

# HOUGH TRANSFORM

Circles

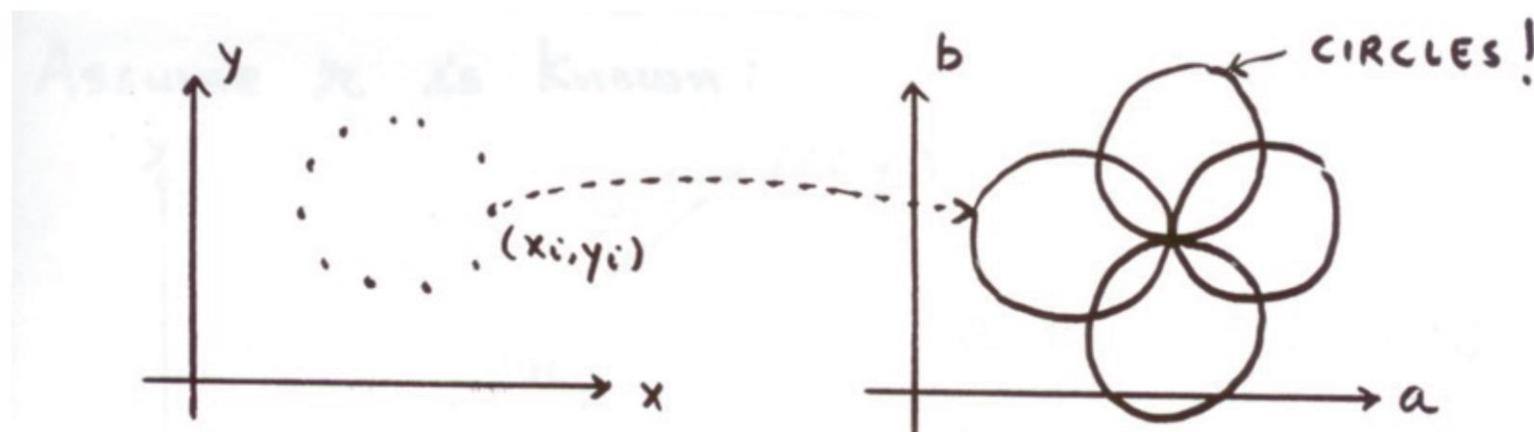
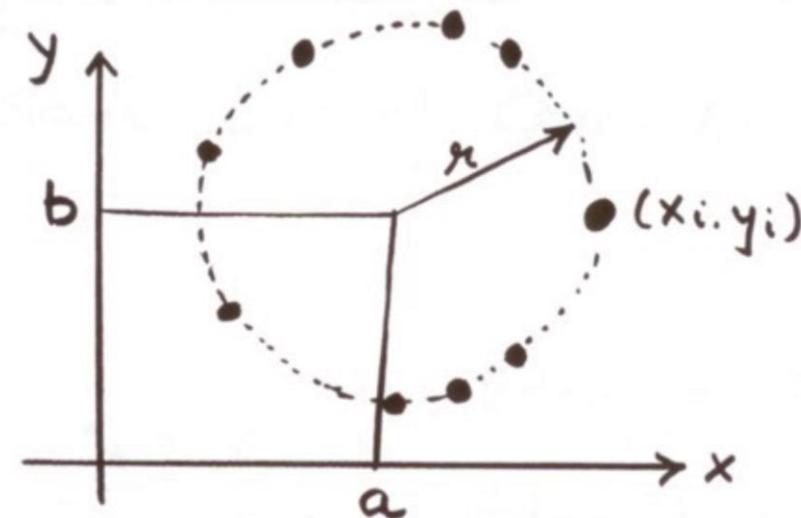
# Finding Circles by Hough Transform

Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

If radius is known: (2D Hough Space)

Accumulator Array  $A(a, b)$



parameters  
 $(x - a)^2 + (y - b)^2 = r^2$   
variables

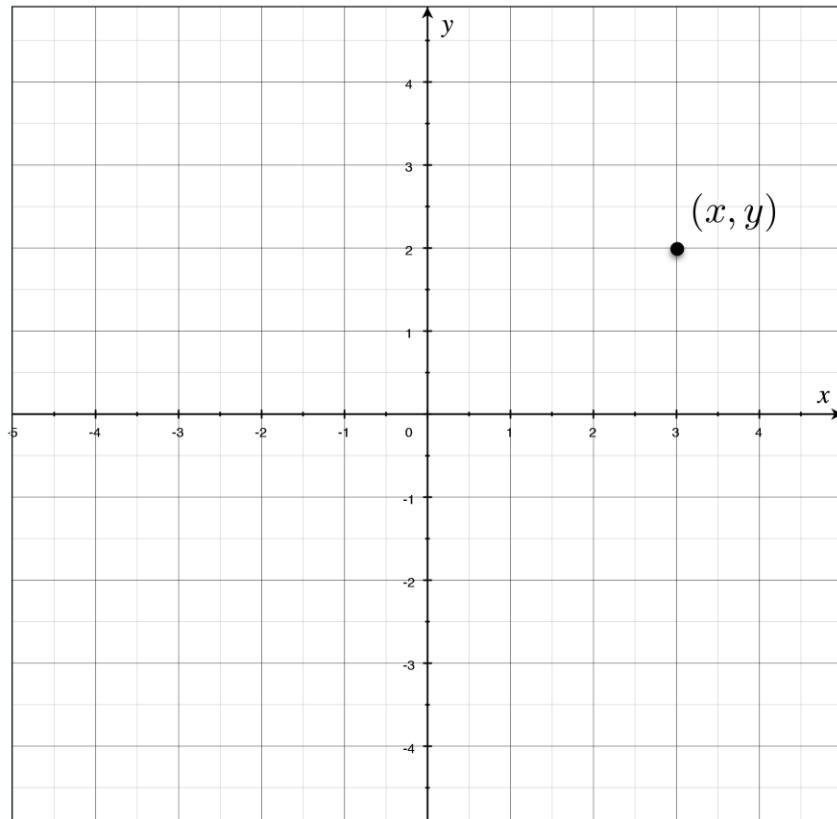
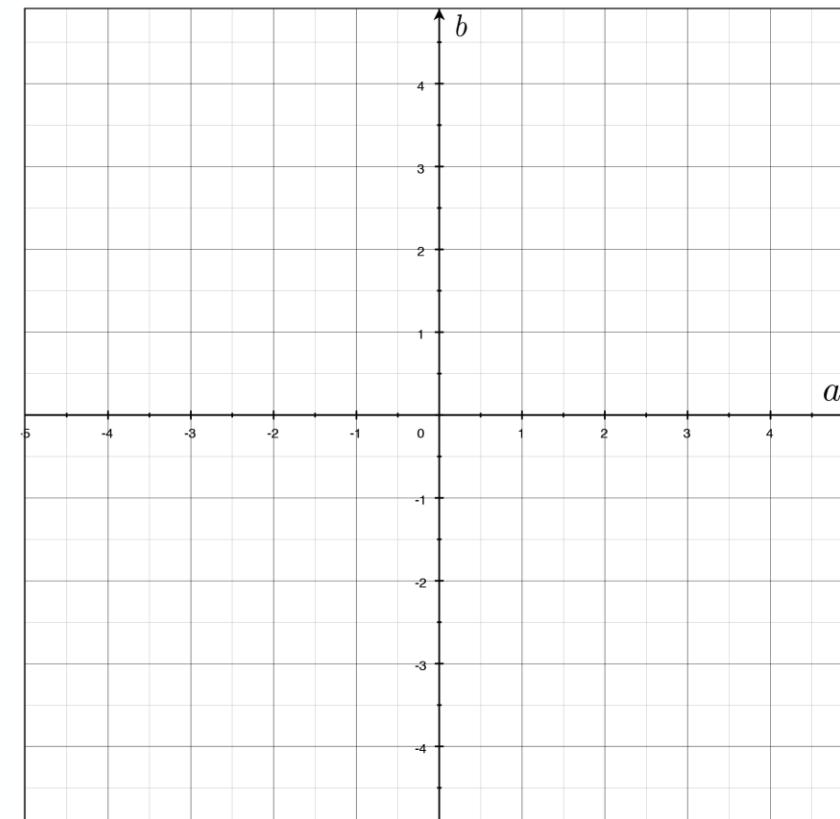


Image space

parameters  
 $(x - a)^2 + (y - b)^2 = r^2$   
variables



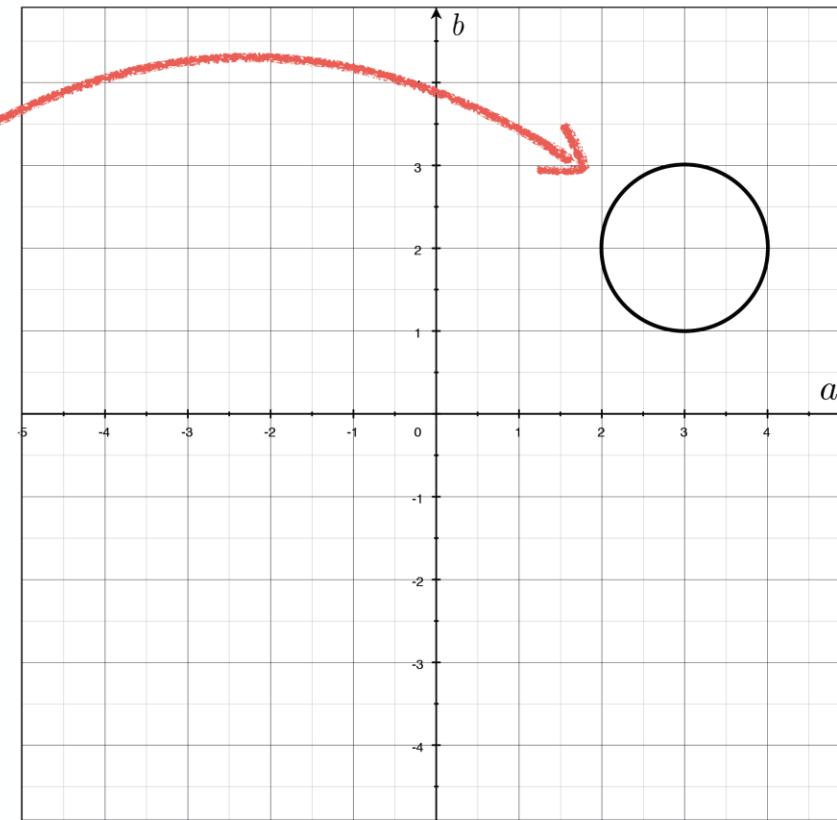
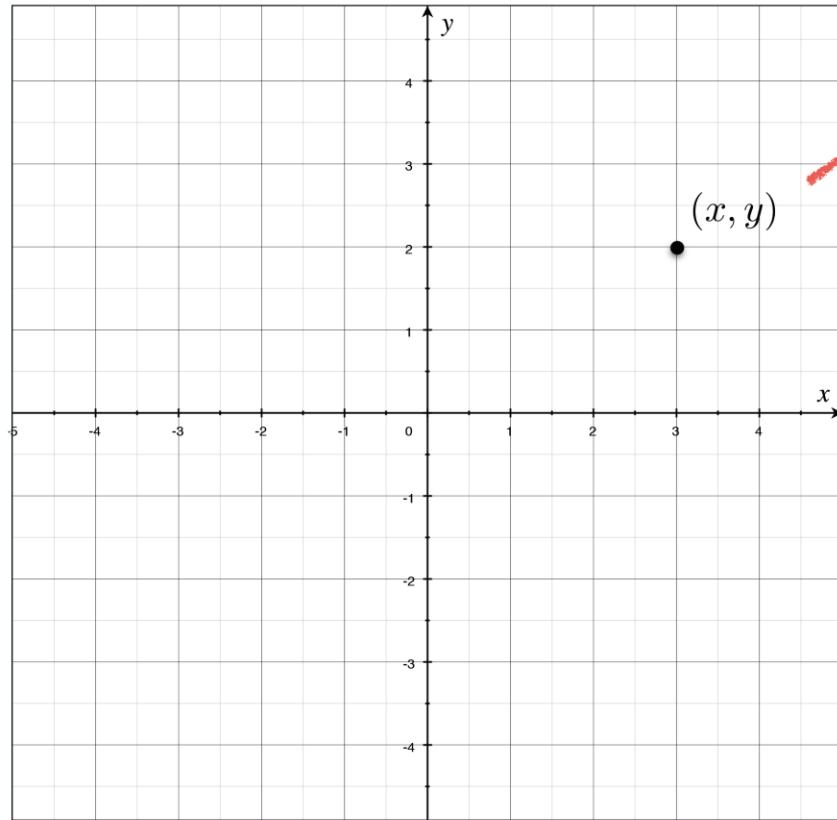
Parameter space

parameters  
variables

$$(x - a)^2 + (y - b)^2 = r^2$$

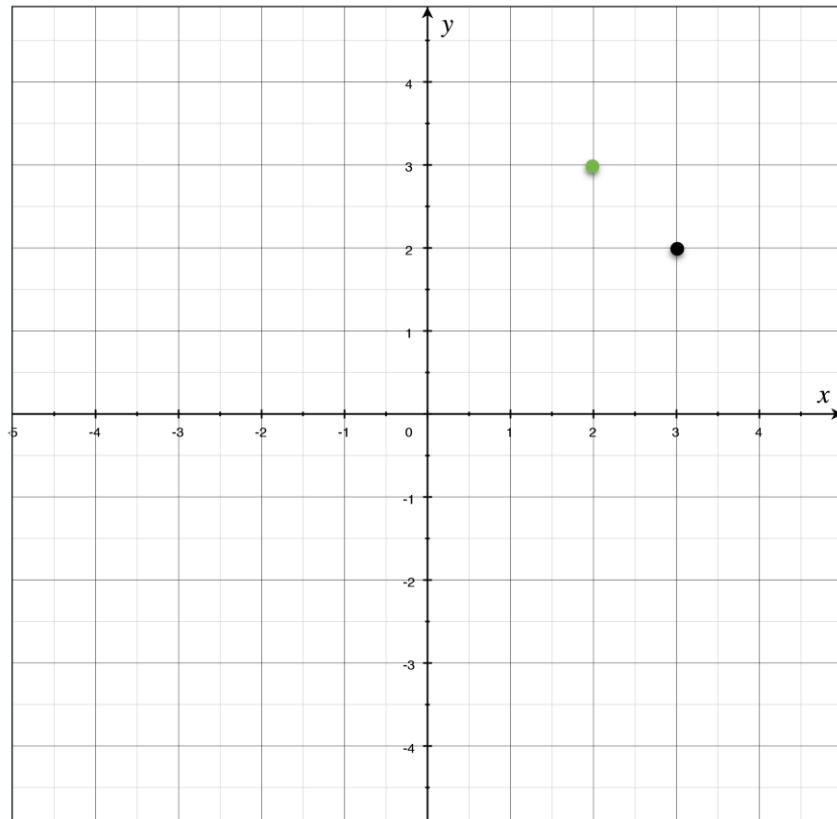
parameters  
variables

$$(x - a)^2 + (y - b)^2 = r^2$$



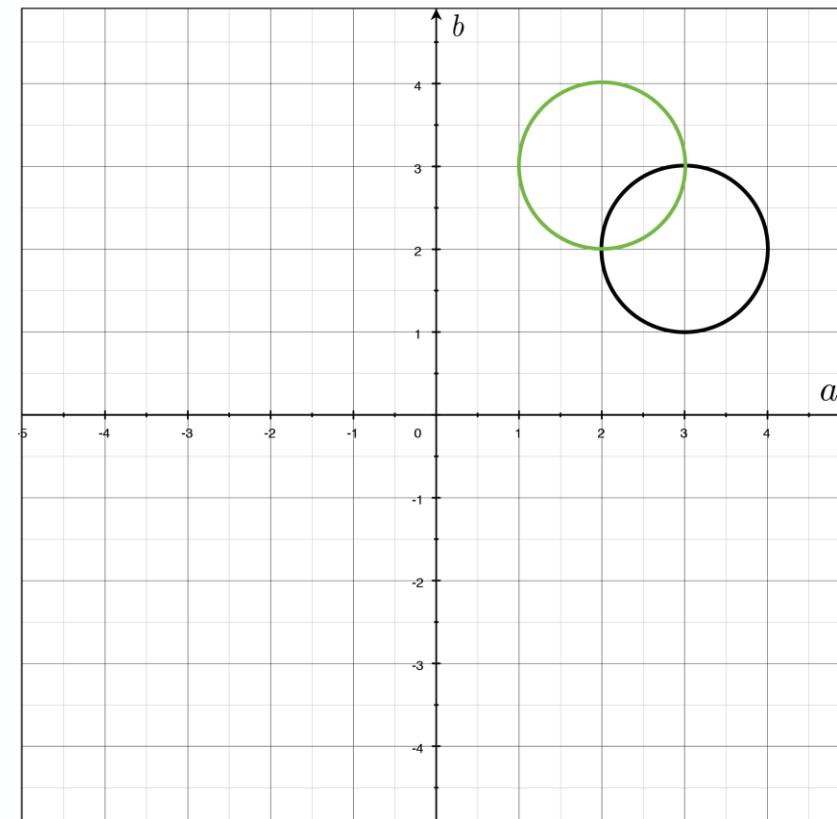
parameters  
variables

$$(x - a)^2 + (y - b)^2 = r^2$$

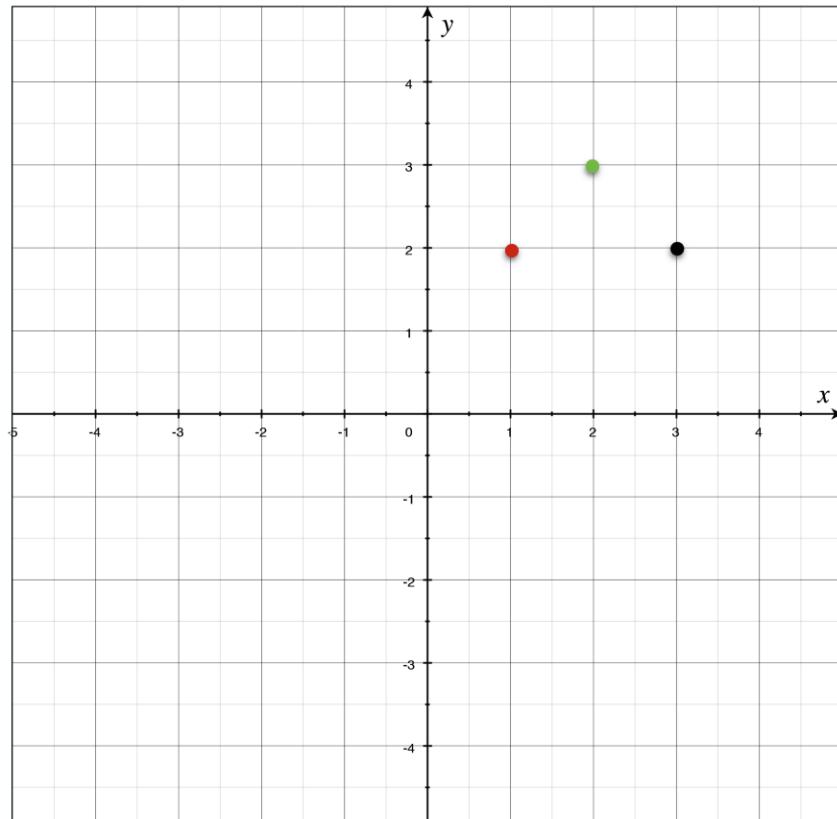


parameters  
variables

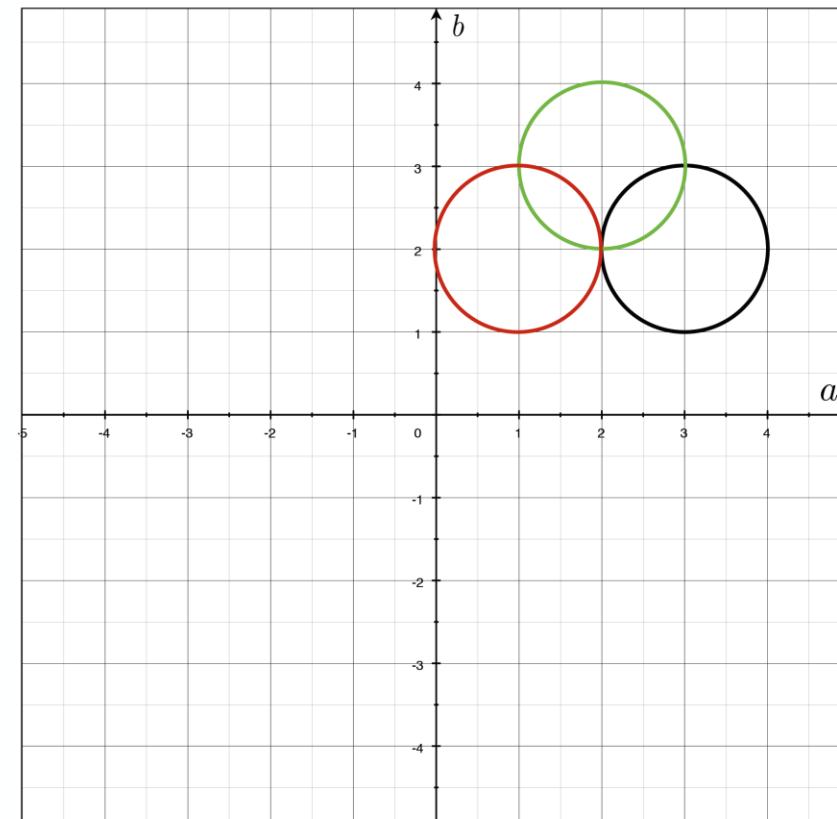
$$(x - a)^2 + (y - b)^2 = r^2$$



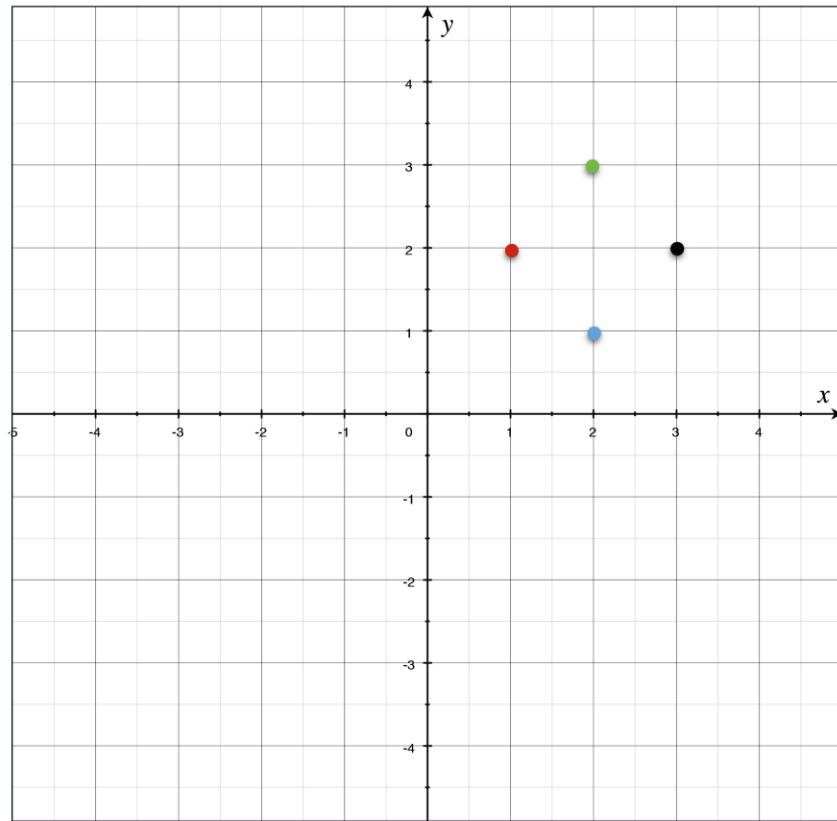
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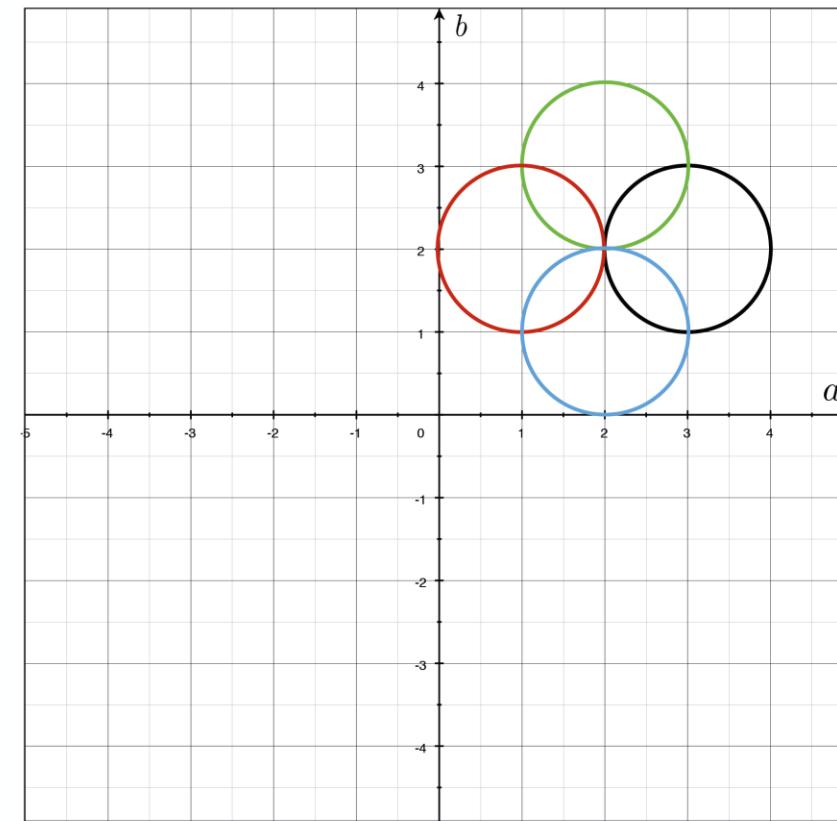
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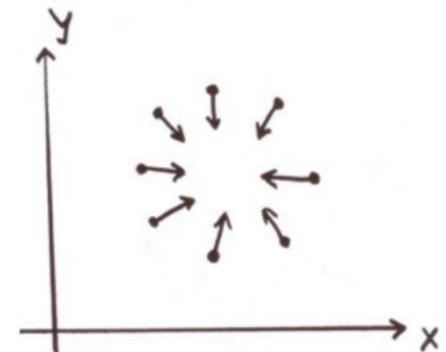


# Using Gradient Information

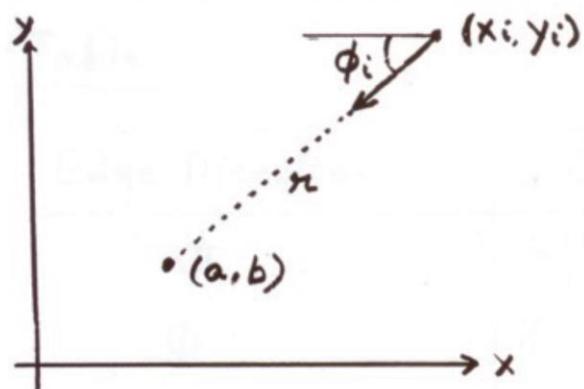
Gradient information can save lot of computation:

Edge Location  $(x_i, y_i)$

Edge Direction  $\phi_i$



Assume radius is known:



$$a = x - r \cos\phi$$

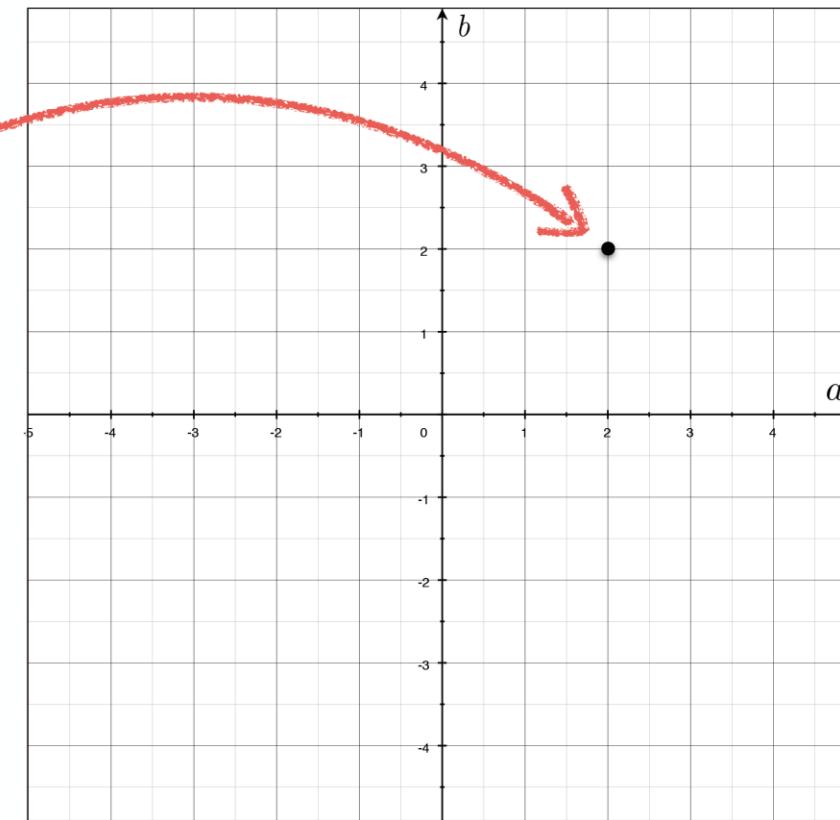
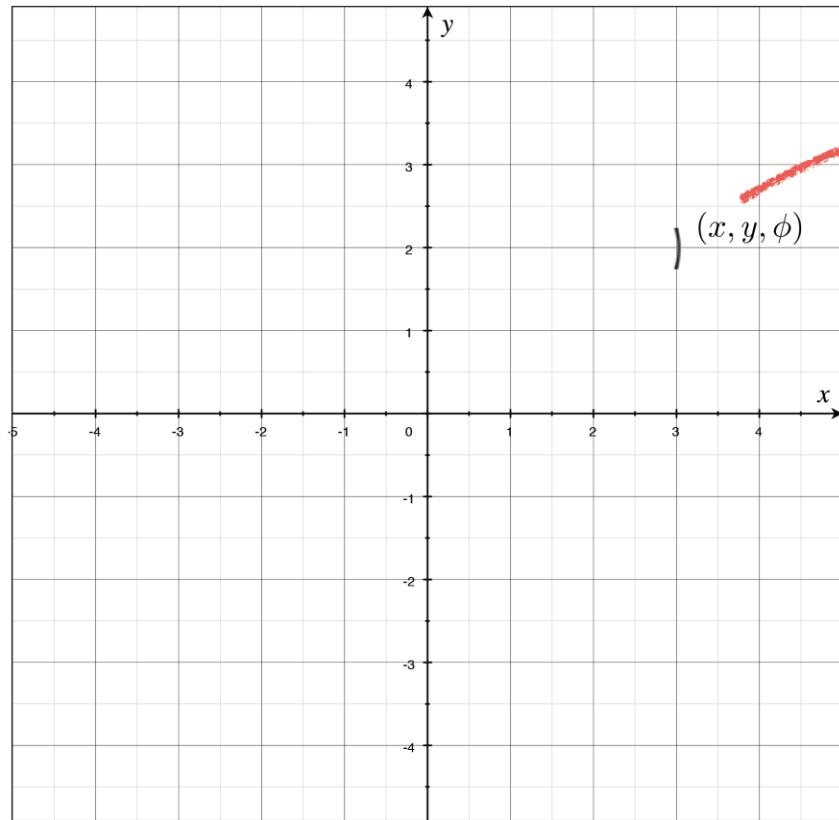
$$b = y - r \sin\phi$$

*Need to increment only one point in accumulator!!*

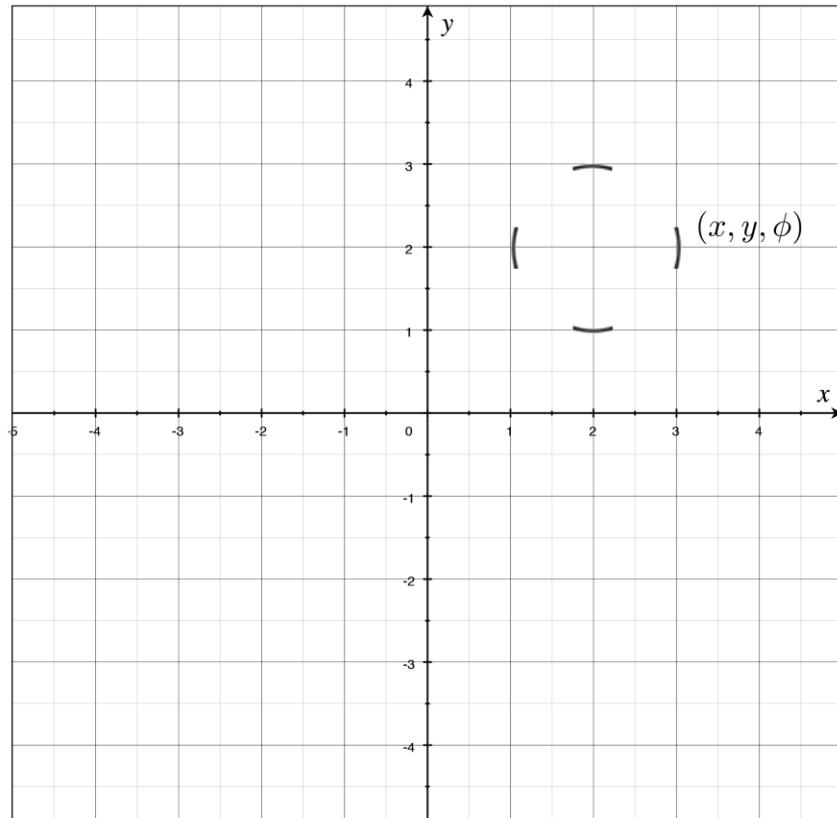


parameters  
 $(x - a)^2 + (y - b)^2 = r^2$   
variables

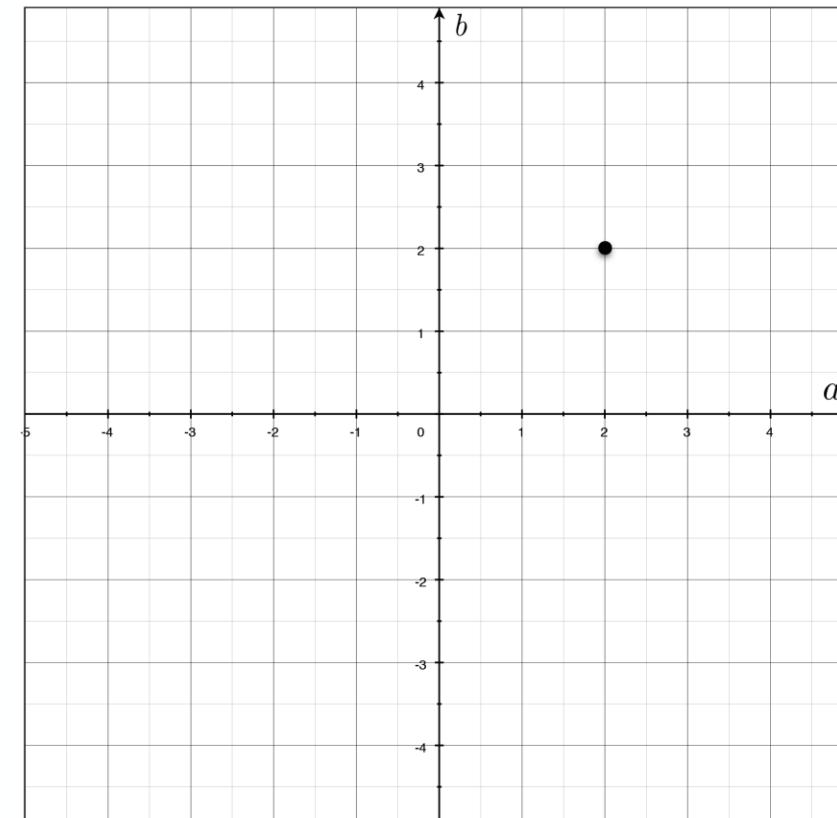
parameters  
 $(x - a)^2 + (y - b)^2 = r^2$   
variables

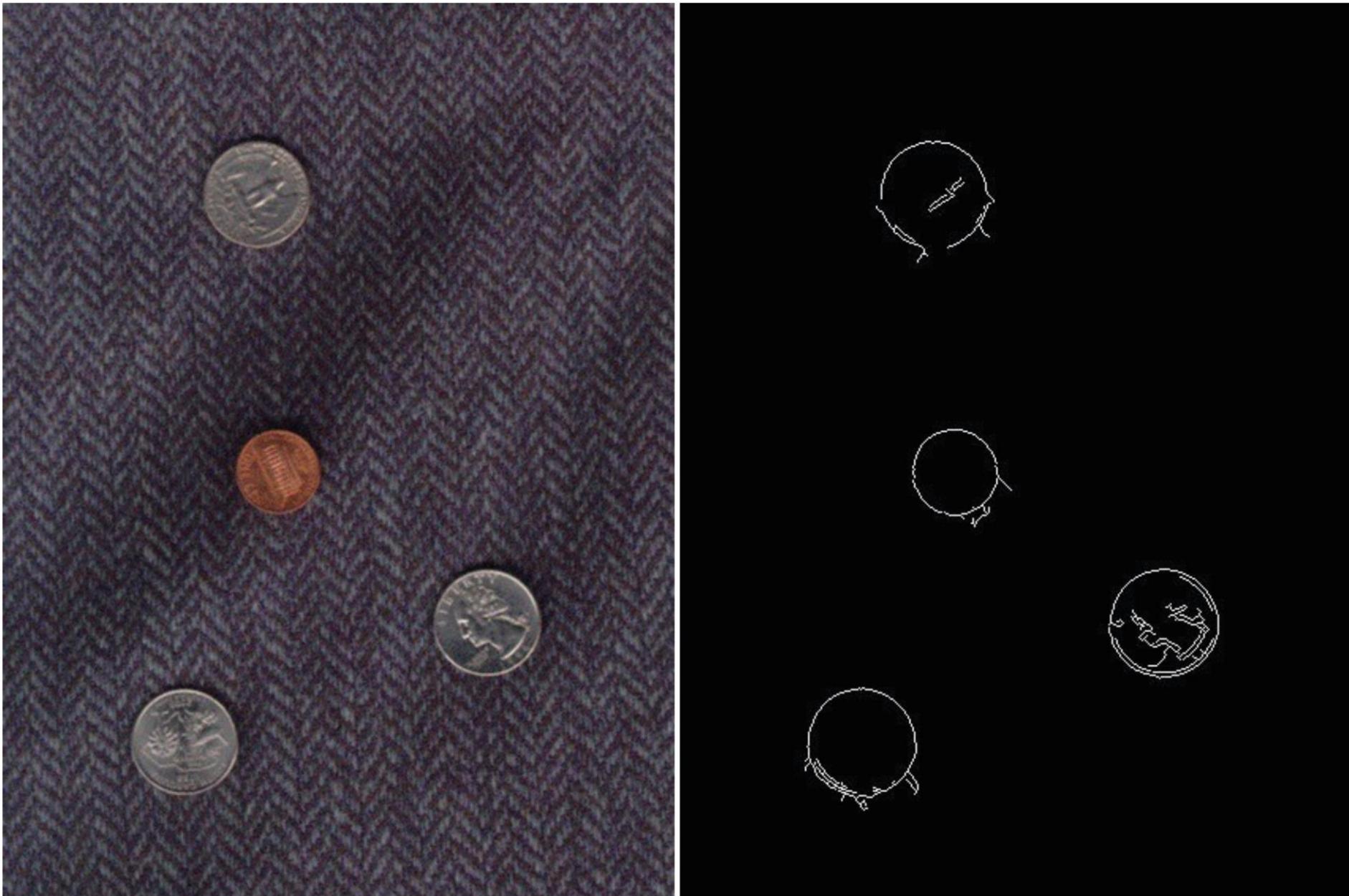


parameters  
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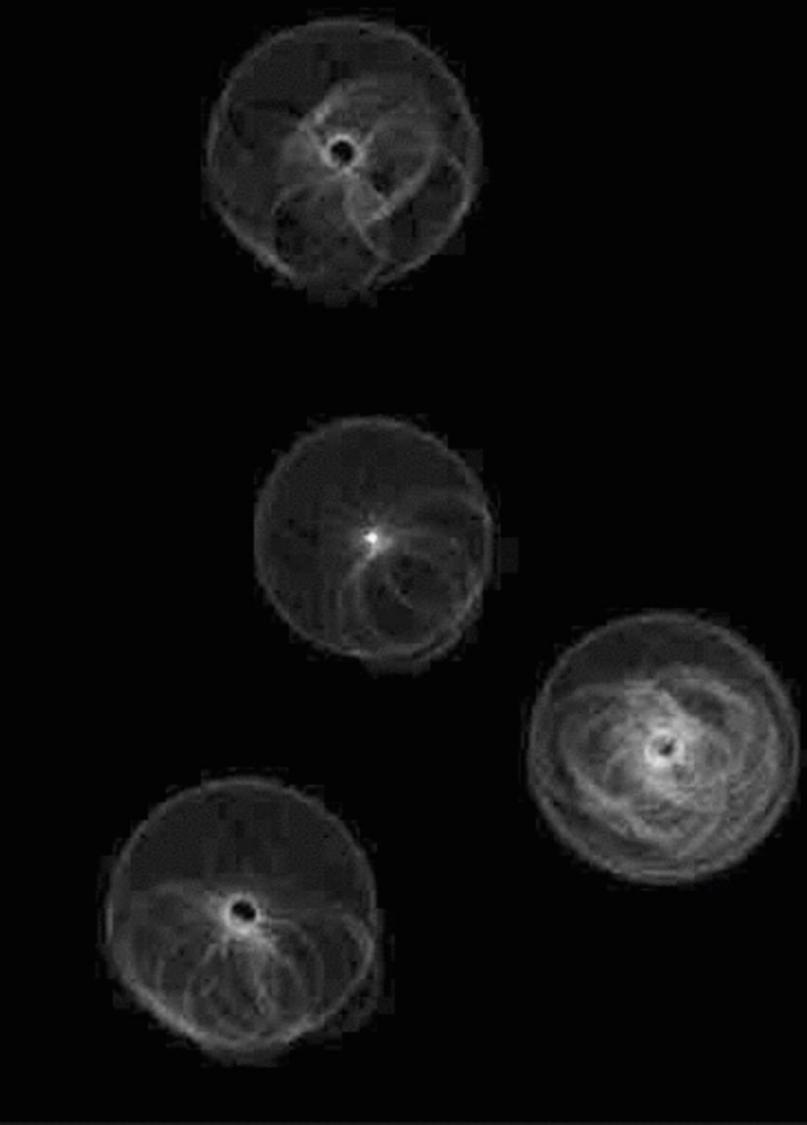
Pennie Hough detector



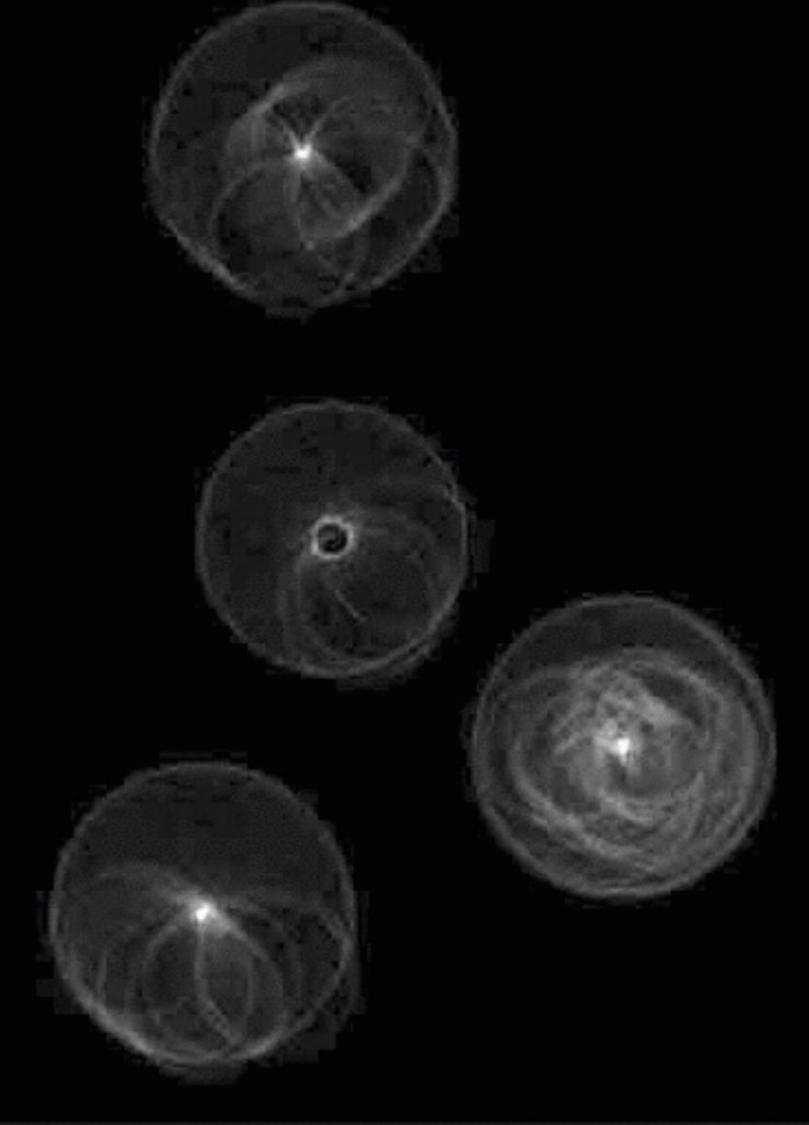
Quarter Hough detector



Pennie Hough detector



Quarter Hough detector

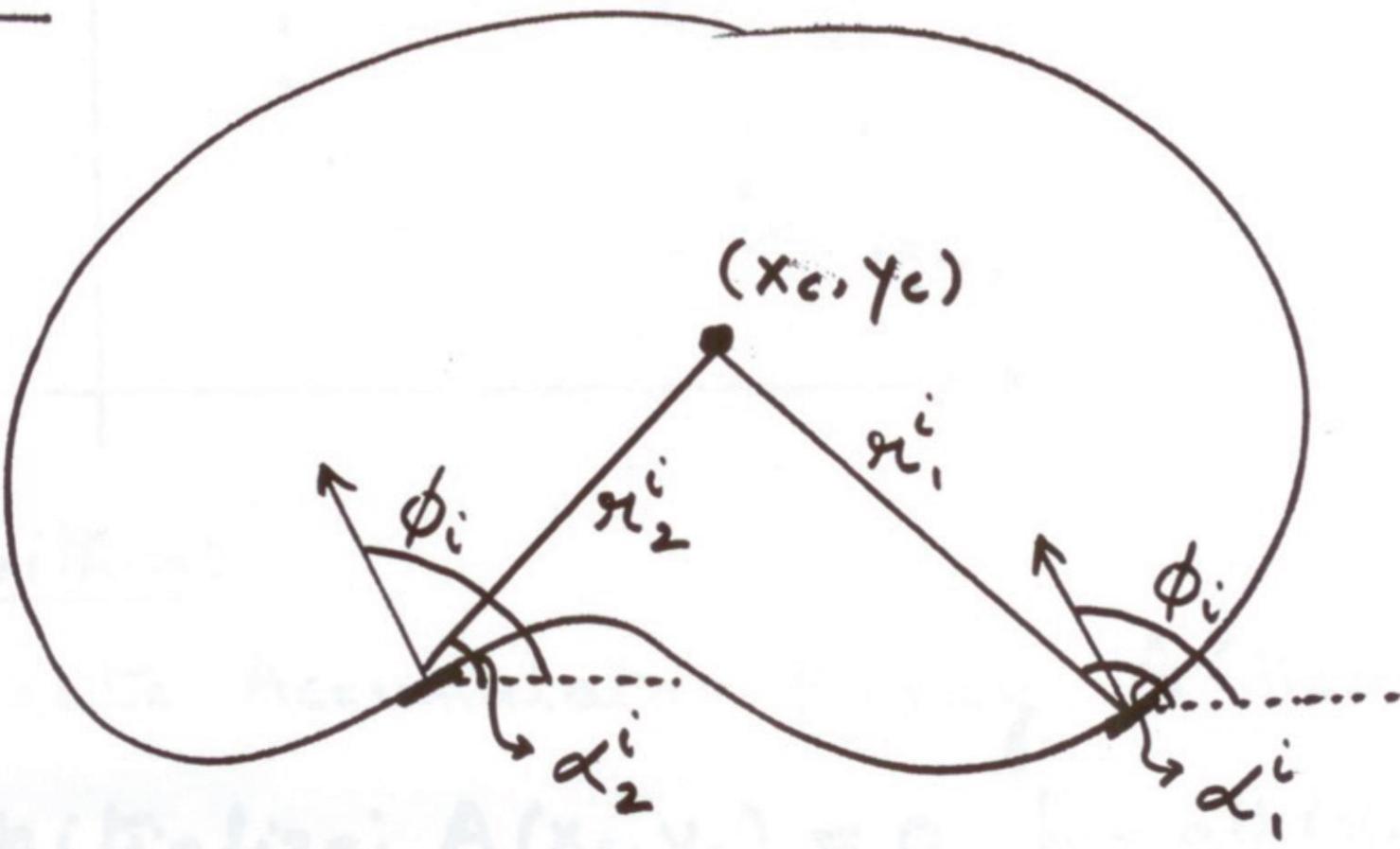


Can you use Hough Transforms for other objects,  
beyond lines and circles?



# Generalized Hough Transform

Model :



## $\phi$ -Table

Edge Direction	$\bar{\pi} = (\pi, \alpha)$
$\phi_1$	$\bar{\pi}_1^1, \bar{\pi}_2^1, \bar{\pi}_3^1$
$\phi_2$	$\bar{\pi}_1^2, \bar{\pi}_2^2$
$\phi_i$	$\bar{\pi}_1^i; \bar{\pi}_2^i$
$\phi_n$	$\bar{\pi}_1^n, \bar{\pi}_2^n$



# Generalized Hough Transform

Find Object Center  $(x_c, y_c)$  given edges  $(x_i, y_i, \phi_i)$

Create Accumulator Array  $A(x_c, y_c)$

Initialize:  $A(x_c, y_c) = 0 \quad \forall (x_c, y_c)$

For each edge point  $(x_i, y_i, \phi_i)$

    For each entry  $\bar{r}_k^i$  in table, compute:

$$x_c = x_i + r_k^i \cos \alpha_k^i$$

$$y_c = y_i + r_k^i \sin \alpha_k^i$$

    Increment Accumulator:  $A(x_c, y_c) = A(x_c, y_c) + 1$

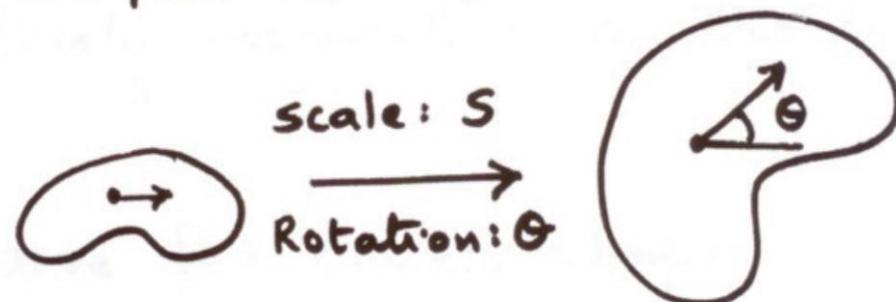
Find Local Maxima in  $A(x_c, y_c)$



## Scale & Rotation:

Use Accumulator Array:

$$A[x_c, y_c, s, \theta]$$



Use:

$$x_c = x_i + r_k^i s \cos(\alpha_k^i + \theta)$$

$$y_c = y_i + r_k^i s \sin(\alpha_k^i + \theta)$$

$$A(x_c, y_c, s, \theta) = A(x_c, y_c, s, \theta) + 1.$$



Do you have to use edge detectors  
to vote in Hough Space?



**A. Train phase:**

1. Get features

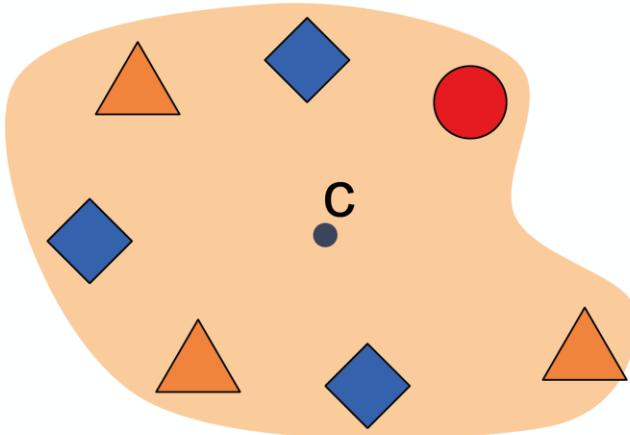
2. Store all displacements of feature from center

**B. Test phase:**

1. Get features & lookup displacements

2. Vote for center location

## Template



**A. Train phase:**

1. Get features

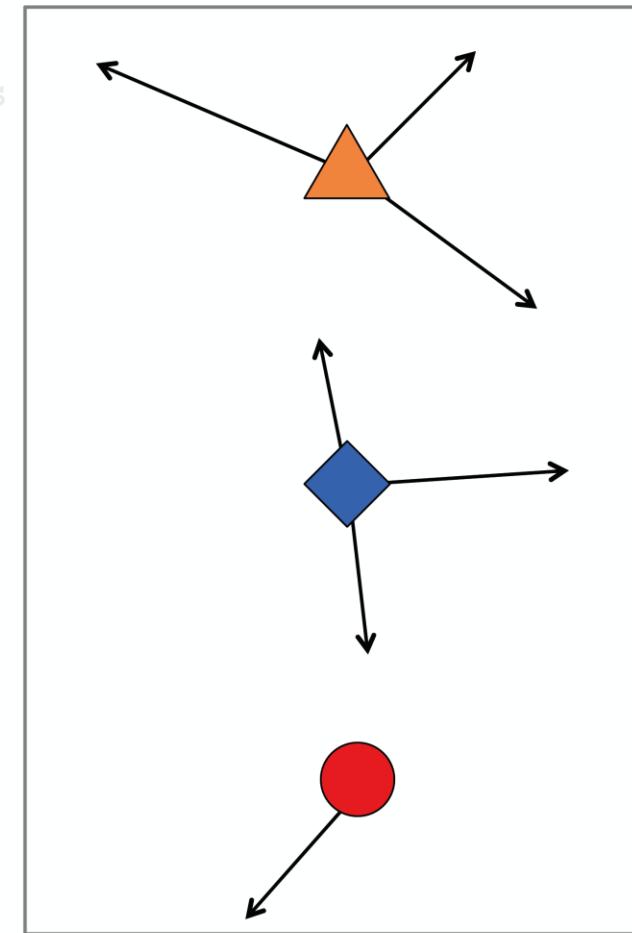
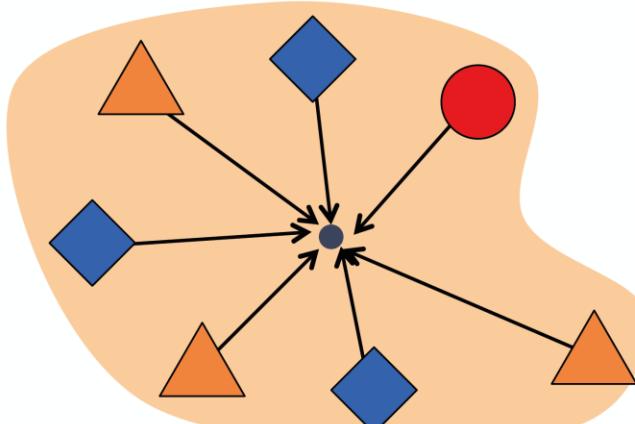
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Template



A. Train phase:

1. Get features

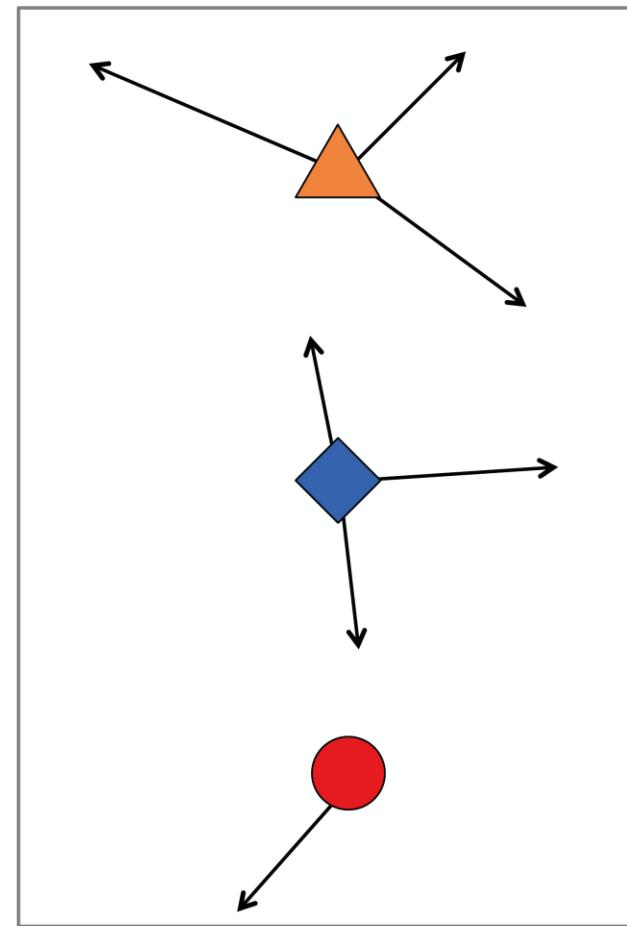
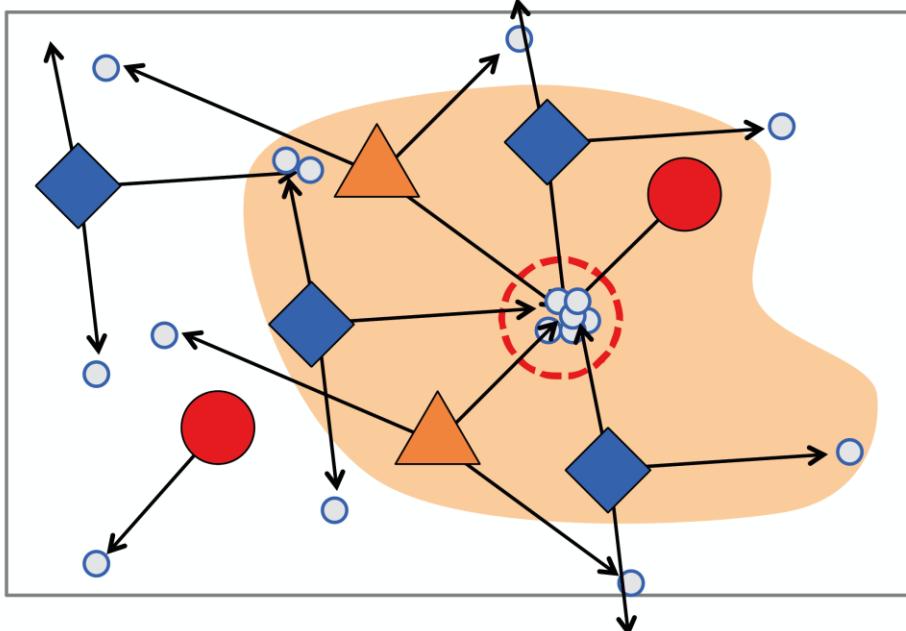
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B. Test phase:

1. Get features & lookup displacements

2. Vote for center location

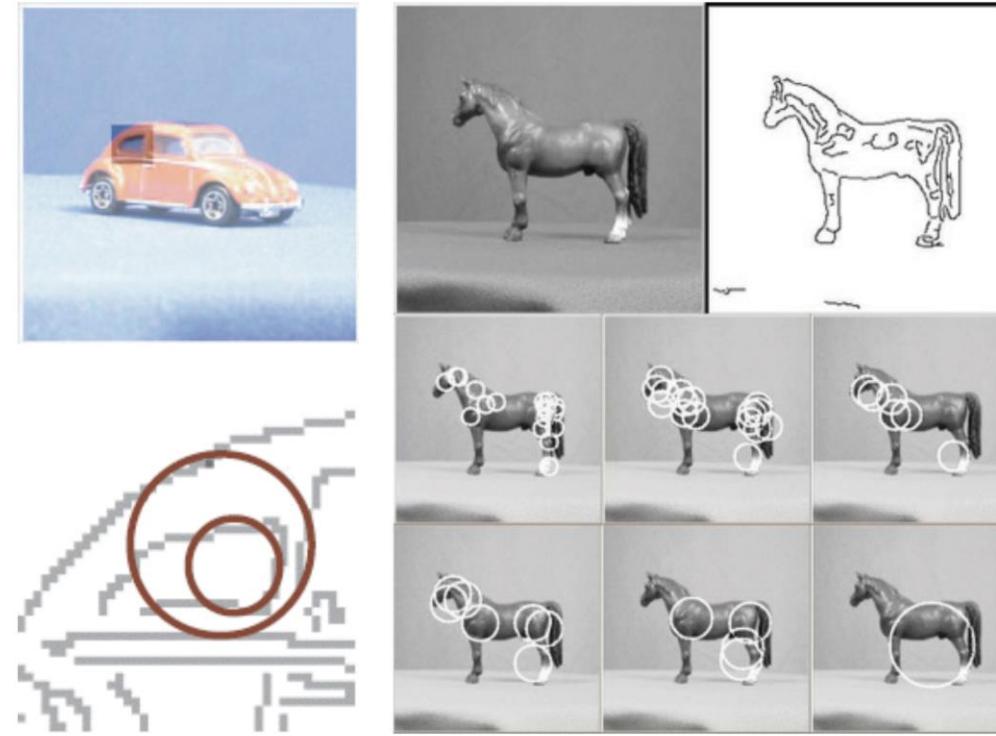
Test image



# Application of Hough Transforms

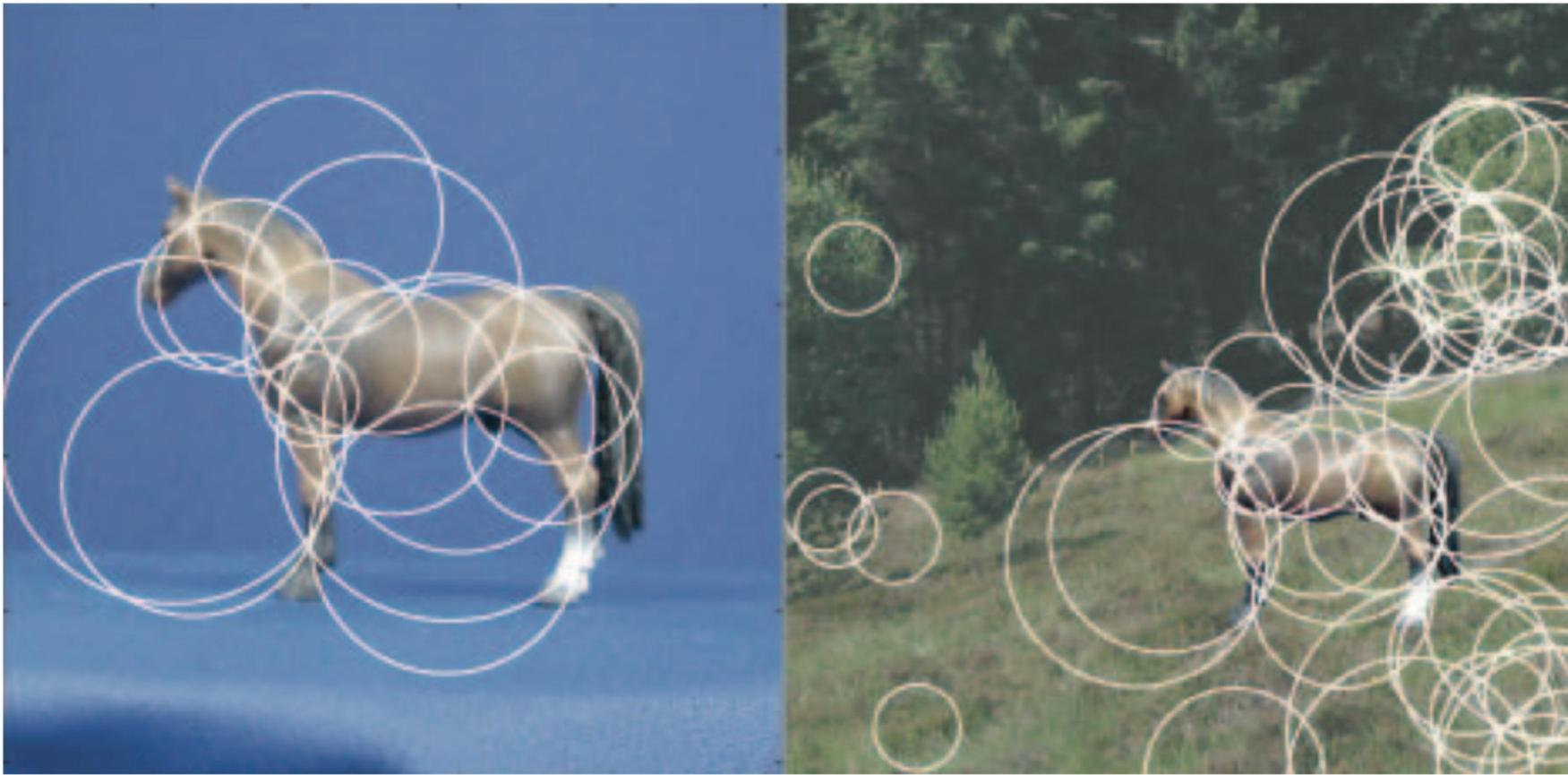


# Detecting shape features



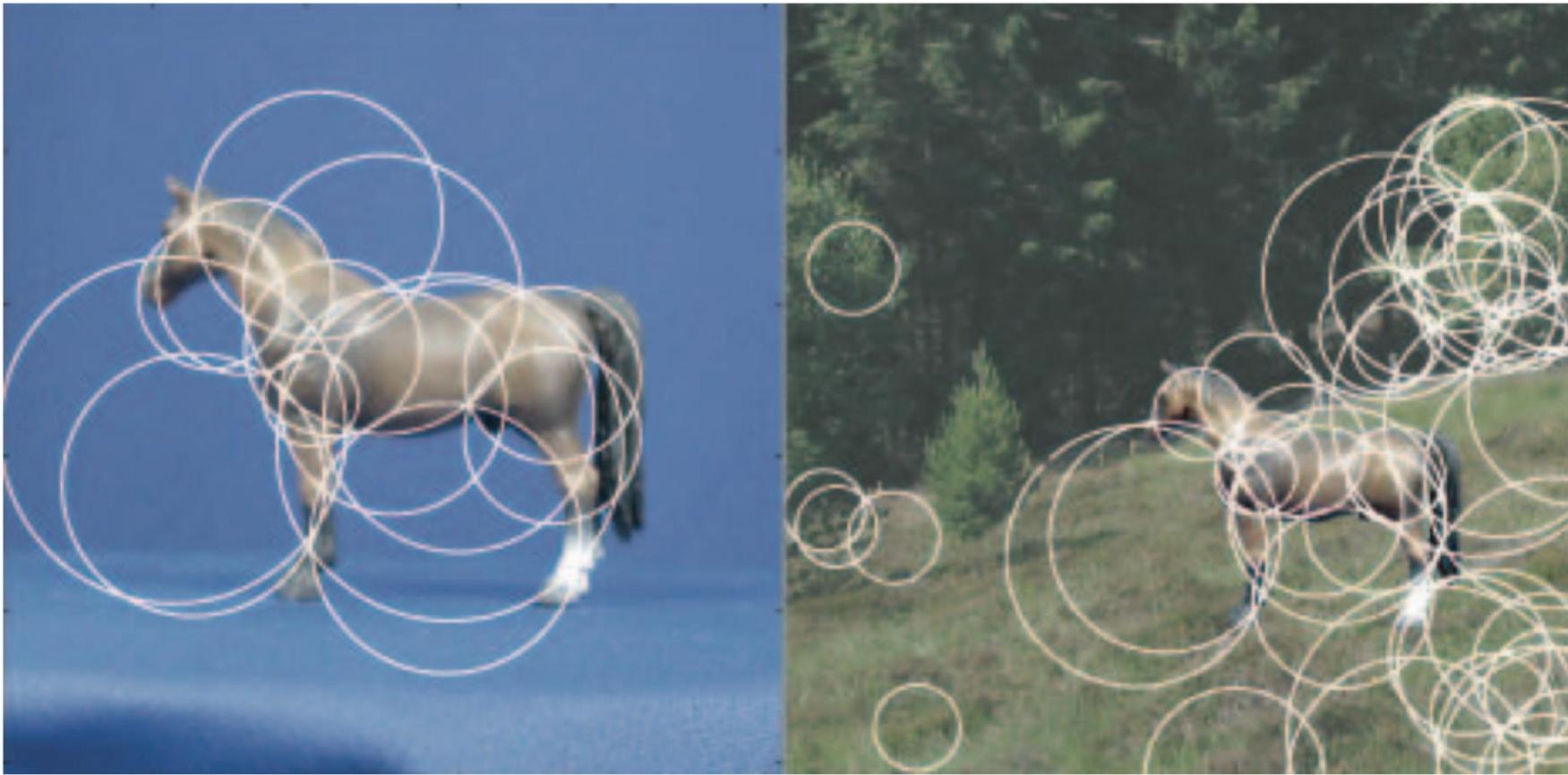
F. Jurie and C. Schmid, Scale-invariant shape features for recognition of object categories, CVPR 2004





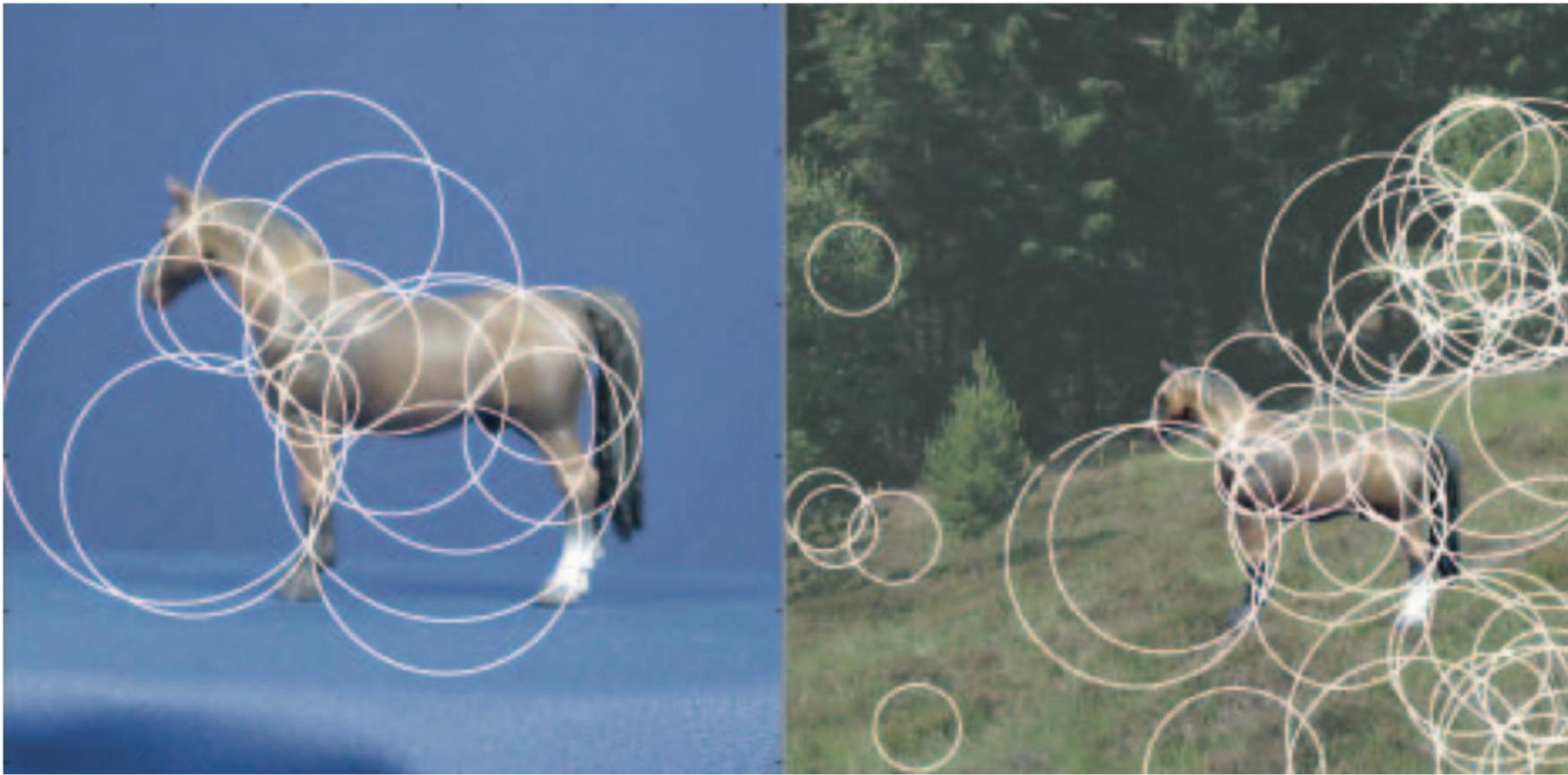
Robustness to scale and clutter





Robustness to scale and **clutter**





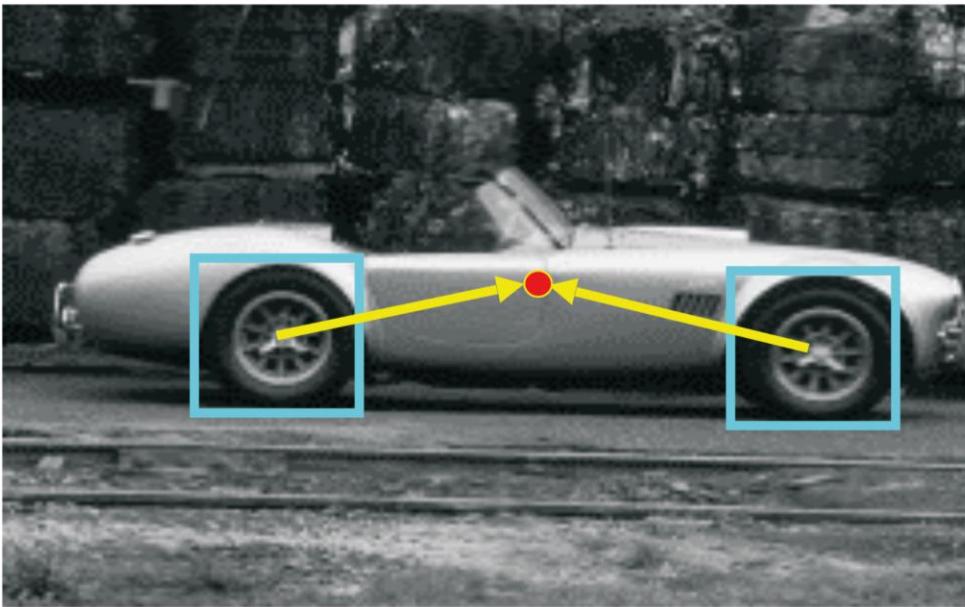
Robustness to scale and **clutter**

Clutter: When the object and the background are similar.

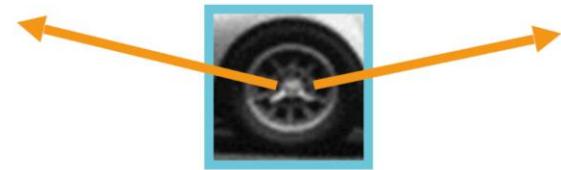


# Object detection

Index displacements by “visual codeword”



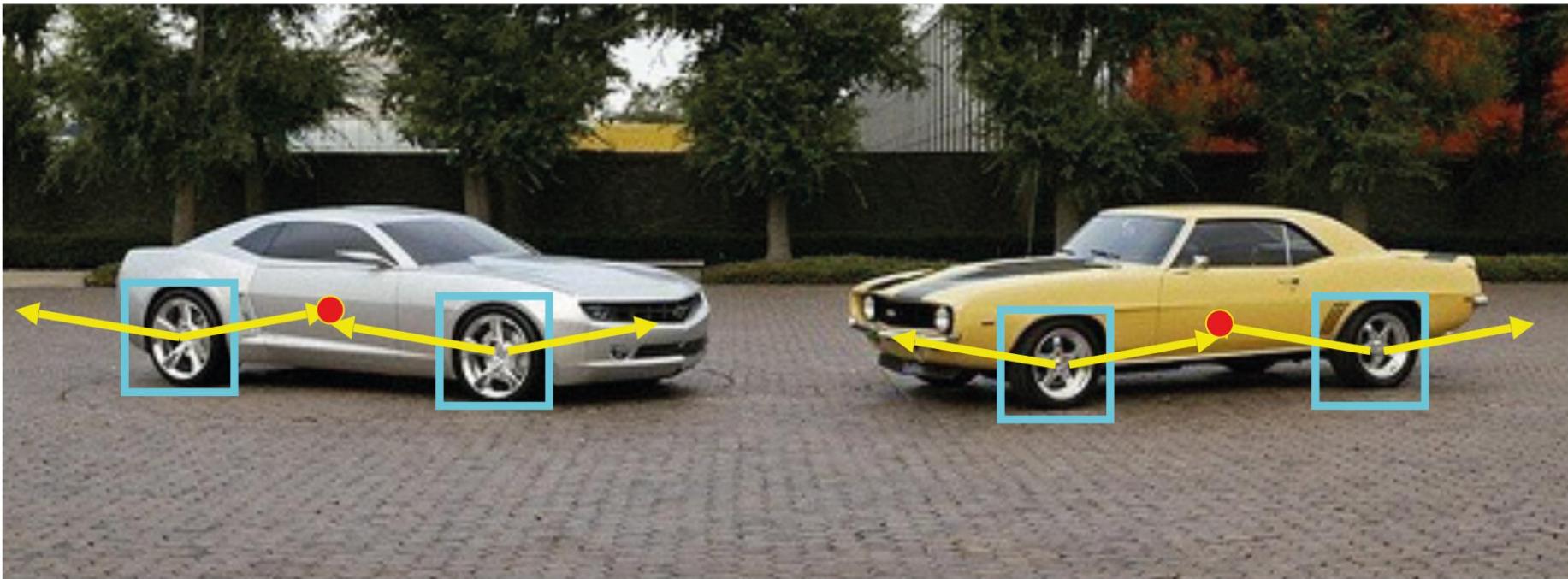
training image



visual codeword with  
displacement vectors

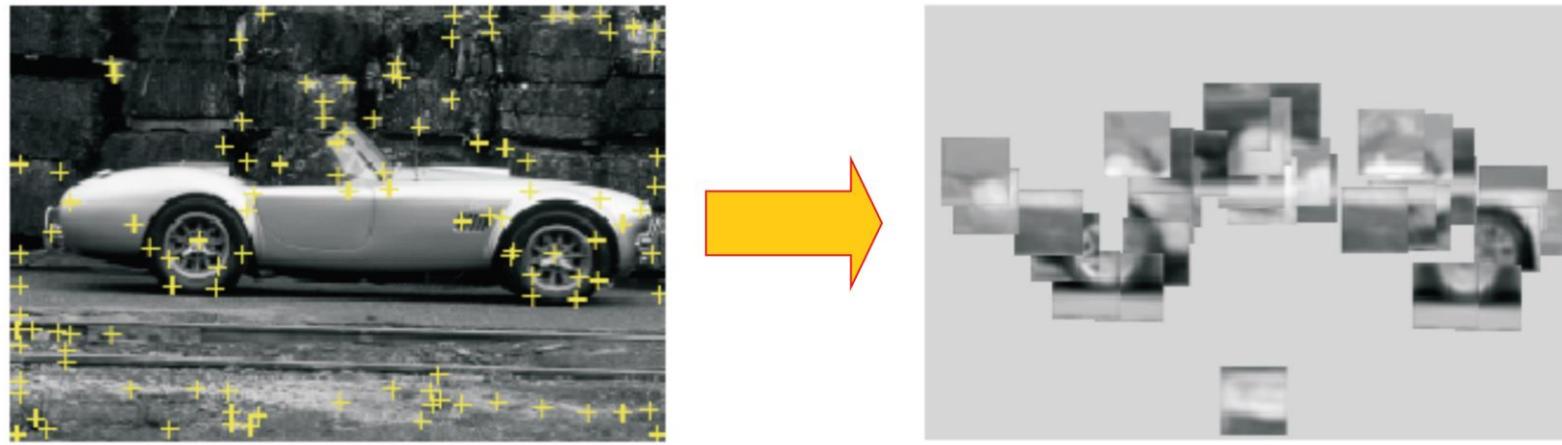
B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model,  
ECCV Workshop on Statistical Learning in Computer Vision 2004





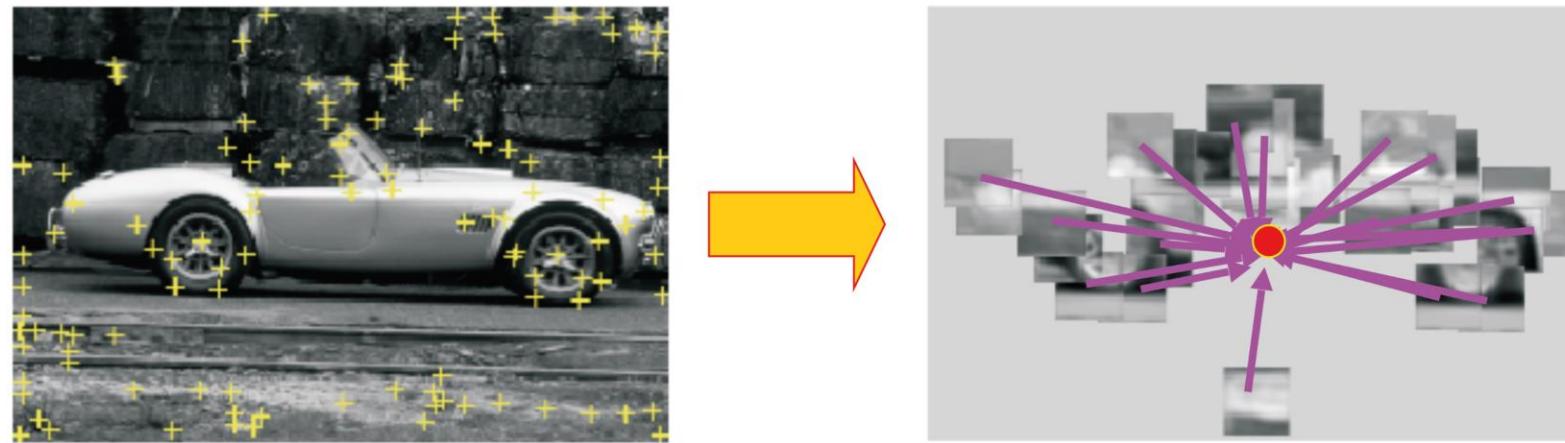
## Train phase

# 1. get features

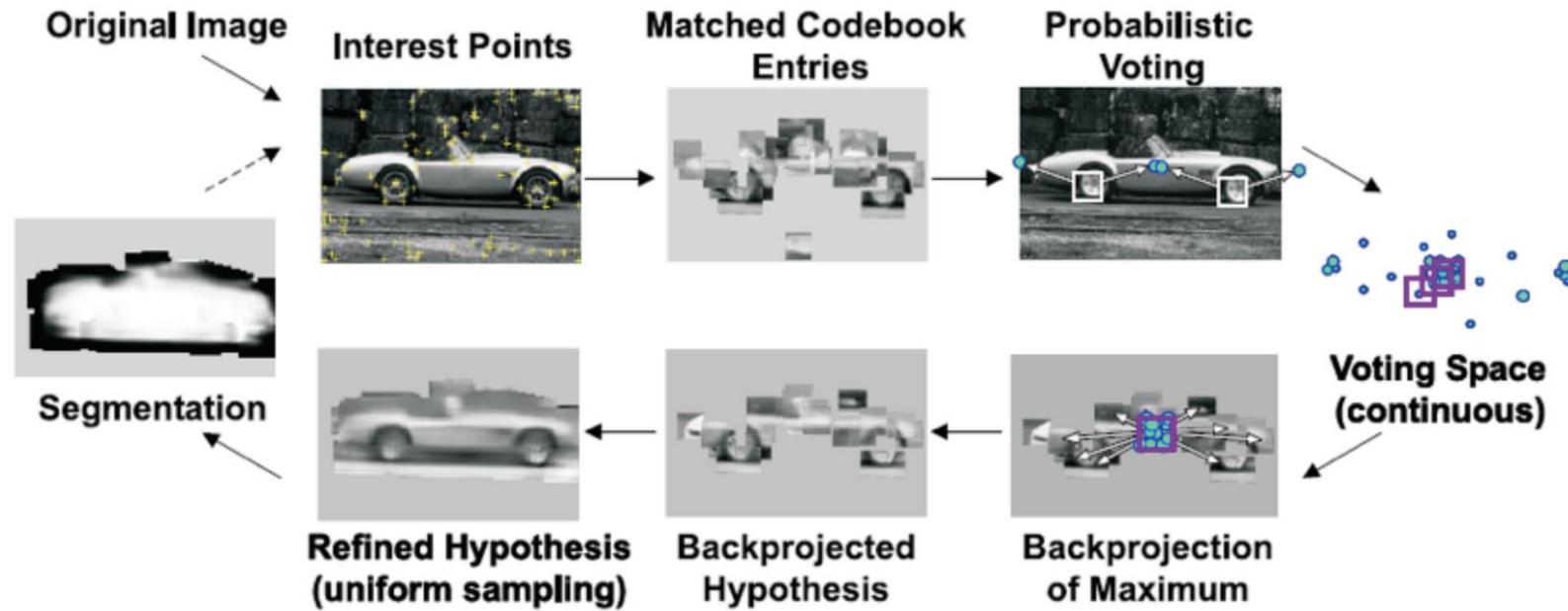


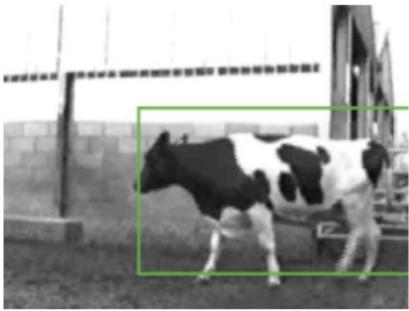
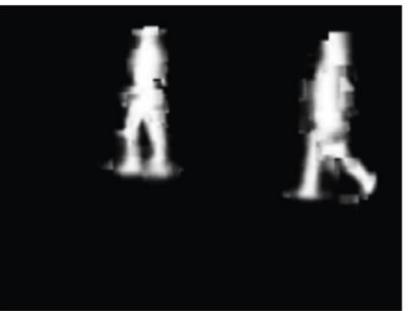
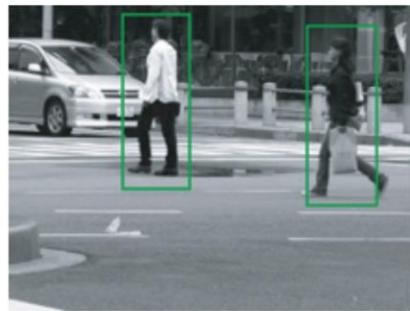
## Train phase

# 2. store displacements



# Test phase





# The Hough transform ...

Deals with **occlusion** well?



Detects multiple instances?



Robust to noise?



Good computational complexity?



Easy to set parameters?



# The Hough transform ...

Deals with **occlusion** well?



Detects multiple instances?



Robust to noise?



Good computational complexity?



Easy to set parameters?



Occlusion: When only part of the object we need to detect is there in the image.

# Thank You

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