CHAPTER 5

RISK AND RETURN

Any investment should be evaluated based on two dimensions:

- Return
- Risk

Return:

There are two types of return that should be calculated by the investor:

1. *Expected return* (R^): the ex-ante return before the investment
2. *Realized return* ($\bar{R}$): the ex-post return after the investment.

- ***Realized return is calculated***

A. *Evaluate the performance*

B. *Develop expectations about future return*

what are the components of return?

I. *Periodic income: dividends and interest income*

II. *Price appreciation: capital gains*

Risk:

The chance that realized return deviates from expected return

Risk return trade off:

Refers to the existence of positive relationship between risk and return.

The higher is the risk, the higher is the expected return

There are different measures of risk:

1. *Variance*
2. *Standard deviation*
3. *Coefficient of variation*
4. *Beta*

Risk can be classified as:

1- stand alone risk

2- portfolio risk

1- The stand alone risk

- The risk associated with holding an asset in isolation.
- The measures of the stand alone risk are:-
  - The variance
  - The standard deviation
  - The coefficient of variation

The variance

- The value of the squared deviation from the mean → % squared

The standard deviation

- The square root of the variance → to give risk the same dimension of return%, σ is calculated.

The coefficient of correlation

- The risk per unit of return.

Calculating Risk:

- ☐ the calculation of risk entails calculating return
- ☐ Risk is calculated without probability distribution and with probability distribution

A- estimating risk without probability distribution:

Problem 1:

you have the following information about the past returns on stock (A) for the last 3 years.

Year	R
1	30%
2	20%

1- Calculate average returns

2- Calculate the variance and the standard deviation

3- Coefficient of variation

<u>Solution:</u>

Average realized returns $(\bar{R}) = \frac{\Sigma \text{ returns}}{\text{number}}$

$= \frac{0.30+0.2-0.10}{3} = 0.2 = 20\%$

Year	R	$R - \bar{R}$ 0.20	$(R - \bar{R})2$
1 2 3	0.30 0.20 0.10	0.30 - 0.20 = 0.10 0.20 - 0.20 = 0 0.10 - 0.20 = -0.10	0.01 0 0.01
total		Always zero	0.02

$$\sigma 2 = (\textit{variance}) = \frac{\Sigma (R - \bar{R})^2}{n-1} = \frac{0.02}{3-1} = \frac{0.02}{2}$$

$$= 0.01$$

$$\sigma = (\textit{standard deviation}) = \sqrt{\sigma^2} = \sqrt{0.01} = 0.1$$

$$\textit{coefficient of variation} = \delta / \bar{R}$$

$$= 0.1/0.2 = 0.5$$

➢ **Whenever faced with different alternative investments to undertake:**

- *If they have the same average return, select the one with the lowest standard deviation.*
- *If investments opportunities have the same σ, select the one that has the highest average returns.*
- *If the investment opportunities have different average returns and different standard deviations, select the one that has the lowest coefficient of variation.*

➢ Problem 2:

- You are given the following information regarding the expected returns on the stocks of Martin products and U.S electric

Year	Martin products	U.S electric
2017	30%	6%
2018	6%	9%
2019	-21%	3%

Required

- **Calculate and comment on your results regarding average expected returns, standard deviation and coefficient if variation.**

Solution

$$\hat{R} = \frac{\Sigma \text{ expected returns}}{\text{number}}$$

$$\hat{R} \textit{ martin products} = \frac{0.30+0.06+(-0.21)}{3}$$

$$= 0.05 = 5\%$$

$$\hat{R} \textit{ U.S electric} = \frac{0.06+0.09+0.03}{3} = \frac{0.18}{3}$$

$$= 0.06 = 6\%$$

- **Based on average expected returns, the stock of U.S *electric gives* higher average expected returns.**

Year	R	R – R	(R – R̂)2	R	R – R̂	(R – R̂)2
2017	0.3	(0.3 - 0.05) = 0.25	0.0625	0.06	(0.06 – 0.06)= 0	0
2018	0.06	(0.06 - 0.05)= 0.01	0.0001	0.09	(0.09 – 0.06)= 0.03	0.0009
2019	-0.21	(-0.21 - 0.05) = -0.26	0.0676	0.03	(0.03 – 0.06)= -0.03	0.0009
		Zero	0.1302			0.0018

$$\sigma 2 \text{ martin products} = \frac{0.1302}{n-1} = \frac{0.1302}{2} = 0.0651$$

$$\sigma = \sqrt{0.0651} = 0.255$$

$$\sigma 2 \text{ U.S electric} = \frac{0.0018}{2} = 0.0009$$

$$\sigma = \sqrt{0.0009} = 0.03$$

based on risk, U.S electric is less risky because it has lower standard deviation.

*coefficient of variation CV =δ/R^*

CV martin products=0.255/0.05=5.1

CV U.S electric= 0.03/0.06=0.5

- *Based on coefficient of variation, U.S electric has lower risk per unit of return.*
- *So, the investor is advised to invest in U.S electric stocks.*

2) Calculating risk with probability distribution

- Probability Distribution

- The probability of occurrence of a set of all possible outcomes.
- Σ probabilities of occurrences = 1

- Scenario Analysis

- The process of developing a list of all possible economic events with their probabilities of occurrence and the return associated with each economic state.

$$\hat{R} \text{ or } \bar{R} = \sum_{i=1}^{n} P_i R_i$$

$$\sigma^2 = \sum_{i=1}^{n} (R_i - \bar{R})^2 * P_i$$

$$\sigma = \sqrt{\sigma^2}$$

Problem 3

- You are given the following information about the expected returns associated with different states of demand.

demand	probability	Return
Very weak	0.2	0.01
Weak	0.2	0.07
Normal	0.3	0.08
Strong	0.1	0.10
Very strong	0.2	0.15
	1.0	

- Calculate $\hat{R}$
- Standard deviation and coefficient of variation

Solution

$\hat{R} = \sum_{i=1}^{n} P_i R_i$

$= (0.2 * 0.01) + (0.2 * 0.07) + (0.3 * 0.08) + (0.1 * 0.10) + (0.2 * 0.15) = 0.08 = 8\%$

demand	R	(R - $\hat{R}$)	(R - $\hat{R}$)2	Pi	(R - $\hat{R}$)2 * Pi
V.weak	0.01	(0.01 – 0.08) = -0.07	0.0049	0.2	0.00098
Weak	0.07	(0.07 – 0.08) = -0.01	0.0001	0.2	0.00002
Normal	0.08	(0.08 – 0.08) = 0	0	0.3	0
Strong	0.10	(0.10 – 0.08) = 0.02	0.0004	0.1	0.00004
Very strong	0.15	(0.15 – 0.08) = 0.07	0.0049	0.2	0.00098
					0.00202

$$\sigma 2 = \sum_{i=1}^{n} (R - \hat{R})2 * P_i$$

$$= 0.00202$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{0.00202} = 0.0449 = 4.49\%$$

2- Portfolio risk:

Portfolio:

Refers to the collection of assets held by the investor.

Portfolio updating and rebalancing:

- ❑ It refers to the process of selling some of the assets held by the investor in his portfolio and using the proceeds to buy other assets to be added to the portfolio of the investor.
- ❑ **OR:** Increasing the size of the portfolio by adding up more assets.
- ❑ **OR:** Reducing the size of the portfolio by selling some of the assets.

- The investor selects securities that seem to be underpriced to be added to his portfolio without making the asset allocation decision.
- It results in undiversified portfolio.

Active portfolio management versus passive portfolio management:

Active portfolio management:

- investors select well diversified portfolios without paying attention to underpriced securities.

Passive portfolio management:

- Investors attempt to identify underpriced securities to be added to their portfolios

- <u>how to calculate the expected returns and the risk of the portfolio?</u>

- ***The expected returns on the portfolio is the weighted average of the returns on each individual asset in the portfolio.***

$$\hat{R_p} = \sum_{i=1}^{n} W_i \hat{R_i}$$

- $\hat{R_p}$: the expected returns on the portfolio
- Wi : the proportion of funds invested in security i
- $\hat{R_i}$: the expected returns on security i

- **W1 + W2 + W3 + ...Wn = 1**
- **Equally weighted portfolio, means that all assets in the portfolio have equal weights.**
- **The return on the portfolio is closer to the return of the security *that has* the largest weight**

Problem 4

- Four securities have the following expected returns

A = 15%	B = 12%	C = 30%	D = 22%

1. Calculate the expected returns on an equally weighted portfolio
2. If the total funds invested by the investor is $10,000 of which $2,500 are invested in stock (A), $1,000 in stock (B), and the remainder equally divided between the remaining two stocks.
3. The portfolio weights are 10% in (A), 10% in (B), and 40% in each of (C) and (D).

Solution

$$\hat{R_p} = \sum_{i=1}^{n} W_i \hat{R_i}$$

1) Equally weighted portfolio

$$WA = WB = WC = WD = \frac{1}{4} = 0.25$$

$$\hat{R_p} = (0.25 * 15\%) + (0.25 * 12\%) + (0.25 * 30\%) + (0.25 * 22\%) = 19.75\%$$

2) Total funds = $10,000

$$WA = \frac{2,500}{10,000} = 0.25$$

$$WB = \frac{1,000}{10,000} = 0.10$$

So, remaining funds = 10,000 – (2,500 + 1,000) = $6,500

Amount of funds invested in security (C) and (D) = 6500/2 = 3250

$$WC = WD = \frac{3,250}{10,000} = 0.325$$

$$\hat{R_p} = (0.25 * 15\%) + (0.10 * 12\%) + (0.325 * 30\%) + (0.325 * 22\%) = 21.85\%$$

3) WA = 10%, WB = 10%, WC = 40%, WD = 40%

$$\hat{R_p} = (0.10 * 15\%) + (0.10 * 12\%) + (0.40 * 30\%) + (0.40 * 22\%) = 23.5\%$$

The Portfolio Risk:

- ***Measured by variance and standard deviation.***
- ***The risk of the portfolio is not the weighted average of the risk of individual assets in the portfolio***

$$\sigma P \neq \sum_{i=1}^{n} W_i \sigma_i \qquad \textit{why?}$$

- ***Because through diversification the investor can reduce the risk of the portfolio without affecting returns.***
- ***The concept of diversification is based on the "law of large numbers", as more assets are added to the portfolio, the risk of the portfolio will decrease.***

> ***What are the different types of diversification?***

- Naive (random) diversification
- Markowitz diversification
- International diversification

1) Naive (random) diversification

- *The investor adds stocks to his portfolio that promise higher expected returns only without paying attention to the co-movements of the returns of the stocks.*

2) Markowitz diversification

- *When constructing a portfolio, the investor should pay attention to the expected return and the total risk of the portfolio.*
- *The portfolio should include firms from different industries because the correlation between their returns tend to be low.*

The correlation coefficient (ρ)

- *Is a relative measure of co-movement between the returns on securities.*
- The correlation coefficient is a number between *[-1, 1]*

$\rho_{A,B} = +1$ "perfect positive correlation"

- ***when the return on stock (A) increases, the return on stock (B) will increase by exactly the same amount.***
- ***when the return on stock (A) decreases, the return on stock (B) will decrease by exactly the same amount.***
- ***Combining securities of perfect positive correlation will not decrease the total risk of the portfolio.***

So that: $\sigma P = \sum_{i=1}^{n} W_i \sigma_i$

- ***The total risk of the portfolio is simply the weighted average of the risk of each individual asset in the portfolio.***

$\rho_{A,B}$ = zero "zero correlation"

- ***there is no relation between the returns on the two securities.***
- ***Combining securities of zero correlation will decrease the risk of the portfolio but it will never be eliminated.***

$\rho_{AB} = -1$ perfect negative correlation

- when the return on security (A) increases , the return on security (B) will decrease by exactly the same amount.
- when the return on security (A) decrease, the return on security (B) will increase by exactly the same amount.
- Combining securities of perfect negative correlation will eliminate the total risk of the portfolio.

❑ *In Reality*

- *Securities tend to have positive correlation with each other, the investor should select securities with the least positive correlation to reduce the total risk of the portfolio.*
- *We need the correlation coefficient to calculate the covariance that is used in the equation of calculating the total risk of the portfolio.*

The Covariance σ_{AB}

- *Is the* **absolute measure of co-movements between the returns of securities.**

$$\sigma_{AB} = \sigma_A \sigma_B \rho_{A,B}$$

- σ_{AB}: the covariance between the two securities A, B
- σ_A: the standard deviation of security (A)
- σ_B: the standard deviation of security (B)
- $\rho_{A,B}$: correlation between the returns on securities (A) and (B)

σ_{AB} will be positive when $\rho_{A,B}$ is positive	σ_{AB} will equal zero, when $\rho_{A,B}$ equal zero	σ_{AB} will be negative if $\rho_{A,B}$ is also negative

- The two security case

$$\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2\, W1\, W2\, \sigma1\, \sigma2\, \rho1,2$$

$$\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2\, W1\, W2\, \sigma1,2$$

$$\sigma p = \sqrt{W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_{1,2}}$$

$$\rho1,2 = \frac{\sigma_{1,2}}{\sigma_1\sigma_2}$$

- When $\rho_{1,2} = +1$

$$\sigma_p = {}_{W_1}\sigma_1 + {}_{W_2}\sigma_2$$

- When $\rho_{1,2} = -1$

$$\sigma_p = {}_{W_1}\sigma_1 - {}_{W_2}\sigma_2$$

- $\rho_{1,2}$ = zero

$$\sigma_p = \sqrt{W_1^2\sigma_1^2 + W_2^2\sigma_2^2}$$

- *as more securities are added to the portfolio, the covariance terms will increase at a rate higher than the variance terns.*
- ***So, the risk of the portfolio highly depends on the covariance between the returns on securities.***

➢ **Three Securities Case**

$$\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + W_3^2\sigma_3^2 + 2\,W_1\,W_2\,\sigma_{1,2} + 2\,W_1\,W_3\,\sigma_{1,3} + 2\,W_2\,W_3\,\sigma_{2,3}$$

➢ **International diversification**

- ***The portfolio of the investor includes both domestic and foreign stocks.***
- ***The total risk of the portfolio (σp) is lower for internationally diversified portfolios than for domestic portfolios for all portfolio sizes.***

➢ **What are the determinants of portfolio risk?**

- ***The relative weights of the securities in the portfolio***
- ***The variance of each individual security***
- ***The covariance between each pair of securities in the portfolio***

Problem 5

- A portfolio is constructed from two securities (A) and (B)

	A	B
R^	26.3%	11.6%
σ	37.3%	23.3%
weight	50%	50%

Required

- Calculate the expected return on the portfolio
- Calculate the total risk of the portfolio at ρA,B = -1, 0, +1, 0.5, 1.5
- What do you observe based on your calculations in *number 2?*

Solution

- $\hat{R_p} = \sum_{i=1}^{n} W_i \hat{R_i}$

$$= (0.5 * 26.3\%) + (0.5 * 11.6\%) = 19\%$$

- At $\rho_{A,B} = -1$

$$\sigma_p = {}_{W_1}\sigma_1 - {}_{W_2}\sigma_2$$

$$\sigma_p = 0.5\,(37.3) - (0.5 * 23.3\%) = 7\%$$

- At $\rho_{A,B} = 0$

$$\sigma p = \sqrt{W_A^2\sigma_A^2 + W_B^2\sigma_B^2}$$

$$\sigma p = \sqrt{(0.5)^2(37.3)^2 + (0.5)^2(23.3)^2}$$

$$\sigma p = \sqrt{(0.25 * 1391.29) + (0.25 * 542.9)}$$

$$\sigma p = \sqrt{347.8 + 135.7} = \mathbf{22\%}$$

- At $\rho_{A,B} = +1$

$$\sigma p = W_A\sigma_A + W_B\sigma_B$$

$$\sigma p = (0.5 * 37.3) + (0.5 * 23.3\%) = 30.3\%$$

- At $\rho_{A,B} = 0.5$

$$\sigma p = \sqrt{W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_AW_B\sigma_A\sigma_B\rho_{A,B}}$$

$$\sigma p = \sqrt{347.8 + 135.7 + 2(0.5 * 0.5 * 37.3 * 23.3 * 0.5)}$$

$$\sigma p = \sqrt{347.8 + 135.7 + 317.3}$$

$$\sigma p = 26.3\%$$

- At $\rho_{A,B} = 1.5$

$$\sigma p = \sqrt{347.8 + 135.7 + 2(0.5 * 0.5 * 37.3 * 23.3 * 1.5)}$$

$$\sigma p = \sqrt{347.8 + 135.7 + 651.8}$$

$$\sigma p = 34\%$$

- The lower is the correlation coefficient between the two securities, the lower is the total risk of the portfolio

You are given the following information about 3 stocks

	A	B	C
σ $\rho_{A,B} = 0.55$ $\rho_{A,C} = 0.6$ $\rho_{B,C} = 0.67$	0.20	0.25	0.30
Portfolio weight	30%	20%	50%
$\hat{R}$	6%	7%	12%

Required

- Calculate the expected return on the portfolio
- Calculate the standard deviation of the portfolio

Solution

- Calculate the expected return on the portfolio
- Calculate the standard deviation of the portfolio

1) $\hat{R}_p = \sum_{i=1}^{n} W_i \hat{R_i}$

$$\hat{R}_p = (0.30 * 6\%) + (0.20 * 7\%) + (0.50 * 12\%)$$

$$\hat{R}_p = 9.2\%$$

2) $\sigma_p^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + W_C^2\sigma_C^2 + 2\, W_A\, W_B\, \sigma_A\, \sigma_B\, \rho_{A,B} + 2\, W_A\, W_C\, \sigma_A\, \sigma_C\, \rho_{A,C} + 2\, W_B\, W_C\, \sigma_B\, \sigma_C\, \rho_{B,C}$

$$\sigma_p^2 = (0.30)^2 (0.20)^2 + (0.20)^2 (0.25)^2 + (0.50)^2 (0.30)^2 + 2\,(0.3 * 0.20 * 0.20 * 0.25 * 0.55) + 2\,(0.30 * 0.50 * 0.20 * 0.30 * 0.6) + 2\,(0.20 * 0.50 * 0.25 * 0.30 * 0.67)$$

$$= (0.09 * 0.04) + (0.04 * 0.0625) + (0.25 * 0.09) + (0.0033) + (0.0108) + (0.01005) = 0.053$$

$$\sigma_p = \sqrt{\sigma_P^2} = \sqrt{0.053} = 0.23 = 23\%$$

- you are given the following information about three stocks

	A	B	C
$\widehat{R}$	10%	15%	30%
Investment value	$10000	$20000	$70000
σ	0.53	0.64	0.72
$\sigma_{A,B} = 0.7$ $\sigma_{A,C} = 0.3$ $\sigma_{B,C} = 0.5$			

- Calculate the expected returns on the portfolio and comment?
- What is the expected returns on the portfolio if $70000 was invested in stock (A), $20000 in (B), and $10000 in (C). comment?
- Based on the givens, calculate the standard deviation of the portfolio?

Solution

1) $\widehat{R}_p = \sum_{i=1}^{n} W_i \widehat{R}_i$

$$WA: \frac{10000}{70000+20000+10000} = \frac{10000}{100000} = 10\%$$

$$WB: \frac{20000}{70000+20000+10000} = 20\%$$

$$WC: \frac{70000}{70000+20000+10000} = 70\%$$

$$\widehat{R}_p = (0.10 * 10\%) + (0.20 * 15\%) + (0.70 * 30\%)$$

$$\widehat{R}_p = 1 + 3 + 21\% = 25\%$$

2)

A	B	C
70000	20000	10000
$W_A: \frac{70000}{100000} = 70\%$	$W_B: \frac{20000}{100000} = 20\%$	$W_C: \frac{10000}{100000} = 10\%$

$$\widehat{R}_p = (0.7 * 10) + (0.20 * 15) + (0.10 * 30\%)$$

$$\widehat{R}_p = 7\% + 3\% + 3\% = 13\%$$

Comment

$\hat{R}_p$ is closer to the expected return on the stock that has the highest weight

2) $\sigma_p^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + W_C^2\sigma_C^2 + 2W_AW_B\sigma_{A,B} + 2W_AW_C\sigma_{A,C} + 2W_BW_C\sigma_{B,C}$

σ_p^2 = [(0.10)2 (0.53)2] + [(0.20)2 (0.64)2] + [(0.7)2 (0.72)2] + [2 * 0.10 * 0.20 * 0.7] + [2 * 0.10 * 0.70 *0.3] + [2 * 0.7 * 0.20 * 0.5]

= [0.01 * 0.281] + [0.04 * 0.41] + [0.49 * 0.52] + [0.028] + [0.042] + [0.14]

= 0.00281 + 0.0164 + 0.2548 + 0.028 + 0.042 + 0.14 = 0.48

$\sigma p = \sqrt{\sigma_P^2} = \sqrt{0.48} = 0.695$