

Multi_Variable_Optimization_Methods_Examples

September 24, 2022

```
[15]: !pip install -U mloptm
```

```
Requirement already satisfied: mloptm in
/home/moaz/anaconda3/lib/python3.9/site-packages (1.0.6)
Requirement already satisfied: Pillow in
/home/moaz/anaconda3/lib/python3.9/site-packages (from mloptm) (9.0.1)
Requirement already satisfied: numpy in /home/moaz/anaconda3/lib/python3.9/site-
packages (from mloptm) (1.21.5)
Requirement already satisfied: tabulate in
/home/moaz/anaconda3/lib/python3.9/site-packages (from mloptm) (0.8.9)
Requirement already satisfied: sympy in /home/moaz/anaconda3/lib/python3.9/site-
packages (from mloptm) (1.10.1)
Requirement already satisfied: matplotlib in
/home/moaz/anaconda3/lib/python3.9/site-packages (from mloptm) (3.5.1)
Requirement already satisfied: cycycler>=0.10 in
/home/moaz/anaconda3/lib/python3.9/site-packages (from matplotlib->mloptm)
(0.11.0)
Requirement already satisfied: kiwisolver>=1.0.1 in
/home/moaz/anaconda3/lib/python3.9/site-packages (from matplotlib->mloptm)
(1.3.2)
Requirement already satisfied: python-dateutil>=2.7 in
/home/moaz/anaconda3/lib/python3.9/site-packages (from matplotlib->mloptm)
(2.8.2)
Requirement already satisfied: pyparsing>=2.2.1 in
/home/moaz/anaconda3/lib/python3.9/site-packages (from matplotlib->mloptm)
(3.0.4)
Requirement already satisfied: fonttools>=4.22.0 in
/home/moaz/anaconda3/lib/python3.9/site-packages (from matplotlib->mloptm)
(4.25.0)
Requirement already satisfied: packaging>=20.0 in
/home/moaz/anaconda3/lib/python3.9/site-packages (from matplotlib->mloptm)
(21.3)
Requirement already satisfied: six>=1.5 in
/home/moaz/anaconda3/lib/python3.9/site-packages (from python-
dateutil->matplotlib->mloptm) (1.16.0)
Requirement already satisfied: mpmath>=0.19 in
/home/moaz/anaconda3/lib/python3.9/site-packages (from sympy->mloptm) (1.2.1)
```

```
[16]: import numpy as np
      np.random.seed(42)

      from mloptm.grads import SteepestDescent, NewtonND

      %matplotlib qt
```

```
[17]: import warnings
      warnings.filterwarnings(action="ignore")
```

1 Define Some Functions To Use

$$f_1(x, y) = x^2 + y^2$$

$$f_2(x, y) = \cos(x^2 - 3y) + \sin(x^2 + y^2)$$

$$f_3(x_1, \dots, x_{30}) = \sum_{i=1}^{30} N_i x_i^C = 2x_1^4 + 5x_2^2 + \dots + 9x_{30}^4$$

```
[18]: f1 = "x^2+y^2"
      f2 = "cos(x^2 - 3*y) + sin(x^2 + y^2)"

      ## Foreget about this, only used to create the third function
      N_VARS = 30
      f3_vars = [f"x{i}" for i in range(1, N_VARS+1)]
      f3 = "+".join( [ f"{np.random.randint(1, 10)}*x{i}^{np.random.choice([2, 4])}"
                      ↪for i in range(1, N_VARS+1) ] )
      f3
```

```
[18]: '7*x1^4+8*x2^2+5*x3^2+3*x4^2+8*x5^2+4*x6^4+8*x7^2+6*x8^2+2*x9^4+6*x10^4+5*x11^2+
      6*x12^2+9*x13^2+3*x14^4+7*x15^4+9*x16^2+5*x17^2+7*x18^2+9*x19^2+2*x20^4+9*x21^4+
      2*x22^4+9*x23^4+5*x24^4+4*x25^4+7*x26^4+8*x27^2+3*x28^4+1*x29^4+2*x30^4'
```

2 Using Steepest Descent Method

```
[19]: s1 = SteepestDescent(f1, ("x", "y"))
      minima = s1.Minimize(x0=(10, 10), eps=1e-5, verbose=True)
      display(minima)

      ## Plot Error Over Iterations
      s1.PlotError()
```

```
## Plot Function in Contour Form  
s1.PlotContour(xdomain=(-15, 15), ydomain=(-15, 15), contours=200)  
  
## Plot Function in 3D Form  
s1.Plot3D(xdomain=(-15, 15), ydomain=(-15, 15), alpha=0.8, contours=200)
```

Running with Tolerance of [0.0000100000] for Max Iterations of [50]

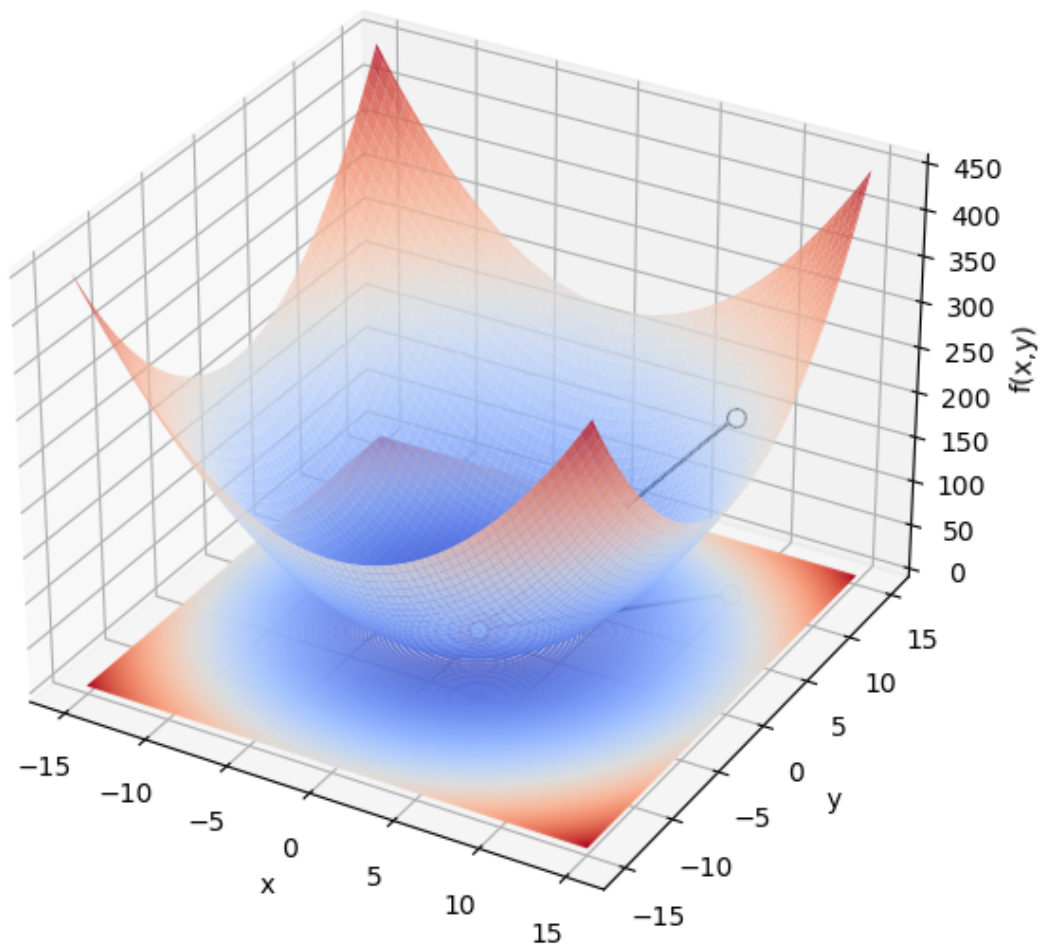
Error at Iter [1] = 1.000002276846

Error at Iter [2] = 0.000032199542

```
array([[ -1.18032373e-16],  
       [ -1.18032373e-16]])
```

[19]:

Minimization using SteepestDescent



```
[25]: s2 = SteepestDescent(f2, ("x", "y"))
minima = s2.Minimize(x0=(1, 1), eps=1e-5, verbose=True)
display(minima)

## Plot Error Over Iterations
s2.PlotError()

## Plot Function in Contour Form
s2.PlotContour(xdomain=(-3, 3), ydomain=(-3, 3), contours=200)

## Plot Function in 3D Form
s2.Plot3D(xdomain=(-3, 3), ydomain=(-3, 3), alpha=0.7, contours=200)
```

Running with Tolerance of [0.0000100000] for Max Iterations of [50]

Error at Iter [1] = 0.424159602445

Error at Iter [2] = 0.300241800580

Error at Iter [3] = 0.020752222087

Error at Iter [4] = 0.003268755730

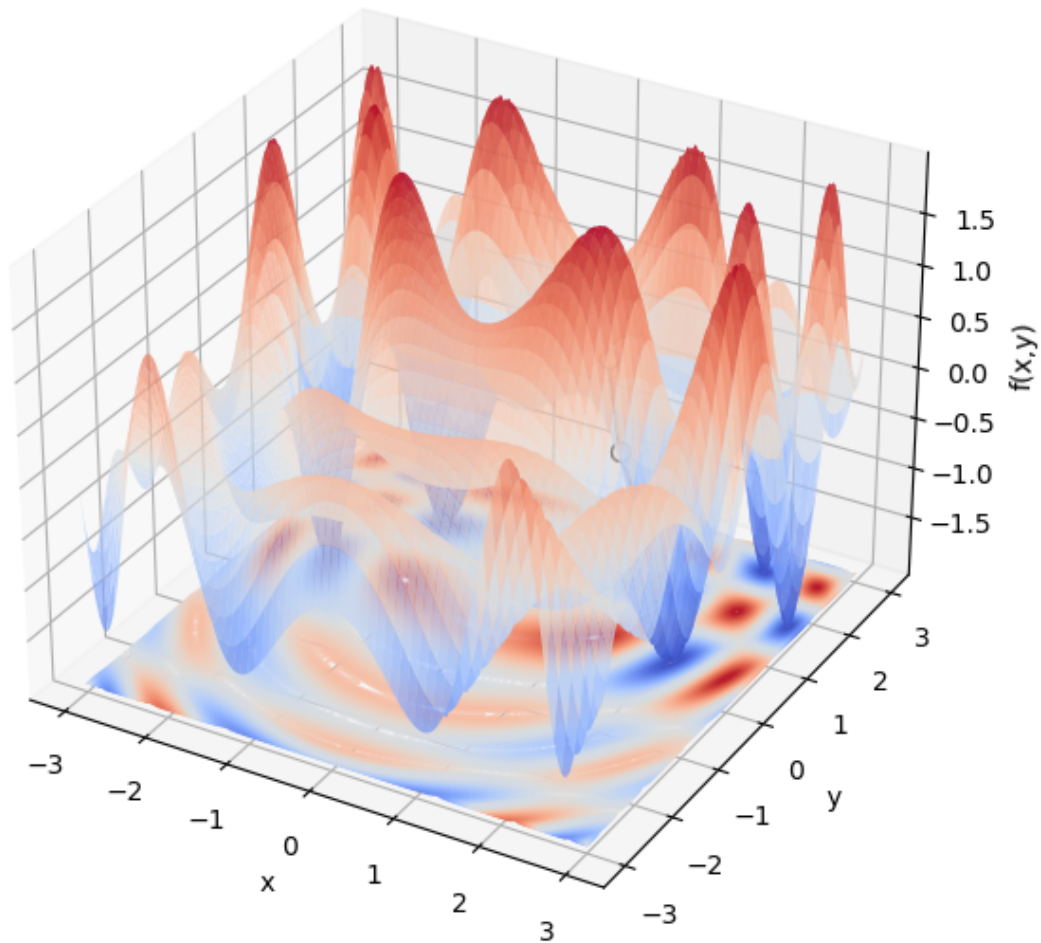
Error at Iter [5] = 0.000366715852

Error at Iter [6] = 0.000049562070

```
array([[1.37638334],
       [1.67867582]])
```

[25]:

Minimization using SteepestDescent



```
[21]: s3 = SteepestDescent(f3, f3_vars)
      minima = s3.Minimize(x0=[1 for _ in range(N_VARS)], eps=1e-3, verbose=True)
      display(minima)

      ## Plot Error Over Iterations
      s3.PlotError()
```

Running with Tolerance of [0.0010000000] for Max Iterations of [50]

Error at Iter [1] = 0.845243827698

Error at Iter [2] = 0.756205194358

Error at Iter [3] = 0.311275878770

Error at Iter [4] = 0.186243032613

Error at Iter [5] = 0.142405525558

Error at Iter [6] = 0.103968914313

```

Error at Iter [ 7] = 0.086144588546
Error at Iter [ 8] = 0.066260317396
Error at Iter [ 9] = 0.059112932556
Error at Iter [10] = 0.047607361804
Error at Iter [11] = 0.044068223683
Error at Iter [12] = 0.036439085723
Error at Iter [13] = 0.034544712201
Error at Iter [14] = 0.029080829358
Error at Iter [15] = 0.028044089360
Error at Iter [16] = 0.023918937603
Error at Iter [17] = 0.023364642453
Error at Iter [18] = 0.020128962228
Error at Iter [19] = 0.019862115405
Error at Iter [20] = 0.017248347486
Error at Iter [21] = 0.017158253665
Error at Iter [22] = 0.014996810553
Error at Iter [23] = 0.015019106351
Error at Iter [24] = 0.013197442410
Error at Iter [25] = 0.013291830415
Error at Iter [26] = 0.011732395519
Error at Iter [27] = 0.011872703846
Error at Iter [28] = 0.010520526932
Error at Iter [29] = 0.010690624973
Error at Iter [30] = 0.009504513428
Error at Iter [31] = 0.009692622133
Error at Iter [32] = 0.008642687183
Error at Iter [33] = 0.008841222912
Error at Iter [34] = 0.007904074950
Error at Iter [35] = 0.008107826491
Error at Iter [36] = 0.007265134002
Error at Iter [37] = 0.007471144024
Error at Iter [38] = 0.006708134289
Error at Iter [39] = 0.006913879793
Error at Iter [40] = 0.006219122315
Error at Iter [41] = 0.006422693735
Error at Iter [42] = 0.005786859479
Error at Iter [43] = 0.005987314179
Error at Iter [44] = 0.005402645391
Error at Iter [45] = 0.005598857023
Error at Iter [46] = 0.005059213842
Error at Iter [47] = 0.005250996779
Error at Iter [48] = 0.004750886929
Error at Iter [49] = 0.004937713047
Error at Iter [50] = 0.004472725359

```

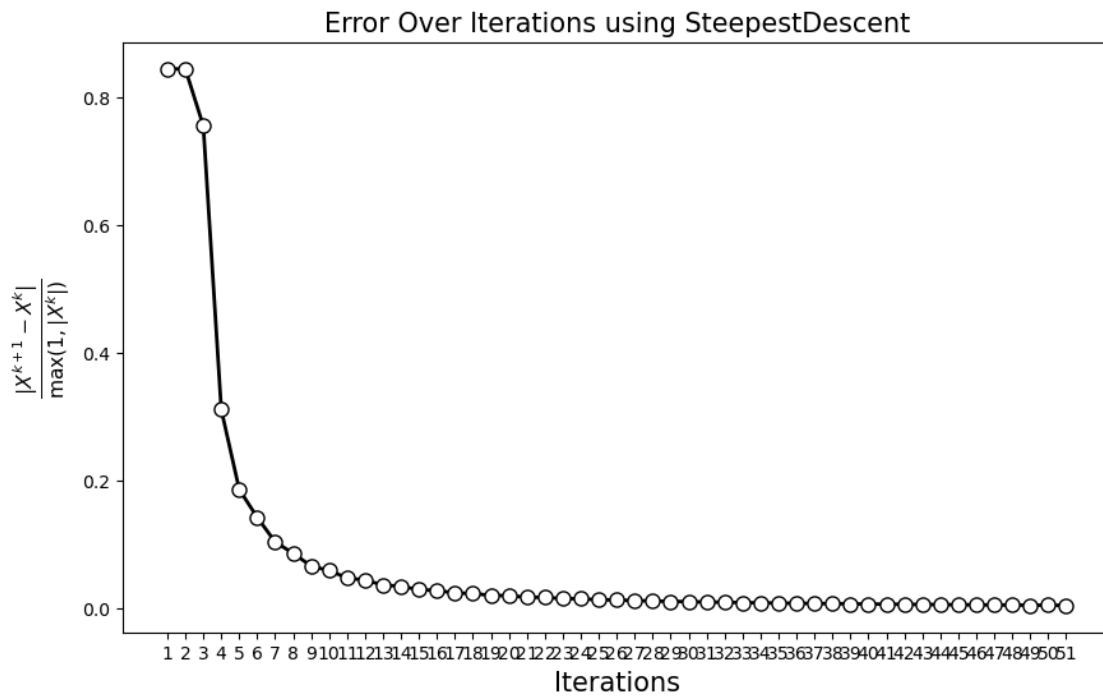
```

array([[ -5.61027632e-02],
       [ -2.30674445e-09],
       [  4.56618281e-66],

```

```
[ 9.89666950e-23],
[-2.30674445e-09],
[ 7.27674162e-02],
[-2.30674445e-09],
[ 3.74482208e-30],
[ 1.05081414e-01],
[-5.56018654e-02],
[ 4.56618281e-66],
[ 3.74482208e-30],
[-8.64860205e-04],
[ 8.55723395e-02],
[-5.61027632e-02],
[-8.64860205e-04],
[ 4.56618281e-66],
[-2.77753992e-17],
[-8.64860205e-04],
[ 1.05081414e-01],
[ 4.95873134e-02],
[ 1.05081414e-01],
[ 4.95873134e-02],
[ 4.73117151e-02],
[ 7.27674162e-02],
[-5.61027632e-02],
[-2.30674445e-09],
[ 8.55723395e-02],
[ 1.48420047e-01],
[ 1.05081414e-01]])
```

[21] :



3 Using Newton Method

```
[22]: s1 = NewtonND(f1, ("x", "y"))
minima = s1.Minimize(x0=(10, 10), eps=1e-5, verbose=True)
display(minima)

## Plot Error Over Iterations
s1.PlotError()

## Plot Function in Contour Form
s1.PlotContour(xdomain=(-15, 15), ydomain=(-15, 15), contours=200)

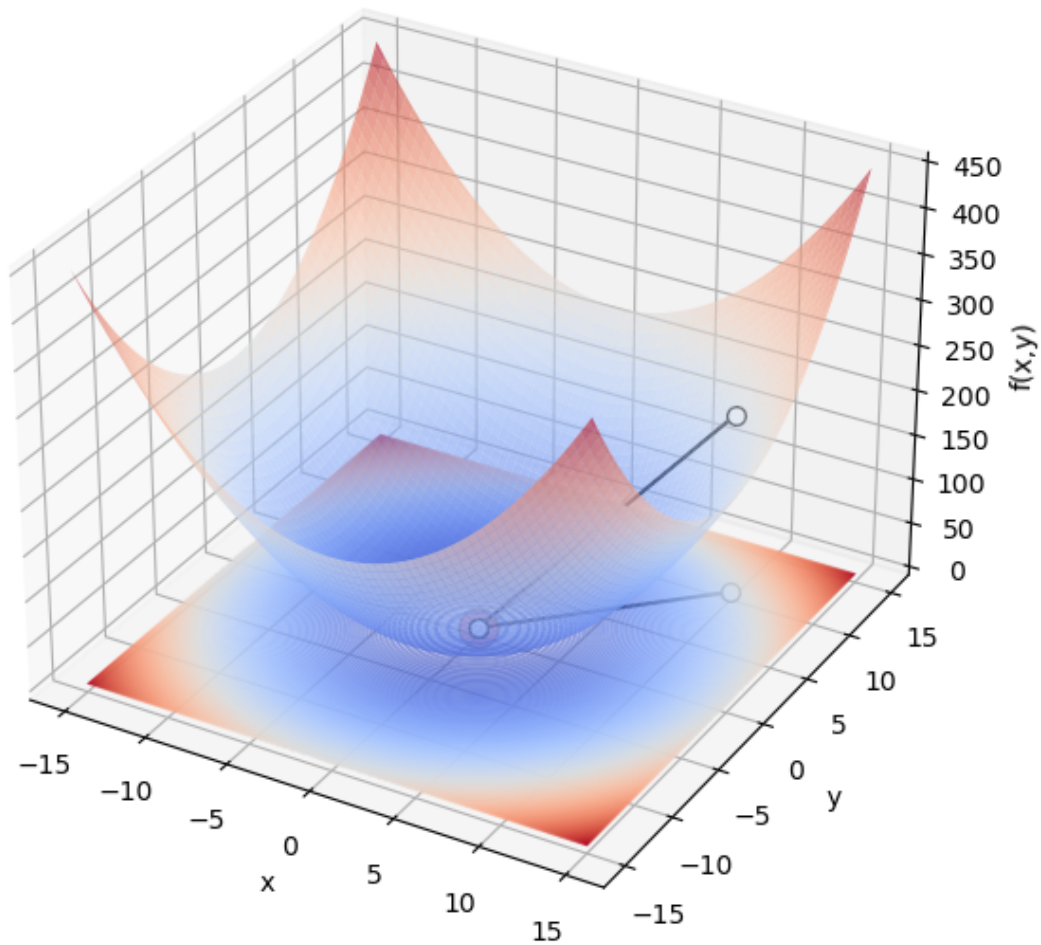
## Plot Function in 3D Form
s1.Plot3D(xdomain=(-15, 15), ydomain=(-15, 15), alpha=0.6, contours=200)
```

```
Error at Iter [ 1] = [1.0000000000000000]
Error at Iter [ 2] = [0.0000000000000000]

array([[0.],
       [0.]])
```

[22]:

Minimization using NewtonND



```
[23]: s2 = NewtonND(f2, ("x", "y"))
      minima = s2.Minimize(x0=(1, 1), eps=1e-2, verbose=True)
      display(minima)

      ## Plot Error Over Iterations
      s2.PlotError()

      ## Plot Function in Contour Form
      s2.PlotContour(xdomain=(-3, 3), ydomain=(-3, 3), contours=200)

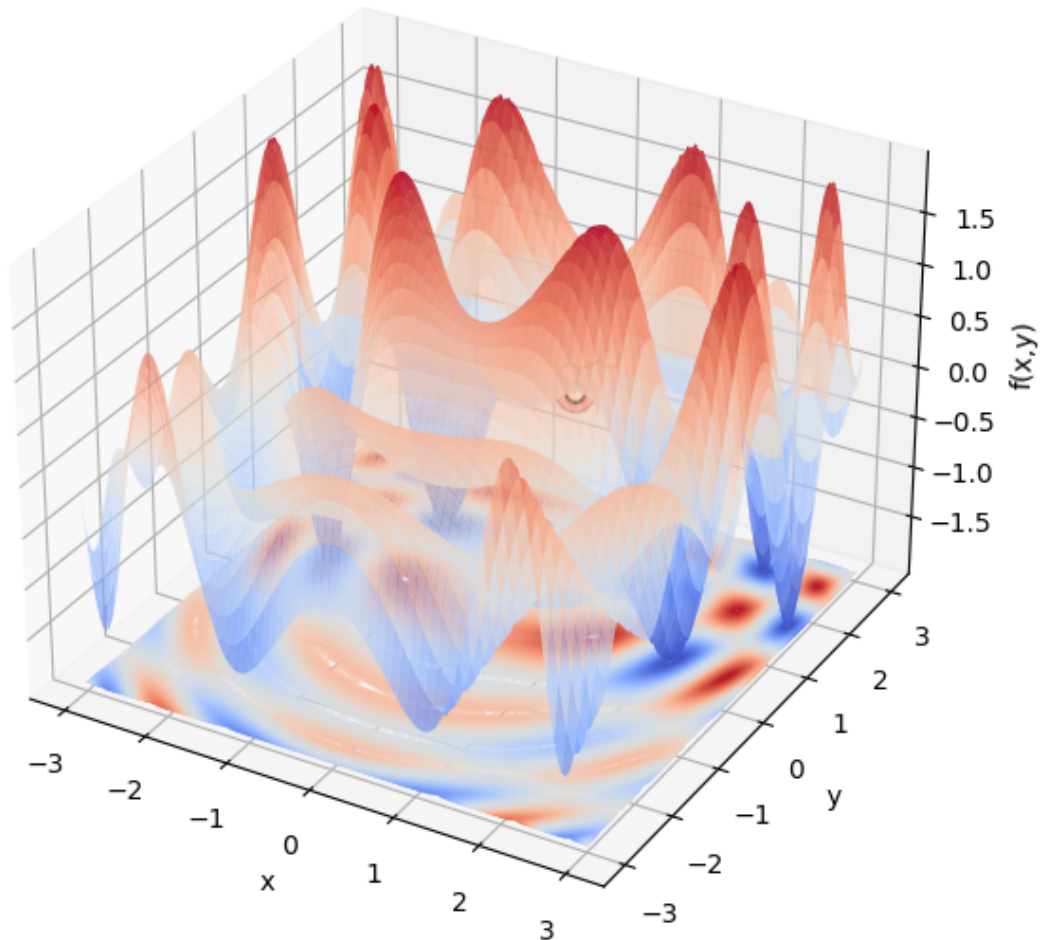
      ## Plot Function in 3D Form
      s2.Plot3D(xdomain=(-3, 3), ydomain=(-3, 3), alpha=0.7, contours=200)
```

Error at Iter [1] = [0.469247893098626301]

```
Error at Iter [ 2] = [0.195890022888327908]
Error at Iter [ 3] = [0.061925083714573843]
Error at Iter [ 4] = [0.002853506229684132]
array([[0.52376039],
       [1.13863407]])
```

[23]:

Minimization using NewtonND



```
[24]: s3 = NewtonND(f3, f3_vars)
minima = s3.Minimize(x0=[1 for _ in range(N_VARS)], eps=1e-3, verbose=True)
display(minima)

## Plot Error Over Iterations
s3.PlotError()
```

```
Error at Iter [ 1] = [0.725207505425809806]
```

```

Error at Iter [ 2] = [0.333333333333333259]
Error at Iter [ 3] = [0.333333333333333426]
Error at Iter [ 4] = [0.333333333333333259]
Error at Iter [ 5] = [0.263374485596707841]
Error at Iter [ 6] = [0.175582990397805228]
Error at Iter [ 7] = [0.117055326931870152]
Error at Iter [ 8] = [0.078036884621246735]
Error at Iter [ 9] = [0.052024589747497837]
Error at Iter [10] = [0.034683059831665225]
Error at Iter [11] = [0.023122039887776818]
Error at Iter [12] = [0.015414693258517879]
Error at Iter [13] = [0.010276462172345253]
Error at Iter [14] = [0.006850974781563502]
Error at Iter [15] = [0.004567316521042333]
Error at Iter [16] = [0.003044877680694889]
Error at Iter [17] = [0.002029918453796593]
Error at Iter [18] = [0.001353278969197729]
Error at Iter [19] = [0.000902185979465152]

```

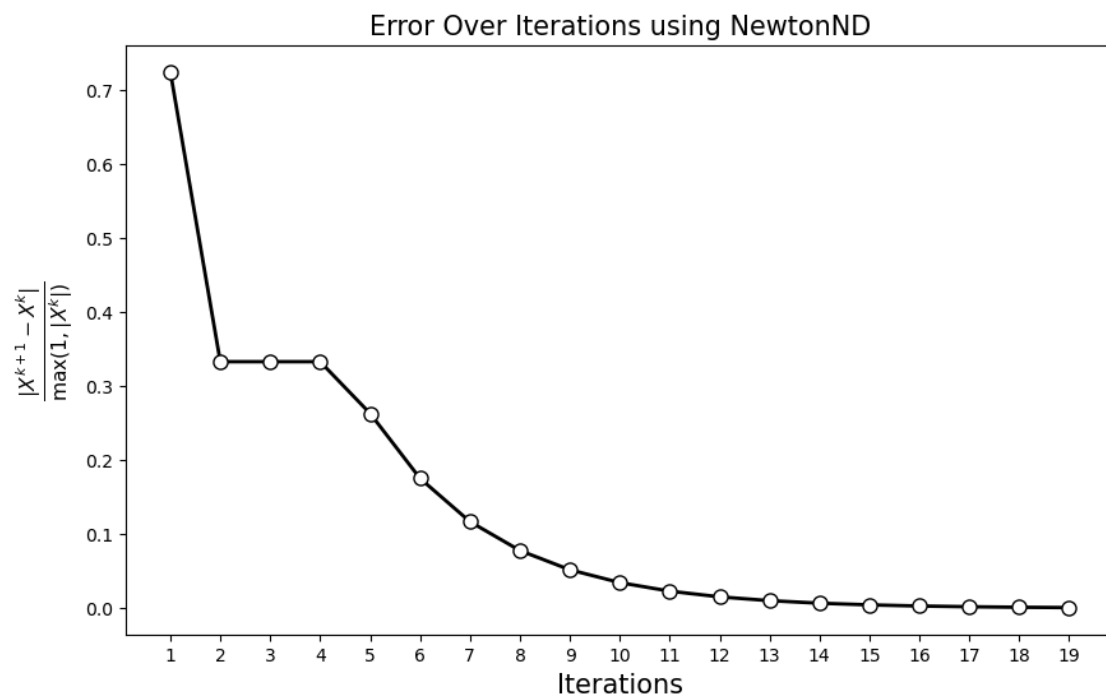
```

array([[0.00045109],
       [0.         ],
       [0.         ],
       [0.         ],
       [0.00045109],
       [0.         ],
       [0.         ],
       [0.00045109],
       [0.00045109],
       [0.         ],
       [0.         ],
       [0.         ],
       [0.00045109],
       [0.00045109],
       [0.         ],
       [0.         ],
       [0.         ],
       [0.         ],
       [0.00045109],
       [0.00045109],
       [0.00045109],
       [0.00045109],
       [0.00045109],
       [0.00045109],
       [0.00045109],
       [0.         ],
       [0.00045109],
       [0.00045109],

```

[0.00045109]])

[24]:



[]: