \omega_d)t + \phi_j]$$

… and summation on all the particles

$$I(t) = \sum_j C_j D_j \cos[\omega_d t + \phi_j] = \sum_j A_j(\cos[\omega_d t] + B_j \sin[\omega_d t])C_j$$

$$Q(t) = \sum_j C_j D_j \sin[\omega_d t + \phi_j]$$

Introduction to Doppler signal

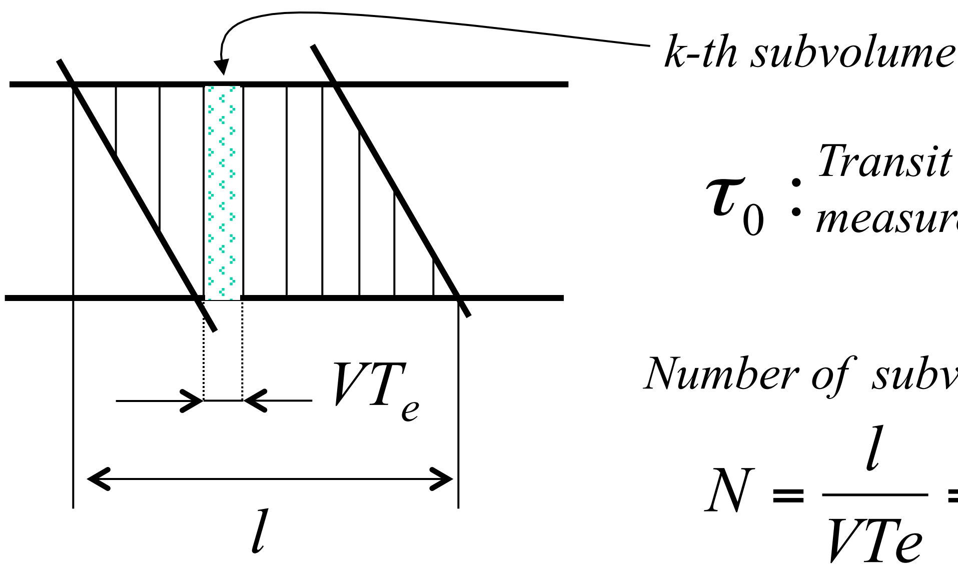
Instantaneous Doppler angular frequency

$$\omega = \frac{d}{dt}\Phi(t) = \frac{d}{dt}(\omega_d t + \phi) = \omega_d + \frac{d}{dt}\phi(t)$$

particles movement

Doppler Ambiguity (transit time)

spatial et time sampling



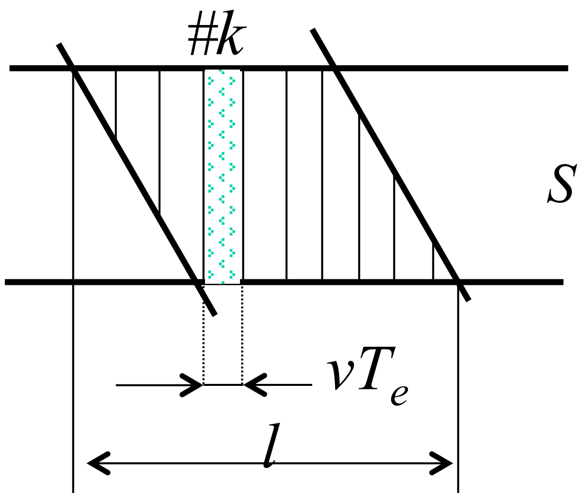
τ_0 : *Transit time of the particles in the measurement volume*

Number of subvolumes in the measurement volume

$$N = \frac{l}{VTe} = \frac{lFe}{V}$$

Introduction to Doppler signal

spatial et time sampling



$$S_i(t) = S(t_i) = \sum_k S^{(k)}(t_i)$$

$$S^{(k)}(t_i) = \sum_j [A_{ij}^{(k)} \cos(\omega_d t_i) + B_{ij}^{(k)} \sin(\omega_d t_i)] C_j^{(k)}$$

If $C_j^{(k)}$ is quite constant in each subvolume (k) :

$$C_j^{(k)} = C^{(k)}$$

Then $S^{(k)}(t_i) = [\alpha_i^{(k)} \cos(\omega_d t_i) + \beta_i^{(k)} \sin(\omega_d t_i)] C^{(k)}$

So $S_i = \sum_k S^{(k)}(t_i) = \sum_k [\alpha_i^{(k)} \cos(\omega_d t_i) + \beta_i^{(k)} \sin(\omega_d t_i)] C^{(k)}$ with $\alpha_i^{(k)} = \sum_j A_{ij}^{(k)}$ et $\beta_i^{(k)} = \sum_j B_{ij}^{(k)}$

$S_i = \alpha_i \cos(\omega_d t_i) + \beta_i \sin(\omega_d t_i)$ avec $\alpha_i = \sum_k \alpha_i^{(k)} C^{(k)}$ et $\beta_i = \sum_k \beta_i^{(k)} C^{(k)}$

α_i et β_i are zero mean random variabes with variance σ^2

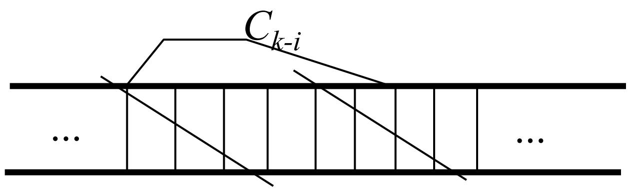
Introduction to Doppler signal

Principe of simulation of complex ultrasound Doppler signal

1. Generate deux vectors $g1_k$ et $g2_k$ of zeros mean random numbers with variance α^2 (exple 1024+N number)

2. Apply a ponderation C_{k-i} to the two vectors

$$\alpha_i = \sum_{k=i}^{i+N} g1_i C_{k-i} \quad et \quad \beta_i = \sum_{k=i}^{i+N} g2_i C_{k-i}$$



3. Use α_i and β_i to generate I_i the real part of the Doppler signal (do the same for Q_i the imaginary part)

$$I_i = \alpha_i \cos(\omega_d t_i) + \beta_i \sin(\omega_d t_i) = K_i \cos(\omega_d t_i + \phi_i)$$ avec $K_i = \sqrt{\alpha_i^2 + \beta_i^2}$ et $\phi_i = -arctg(\frac{\beta_i}{\alpha_i})$

4. Add noise (white gaussian noise, with zero mean and variance α_{bruit}^2 from a signal to noise ratio RSB

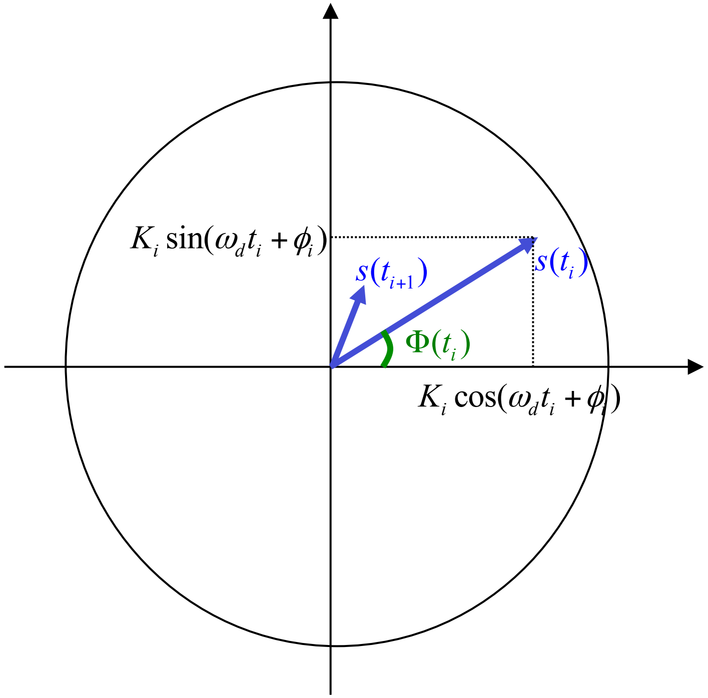
$$RSB_{dB} = 10\log(\frac{\alpha^2}{\alpha_{bruit}^2})$$

Introduction to Doppler signal

Analytic Doppler signal and its geometric interpretation

$$I_i = I(t_i) = K_i \cos(\omega_d t_i + \phi_i) \quad et \quad Q_i = Q(t_i) = K_i \sin(\omega_d t_i + \phi_i)$$

$$\boxed{s(t_i) = K_i \exp\left[j(\omega_d t_i + \phi_i) \right] \quad = \lambda_i \exp\left[j(\omega_d t_i) \right]}$$



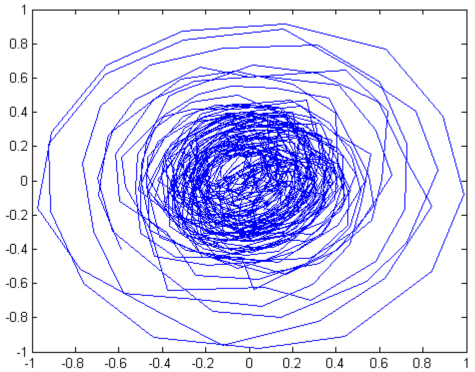
$$\Phi(t_i) = \omega_d t_i + \phi_i$$

Introduction to Doppler signal

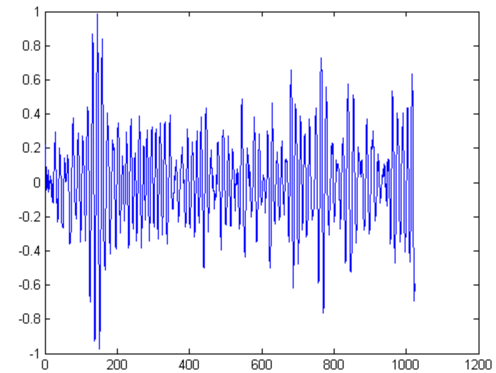
Analytic Doppler signal and its geometric interpretation

Case : constant velocity

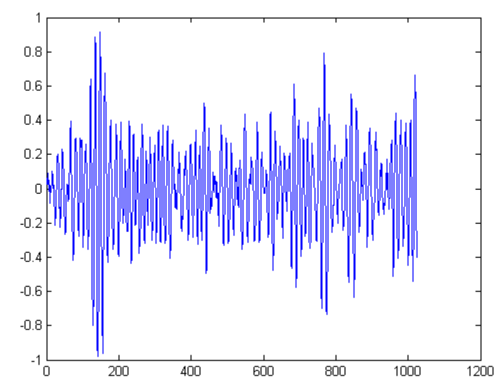
complex signal :$s(t_i)$



Real part: $I(t_i)$

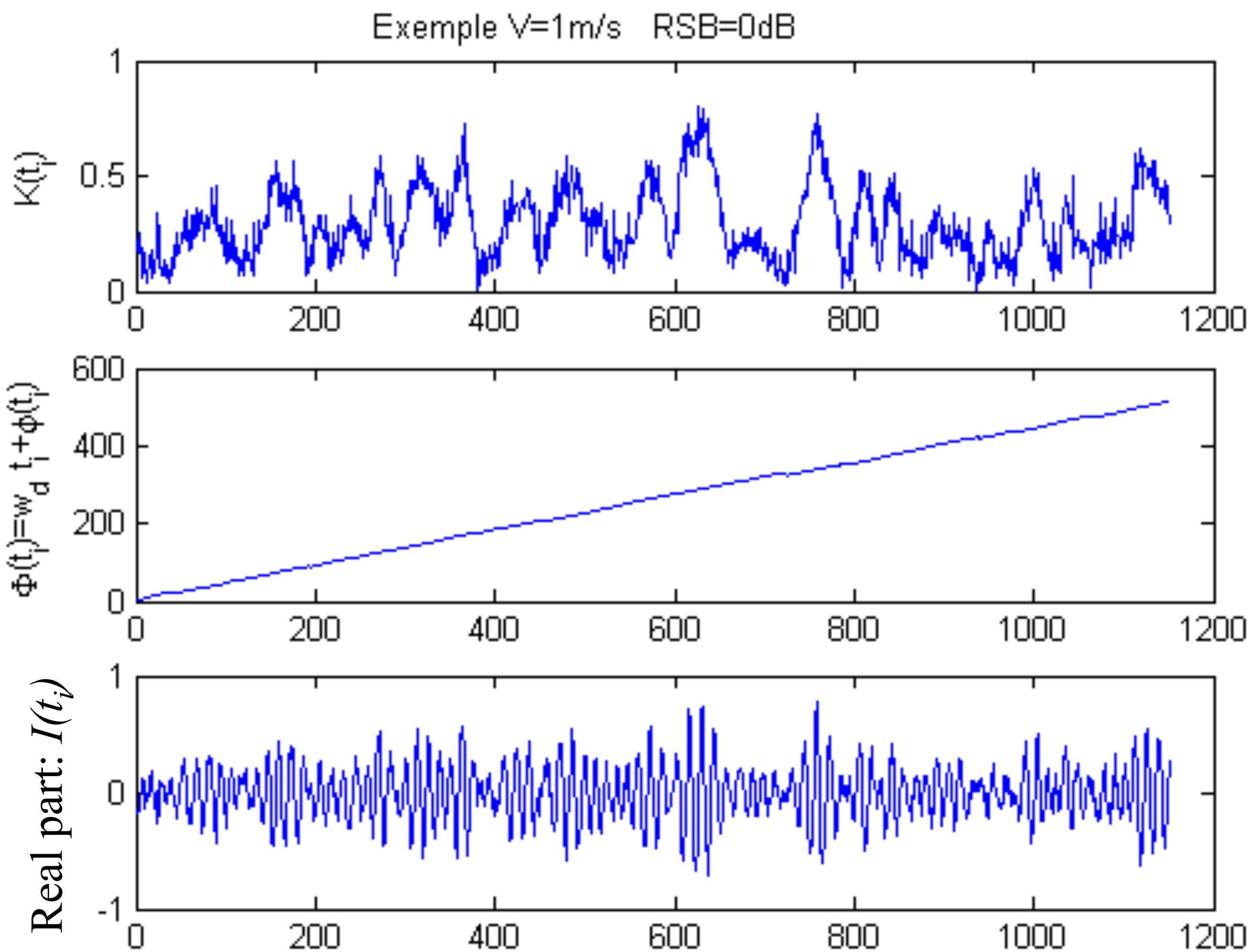


Imaginary part : $Q(t_i)$



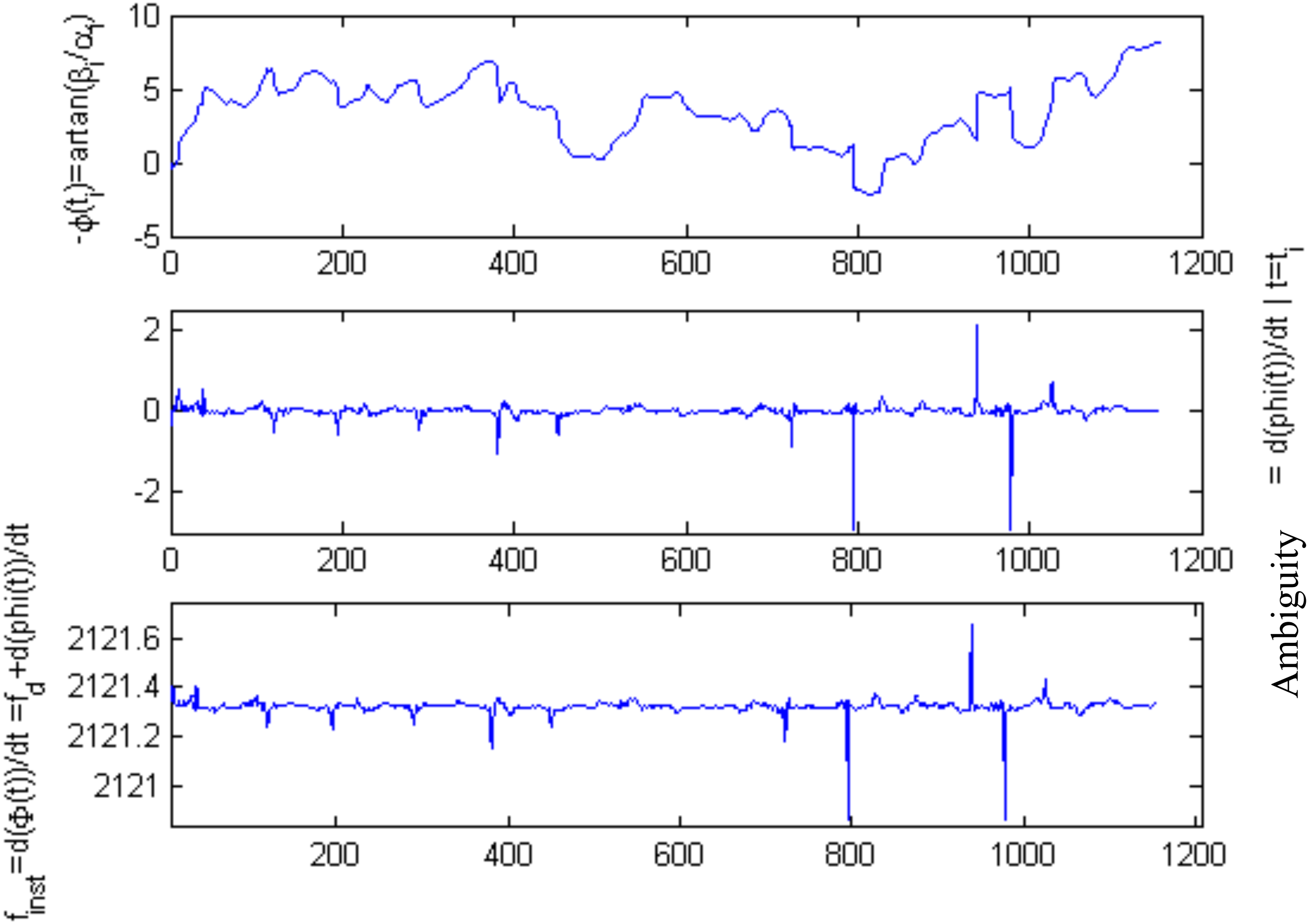
Introduction to Doppler signal

Illustration



Introduction to Doppler signal

Illustration



Introduction to Doppler signal

Enhancement of Doppler Spectrum ☞ main causes:

Transit time effect

$$s(t_i) = K_i \exp\left[j(\omega_d t_i + \phi_i)\right] \quad = \lambda(t_i)\exp\left[j(\omega_d t_i)\right]$$

$$avec \quad \lambda_i = \alpha_i + j\beta_i = \sqrt{\alpha_i^2 + \beta_i^2}\exp(j\phi_i)$$

Doppler signal autocorrelation $R_{ss}(\tau)$

$$R_{ss}(\tau) = E\{s^*(t)s(t+\tau)\} = \frac{1}{N-\tau+1}\sum_{n=0}^{N-\tau+1} s^*(n)s(n+\tau)$$

$$= E\{\lambda^*(t)\lambda(t+\tau)\exp(-j\omega_d t)\exp(j\omega_d(t+\tau)\}$$

$$R_{ss}(\tau) = R_{\lambda\lambda}(\tau)\exp(j\omega_d\tau)$$

Introduction to Doppler signal

Transit time effect

Power spectrum density(PSD) $P_{ss}(f) = TF\{R_{ss}(\tau)\} = P_{\lambda\lambda}(f) * \delta(f - f_d) = P_{\lambda\lambda}(f - f_d)$

Doppler PSD = PSD $P_{\lambda\lambda}(f)$ of its envelop λ, shifted by f_d

Width of $P_{\lambda\lambda}(f)$ is determined by α_i *et* β_i

☞**if V is low**

At constant sampling frequency

+ V ↘ , + N (number of de particles) ↗ , + τ_0 ↗ , so + the correlation window C_{k-i} ↗

$\Rightarrow R_{ss}(\tau)$ is large ⇒ It Fourier transform is narrow band

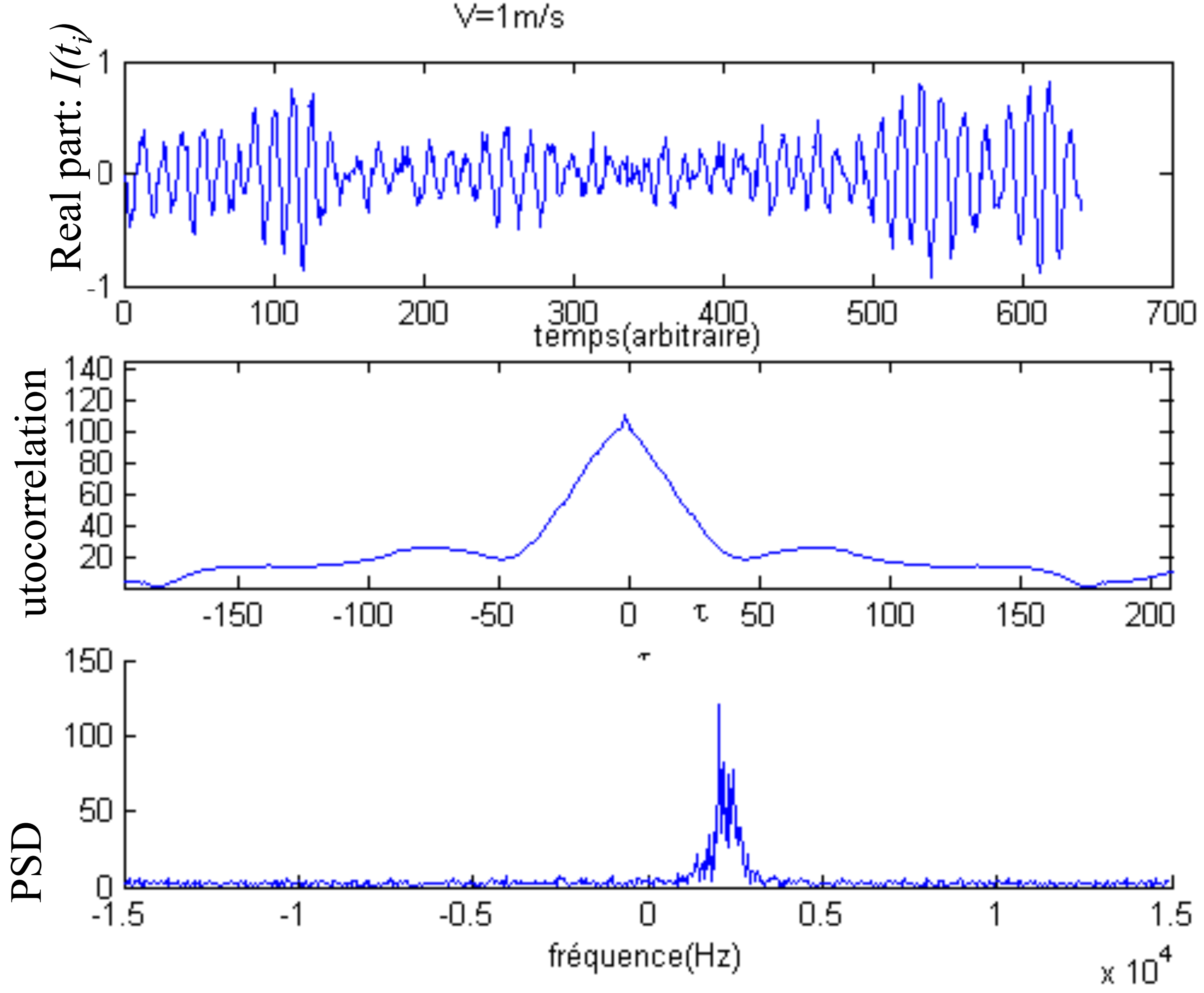
☞**if V is large**

N is small ; τ_0 is small, + the correlation window is small

$\Rightarrow R_{ss}(\tau)$ is narrow⇒ It Fourier transform is large bande

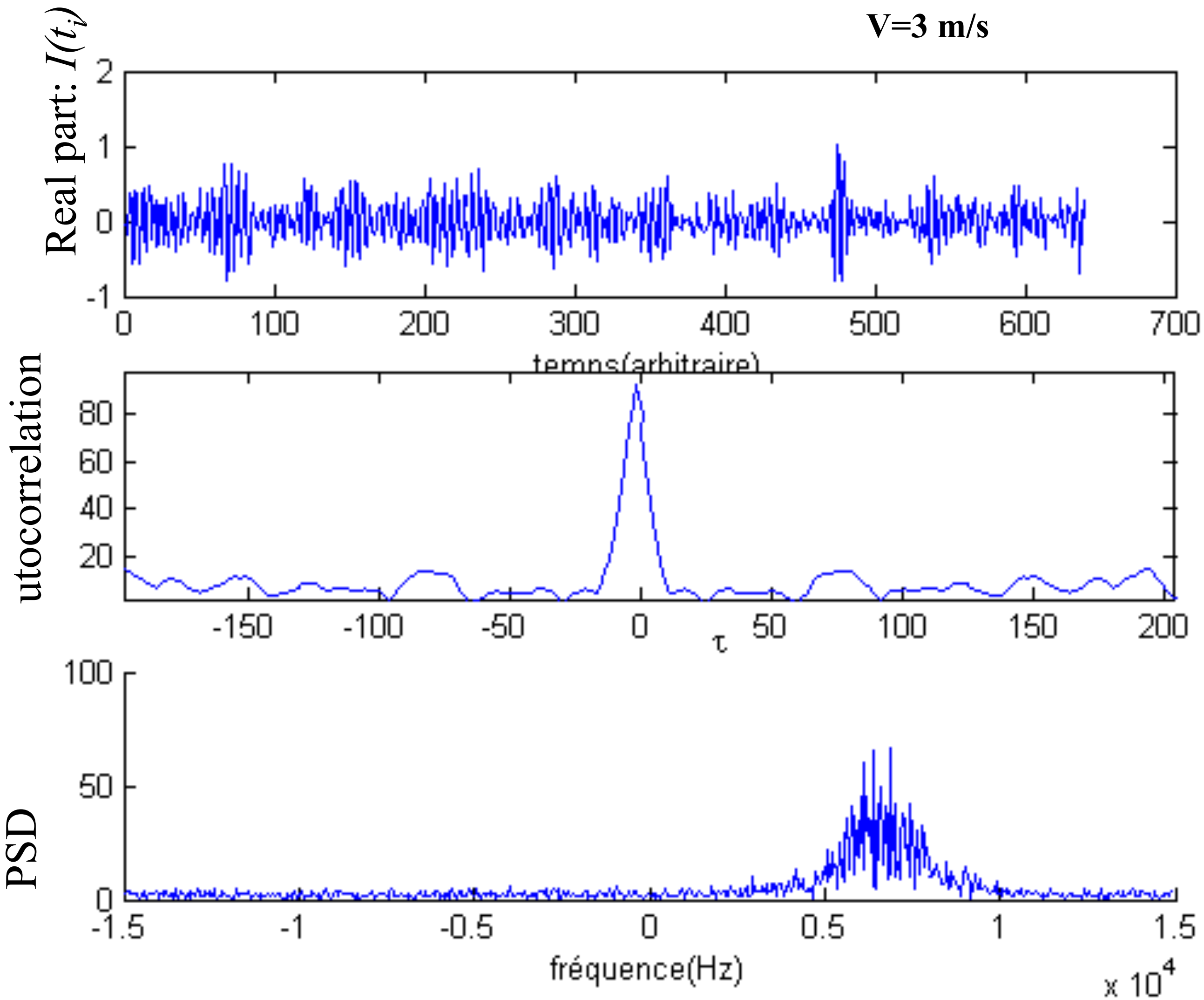
Introduction to Doppler signal

Transit time effect



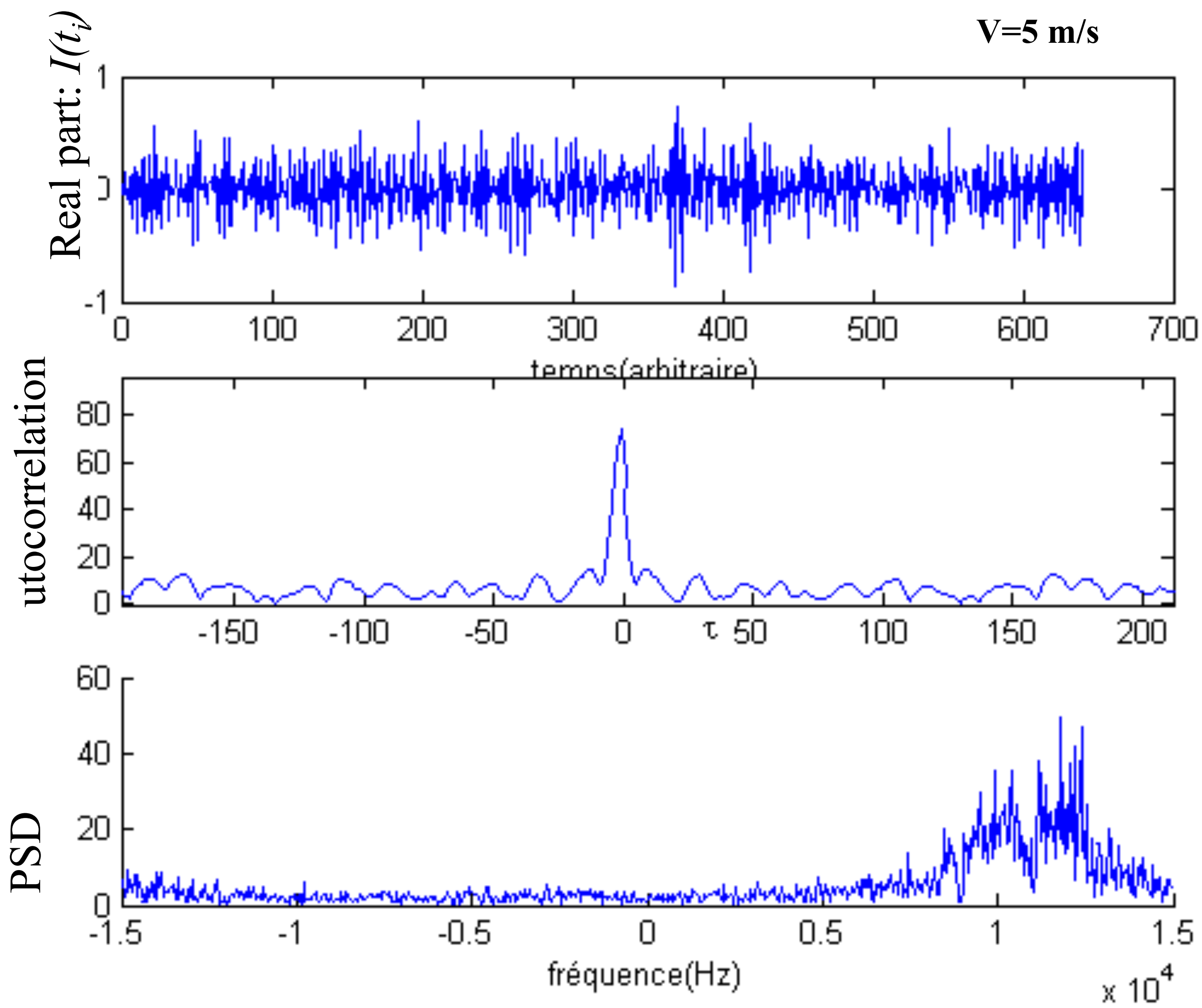
Introduction to Doppler signal

Transit time effect



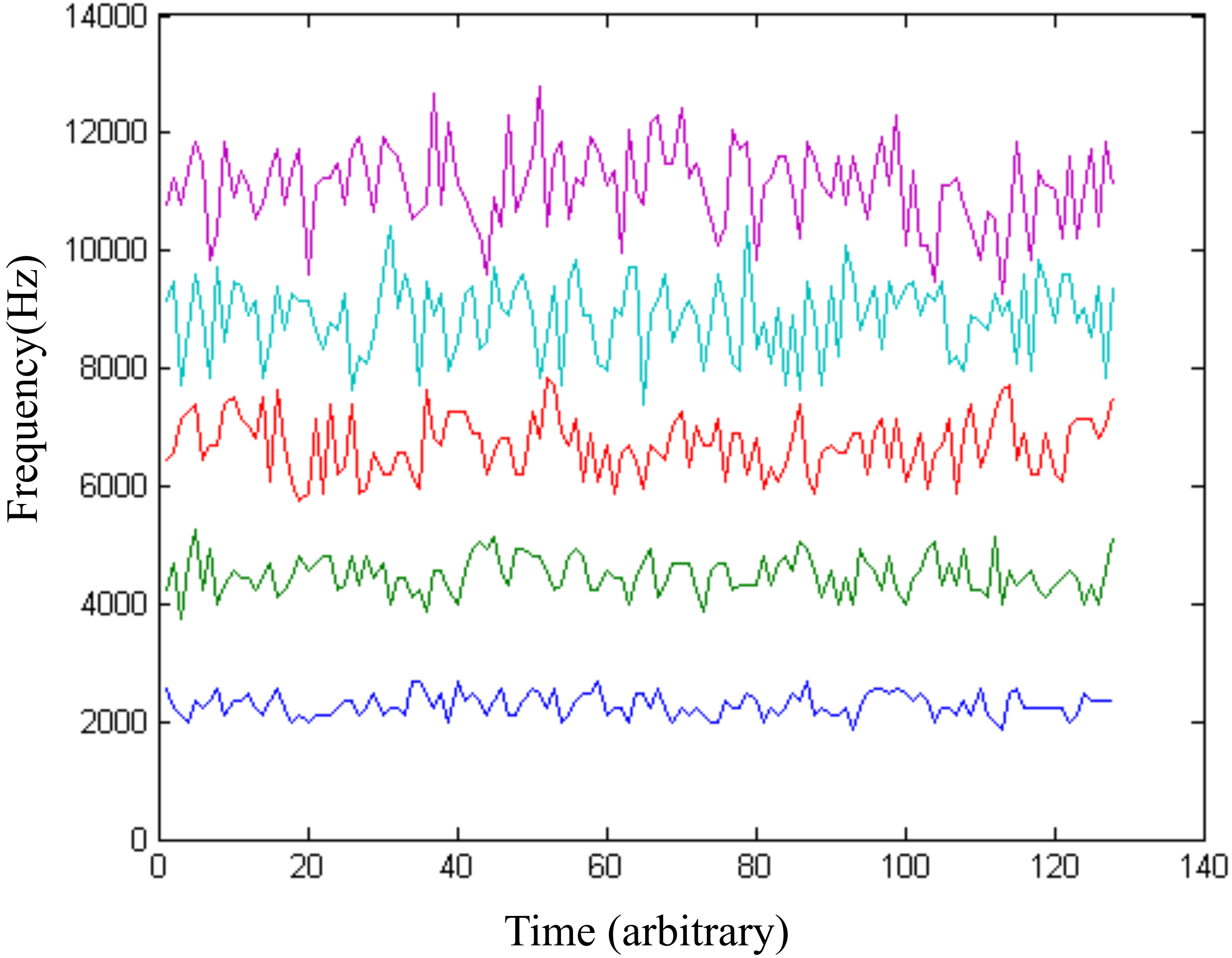
Introduction to Doppler signal

Transit time effect



Introduction to Doppler signal

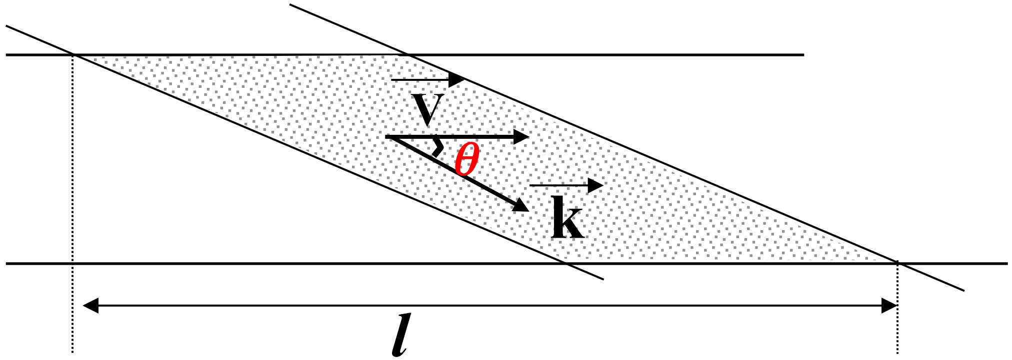
Transit time effect



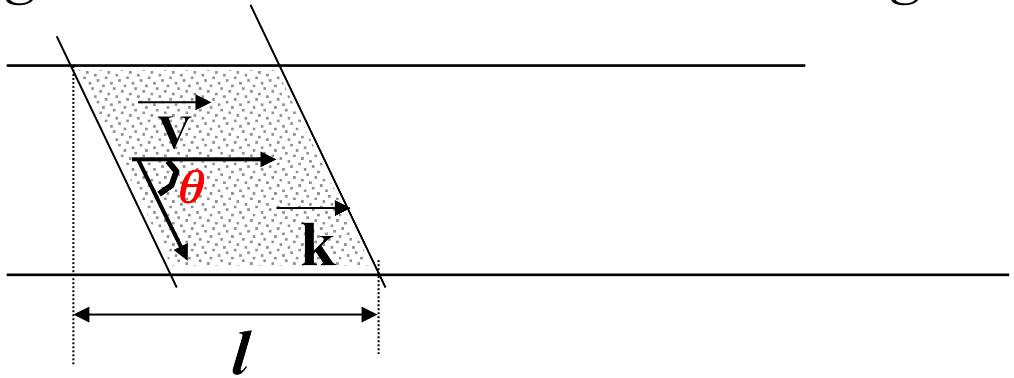
Introduction to Doppler signal

Effect of the Doppler angle θ on the transit time

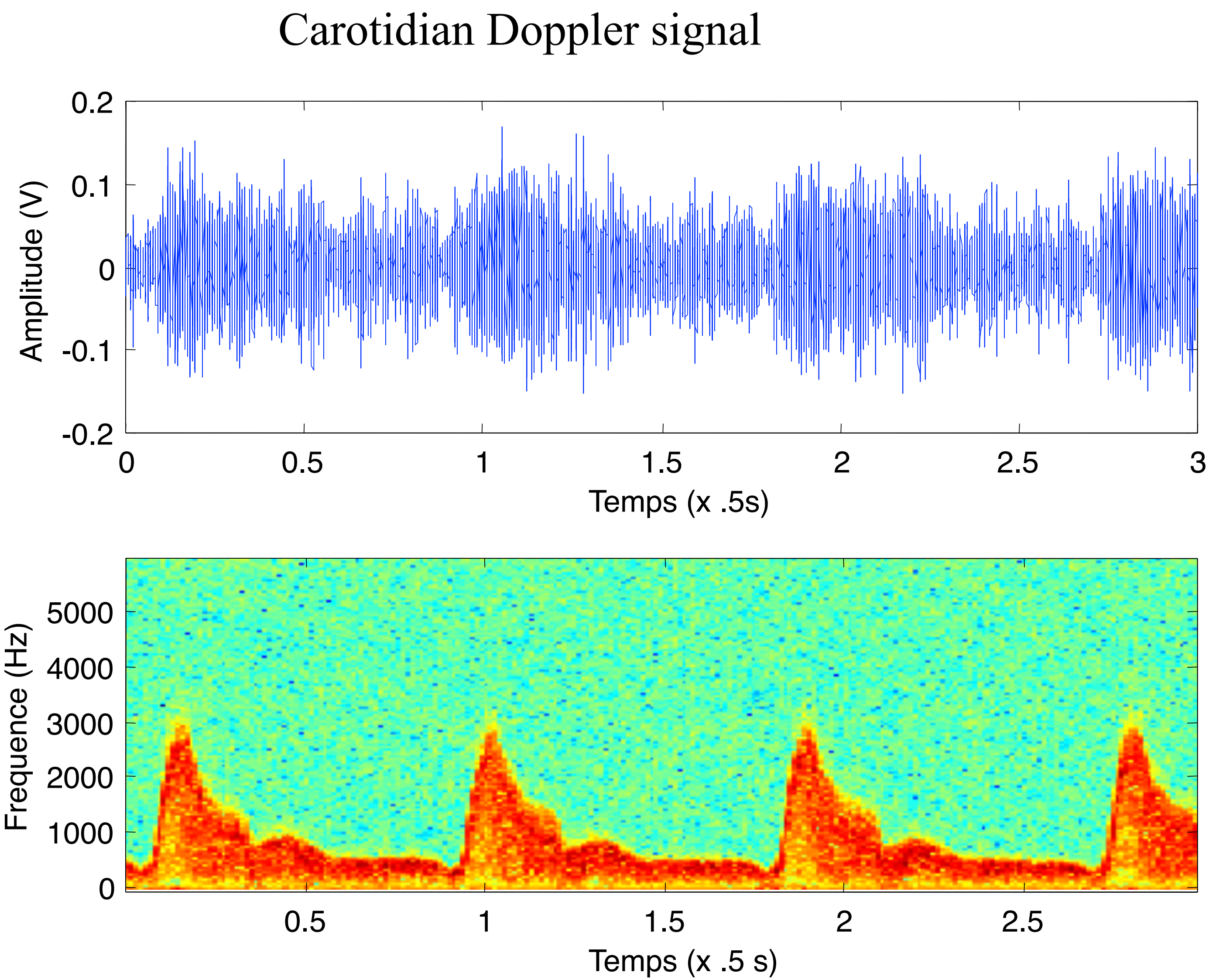
θ small → *large transit time and, narrow band spectrum*



θ large → *small transit time and large band spectrum*



Exemple of carotid Doppler signal



- Once the Doppler signal is obtained,
- The next step is to estimate the Doppler frequency
- Some examples of estimation

Time based estimation methods for Doppler frequency estimation

Time based methods

Recall

$$Let \quad x(t) = I(t) + jQ(t)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\exp(-j\omega t)dt \quad et \quad P(\omega) = X^*(\omega)X(\omega) = \left|X(\omega)\right|^2$$

Normalized PSD

$$p(\omega) = \frac{P(\omega)}{\int_{-\infty}^{\infty} P(\omega)d\omega}$$

Mean Frequency (first order moment or, centroïd) et variance of Normalized PSD of the signal Doppler signal :

$$\overline{\omega} = E\{\omega\} = \int_{-\infty}^{\infty} \omega p(\omega)d\omega \quad et \quad \mathrm{var}\{\omega\} = E\{\left|\omega - E\{\omega\}\right|^2\} = \int_{-\infty}^{\infty} \omega^2 p(\omega)d\omega - \overline{\omega}^2$$

Time based methods

$$\bar{\omega} = \int_{-\infty}^{\infty} \omega p(\omega) d\omega = \frac{\int_{-\infty}^{\infty} \omega |X(\omega)|^2 \, d\omega}{\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega} = \frac{\int_{-\infty}^{\infty} \omega_{inst} |x(t)|^2 \, dt}{\int_{-\infty}^{\infty} |x(t)|^2 \, dt}$$

$$\omega_{inst} = \frac{d\Phi(t)}{dt} \quad avec \quad \Phi(t) = artg\left(\frac{Q(t)}{I(t)}\right)$$

$$\frac{d\Phi(t)}{dt} = \frac{\Delta\Phi_i}{T_e} \quad avec \ \Delta\Phi_i = \Phi_i - \Phi_{i-1}$$

In its T_e-sampled version

$$\Delta\Phi_i = \Phi_i - \Phi_{i-1} = artg\left(\frac{Q_i}{I_i}\right) - artg\left(\frac{Q_{i-1}}{I_{i-1}}\right) = artg\left(\frac{Q_i I_{i-1} - Q_{i-1} I_i}{I_i I_{i-1} + Q_i Q_{i-1}}\right)$$

$$artg(a) - artg(b) = artg\left(\frac{a - b_i}{1 + ab}\right)$$

Time based methods

Thus

$$\bar{\omega} = F_e \frac{\sum_{i=1}^{M} \left(I_i^2 + Q_i^2\right) artg\left(\frac{Q_i I_{i-1} - Q_{i-1} I_i}{I_i I_{i-1} + Q_i Q_{i-1}}\right)}{\sum_{i=1}^{M} \left(I_i^2 + Q_i^2\right)} \; avec \;\; F_e = 1/T_e$$

If M the number of the samples of the signal is low(8 to 16 samples), the amplitude of the Doppler signal is almost constant

$$\bar{\omega} \approx F_e Arctg\left(\frac{Q_i I_{i-1} - Q_{i-1} I_i}{I_i I_{i-1} + Q_i Q_{i-1}}\right)$$

Time based methods

Correlation angle (Aloka)

$$R[\tau] = \int_{-\infty}^{\infty} P(\omega)\exp(j\omega\tau)d\omega \quad so \quad R[0] = \int_{-\infty}^{\infty} P(\omega)d\omega$$

$$\frac{dR[\tau]}{d\tau} = \int_{-\infty}^{\infty} j\omega P(\omega)\exp(j\tau\omega)d\omega \Longrightarrow \frac{dR[0]}{d\tau} = \int_{-\infty}^{\infty} j\omega P(\omega)d\omega$$

Since $\bar{\omega} = \int_{-\infty}^{\infty} \omega p(\omega)d\omega$ and $p(\omega) = \frac{P(\omega)}{\int_{-\infty}^{\infty} P(\omega)d\omega}$

$$\Longrightarrow \quad \bar{\omega} = \frac{1}{j}\frac{\frac{d}{d\tau}R[0]}{R[0]}$$

Time based methods

Correlation angle (Aloka)

$$\overline{\omega}=\frac{1}{j}\frac{\frac{d}{d\tau}R[0]}{R[0]}$$

Denoting: $R[\tau]=A[\tau]\exp(j\varphi[\tau])$ *on a* $\frac{dR[\tau]}{d\tau}=(\frac{dA[\tau]}{d\tau}+jA[\tau]\frac{d\varphi[\tau]}{d\tau})\exp(j\varphi[\tau])$

A *is an even function and* ***φ*** *an odd function : So*

$$\frac{dR[0]}{d\tau}=(jA[0]\frac{d\varphi[0]}{d\tau}) \text{ et } R[0]=A[0] \Longrightarrow \overline{\omega}=\frac{d}{d\tau}\varphi[0] \text{ so } \boxed{\overline{\omega}=\frac{\varphi[T_e]-\varphi[0]}{T_e}=\frac{\varphi[T_e]}{T_e}}$$

Using discrete time notation

Thusi

$$\boxed{\bar{f}=\frac{F_e}{2\pi}Arg\{R[T_e]\}=\frac{F_e}{2\pi}Arctg\left\{\frac{\mathrm{Im}(R[T_e])}{\mathrm{Re}(R[T_e])}\right\}}$$

The Doppler frequency is the first lag of the autocorrelation of Doppler signal, multiply by $F_e/2\pi$

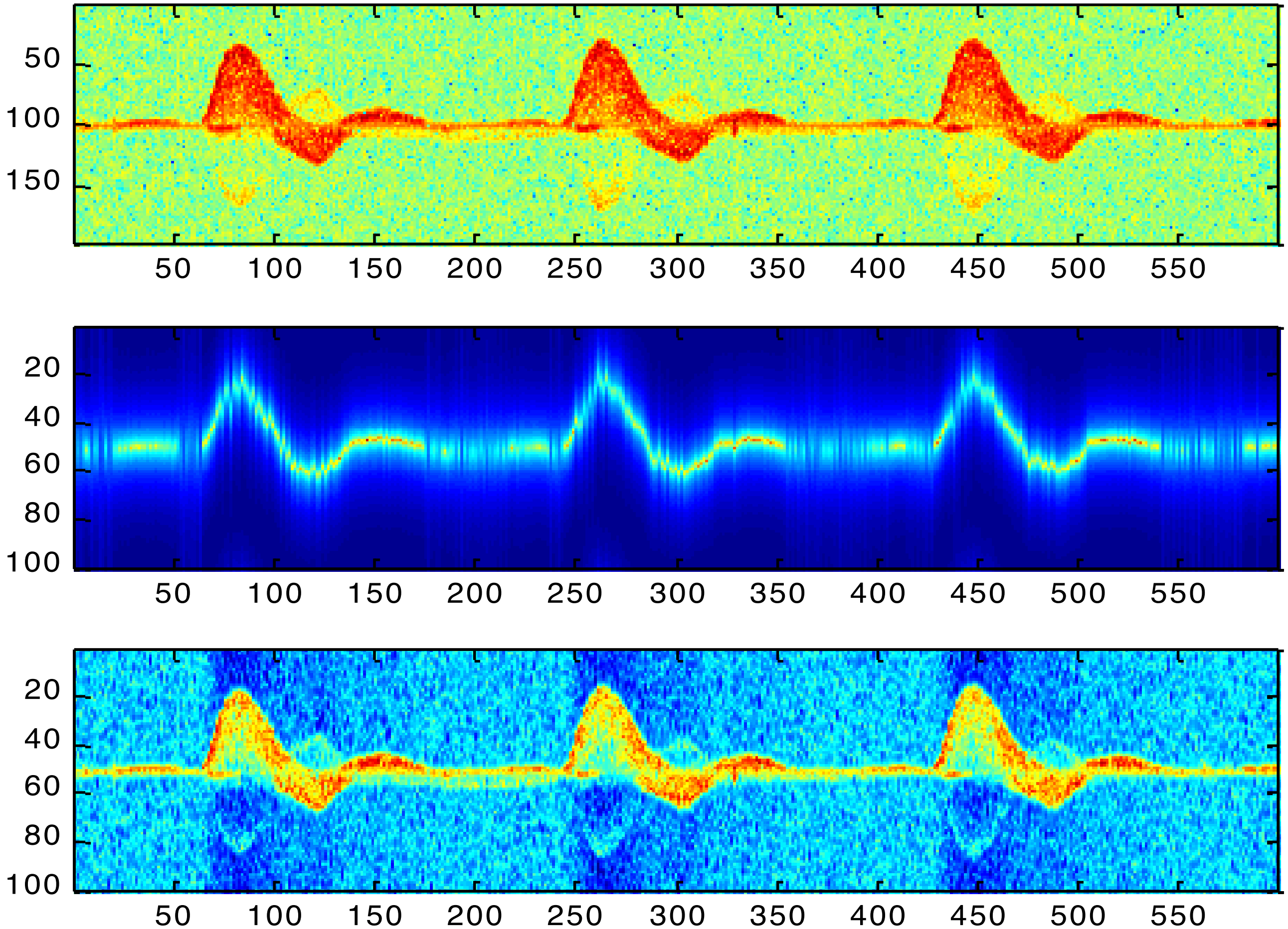
These methods are fast, but unaccurate.

So many spectral analysis have been investigates

Some basic examples

Example

femoral artery

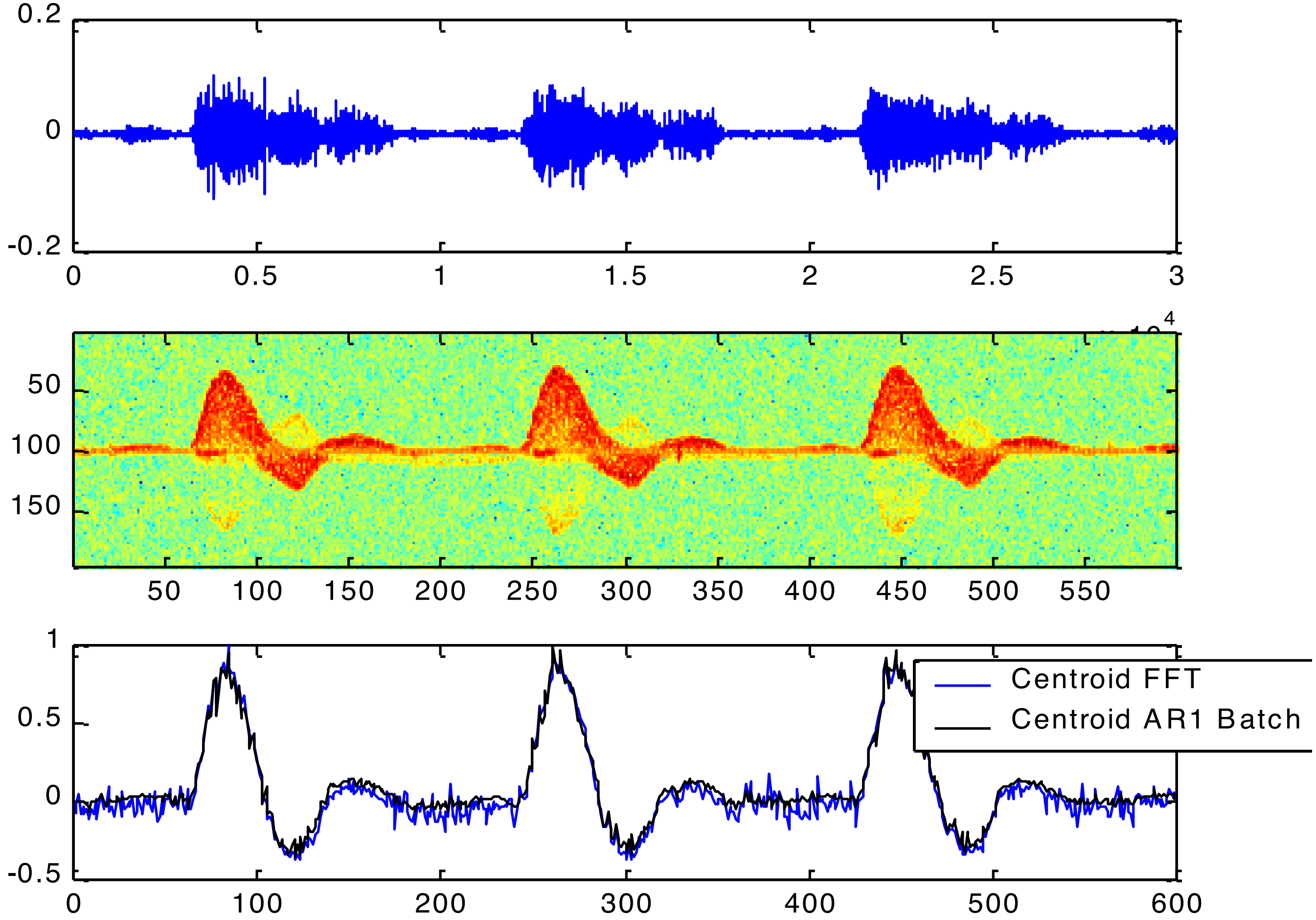


AR Spectrum (AR1)

AR Spectrum (AR30)

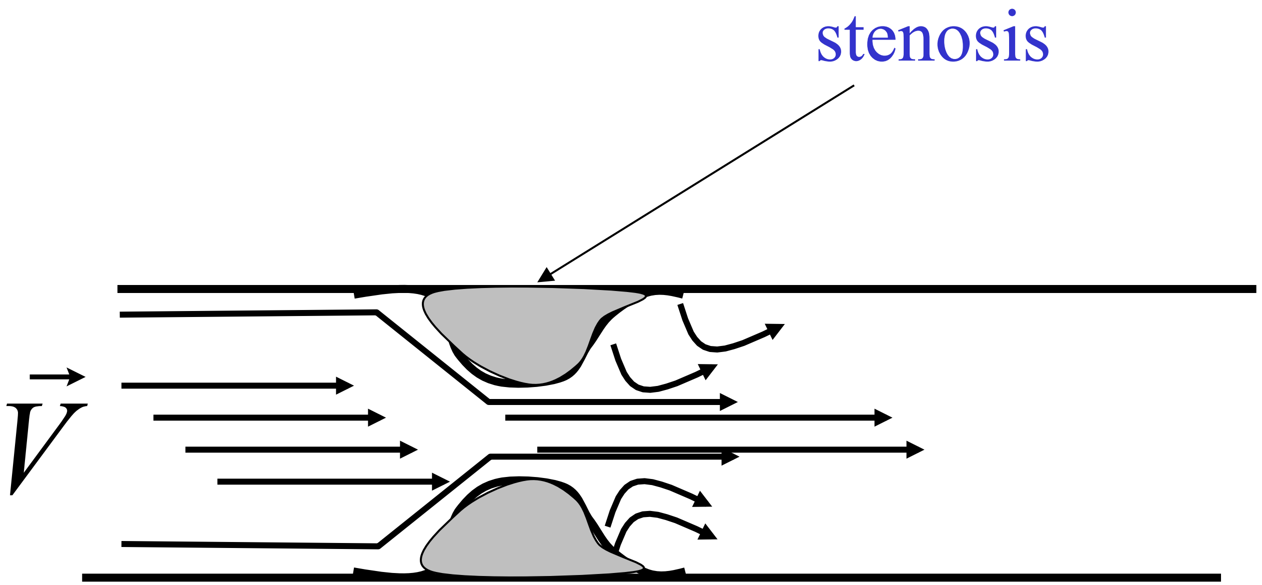
Exemple

Centroid(femoral artery)



Application : stenosis

Blood fow in the presence of stenosis



Application : sténose

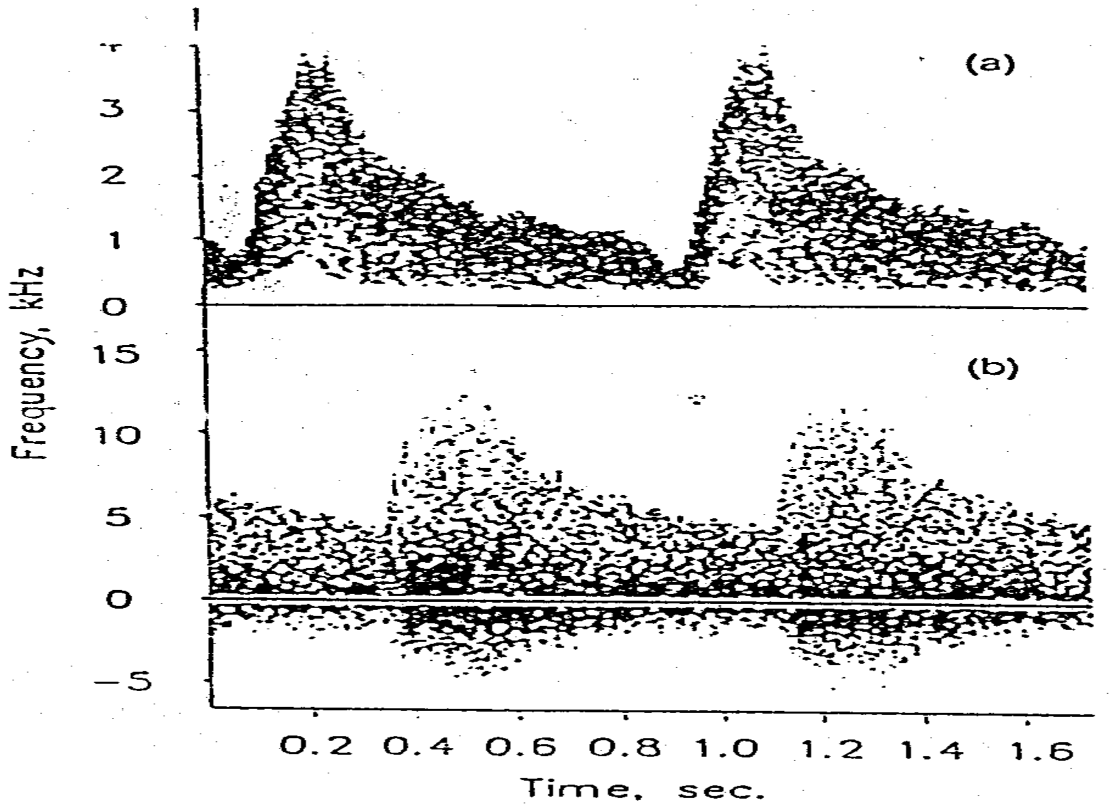
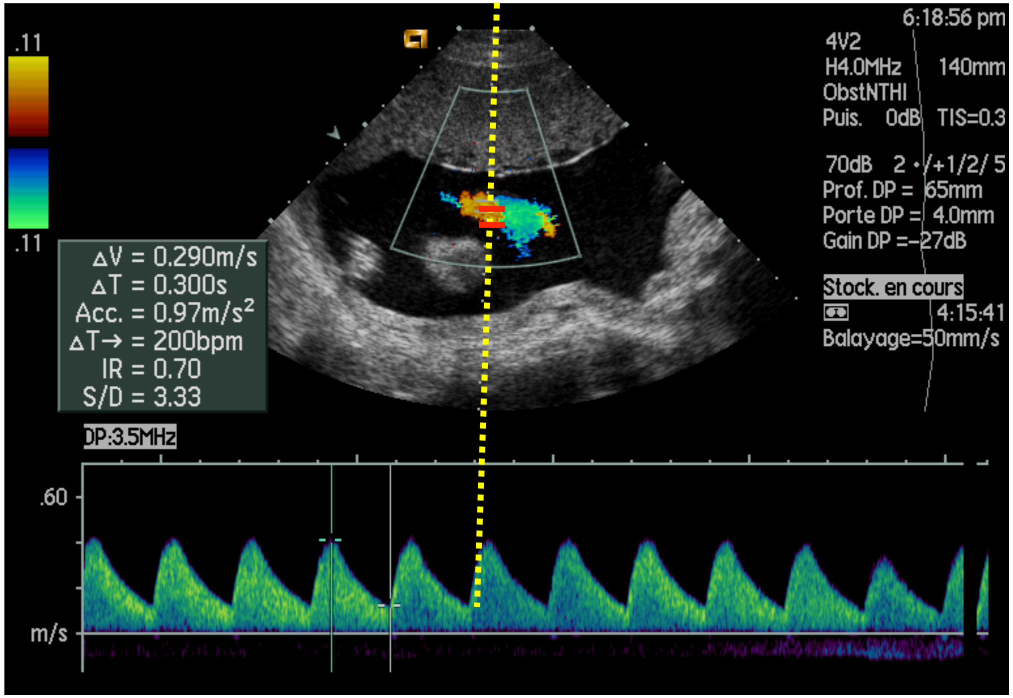


Fig. 1. Examples of CW Doppler grey-scale spectra recordings from human internal carotid arteries. (a) Normal vessel showing the presence of a clear window in the region of peak systole. (b) Recording taken distal to an 80% area stenosis, showing the presence of spectral broadening.

Application : Blood flow in placenta



Conclusion and discussions

- *The origin of ultrasound signal and image have been presented*
- *This origin is a very important since it explains the nature of the ultrasound images*

- *Unlike other medical imaging modalities where signal and image processing techniques belong to the acquisition process, in ultrasound imaging, signal and image need to be developed.*

- *Different challenges have been presented and require the development of dedicated signal and image processing techniques*
- *Currently, among the most important ones fast acquisition, tissue characterization and ultrasound image resolution enhancement are very warm problems*

- *Although not underlined in this presentation, we shall also mention that many works are also being developed in the nonlinear ultrasound imaging field.*

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