

Parallel and Distributed Programming $\mathbf{Assignment}\ \mathbf{3}$

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Contents

1	Introduction	1
	1.1 Theory	1
	1.1.1 The Quicksort Algorithm	1
	1.1.2 The Parallel Quicksort Algorithm	
	1.1.3 Strong and Weak Scaling	1
2	Implementation	2
3	Numerical Experiments	4
4	Results	5
5	Discussion	10
6	Appendix	11
	6.1 Parallelized Code	11
	6.2 Performance Evaluation Code	18

1 Introduction

Fast and efficient algorithms for sorting lists are of great importance in computer science. Quicksort is a widely used sorting algorithm that also can be parallelized in order to increase its efficiency. In this assignment the quicksort algorithm is parallelized using C and MPI, and the performance of the program is analyzed by examining its strong and weak scaling.

1.1 Theory

1.1.1 The Quicksort Algorithm

The quicksort algorithm is based on the divide and conquer approach. The first step is to choose a pivot element in the list and thereafter partition the list so that all the elements smaller than the pivot are positioned to the left of the pivot, and the larger elements to the right of the pivot. Then, this process is repeated recursively for the part of the list to the left of the pivot and for the part to the right of the pivot.

1.1.2 The Parallel Quicksort Algorithm

In parallelized quicksort, the first step is to divide the list into equally sized parts and distribute one part to every process. The local list must then be sorted and thereafter a pivot element is chosen. On every local list, the data is partitioned into two parts, one containing the elements smaller than the pivot and one containing the elements larger than the pivot. The processes are then split into two groups and processes exchange data pairwise so that only one of the groups has the data less than the pivot and the other only has data larger than the pivot. This course of action is then repeated recursively for every group of processes until no more splits of the process groups can be made. Finally, the local lists of every process are then put together to form the final sorted list.

1.1.3 Strong and Weak Scaling

When analyzing the performance of a program, its strong and weak scaling can be taken into account. Strong scaling implies that the size of the problem is kept constant while the number of processes used increases. On the contrary, for weak scaling both the size of the problem and the number of processes is increased in a way so that the workload for the processes is kept constant. To calculate the strong scalability the expression presented in equation (1) can be used, where N is the size of the problem, p is the number of processes, S is the speed-up and T_{serial} and $T_{parallel}$ are the timings for the serial and parallelized codes, respectively.

$$S(N,p) = \frac{T_{serial}(N)}{T_{parallel}(N,p)} \tag{1}$$

Similarly, the weak scaling can be computed with equation (2).

$$S(N,p) = \frac{T_{serial}(N)}{T_{parallel}(N,p)} \times p$$
 (2)

The parallel efficiency of a program is also something that can be analyzed, which can be thought of as the speed-up per process. The expression for computing the efficiency is presented in equation (3).

$$E(N,p) = \frac{S(N,p)}{p} = \frac{T_{serial}(N)}{p \times T_{parallel}(N,p)}$$
(3)

A strongly scalable program has a constant efficiency when the size of the problem stays the same, and a weakly scalable program has a constant efficiency when the problem size increases.

2 Implementation

The implemented program is presented in Appendix section 6.1. The program takes three input arguments, where the first one determines how the elements in the list are generated. This input argument is stored in the variable seq (for "sequence") and the list is given uniform random numbers if 0 is entered, an exponential distribution if 1 is entered, a normal distribution if 2 is entered and a list with decreasing values if 3 is entered. The second argument specifies the length of the list which is stored in the variable len. The last input argument which is stored in strategy determines according to which strategy the pivot element is chosen in the algorithm. If 0 is chosen then the median in one of the processes is used as pivot, if 1 is chosen the median in every process is computed and the median of those values is chosen as pivot. If 2 is entered the mean of all process's medians is chosen as the pivot instead.

The first part of the code initializes the MPI and then the input format and the assumption that the number of processes can be expressed as 2^k , where k is an integer, are checked. The list data is filled with values on the process of rank 0, according to the chosen sequence. Then, the number of values that are to be distributed to every process is computed. If the length of the list is not equally divisible with the number of processes, the process of rank 0 will receive the remaining values. Thereafter, the parts of the list are distributed to all the processes with the use of the function MPI_Scatter, and the scattered values are stored in a local list for every process. Every local list is then sorted using the serial quicksort function quicksort, and thereafter the recursive parallel quicksort function $p_{quicksort}$ is called for the local list on every process.

The parallel quicksort function p_quicksort is implemented so that it takes a list and its length together with a communicator, the chosen strategy and the rank of the process in the MPI_COMM_WORLD communicator as input. At first, the size of the input communicator and the rank of the process calling the function (according to the input communicator) are stored in the variables local_size and local_rank. Then, the base case of the recursive function presents itself, which checks if the size of the input communicator is equal to 1, meaning that no more splits of the current communicator can be made. If so, the process will send its local list to rank 0 (according to the MPI_COMM_WORLD communicator) using MPI_Isend and then return. However if the base case condition is not fulfilled, the pivot element will be computed according to the chosen strategy. Then, the local list will be partitioned, i.e. the number of elements smaller and larger than the pivot are found. Using this information, the pairwise communication described in section 1.1.2 can be executed. This is done by assigning the processes into two groups, one that contains the processes that are to store the elements smaller than the pivot (the left group), and the other the elements larger than the pivot (the right group). From this, the processes in the left group will send their local values that are larger than the pivot to a process in the right group. Vice versa, the processes in the right group will send their values that are smaller than the pivot to its pair process in the left group. In order to be able to know how much memory to allocate for the incoming elements, the functions MPI_Probe and MPI_Get_count are used to obtain the number of values that are to be received on every process. Using this information, the values are then received in the allocated list temp_data. The values that were not sent (that are to be kept in the local list of the process) are transferred to the allocated list old_data, which then is merged together with the temp_data array using the function merge. Right before the merge function is called, the old local_data array is freed so that the resulting list that is produced by merge can be assigned to local_data. After this, the input communicator is split into two parts (defined by the groups described above) by the function MPI_Comm_split. The temporary arrays temp_data and old_data are then freed and thereafter a recursive call to the p_quicksort function is made, where the updated local_data list and the new communicator are passed as input.

Back in the main function, the next step is to receive the data that is sent by the processes from the p_quicksort function. Firstly, the number of elements that are to be received from every process is obtained and stored in the allocated array recv_sizes, by using the functions MPI_Probe and MPI_Get_count. Using this information, the elements are then received and put in the original list data using MPI_Irecv

and MPI_Wait calls. Lastly, the result is checked and all allocated memory is freed before a call to the function MPI_Finalize is made and the program terminates.

3 Numerical Experiments

A set of experiments was set up on the UPPMAX system Snowy for the purpose of analyzing the strong and weak scaling of the program, for combinations of the different sequences and strategies. The UPPMAX system enabled a total of 40 processes to be used in the experiments. When measuring the time for the parallel quicksort algorithm, time measurements were made on every process, and the longest measured time was considered as the final execution time.

A list of length 200000000 was used when the strong scaling experiments were carried out. As the assumption described in section 2 had to be fulfilled, time measurements could be made for when 1, 2, 4, 8, 16 and 32 processes were used. This was done for all possible combinations of the chosen sequence and strategy, giving a total of 12 strong scaling experiments where the number of processes was varied. The results are presented in section 4 where the outcome of the experiments for the different sequences are presented together in a plot for every strategy.

When the weak scaling experiments were carried out, the length of the local list (the workload) for every process was kept constant with the value 25000000. Hence, the program was run with list lengths of 25000000, 50000000, 100000000, 200000000, 400000000 and 800000000 for the number of processes 1, 2, 4, 8, 16 and 32 respectively. Thus, a constant list-length-per-process was kept through the different measurements. Also, the weak scaling experiments were carried out for all possible combinations of the sequence and strategy options, and the results for the different sequences are presented together in a plot for every strategy, which are displayed in section 4.

4 Results

The speed-up of the strong scaling analysis was calculated with the help of equation (1). In Figures 1, 2 and 3 the results of the strong scaling experiments for strategies 0, 1 and 2, respectively, can be seen. The numerical values of the calculated speed-up for each strategy can be found in Table 1.

Table 1: Table over the number of processes, the problem sizes and the timings acquired during the strong scaling analysis and the calculated speed-up for each sequence and strategy. All values have been rounded to four significant digits.

Strong Scaling Speed-up - Strategy = 0	Strong	Scaling	Speed-up -	Strategy	= 0
--	--------	---------	------------	----------	-----

Processes	Problem size	Sequence $= 0$	Sequence = 1	Sequence = 2	Sequence = 3
1	200000000	1.000	1.000	1.000	1.000
2	200000000	1.570	1.014	0.753	1.385
4	200000000	2.364	1.294	0.909	1.460
8	200000000	3.271	1.515	0.988	1.820
16	200000000	5.221	1.751	1.062	3.144
32	200000000	6.227	1.852	1.086	3.802

Strong Scaling Speed-up - Strategy = 1

Processes	Problem size	Sequence = 0	Sequence = 1	Sequence = 2	Sequence = 3
1	200000000	1.000	1.000	1.000	1.000
2	200000000	1.572	1.013	0.756	1.379
4	200000000	2.367	1.291	0.900	1.617
8	200000000	3.300	1.512	0.988	2.034
16	200000000	5.199	1.744	1.059	3.435
32	200000000	6.165	1.840	1.094	4.014

Strong Scaling Speed-up - Strategy = 2

Processes	Problem size	Sequence $= 0$	Sequence = 1	Sequence = 2	Sequence = 3
1	200000000	1.000	1.000	1.000	1.000
2	200000000	1.571	1.015	0.761	1.480
4	200000000	2.390	1.295	0.912	1.727
8	200000000	3.318	1.516	0.988	2.230
16	200000000	5.216	1.748	1.065	3.906
32	200000000	6.182	1.850	1.098	4.157

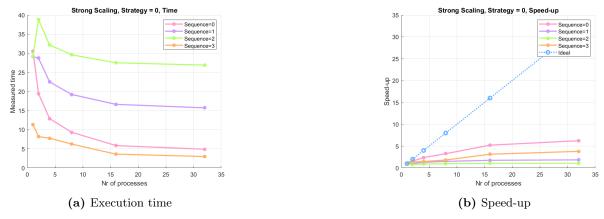


Figure 1: Plots over the strong scaling analysis, including the measured execution time and the calculated speed-up, for Strategy 0.

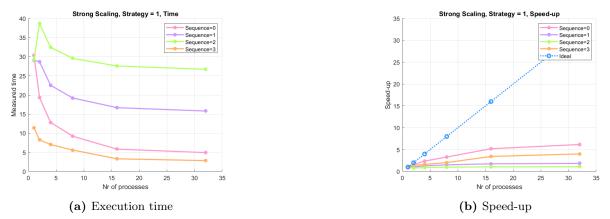


Figure 2: Plots over the strong scaling analysis, including the measured execution time and the calculated speed-up, for Strategy 1.

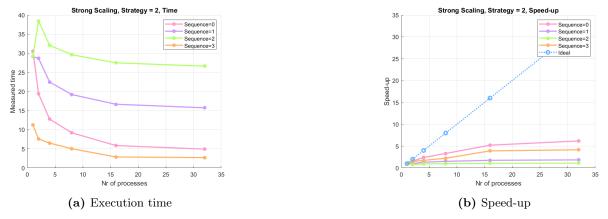


Figure 3: Plots over the strong scaling analysis, including the measured execution time and the calculated speed-up, for Strategy 2.

Moreover, the speed-up of the weak scaling analysis was calculated with the help of equation (2). In Figures 4, 5 and 6 the results of the weak scaling experiments for strategies 0, 1 and 2, respectively, can be observed. The numerical values of the calculated speed-up for each strategy are displayed in Table 2.

Table 2: Table over the number of processes, the problem sizes and the timings acquired during the weak scaling analysis and the calculated speed-up for each sequence and strategy. All values have been rounded to four significant digits.

Weak Scaling Speed-up -	Strategy =	= 0
-------------------------	------------	-----

Processes	Problem size	Sequence $= 0$	Sequence = 1	Sequence = 2	Sequence = 3
1	25000000	1.000	1.000	1.000	1.000
2	50000000	1.461	0.938	0.700	1.135
4	100000000	2.051	1.185	0.830	1.420
8	200000000	2.855	1.347	0.878	1.461
16	400000000	4.501	1.560	0.945	2.547
32	800000000	5.426	1.665	0.991	3.011

Weak Scaling Speed-up - Strategy = 1

Processes	Problem size	Sequence $= 0$	Sequence = 1	Sequence = 2	Sequence = 3
1	25000000	1.000	1.000	1.000	1.000
2	50000000	1.475	0.939	0.701	1.138
4	100000000	2.083	1.195	0.832	1.514
8	200000000	2.8812	1.353	0.880	1.615
16	400000000	4.475	1.562	0.948	2.755
32	800000000	5.444	1.662	0.991	3.248

Weak Scaling Speed-up - Strategy = 2

Processes	Problem size	Sequence = 0	Sequence = 1	Sequence $= 2$	Sequence = 3
1	25000000	1.000	1.000	1.000	1.000
2	50000000	1.486	0.916	0.699	1.258
4	100000000	2.111	1.194	0.825	1.729
8	200000000	2.897	1.351	0.888	1.731
16	400000000	4.486	1.560	0.956	3.053
32	800000000	5.427	1.655	0.988	3.303

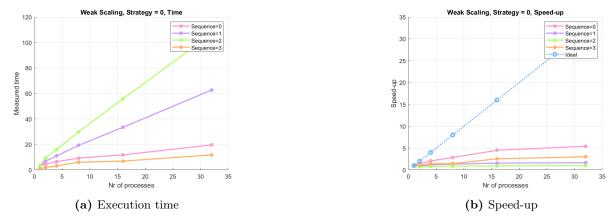


Figure 4: Plots over the weak scaling analysis, including the measured execution time and the calculated speed-up, for Strategy 0.

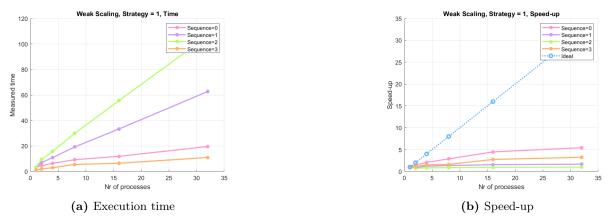


Figure 5: Plots over the weak scaling analysis, including the measured execution time and the calculated speed-up, for Strategy 1.

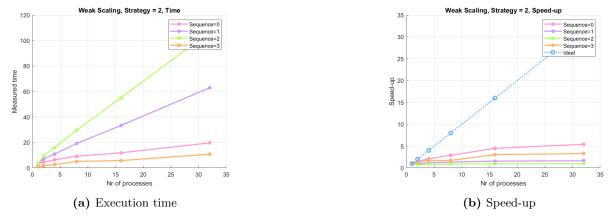


Figure 6: Plots over the weak scaling analysis, including the measured execution time and the calculated speed-up, for Strategy 2.

Lastly, the efficiency of the strong and weak scaling experiments was calculated with equation (3). Figure 7 shows the efficiency of the strong scaling analyses while Figure 8 depicts the efficiency of the weak scaling analyses. The Matlab code responsible for generating all the presented plots can be found in the Appendix, in section 6.2.

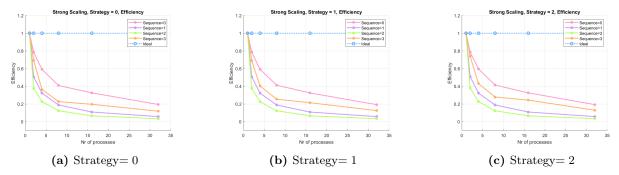


Figure 7: Plots over the efficiency of the strong scaling analyses for Strategies 0, 1 and 2.

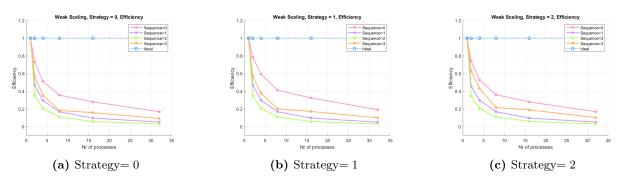


Figure 8: Plots over the efficiency of the weak scaling analyses for Strategies 0, 1 and 2.

5 Discussion

Figures 1a, 2a and 3a show that the execution time of the strong scaling experiments decreases as the the number of processes increases, which can be expected. From Figures 1b, 2b and 3b, it is implied that there is a minimal increase in the speed-up with an increasing number of processes. The same plots also suggest that none of the available strategies achieve a speed-up as high as the ideal speed-up. The above conclusions indicate that this quicksort implementation is not strongly scalable.

By studying Figures 4a, 5a and 6a, it is obvious that the execution time increases while the number of processes and the problem size increases, even though the workload is kept constant. This results in the implementation obtaining only a minimal speed-up, which is depicted in Figures 4b, 5b and 6b. As a conclusion, the results of the weak scaling analyses suggest that this quicksort implementation is not weakly scalable.

Figures 7 and 8 illustrate the calculated strong and weak efficiency of the different sequence and strategy combinations. It is clear that this algorithm does not produce an efficiency close to the ideal one. The conclusions that this implementation is neither strongly nor weakly scalable are therefore verified.

A general observation was made when studying the different generated plots. There is a trend in the performance of the algorithm regarding the choice of sequence. Regardless the choice of strategy, the implementation seems to always perform best with sequence 0, which entails filling the initial list with uniformly random numbers. On the other hand the implementation seems to perform the worst when sequence 2 is chosen, which involves filling the initial list with values of a normal distribution. Also, a list with values generated according to an exponential distribution (sequence 1) seems to have a slightly better performance than for a normal distribution (sequence 2). This can be due to the fact that an exponentially generated list will have a larger amount of sorted elements in its original list than a list with a normal distribution, as the latter will have larger values in the middle of the list and smaller on the edges. Thus, less communications might have to be made in the algorithm for the exponential sequence which can be a cause of its better performance. Additionally, sequence 3 has the lowest execution times in all experiments. This can be explained by the fact that this list already is sorted, but in the wrong direction (values go from larger to smaller instead of smaller to larger). Hence, for sequence 3 the list only has to be changed to the right direction, which requires less work in the algorithm.

When observing the result of different strategies for choosing the pivot value, there seems to be no distinct difference in the performance of the program. Hence, it can be concluded that the method of choosing a pivot value in the parallel quicksort algorithm does not have a great importance when it comes to the efficiency of the algorithm.

6 Appendix

6.1 Parallelized Code

```
* Parallel Ouick sort
    * Usage: ./a.out sequence length strategy
5 #define PI 3.14159265358979323846
6 #include <stdio.h>
  #include <stdlib.h>
  #include <math.h>
9 #include <mpi.h>
int partition(double *data, int left, int right, int pivotIndex);
void quicksort(double *data, int left, int right);
double *merge(double *v1, int n1, double *v2, int n2);
void printList(double* data, int len);
15 void p_quicksort(double *local_data, int len, MPI_Comm comm, int strategy, int world_rank);
void writeOutput(char* filename, double* data, int n);
   int main(int argc, char *argv[]) {
18
       double start_time, execution_time, max_time, *data, *local_data, pivot, *temp_data;
19
       MPI_Status status;
20
       MPI_Request request;
21
       int size, rank, len, seq, strategy, num_splits, chunk, last_chunk = 0, ...
22
           uneven_distribution = 0;
23
24
       if(argc != 4){
           printf("ERROR! Expected input: quicksort sequence length strategy\n");
25
26
       seq=atoi(argv[1]);
27
       len=atoi(argv[2]);
28
       strategy=atoi(argv[3]);
29
30
                                              // Initialize MPI
31
       MPI_Init(&argc, &argv);
       MPI_Comm_size(MPI_COMM_WORLD, &size); // Get the number of processors
32
       MPI_Comm_rank(MPI_COMM_WORLD, &rank); // Get my number
34
       // Check assumptions
35
36
       if((seq < 0) || (seq > 3)){}
           if(rank == 0) printf("ERROR! Invalid sequence selection. Use 0, 1 or 2!\n");
37
39
       if((strategy < 0) || (strategy > 2)){
40
           if(rank == 0) printf("ERROR! Invalid strategy selection. Use 0, 1, 2 or 3!\n");
41
42
       if(len \leq 0){
44
           if(rank == 0) printf("ERROR! Invalid length selection. Please use a positive ...
               integer!\n");
           exit(0);
46
       num_splits = log(size) / log(2);
48
       if(pow(2, num_splits) != size){
49
           if(rank == 0) printf("ERROR! Invalid number of processes.\n");
50
           exit(0);
51
52
       }
53
       // Fill list
54
       if(rank==0){
55
           data = (double *)malloc(len*sizeof(double));
56
           if(seq == 0) { // Uniform random numbers
57
```

```
for (int i = 0; i < len; i++)</pre>
58
                     data[i] = drand48();
             }else if(seq == 1){ // Exponential distribution
60
                 double lambda = 10;
                 for (int i = 0; i < len; i++)</pre>
62
                     data[i] =- lambda*log(1-drand48());
63
             }else if(seq == 2){ // Normal distribution
64
                 double x, y;
65
                 for (int i = 0;i < len; i++) {</pre>
                     x = drand48(); y = drand48();
67
                     data[i] = sqrt(-2*log(x)) * cos(2*PI*y);
68
69
            }else if(seq == 3){
70
                 for(int i = 0; i < len; i++)</pre>
72
                     data[i] = len - i;
73
            // Print initial list
74
            /*
75
            printf("Initial list\n");
            printList(data, len);
77
78
        }
79
80
        // Calculate local_data size and allocate memory
81
        chunk = len/size;
82
        if(len % size != 0){
83
            uneven_distribution = 1;
84
            if (rank==0) {
85
                 last_chunk = len % size;
86
87
        local_data = (double *) malloc((chunk+last_chunk)*sizeof(double));
89
91
        // Start timer
        start_time = MPI_Wtime();
92
93
        // Scatter data to local data arrays
94
        MPI_Scatter(&data[0], chunk, MPI_DOUBLE, &local_data[0], chunk, MPI_DOUBLE, 0, ...
             MPI_COMM_WORLD);
96
        if(uneven_distribution == 1) { // add rest to rank 0
97
            if(rank == 0){
                 int counter = 0;
98
                 for(int i = size*chunk; i < size*chunk+last_chunk; i++) {</pre>
                     local_data[chunk+counter] = data[i];
100
101
                     counter++;
102
103
            }
104
105
        // Sort local lists
106
        quicksort(local_data, 0, chunk+last_chunk-1);
107
108
109
        // Call parallel quicksort
        p_quicksort(local_data, chunk+last_chunk, MPI_COMM_WORLD, strategy, rank);
110
111
        // Receive data
112
        if(rank == 0){
113
114
            int* recv_sizes = (int*)malloc(size*sizeof(int));
115
            int position = 0;
             for(int i = 0; i < size; i++) {</pre>
116
117
                MPI_Probe(i, 200+i, MPI_COMM_WORLD, &status); // fetch status thing for ...
                     communication
                MPI_Get_count(&status, MPI_DOUBLE, &recv_sizes[i]); // find number of elements ...
118
                     to receive
119
```

```
120
                 MPI_Irecv(&data[position], recv_sizes[i], MPI_DOUBLE, i, 200+i, ...
                     MPI_COMM_WORLD, &request);
                 MPI_Wait(&request, &status);
121
                 position += recv_sizes[i];
123
124
             free(recv_sizes);
125
126
         // Print final list
127
        /*
128
129
         if(rank == 0) {
             printf("FINAL LIST \n");
130
             printList(data, len);
131
132
133
         */
134
135
        // Compute time
136
        execution_time = MPI_Wtime()-start_time; // stop timer
137
        MPI_Reduce(&execution_time, &max_time, 1, MPI_DOUBLE, MPI_MAX, 0, MPI_COMM_WORLD);
138
139
         // Display timing results
        if (rank==0) {
140
141
             printf("%f\n", max_time);
142
143
         // Check results
144
        if(rank == 0){
145
             int OK = 1;
146
             for (int i = 0; i < len-1; i++) {</pre>
147
                 if(data[i] > data[i+1]) {
148
                     printf("Wrong result: data[%d] = %f, data[%d] = %f\n", i, data[i], i+1, ...
149
                          data[i+1]);
                     OK = 0;
150
151
                 }
152
             if (OK) printf("Data sorted correctly!\n");
153
154
155
        // Clean up
156
157
        if (rank == 0) {
158
             free (data);
159
        MPI_Finalize();
160
        return 0;
161
162
163
164
    void p_quicksort(double *local_data, int len, MPI_Comm comm, int strategy, int world_rank) {
165
        int local_size, local_rank;
        double pivot;
166
167
        MPI_Request request, request_send, request_recv;
        MPI_Status status;
168
169
170
        MPI_Comm_size(comm, &local_size);
171
        MPI_Comm_rank(comm, &local_rank);
172
        // Basecase
173
         if(local_size == 1){
174
175
             MPI_Isend(&local_data[0], len, MPI_DOUBLE, 0, 200+world_rank, MPI_COMM_WORLD, ...
                 &request);
176
             return;
177
178
         // Select pivot element according to chosen strategy
179
        if(strategy == 0){ // median in one process
180
181
             if(local_rank == 0){
```

```
pivot = local_data[len/2];
182
183
             MPI_Bcast(&pivot, 1, MPI_DOUBLE, 0, comm);
184
185
         } else if(strategy == 1) { // median of all medians
             double *medians;
186
             int *lengths;
187
188
             // Find how many local lists that are NOT empty
189
             if(local_rank == 0){
190
                 lengths = (int*)malloc(local_size*sizeof(int));
191
192
             MPI_Gather(&len, 1, MPI_INT, &lengths[0], 1, MPI_INT, 0, comm);
193
194
             if(local_rank == 0){
195
                 int num_medians = 0;
196
                 for(int i = 0; i < local_size; i++) {</pre>
197
198
                     if(lengths[i]>0){
                          num_medians++;
199
200
                 }
201
202
                 medians = (double *)malloc(local_size*sizeof(double));
203
204
             // If local list not empty: send median
205
             if(len > 0){
206
                 MPI_Isend(&local_data[len/2], 1, MPI_DOUBLE, 0, 500+local_rank, comm, &request);
207
208
209
             // Receive medians
210
             if(local_rank == 0){
211
                 int counter = 0;
212
                 for(int i = 0; i < local_size; i++) {</pre>
213
214
                     if(lengths[i] > 0){
                          MPI_Irecv(&medians[counter], 1, MPI_DOUBLE, i, 500+i, comm, &request);
215
                          MPI_Wait(&request, &status);
216
217
                          counter++;
                     }
218
219
                 }
220
221
                 // Compute medians of medians
                 quicksort (medians, 0, counter-1);
222
223
                 pivot = medians[counter/2];
224
                 free (medians);
225
                 free (lengths):
226
             MPI_Bcast(&pivot, 1, MPI_DOUBLE, 0, comm);
227
228
229
         } else if (strategy == 2) { // mean of all medians
             double median = local_data[len/2];
230
231
             MPI_Reduce(&median, &pivot, 1, MPI_DOUBLE, MPI_SUM, 0, comm);
             pivot = pivot/local_size;
232
             MPI_Bcast(&pivot, 1, MPI_DOUBLE, 0, comm);
233
234
235
236
         // Find split in local list
        int num_smaller_elements = 0;
237
         for(int i = 0; i < len; i++){</pre>
238
239
             if(local_data[i] < pivot){</pre>
240
                 num_smaller_elements++;
241
             } else {
242
                 break;
243
244
        int num_larger_elements = len - num_smaller_elements;
245
246
```

```
// EXCHANGE DATA PAIRWISE
247
248
         // Split processes in two groups
        int rank_friend; // rank to exchange with
249
250
        int group;
251
        if(local_rank < local_size/2) {</pre>
             group = 0; // left part of list
252
             rank_friend = local_rank + local_size/2;
253
254
        } else {
255
             group = 1; // right part of list
             rank_friend = local_rank - local_size/2;
256
257
258
         // Send elements
259
         if(group == 0){ // left group
260
261
             MPI_Isend(&local_data[num_smaller_elements], num_larger_elements, MPI_DOUBLE, ...
                 rank_friend, 100+rank_friend, comm, &request_send);
         } else { // right group
262
             MPI_Isend(&local_data[0], num_smaller_elements, MPI_DOUBLE, rank_friend, ...
263
                 100+rank_friend, comm, &request_send);
        }
264
265
        MPI_Probe(rank_friend, 100+local_rank, comm, &status); // fetch status thing for ...
266
             communication
267
        int recv_size;
        MPI_Get_count(&status, MPI_DOUBLE, &recv_size); // find number of elements to receive
268
269
        double *temp_data, *old_data;
270
271
        int new_len;
272
        if(group == 0){
             temp_data = (double*)malloc(recv_size*sizeof(double));
273
274
             MPI_Irecv(&temp_data[0], recv_size, MPI_DOUBLE, rank_friend, 100+local_rank, comm, ...
                 &request recv):
            MPI_Wait(&request_recv, &status);
275
276
             old_data = (double*)malloc(num_smaller_elements*sizeof(double));
277
             for(int i = 0; i < num_smaller_elements; i++) {</pre>
278
                 old_data[i] = local_data[i];
279
280
281
282
             // Merge
            MPI_Wait(&request_send, &status);
283
284
             free (local data);
             local_data = merge(temp_data, recv_size, old_data, num_smaller_elements);
285
286
             new_len = recv_size+num_smaller_elements;
287
         } else {
             temp_data = (double*)malloc(recv_size*sizeof(double));
288
289
             MPI_Irecv(&temp_data[0], recv_size, MPI_DOUBLE, rank_friend, 100+local_rank, comm, ...
                 &request_recv);
             MPI_Wait(&request_recv, &status);
290
291
             old_data = (double*)malloc(num_larger_elements*sizeof(double));
292
293
294
             int counter = 0;
             for(int i = num_smaller_elements; i < len; i++) {</pre>
295
296
                 old_data[counter] = local_data[i];
297
                 counter++;
298
             // Merge
299
             MPI_Wait(&request_send, &status);
300
             free(local_data);
301
             local_data = merge(temp_data, recv_size, old_data, num_larger_elements);
302
             new_len = recv_size+num_larger_elements;
304
305
306
        // Split comm into two parts
```

```
307
        MPI Comm comm new;
308
        MPI_Comm_split(comm, group, local_rank, &comm_new);
309
310
        // Local clean up
        free(temp_data);
311
        free(old_data);
312
313
         // Recursive call
314
315
        p_quicksort(local_data, new_len, comm_new, strategy, world_rank);
316
317
    int partition(double *data, int left, int right, int pivotIndex){
318
319
        double pivotValue, temp;
320
        int storeIndex, i;
321
        pivotValue = data[pivotIndex];
        temp = data[pivotIndex];
322
        data[pivotIndex] = data[right];
323
324
        data[right] = temp;
325
        storeIndex = left;
        for (i = left; i < right; i++)</pre>
326
327
         if (data[i] < pivotValue) {</pre>
             temp = data[i];
328
329
             data[i] = data[storeIndex];
330
             data[storeIndex] = temp;
331
332
             storeIndex = storeIndex + 1;
333
        temp = data[storeIndex];
334
        data[storeIndex] = data[right];
335
        data[right] = temp;
336
337
        return storeIndex;
338
339
    void quicksort(double *data, int left, int right) {
340
        int pivotIndex, pivotNewIndex;
341
        if (right > left){
342
             pivotIndex = left+(right-left)/2;
343
344
             pivotNewIndex = partition(data, left, right, pivotIndex);
             quicksort(data, left, pivotNewIndex - 1);
345
346
             quicksort(data, pivotNewIndex + 1, right);
347
348
349
    double *merge(double *v1, int n1, double *v2, int n2){
350
        int i,j,k;
351
        double *result;
352
353
        result = (double *) malloc((n1+n2) *sizeof(double));
354
355
356
        i=0; j=0; k=0;
        while(i<n1 && j<n2)
357
             if(v1[i] < v2[j])</pre>
358
359
                 result[k] = v1[i];
360
361
                 i++; k++;
             }
362
             else
363
364
                 result[k] = v2[j];
365
366
                 j++; k++;
367
        if (i==n1)
368
             while(j<n2)</pre>
369
370
371
                 result[k] = v2[j];
```

```
372
                 j++; k++;
373
374
         else
375
            while(i<n1)</pre>
             {
376
                  result[k] = v1[i];
i++; k++;
377
378
379
             }
         return result;
380
381
382
    void printList(double* data, int len) {
383
         for (int i = 0; i < len; i++) {</pre>
384
            printf("%10f ", data[i]);
385
386
         printf("\n");
387
388 }
```

6.2 Performance Evaluation Code

```
close all;
clear all;
   %----- PDP ASSIGNMENT 3 -----
   %----- Strong Scaling -----
6
   cores = [1, 2, 4, 8, 16, 32];
   % ----- STRATEGY = 0 -----
10
11
12 % Sequence = 0 Strategy = 0
13 time0_0 = zeros(1,length(cores));
14 \text{ time0}_0(1) = 30.525632; % 1 core
15 \text{ time0}_0(2) = 19.438958; % 2 cores
16 time0_0(3) = 12.912413; % 4 cores
17 \text{ time0}_0(4) = 9.332403; % 8 cores
18 time0_0(5) = 5.847092; % 16 cores
  time0_0(6) = 4.902279; % 32 cores
19
20
21 % Calculating speed-up, efficiency and ideal time
speedup0_0 = zeros(1,length(cores));
23 efficiency0_0 = zeros(1,length(cores));
t0_0 = zeros(1, length(cores));
   for i=1:length(cores)
       speedup0_0(i) = time0_0(1)/time0_0(i);
26
       efficiency0_0(i) = speedup0_0(i)/cores(i);
       t0_0(i) = time0_0(1)/cores(i);
28
29
30
31 % Sequence = 1 Strategy = 0
32 time1_0 = zeros(1,length(cores));
33 time1_0(1) = 29.117637; % 1 core
   time1_0(2) = 28.721584; % 2 cores
35 \text{ time1}_0(3) = 22.506445; % 4 cores
36 time1_0(4) = 19.215338; % 8 cores
37 time1_0(5) = 16.627161; % 16 cores
38 time1_0(6) = 15.724588; % 32 cores
40 % Calculating speed-up, efficiency and ideal time
41 speedup1_0 = zeros(1,length(cores));
42 efficiency1_0 = zeros(1,length(cores));
t1_0 = zeros(1, length(cores));
   for i=1:length(cores)
44
       speedup1_0(i) = time1_0(1)/time1_0(i);
45
       efficiency1_0(i) = speedup1_0(i)/cores(i);
46
       t1_0(i) = timel_0(1)/cores(i);
47
48
49
50 % Sequence = 2 Strategy = 0
51 time2_0 = zeros(1,length(cores));
52 \text{ time2}_0(1) = 29.218643; % 1 core
53 \text{ time2}_0(2) = 38.792964; % 2 cores
54 time2_0(3) = 32.160024; % 4 cores
55 \text{ time2}_0(4) = 29.581625; % 8 cores
56 time2_0(5) = 27.506236; % 16 cores
57 time2_0(6) = 26.894346; % 32 cores
58
59 % Calculating speed-up, efficiency and ideal time
speedup2_0 = zeros(1,length(cores));
61 efficiency2_0 = zeros(1, length(cores));
```

```
t2 0 = zeros(1, length(cores));
    for i=1:length(cores)
         speedup2_0(i) = time2_0(1)/time2_0(i);
64
         efficiency2_0(i) = speedup2_0(i)/cores(i);
         t2_0(i) = time2_0(1)/cores(i);
66
    end
67
68
   % Sequence = 3 Strategy = 0
69
70 time3_0 = zeros(1,length(cores));
71 time3_0(1) = 11.361397; % 1 core
    time3_0(2) = 8.201077; % 2 cores
72
73 time3_0(3) = 7.781006; % 4 cores
74 \text{ time3}_0(4) = 6.243395; % 8 cores
75 \text{ time3}_{0}(5) = 3.614225; % 16 cores
76 \text{ time3}_{0}(6) = 2.988533; % 32 cores
    % Calculating speed-up, efficiency and ideal time
78
79 speedup3_0 = zeros(1,length(cores));
    efficiency3_0 = zeros(1, length(cores));
   t3_0 = zeros(1,length(cores));
81
82
    for i=1:length(cores)
         speedup3_0(i) = time3_0(1)/time3_0(i);
83
         efficiency3_0(i) = speedup3_0(i)/cores(i);
84
         t3_0(i) = time3_0(1)/cores(i);
85
86
   % Generating figures
88
89 figure
90 hold on
    grid on
91
    plot(cores, time0_0, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
93 plot(cores, time1_0, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
94 plot(cores, time2_0, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
95 plot(cores, time3_0, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
    %plot(cores, t0_0, ':o', 'Color', [255/255 153/255 204/255], 'LineWidth', 2); %plot(cores, t1_0, ':o', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
    %plot(cores, t2_0, ':o', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
99 %plot(cores, t3_0, ':o', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3');
    title('Strong Scaling, Strategy = 0, Time');
102 xlabel('Nr of processes');
    ylabel('Measured time');
103
104 hold off
105
106
    figure
107 hold on
108 grid on
plot (cores, speedup0_0, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);

plot (cores, speedup1_0, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);

plot (cores, speedup2_0, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
112 plot(cores, speedup3_0, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
113 plot(cores, cores, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
legend('Sequence=0','Sequence=1','Sequence=2','Sequence=3','Ideal');
115 title('Strong Scaling, Strategy = 0, Speed-up');
    xlabel('Nr of processes');
    ylabel('Speed-up');
117
   hold off
119
    % ----- STRATEGY = 1 -----
120
121
122 % Sequence = 0 Strategy = 1
time0_1 = zeros(1, length(cores));
124 time0_1(1) = 30.387830; % 1 core
125 time0_1(2) = 19.329221; % 2 cores
126 time0_1(3) = 12.838916; % 4 cores
```

```
127 time0 1(4) = 9.207917; % 8 cores
   time0_1(5) = 5.844428; % 16 cores
129 \text{ time0}_1(6) = 4.929357; % 32 cores
131 % Calculating speed-up, efficiency and ideal time
   speedup0_1 = zeros(1,length(cores));
132
    efficiency0_1 = zeros(1,length(cores));
133
    t0 1 = zeros(1,length(cores));
134
    for i=1:length(cores)
135
        speedup0_1(i) = time0_1(1)/time0_1(i);
136
137
        efficiency0_1(i) = speedup0_1(i)/cores(i);
        t0_1(i) = time0_1(1)/cores(i);
138
    end
139
141 % Sequence = 1 Strategy = 1
    time1_1 = zeros(1,length(cores));
142
143 time1_1(1) = 29.084993; % 1 core
144 \text{ time1}_1(2) = 28.703074; % 2 cores
145 time1_1(3) = 22.535320; % 4 cores
   time1_1(4) = 19.215622; % 8 cores
146
147
    time1_1(5) = 16.678529; % 16 cores
    time1_1(6) = 15.804138; % 32 cores
148
149
   % Calculating speed-up, efficiency and ideal time
150
   speedup1_1 = zeros(1,length(cores));
151
    efficiency1_1 = zeros(1, length(cores));
    t1_1 = zeros(1, length(cores));
153
    for i=1:length(cores)
154
        speedup1_1(i) = time1_1(1)/time1_1(i);
155
        efficiency1_1(i) = speedup1_1(i)/cores(i);
156
157
        t1_1(i) = timel_1(1)/cores(i);
    end
158
160
   % Sequence = 2 Strategy = 1
   time2_1 = zeros(1,length(cores));
161
162
    time2_1(1) = 29.225064; % 1 core
163 time2 1(2) = 38.682093; % 2 cores
164 time2_1(3) = 32.456669; % 4 cores
165 time2_1(4) = 29.577839; % 8 cores
166
    time2_1(5) = 27.584621; % 16 cores
    time2_1(6) = 26.715707; % 32 cores
167
168
   % Calculating speed-up, efficiency and ideal time
speedup2_1 = zeros(1,length(cores));
    efficiency2_1 = zeros(1,length(cores));
171
    t2_1 = zeros(1, length(cores));
172
173
    for i=1:length(cores)
        speedup2_1(i) = time2_1(1)/time2_1(i);
174
        efficiency2_1(i) = speedup2_1(i)/cores(i);
175
        t2_1(i) = time2_1(1)/cores(i);
176
    end
177
178
179 % Sequence = 3 Strategy = 1
180 time3_1 = zeros(1,length(cores));
   time3_1(1) = 11.379709; % 1 core
182 time3_1(2) = 8.254606; % 2 cores
183 time3_1(3) = 7.038642; % 4 cores
184 time3_1(4) = 5.595951; % 8 cores
    time3_1(5) = 3.312614; % 16 cores
185
   time3_1(6) = 2.834814; % 32 cores
186
187
188 % Calculating speed-up, efficiency and ideal time
speedup3_1 = zeros(1,length(cores));
    efficiency3_1 = zeros(1,length(cores));
190
191 t3_1 = zeros(1, length(cores));
```

```
192
    for i=1:length(cores)
         speedup3_1(i) = time3_1(1)/time3_1(i);
193
         efficiency3_1(i) = speedup3_1(i)/cores(i);
194
         t3_1(i) = time3_1(1)/cores(i);
196
    end
197
    % Generating figures
198
    figure
199
200 hold on
201 grid on
    plot(cores, time0_1, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
202
    plot(cores, time1_1, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
203
204 plot(cores, time2_1, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
plot(cores, time3_1, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
    %plot(cores, t0_1, ':o', 'Color', [255/255 153/255 204/255], 'LineWidth', 2); %plot(cores, t1_1, ':o', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
206
207
    %plot(cores, t2_1, ':o', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
208
    %plot(cores, t3_1, ':o', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
209
210 legend('Sequence=0','Sequence=1','Sequence=2','Sequence=3');
211 title('Strong Scaling, Strategy = 1, Time');
    xlabel('Nr of processes');
    ylabel('Measured time');
213
214
    hold off
215
    figure
216
217 hold on
218 grid on
plot (cores, speedup0_1, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);

plot (cores, speedup1_1, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);

plot (cores, speedup2_1, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);

plot (cores, speedup3_1, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
223 plot(cores, cores, ':o', 'Color', [51/255 153/255 255/255], 'LineWidth', 2);
legend('Sequence=0','Sequence=1','Sequence=2','Sequence=3','Ideal');
225 title('Strong Scaling, Strategy = 1, Speed-up');
    xlabel('Nr of processes');
226
    ylabel('Speed-up');
227
    hold off
228
229
230
231
    % ----- STRATEGY = 2 -----
232
233
234 % Sequence = 0 Strategy = 2
235 time0_2 = zeros(1,length(cores));
    time0_2(1) = 30.534189; % 1 core
236
237 time0_2(2) = 19.441526; % 2 cores
238 \text{ time0}_2(3) = 12.773972; % 4 cores
239 time0_2(4) = 9.202893; % 8 cores
    time0_2(5) = 5.854132; % 16 cores
240
    time0_2(6) = 4.939196; % 32 cores
241
242
243 % Calculating speed-up, efficiency and ideal time
244 speedup0_2 = zeros(1,length(cores));
    efficiency0_2 = zeros(1,length(cores));
245
246
    t0_2 = zeros(1,length(cores));
    for i=1:length(cores)
247
         speedup0_2(i) = time0_2(1)/time0_2(i);
248
249
         efficiency0_2(i) = speedup0_2(i)/cores(i);
         t0_2(i) = time0_2(1)/cores(i);
250
251
    end
252
253 % Sequence = 1 Strategy = 2
254 time1_2 = zeros(1,length(cores));
255 time1_2(1) = 29.114526; % 1 core
256 time1_2(2) = 28.676153; % 2 cores
```

```
257 time1 2(3) = 22.483128; % 4 cores
258 time1_2(4) = 19.200803; % 8 cores
259 time1 2(5) = 16.652579; % 16 cores
260 time1_2(6) = 15.739615; % 32 cores
261
    % Calculating speed-up, efficiency and ideal time
262
263
    speedup1_2 = zeros(1,length(cores));
    efficiency1 2 = zeros(1,length(cores));
264
    t1_2 = zeros(1,length(cores));
    for i=1:length(cores)
266
         speedup1_2(i) = time1_2(1)/time1_2(i);
267
         efficiency1_2(i) = speedup1_2(i)/cores(i);
268
         t1_2(i) = time1_2(1)/cores(i);
269
270
271
    % Sequence = 2 Strategy = 2
272
273 time2_2 = zeros(1,length(cores));
274 time2_2(1) = 29.246983; % 1 core
275 time2_2(2) = 38.418233; % 2 cores
276 time2_2(3) = 32.073953; % 4 cores
277
    time2_2(4) = 29.614933; % 8 cores
    time2 2(5) = 27.487457; % 16 cores
278
279
    time2_2(6) = 26.626454; % 32 cores
280
    % Calculating speed-up, efficiency and ideal time
281
    speedup2_2 = zeros(1,length(cores));
282
    efficiency2_2 = zeros(1,length(cores));
283
    t2_2 = zeros(1,length(cores));
284
285
    for i=1:length(cores)
         speedup2_2(i) = time2_2(1)/time2_2(i);
286
287
         efficiency2_2(i) = speedup2_2(i)/cores(i);
         t2_2(i) = time2_2(1)/cores(i);
288
    end
289
290
    % Sequence = 3 Strategy = 2
291
292
    time3_2 = zeros(1,length(cores));
293 time3 2(1) = 11.238143; % 1 core
294 time3_2(2) = 7.593101; % 2 cores
295 time3_2(3) = 6.507833; % 4 cores
296
    time3_2(4) = 5.040112; % 8 cores
    time3_2(5) = 2.877320; % 16 cores
297
    time3_2(6) = 2.703439; % 32 cores
298
300 % Calculating speed-up, efficiency and ideal time
    speedup3_2 = zeros(1,length(cores));
301
    efficiency3_2 = zeros(1,length(cores));
302
303
    t3_2 = zeros(1, length(cores));
    for i=1:length(cores)
304
         speedup3_2(i) = time3_2(1)/time3_2(i);
305
         efficiency3_2(i) = speedup3_2(i)/cores(i);
306
         t3_2(i) = time3_2(1)/cores(i);
307
308
309
    % Generating figures
310
311
    figure
312 hold on
314 plot(cores, time0_2, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
    plot(cores, time1_2, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2); plot(cores, time2_2, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
315
317 plot(cores, time3_2, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
318 %plot(cores, t0_2, ':o', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
319 %plot(cores, t1_2, ':o', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
320 %plot(cores, t2_2, ':o', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
321 %plot(cores, t3_2, ':o', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
```

```
legend('Sequence=0','Sequence=1','Sequence=2','Sequence=3');
    title('Strong Scaling, Strategy = 2, Time');
324 xlabel('Nr of processes');
325 ylabel('Measured time');
326 hold off
327
328
    figure
    hold on
329
    grid on
    plot(cores, speedup0_2, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
331
    plot(cores, speedup1_2, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2); plot(cores, speedup2_2, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
332
333
334 plot(cores, speedup3_2, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
335 plot(cores, cores, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
    legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3', 'Ideal');
336
    title('Strong Scaling, Strategy = 2, Speed-up');
337
    xlabel('Nr of processes');
338
    ylabel('Speed-up');
339
    hold off
340
341
    %----- Weak Scaling -----
342
343
    cores = [1, 2, 4, 8, 16, 32];
344
    %sizes = [125,250,500,1000,2000,4000];
345
346
347
    % ----- STRATEGY = 0 -----
348
349
    % Sequence = 0 Strategy = 0
350
    wtime0_0 = zeros(1,length(cores));
351
    wtime0_0(1) = 3.330238; % 1 cores
352
    wtime0 0(2) = 4.558199; % 2 cores
353
    wtime0_0(3) = 6.494247; % 4 cores
    wtime0_0(4) = 9.330230; % 8 cores
355
    wtime0_0(5) = 11.837867; % 16 cores
356
    wtime0_0(6) = 19.639124; % 32 cores
357
358
359
    % Calculating speed-up, efficiency and ideal time
    wspeedup0_0 = zeros(1,length(cores));
360
361
    wefficiency0_0 = zeros(1,length(cores));
    wt0_0 = wtime0_0(1) * ones(1, length(cores));
362
    for i=1:length(cores)
363
        wspeedup0_0(i) = wtime0_0(1)/wtime0_0(i)*cores(i);
364
        wefficiency0_0(i) = wspeedup0_0(i)/cores(i);
365
366
367
368
    % Sequence = 1 Strategy = 0
    wtime1_0 = zeros(1,length(cores));
369
    wtime1_0(1) = 3.264187; % 1 cores
370
    wtime1_0(2) = 6.956834; % 2 cores
372 wtime1_0(3) = 11.019184; % 4 cores
    wtime1_0(4) = 19.380542; % 8 cores
373
374
    wtime1_0(5) = 33.474841; % 16 cores
    wtime1_0(6) = 62.729792; % 32 cores
375
376
    % Calculating speed-up, efficiency and ideal time
377
    wspeedup1_0 = zeros(1,length(cores));
378
379
    wefficiency1_0 = zeros(1,length(cores));
380
    wt1_0 = wtime1_0(1) * ones(1, length(cores));
381
    for i=1:length(cores)
382
        wspeedup1 0(i) = wtime1 0(1)/wtime1 0(i)*cores(i);
        wefficiency1_0(i) = wspeedup1_0(i)/cores(i);
383
384
    end
385
386
   % Sequence = 2 Strategy = 0
```

```
387 wtime2 0 = zeros(1,length(cores));
    wtime2_0(1) = 3.300334; % 1 cores
389 wtime2 0(2) = 9.425067; % 2 cores
390 wtime2_0(3) = 15.910506; % 4 cores
391 \text{ wtime2}_0(4) = 30.065450; % 8 cores
    wtime2_0(5) = 55.890569; % 16 cores
392
    wtime2_0(6) = 106.613515; % 32 cores
393
394
    % Calculating speed-up, efficiency and ideal time
395
    wspeedup2_0 = zeros(1,length(cores));
396
    wefficiency2_0 = zeros(1,length(cores));
397
    wt2_0 = wtime2_0(1) * ones(1, length(cores));
398
    for i=1:length(cores)
399
        wspeedup2_0(i) = wtime2_0(1)/wtime2_0(i)*cores(i);
        wefficiency2_0(i) = wspeedup2_0(i)/cores(i);
401
402
403
    % Sequence = 3 Strategy = 0
404
    wtime3_0 = zeros(1,length(cores));
405
    wtime3_0(1) = 1.109118; % 1 cores
406
    wtime3_0(2) = 1.954304; % 2 cores
    wtime3_0(3) = 3.123370; % 4 cores
408
409 wtime3_0(4) = 6.072183; % 8 cores
    wtime3_0(5) = 6.966555; % 16 cores
410
    wtime3_0(6) = 11.786162; % 32 cores
411
412
    \mbox{\ensuremath{\mbox{\$}}} Calculating speed-up, efficiency and ideal time
413
    wspeedup3_0 = zeros(1,length(cores));
414
    wefficiency3_0 = zeros(1,length(cores));
415
    wt3_0 = wtime3_0(1) * ones(1, length(cores));
416
417
    for i=1:length(cores)
        wspeedup3 0(i) = wtime3 0(1)/wtime3 0(i)*cores(i);
418
        wefficiency3_0(i) = wspeedup3_0(i)/cores(i);
419
420
    end
421
422
    % Generating figures
423 figure
424 hold on
425 grid on
    plot(cores, wtime0_0, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
426
    plot(cores, wtime1_0, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
427
428 plot(cores, wtime2_0, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
429 plot(cores, wtime3_0, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
    %plot(cores, wt0_0, ':o', 'Color', [255/255 153/255 204/255], 'LineWidth', 2); %plot(cores, wt1_0, ':o', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
430
431
    %plot(cores, wt2_0, ':o', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
432
    %plot(cores, wt3_0, ':o', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
433
434 legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3');
    title('Weak Scaling, Strategy = 0, Time');
435
    xlabel('Nr of processes');
436
    vlabel('Measured time'):
437
438
   hold off
439
    figure
440
441
    hold on
442 arid on
   plot(cores,wspeedup0_0, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
    plot(cores,wspeedup1_0, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
444
    plot(cores,wspeedup2_0, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
445
    plot(cores, wspeedup3_0, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
446
447 plot(cores, cores, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
1 legend('Sequence=0','Sequence=1','Sequence=2','Sequence=3','Ideal');
449 title('Weak Scaling, Strategy = 0, Speed-up');
450 xlabel('Nr of processes');
451 ylabel('Speed-up');
```

```
452 hold off
453
    % ----- STRATEGY = 1 -----
454
456 % Sequence = 0 Strategy = 1
457 wtime0_1 = zeros(1,length(cores));
   wtime0_1(1) = 3.326104; % 1 cores
458
459 wtime0 1(2) = 4.510797; % 2 cores
460 wtime0_1(3) = 6.385795; % 4 cores
461 wtime0_1(4) = 9.233575; % 8 cores
462
    wtime0_1(5) = 11.891216; % 16 cores
    wtime0_1(6) = 19.550342; % 32 cores
463
464
   % Calculating speed-up, efficiency and ideal time
466 wspeedup0_1 = zeros(1,length(cores));
    wefficiency0_1 = zeros(1,length(cores));
467
    wt0_1 = wtime0_1(1) * ones(1, length(cores));
468
    for i=1:length(cores)
469
470
        wspeedup0_1(i) = wtime0_1(1)/wtime0_1(i)*cores(i);
        wefficiency0_1(i) = speedup0_1(i)/cores(i);
471
472
    end
473
474 % Sequence = 1 Strategy = 1
475 wtime1_1 = zeros(1,length(cores));
   wtime1_1(1) = 3.253699; % 1 cores
476
    wtime1_1(2) = 6.933638; % 2 cores
477
478 wtime1_1(3) = 10.888042; % 4 cores
    wtime1_1(4) = 19.239228; % 8 cores
480 wtime1_1(5) = 33.337780; % 16 cores
    wtime1_1(6) = 62.664098; % 32 cores
481
482
483 % Calculating speed-up, efficiency and ideal time
484 wspeedup1_1 = zeros(1,length(cores));
485 wefficiency1_1 = zeros(1,length(cores));
    wt1_1 = wtime1_1(1) * ones(1, length(cores));
486
487
    for i=1:length(cores)
        wspeedup1 1(i) = wtime1 1(1)/wtime1 1(i)*cores(i);
488
489
        wefficiency1_1(i) = wspeedup1_1(i)/cores(i);
    end
490
491
   % Sequence = 2 Strategy = 1
492
493 wtime2_1 = zeros(1,length(cores));
494 wtime2_1(1) = 3.290981; % 1 cores
495 wtime2_1(2) = 9.385015; % 2 cores
    wtime2_1(3) = 15.825040; % 4 cores
496
   wtime2_1(4) = 29.931524; % 8 cores
497
   wtime2_1(5) = 55.575614; % 16 cores
498
   wtime2_1(6) = 106.233427; % 32 cores
499
500
   % Calculating speed-up, efficiency and ideal time
501
502 wspeedup2_1 = zeros(1,length(cores));
503 wefficiency2_1 = zeros(1,length(cores));
504
    wt2_1 = wtime2_1(1) * ones(1, length(cores));
505
    for i=1:length(cores)
506
        wspeedup2_1(i) = wtime2_1(1)/wtime2_1(i)*cores(i);
        wefficiency2_1(i) = wspeedup2_1(i)/cores(i);
507
508
509
   % Sequence = 3 Strategy = 1
510
511 wtime3_1 = zeros(1,length(cores));
512 wtime3 1(1) = 1.110164; % 1 cores
513 wtime3_1(2) = 1.951809; % 2 cores
514 wtime3_1(3) = 2.932658; % 4 cores
515 wtime3_1(4) = 5.498014; % 8 cores
516 wtime3_1(5) = 6.448554; % 16 cores
```

```
517 wtime3_1(6) = 10.938075; % 32 cores
518
    % Calculating speed-up, efficiency and ideal time
519
520 wspeedup3_1 = zeros(1,length(cores));
521 wefficiency3_1 = zeros(1,length(cores));
    wt3_1 = wtime3_1(1) * ones(1, length(cores));
522
523
    for i=1:length(cores)
         wspeedup3 1(i) = wtime3 1(1)/wtime3 1(i)*cores(i);
524
         wefficiency3_1(i) = wspeedup3_1(i)/cores(i);
525
    end
526
527
    % Generating figures
528
   figure
529
530 hold on
531 grid on
    plot(cores, wtime0_1, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
532
    plot(cores, wtime1_1, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
533
534 plot(cores, wtime2_1, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
    plot(cores, wtime3_1, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
    %plot(cores, wt0_1, ':o', 'Color', [255/255 153/255 204/255], 'LineWidth', 2); %plot(cores, wt1_1, ':o', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
536
    %plot(cores, wt2_1, ':o', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
538
    %plot(cores, wt3_1, ':o', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
539
1540 legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3');
    title('Weak Scaling, Strategy = 1, Time');
541
    xlabel('Nr of processes');
    ylabel('Measured time');
543
    hold off
544
545
546
    figure
547 hold on
548 arid on
549 plot(cores,wspeedup0_1, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
550 plot(cores,wspeedup1_1, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
    plot(cores, wspeedup2_1, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2); plot(cores, wspeedup3_1, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
551
552
553 plot(cores, cores, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
    legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3', 'Ideal');
555 title('Weak Scaling, Strategy = 1, Speed-up');
556
    xlabel('Nr of processes');
557 ylabel('Speed-up');
558
    hold off
559
560
561
    % ----- STRATEGY = 2 -----
562
563
    % Sequence = 0 Strategy = 2
564
    wtime0_2 = zeros(1,length(cores));
565
    wtime0_2(1) = 3.333055; % 1 cores
566
567 wtime0_2(2) = 4.487260; % 2 cores
568 wtime0_2(3) = 6.316947; % 4 cores
569 wtime0_2(4) = 9.205733; % 8 cores
570 wtime0_2(5) = 11.887559; % 16 cores
    wtime0_2(6) = 19.653326; % 32 cores
571
572
573 % Calculating speed-up, efficiency and ideal time
574 wspeedup0_2 = zeros(1,length(cores));
    wefficiency0_2 = zeros(1,length(cores));
575
576
    wt0_2 = wtime0_2(1) * ones(1, length(cores));
577
    for i=1:length(cores)
         wspeedup0_2(i) = wtime0_2(1)/wtime0_2(i)*cores(i);
578
         wefficiency0_2(i) = wspeedup0_2(i)/cores(i);
579
    end
580
581
```

```
582 % Sequence = 1 Strategy = 2
584 wtime1_2(1) = 3.252186; % 1 cores
585 wtime1_2(2) = 7.097667; % 2 cores
586 wtime1_2(3) = 10.894351; % 4 cores
587 wtime1_2(4) = 19.252760; % 8 cores
    wtime1_2(5) = 33.348899; % 16 cores
588
    wtime1 2(6) = 62.891490; % 32 cores
589
590
    \ensuremath{\mbox{\ensuremath{\$}}} Calculating speed-up, efficiency and ideal time
591
592
    wspeedup1_2 = zeros(1,length(cores));
    wefficiency1_2 = zeros(1,length(cores));
593
    wt1_2 = wtime1_2(1) * ones(1, length(cores));
594
    for i=1:length(cores)
        wspeedup1_2(i) = wtime1_2(1)/wtime1_2(i)*cores(i);
596
         wefficiency1_2(i) = wspeedup1_2(i)/cores(i);
597
598
    end
599
    % Sequence = 2 Strategy = 2
600
    wtime2_2 = zeros(1,length(cores));
601
    wtime2_2(1) = 3.290253; % 1 cores
    wtime2_2(2) = 9.416190; % 2 cores
603
    wtime2_2(3) = 15.958708; % 4 cores
604
    wtime2_2(4) = 29.653036; % 8 cores
605
    wtime2_2(5) = 55.075647; % 16 cores
606
    wtime2_2(6) = 106.591060; % 32 cores
607
608
    % Calculating speed-up, efficiency and ideal time
609
610 wspeedup2_2 = zeros(1,length(cores));
    wefficiency2_2 = zeros(1,length(cores));
611
    wt2_2 = wtime2_2(1) * ones(1, length(cores));
612
    for i=1:length(cores)
613
        wspeedup2_2(i) = wtime2_2(1)/wtime2_2(i)*cores(i);
614
         wefficiency2_2(i) = wspeedup2_2(i)/cores(i);
615
616
617
    % Sequence = 3 Strategy = 2
618
619
    wtime3_2 = zeros(1,length(cores));
620 wtime3_2(1) = 1.109987; % 1 cores
621
    wtime3_2(2) = 1.765386; % 2 cores
622 wtime3_2(3) = 2.568317; % 4 cores
623 wtime3_2(4) = 5.130037; % 8 cores
624 wtime3_2(5) = 5.817957; % 16 cores
    wtime3_2(6) = 10.754171; % 32 cores
625
626
    % Calculating speed-up, efficiency and ideal time
627
628 wspeedup3_2 = zeros(1,length(cores));
    wefficiency3_2 = zeros(1,length(cores));
629
    wt3_2 = wtime3_2(1) * ones(1, length(cores));
630
    for i=1:length(cores)
631
        wspeedup3_2(i) = wtime3_2(1)/wtime3_2(i)*cores(i);
632
         wefficiency3_2(i) = wspeedup3_2(i)/cores(i);
633
634
    end
635
636
    % Generating figures
   figure
637
638 hold on
639 grid on
    plot(cores, wtime0_2, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
640
    plot(cores, wtime1_2, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
641
642 plot(cores, wtime2_2, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
643 plot(cores, wtime3_2, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
644 %plot(cores, wt0_2, ':o', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
%plot(cores, wt1_2, ':o', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
646 %plot(cores, wt2_2, ':o', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
```

```
647 %plot(cores, wt3_2, ':o', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
     legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3');
649 title('Weak Scaling, Strategy = 2, Time');
650 xlabel('Nr of processes');
651 ylabel('Measured time');
     hold off
652
653
654
     figure
     hold on
655
     grid on
656
     plot(cores, wspeedup0_2, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
657
     plot(cores, wspeedup1_2, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
658
659 plot(cores,wspeedup2_2, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
660 plot(cores,wspeedup3_2, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
     plot(cores, cores, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
661
     legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3', 'Ideal');
662
     title('Weak Scaling, Strategy = 2, Speed-up');
663
     xlabel('Nr of processes');
664
     ylabel('Speed-up');
665
     hold off
666
     %----- Efficiency ------
668
     e = ones(1,length(cores));
669
670
     figure
671
     hold on
672
673 grid on
    plot(cores, efficiency0_0, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
plot (cores, efficiency1_0, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
676 plot (cores, efficiency2_0, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
677 plot (cores, efficiency3_0, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
678 plot(cores, e, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
679 ylim([-0.1 1.2])
680 legend('Sequence=0','Sequence=1','Sequence=2','Sequence=3','Ideal');
     title('Strong Scaling, Strategy = 0, Efficiency');
681
     xlabel('Nr of processes');
682
     vlabel('Efficiency');
683
684
     hold off
685
686
     figure
687
     hold on
688 arid on
689 plot(cores, efficiency0_1, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
690 plot (cores, efficiency1_1, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
691 plot (cores, efficiency2_1, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
692 plot (cores, efficiency3_1, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
693 plot(cores, e, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
694 ylim([-0.1 1.2])
     legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3', 'Ideal');
695
     title('Strong Scaling, Strategy = 1, Efficiency');
697 xlabel('Nr of processes');
     ylabel('Efficiency');
698
699 hold off
700
701
     figure
702 hold on
plot (cores, efficiency0_2, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);

ros plot (cores, efficiency1_2, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);

ros plot (cores, efficiency2_2, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);

ros plot (cores, efficiency3_2, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
708 plot(cores, e, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
709 ylim([-0.1 1.2])
     legend('Sequence=0','Sequence=1','Sequence=2','Sequence=3','Ideal');
710
711 title('Strong Scaling, Strategy = 2, Efficiency');
```

```
712 xlabel('Nr of processes');
713
      ylabel('Efficiency');
714 hold off
716 figure
717 hold on
718
      grid on
719 plot(cores, wefficiency0_0, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
720 plot(cores, wefficiency1_0, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
721 plot (cores, wefficiency2_0, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
722 plot (cores, wefficiency3_0, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
723 plot (cores, e, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
724 ylim([-0.1 1.2])
r25 legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3', 'Ideal');
726 title('Weak Scaling, Strategy = 0, Efficiency');
      xlabel('Nr of processes');
      ylabel('Efficiency');
728
729 hold off
730
      figure
731
732
      hold on
733 grid on
734 plot(cores, wefficiency0_1, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
735 plot(cores, wefficiency1_1, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
736 plot(cores, wefficiency2_1, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
737 plot(cores, wefficiency3_1, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
      plot(cores, e, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
738
739 ylim([-0.1 1.2])
740 legend('Sequence=0','Sequence=1','Sequence=2','Sequence=3','Ideal');
     title('Weak Scaling, Strategy = 1, Efficiency');
741
742 xlabel('Nr of processes');
743 ylabel('Efficiency');
744 hold off
745
      figure
746
747 hold on
748 grid on
749 plot(cores, wefficiency0_2, '-*', 'Color', [255/255 153/255 204/255], 'LineWidth', 2);
750 plot(cores, wefficiency1_2, '-*', 'Color', [204/255 153/255 255/255], 'LineWidth', 2);
751 plot(cores, wefficiency2_2, '-*', 'Color', [178/255 255/255 102/255], 'LineWidth', 2);
752 plot(cores, wefficiency3_2, '-*', 'Color', [255/255 178/255 102/255], 'LineWidth', 2);
753 plot(cores, e, ':o', 'Color', [102/255 178/255 255/255], 'LineWidth', 2);
754 ylim([-0.1 1.2])
155 legend('Sequence=0', 'Sequence=1', 'Sequence=2', 'Sequence=3', 'Ideal');
      title('Weak Scaling, Strategy = 2, Efficiency');
757 xlabel('Nr of processes');
758 ylabel('Efficiency');
759 hold off
```