Portfolio and Risk Management Project
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Part A:

I used 5-year monthly Netflix and AMC stocks data to plot the efficient frontier and determined the minimum variance portfolio of two. I discussed the risk-free asset's effect to find an optimal portfolio. Then, I compared Metro Inc. and Netflix stocks (negative correlation) and Coca-Cola Co. and PepsiCo stocks (high positive correlation) to compare with my previous selection. Finally, I argued fundamentally what these results could mean.

I used $r_i = \ln \frac{S_{i+1}}{S_i}$ to find each monthly return and then calculated the average of all 60 data for each stock. Next, I found the variance, standard deviation, covariance and correlation for two stocks using excel. I converted all the vales to annual.

$\bar{r}(annual) = \bar{r}(monthly) * 12,$ $\sigma(annual) = \sigma(monthly) * \sqrt{12},$ $\sigma_{12}(annual) = \sigma_{12}(monthly) * 12$

Here are the annual returns and standard deviations of NFLX and AMC stocks.

Table1	AMC(1)	NFLX(2)
Average Return	r_2 =-22.69%	r_1 =34.35%
Std.Dev	σ_2 =118.28%	σ_1 =33.10%
Variance	139.91%	10.95%
Covariance	σ_{12} =-0.072%	
Correlation	ρ = -0.0018~0	

Note: All values are annual.

The portfolio return and standard deviation can be obtained from the below formulas.

$$E(r_p) = \alpha r_1 + (1-\alpha) r_2$$

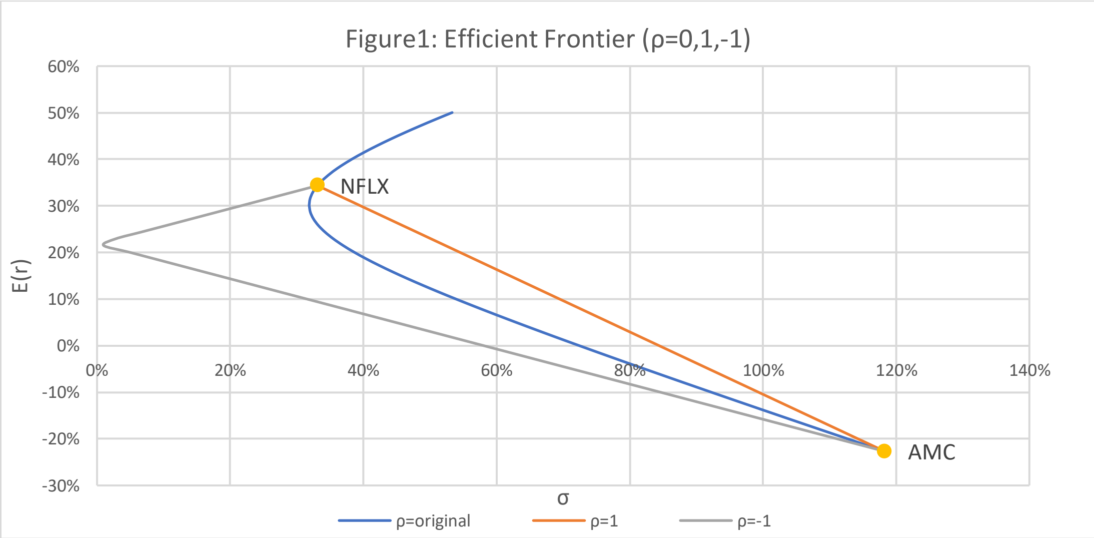
$$\sigma^2 = \alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha(1-\alpha)\sigma_{12}$$

$$\sigma = \sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha(1-\alpha)\sigma_{12}}$$

By putting all $r_1, r_2, \sigma_1, \sigma_2, \sigma_{12}$ values in the above formulas and changing α, we can find a minimum variance portfolio using Excel.

α =7.3%, r=30.19%, σ=31.86%

To draw an efficient frontier diagram using Excel, we should change α to find r_p and σ values. The correlation, ρ between NFLX and AMC are close to zero. I drew two other plots by putting ρ to 1 and -1 to see the effect of positive and negative correlation, respectively. We can see all three scenarios in the below plot.



We have assumed there are only two risky assets (NFLX and AMC). If we add a risk-free asset to our portfolio, what will happen with and without short selling? Adding a risk-free asset can expand the feasible region we want to invest in and help us find the single optimal portfolio. To find it, we need to construct the Sharpe ratio formula and maximize it to become a tangent line to the efficient frontier curve. There are two ways to find the optimal portfolio: One is to use a solver in Excel. To do that, we define the constraints and the Sharpe ratio that we need to maximize to find optimal portfolio weights, return and standard deviation. The second is to use below formulas:

Sharpe ratio: $S_a = \frac{\bar{r} - r_f}{\sigma_p}$

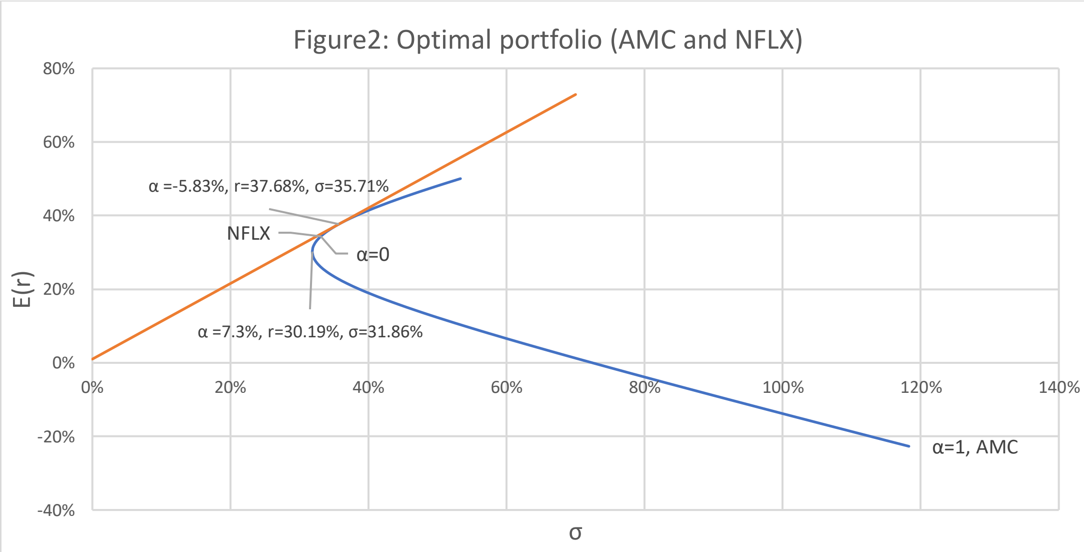
$\sum_{i=1}^{n} \sigma_{ki} v_i = \bar{r} - r_f, \qquad k = 1, 2, \dots, n$

$\omega_i = \frac{v_i}{\sum_{k=1}^{n} v_i}$

ω_1 = -5.83%, ω_2 = 105.83%, r= 37.68%, σ= 35.71%

This result showed us we needed to borrow about 6% of our investment from AMC stock and invest 106% in NFLX stock. If we did not have the option to borrow, we should have allocated 100% of our investment in NFLX stock for having an optimal portfolio.

Below plot is the risk-free line tangent to efficient frontier curve with minimum variance and optimal portfolio weights, return and standard deviation data.

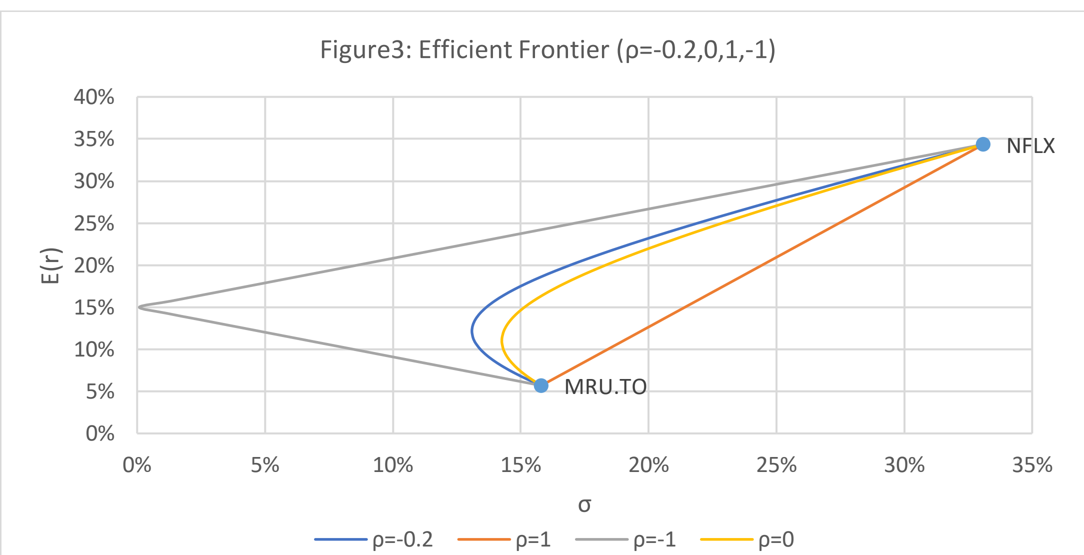


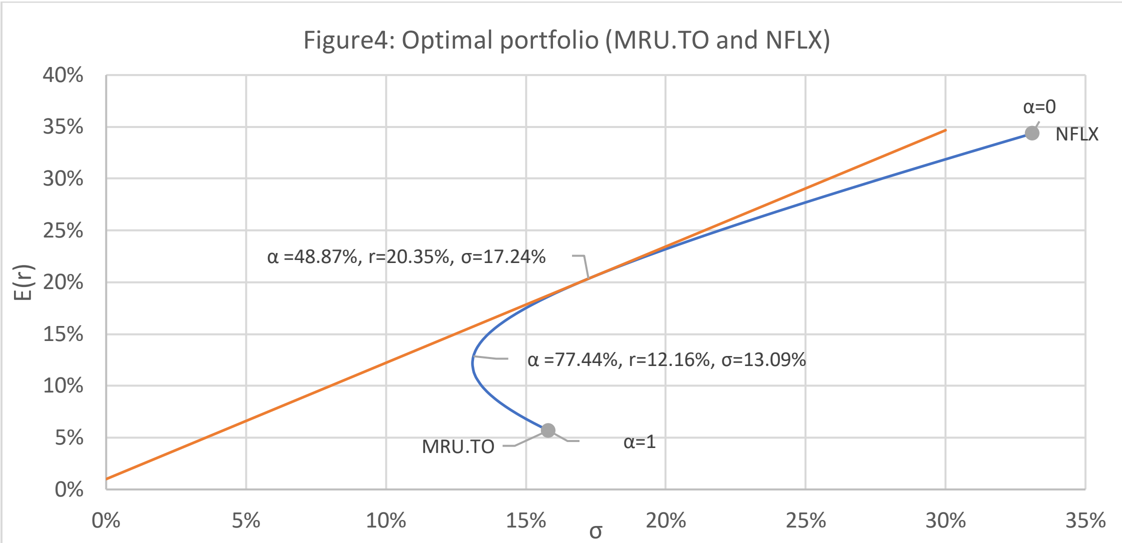
To better understand the negative and positive correlation impact, I picked two other stocks (Metro Inc. and Netflix) as a real-world example of negative correlation and The Coca-Cola Co. and PepsiCo stocks for positive correlation. For concision, I just showed the result.

Metro Inc. and Netflix:

Table2	AMC(1)	NFLX(2)
Average Return	r_2=5.69%	r_1 =34.35%
Std.Dev	σ_2 =15.81%	σ_1 =33.10%
Variance	2.5%	10.95%
Covariance	σ_{12} =-0.98%	
Correlation	ρ = -0.19	

Note: All values are annual.

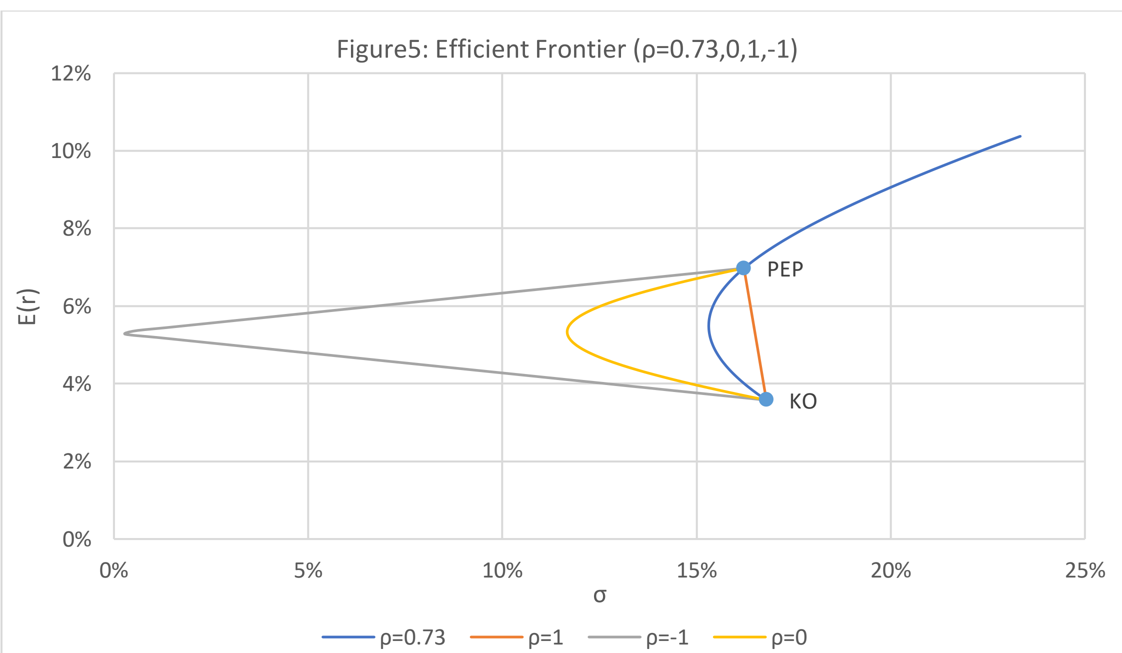


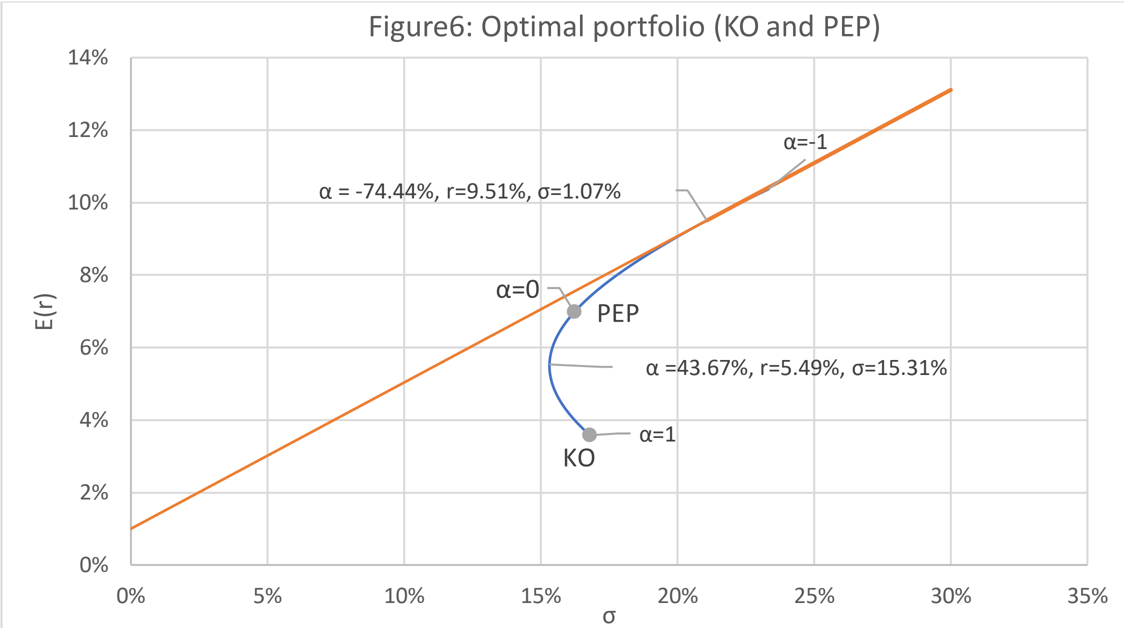


Coca-Cola Co. and PepsiCo:

Table3	KO(1)	PEP(2)
Average Return	r_2=3.58%	r_1 =6.98%
Std.Dev	σ_2 =16.80%	σ_1 =16.22%
Variance	2.82%	2.63%
Covariance	σ_{12} =1.98%	
Correlation	ρ = 0.73	

Note: All values are annual.





After considering all three scenarios, we could see as correlation changes from 1 to -1; the efficient frontier curve moves to the left. As a result, we have a more feasible region in negative correlation assets o construct our portfolio.

Discussion:

AMC is the largest American movie theatre chain in the world. Netflix, Inc. is an American over-the-top content platform and production company. I picked these stocks to see what the effect of online platforms on movie theatre is. Presumably, I believe with the development of online platforms fewer people go to cinemas, so the correlation should be negative. However, the result shows there is no correlation between these two incorporations.
Metro Inc. is a Canadian food retailer which produces non-discretionary goods. People spend money on food and groceries regardless of the economic situation. The company's beta is -0.15, which shows a negative correlation (-0.19) to the whole market. My result also indicates a negative correlation between Netflix and Metro stocks.
My final selection is two stocks (Coca-Cola and Pepsi) that belong to the soft drink industry. The result shows they have a high positive correlation (0.73) which is expected. Having both stocks in our portfolio does not add much diversification, as is evident in their efficient portfolio plot.

Part B:

In this part, I picked 2-year monthly data of five stocks (LOW, TSLA, NEM, GOLD, SPOT). Two of these (Barrick Gold and Newmont) are gold mining companies. Tesla is an American electric vehicle and clean energy company. Spotify is a Swedish audio streaming and media services provider, and Lowe is an American retail company specializing in home improvement.
I found the annual average return, variance, standard deviation, covariance matrix and correlation matrix for these five stocks using the same steps in part A.

Table4	Low(1)	TSLA(2)	NEM(3)	GOLD(4)	SPOT(5)
Average Return	28.11%	132.60%	34.16%	25.56%	36.05%
Std.Dev	34.98%	73.79%	28.25%	39.44%	46.81%

Note: All values are annual.

Covariance Matrix: $$\Sigma = \begin{bmatrix} 12.23\% & 12.09\% & 3.79\% & 6.36\% & 5.79\% \\ 12.09\% & 54.45\% & 5.81\% & 9.29\% & 20.57\% \\ 3.79\% & 5.81\% & 7.98\% & 9.83\% & 3.26\% \\ 6.36\% & 9.29\% & 9.83\% & 15.55\% & 5.69\% \\ 5.79\% & 20.57\% & 3.26\% & 5.69\% & 21.91\% \end{bmatrix}$$

Correlation Matrix: $$\mathbb{P} = \begin{bmatrix} 1 & 0.469 & 0.383 & 0.461 & 0.354 \\ 0.469 & 1 & 0.279 & 0.319 & 0.595 \\ 0.383 & 0.279 & 1 & 0.882 & 0.246 \\ 0.461 & 0.319 & 0.882 & 1 & 0.308 \\ 0.354 & 0.595 & 0.246 & 0.308 & 1 \end{bmatrix}$$

The portfolio return and standard deviation can be obtained from the below formulas.

$$E(r_p) = \sum_{i=1}^{5} \omega_i r_i$$

$$\sigma_p^2 = \sum_{i=1}^{5} \sum_{j=1}^{5} \omega_i \, \omega_j \, \sigma_{ij} = W^T . \Sigma . \mathrm{W}$$

$$\sigma_p = \sqrt{\sigma_p^2}$$

Where $$\Sigma = \begin{bmatrix} {\sigma^2}_1 & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \sigma_{34} & \sigma_{35} \\ \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_4^2 & \sigma_{45} \\ \sigma_{15} & \sigma_{25} & \sigma_{35} & \sigma_{45} & \sigma_5^2 \end{bmatrix}, \quad W = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{bmatrix}$$

Using solver in Excel, I found a minimum variance portfolio of these five stocks with and without short selling.

With short selling:

$$W = \begin{bmatrix} 0.33 \\ -0.08 \\ 1.1 \\ -0.53 \\ 0.19 \end{bmatrix}, \sigma_p^2 = 4.91\%, \quad \sigma_p = 22.16\%, \quad E(r_p) = 29.25\%$$

No short selling:

$$W = \begin{bmatrix} 0.26 \\ 0 \\ 0.61 \\ 0 \\ 0.13 \end{bmatrix}, \sigma_p^2 = 6.28\%, \quad \sigma_p = 25.05\%, \quad E(r_p) = 32.80\%$$

To have the minimum variance, it should not be any Tesla stock in our portfolio. Moreover, NEM and LOW stock allocate the most portion of our portfolio regarding their non-cyclical industry behaviour. It is worth mentioning; we need to borrow GOLD stock to have the minimum variance since it has relatively low profit and high risk compared to low-risk assets in our portfolio.

To draw an efficient frontier with short selling, I constrained the portfolio return to 1% in the solver to find each portfolio's weight and SD. I incremented the value of portfolio return by 5% to find the next point of the efficient frontier portfolio. I repeated these steps and found 25 points of the efficient frontier and drew the risk-return diagram.

In case of no short selling, I constrained the portfolio return from the lowest stock return, which was 25.56%(GOLD), to the highest return, 132.60 (TSLA). Then, I drew the risk-return diagram as is shown below.

By adding a risk-free asset to our portfolio, we can expand the feasible region for investing. Theoretically, we can obtain an optimal portfolio by maximizing the Sharpe ratio.
Here are the simplified formulas that could provide us with the optimal portfolio's weights, return and standard deviation. It is important to note that this method only gives us the result with the assumption of short selling.

Sharpe ratio: $S_a = \frac{\bar{r}-r_f}{\sigma_p}$

$\Sigma\,.\mathrm{V} = \bar{r} - r_f$ or $\mathrm{V} = \Sigma^{-1}(\bar{r} - r_f)$, $\omega_i = \frac{v_i}{\sum_{k=1}^{n} v_i}$

Where $\Sigma = \begin{bmatrix} {\sigma^2}_1 & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \sigma_{34} & \sigma_{35} \\ \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_4^2 & \sigma_{45} \\ \sigma_{15} & \sigma_{25} & \sigma_{35} & \sigma_{45} & \sigma_5^2 \end{bmatrix}$, $W = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{bmatrix}$, $V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$, $\bar{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix}$, $r_f = \begin{bmatrix} r_f \\ r_f \\ r_f \\ r_f \\ r_f \end{bmatrix}$

If we want to have the result with no short selling, we can use a solver in Excel. The results for the two scenarios are shown below.

With short selling:

$$W = \begin{bmatrix} -0.003 \\ 0.471 \\ 1.717 \\ -1.036 \\ -0.149 \end{bmatrix}, \sigma_p^2 = 15.32\%, \quad \sigma_p = 39.14\%, \quad E(r_p) = 89.13\% ,$$

With short selling:

$$W = \begin{bmatrix} 0 \\ 0.452 \\ 0.548 \\ 0 \\ 0 \end{bmatrix}, \sigma_p^2 = 16.39\%, \quad \sigma_p = 40.48\%, \quad E(r_p) = 78.63\%$$

We can see from this result; when we have an option to short sell, we can get a return close to 90% with the standard deviation of about 40%, which improves a lot, comparing with allocating 100% investment to every single stock in our portfolio.

I plotted the efficient portfolio with a risk-free asset line for two cases: with and without short selling.

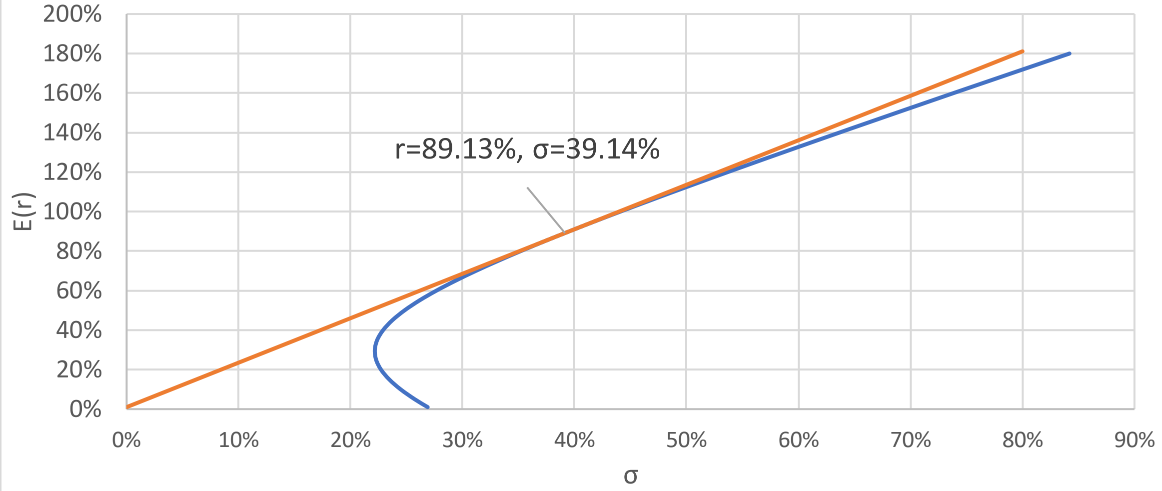


Figure8: Optimal portfolio (with short selling)

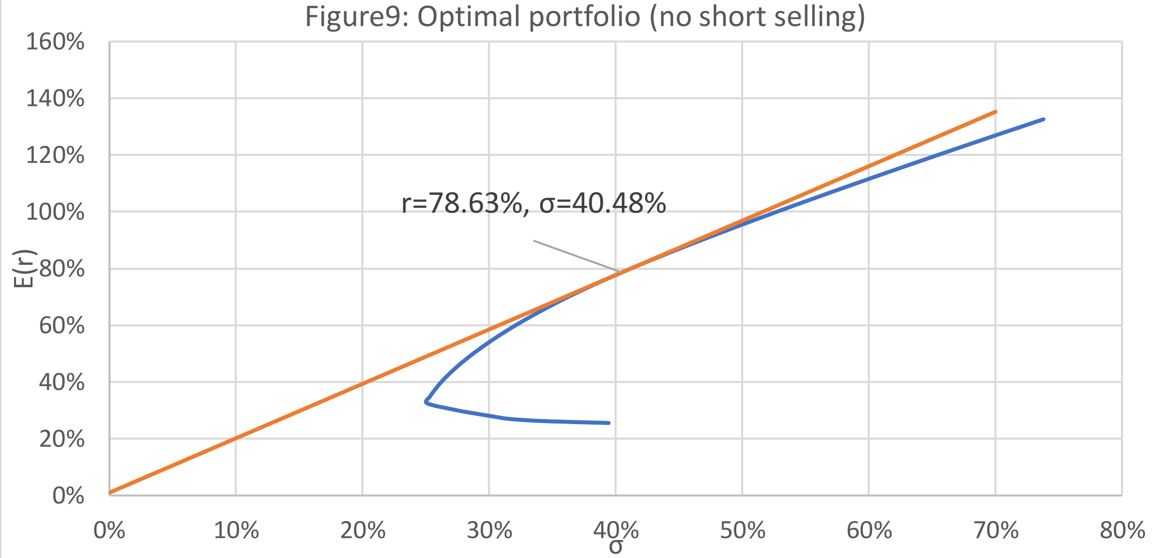
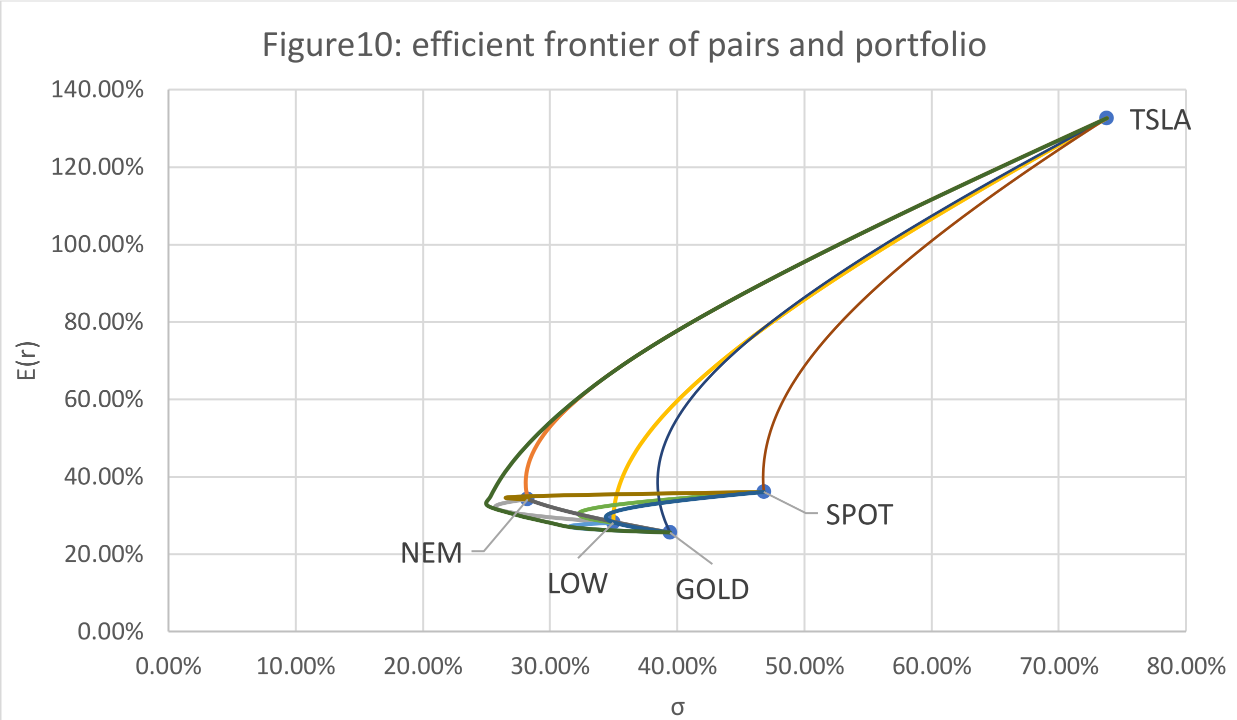


Figure9: Optimal portfolio (no short selling)

Figure 10 shows the efficient frontier of each pair and portfolio efficient frontier in one figure. It is indicated for the profit of more than 65%; we only need to allocate our portfolio to TSLA and NEM stocks.



Discussion:

After investigating the result, we could argue that Lowe stock is not helpful for our portfolio because it has the least portion in our basket of stocks when we find the minimum variance portfolio and optimal portfolio weights. Tesla stock is beneficial if we want to maximize our profit while having a reasonable risk. Newmont stock is also a great choice to decrease our portfolio risk; however, Barrick Gold stock is a horrible option because it is in the same sector as Newmont and has lower return and higher risk. We cannot certainly argue the benefit of Spotify stock since it is in a different industry and might have a bright future due to audio streaming development.