Belief Base Revision for Qualitative Reasoning Models

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Abstract

We present a principled approach to revising Qualitative Reasoning Models based on observations. There are already approaches that integrate unexplained observations into the model by reasoning about potential hypotheses and modifications to the model. We pose this process as *belief base revision*, adapting classic postulates of belief revision theory to suit the non-monotonic and non-deterministic nature of qualitative simulation. As a result, we obtain an operator that uses logic-based abduction to accomplish model update.

1 Introduction

Humans perceive, understand, and explain the physical world intuitively by identifying causal relationships and formulate general principles. Creating and correcting the understanding, i.e., the *model* of the world, is a continuous cognitive task, evident in a child's development that leads to more precise predictions and explanations [Johnson-Laird, 1983].

Being able to reason beyond what has been directly experienced is an important part of intelligence. It helps us predict and plan for specific outcomes. Particularly, in a naive or common-sense setting, agents behave in accordance with their current beliefs about their environment. If conflicting evidence arises these beliefs are challenged and may be changed to account for the new insights.

The field of *belief change* models how rational agents could change their beliefs. The most prominent logical framework is AGM Belief Revision, named after the authors Alchourron, Gärdenfors and Makinson [Alchourrón *et al.*, 1985], where beliefs are represented as epistemic inputs of sets in propositional logic, so called belief sets [Alchourrón *et al.*, 1985]. Representing beliefs with deductively closed belief sets is computationally often impractical, as maintaining all possible beliefs is intractable [Nebel, 1998]. Belief base revision addresses this problem by maintaining a non-deductively closed set of core beliefs, the belief base, from which other beliefs can be derived [Hansson, 1999a].

However, neither approach captures the inductive reasoning that occurs, when an agent tries to make sense of what it observed. Technically speaking, making sense of observations means to ground them in the agent's model of the world We argue that to understand, predict, and explain beliefs one needs to incorporate abductive and inductive reasoning to form and to integrate hypotheses into an explicit qualitative and causal representation.

In this paper we propose a belief base revision operator that aims to induce knowledge from observation. Specifically, we apply a belief revision approach to the revision of QR models, which have long played a key role in representing mental models for common-sense reasoning, conceptual modeling and explanation [Bredeweg, 1992; Forbus, 2019].

2 Preliminaries

We introduce concepts of Qualitative Reasoning and Belief Revision important for this paper.

2.1 Qualitative Reasoning

Humans tend to base explanations on causal processes between physical entities [Halpern and Pearl, 2005], while dramatically abstracting from numerical details. A Qualitative Simulation Model (QSM) is an explicit, graph-like representation that makes causal effects and constraints of the domain it represents explicit [Forbus, 1984]. Nodes represent quantities that are influenced by other quantities by explicit links, for example the effects of applying physical forces.

Qualitative Simulation Model (QSM)
We represent a QSM according to the Garp3 modeling toolkit [Bredeweg *et al.*, 2009] as a graph $\mathcal{M} = \langle Q, P, C \rangle$, where:

- Q is a set of nodes representing *quantities* associated with physical entities;
- P is a set of (directed) edges representing processes, which indicate *causal dependencies* between quantities;
- C is a set of relations *inequalities*[1] between quantities, acting as static constraints.

The vector of all QSM quantities that hold at a given point in time can be regarded as *state*. Simulation of QSMs considers how states evolve over discrete time points t_i, within a finite horizon h, where $1 \leq i \leq h, h \in \mathbb{N}$. The progression of states can be organized as a trajectory of the system,

[1]typically, inequalities including $\{=_p, \leq_p, \geq_p\}$ in point calculus [Vilain *et al.*, 1990] are considered, but the set may be generalized to other types of relational algebra [Dylla *et al.*, 2013]

represented either as a branching tree or general graph. Trees rooted at a specifc state can be used to capture multiple possible outcomes, while graphs can portray the set of all states and their potential transitions.

Quantities Q are variables and can take values from the finite and discrete domain $\mathbb{D}(q), q \in Q$. These domains are called *quantity spaces*. Continuous variables are turned into quantities by discretization, typically using a step function, e.g., from the reals to the quantity space. At any given discrete time point t_i each quantity q has a value (magnitude) $\text{val}(q, t_i) \in \mathbb{D}(q)$ and a derivative $\delta(q, t_i) \in \{-, 0, +\}$, which is also denoted using arrows to indicate the trend. The derivative indicates the trend of the quantity at the next time point t_{i+1}. Thus, a time point during simulation is associated with a qualitative state $\sigma \in \Sigma_Q$, which itself is a configuration of magnitudes and derivations of the quantities.

Causal dependencies P between two quantities q_i, q_k are dynamic constraints that determine the trajectory of a simulation by constraining and influencing the values of the quantities. Linking a cause q_i to a target q_j, causal dependencies take the form of positive or negative *influences* $I^{\pm}(q_i,\ q_j)$, which change the derivative of target quantity q_j based on the magnitude of q_i. *Proportionalities* $P^{\pm}(q_i, q_j)$ are indirect influences that propagate the effect of a process from q_i to q_j. Finally, *Correspondences* $C(q_i, q_j)$ are constraints which state that the magnitudes of quantities correspond. Causal dependencies can be conditional, such that it only takes effect if a certain condition is met, concerning magnitude, relation and derivation of a quantity[2]. Any transition between states σ_i and σ_{i+1} must be justified. In other words, for the derivative of a quantity to change from t_i to t_{i+1} there has to exist a causal dependency for that change.

Inequalities C between quantities q_i, q_k express static constraints, e.g., that the value of one quantity must be less than that of another, written $\text{val}(q_i, t) <_p \text{val}(q_k, t)$, or feature simple operations over signs such as subtraction and multiplication [Travé-Massuyès *et al.*, 2003].

Simulation of QSMs is performed by generating all possible future states for a given state. Any state transition must be justified by causal dependencies, i.e., dynamic constraints, and consistent with inequalities, i.e., static constraints. Linking states to possible future states, a graph is obtained that encompasses possible trajectories (paths) through the state space. QSM are *explainable* in the sense that all state transitions are justified by the constraints that permit or inhibit the transition. QSM are *explainable* in the sense that all state transitions are justified by the constraints that permit or inhibit a transition.

Example 1 (Bathtub)**.** *Consider a bathtub filling up with water while the drain is open. The amount of water in the tub, as well as the inflow and outflow streams, are relevant quantities* $Q = \{flow_{in}, flow_{out}, level\}$*, which are all at least zero and have a maximum magnitude, but can take any value in between. This determines their quantity space:* $\mathbb{D}(flow_{in}) = \mathbb{D}(flow_{out}) = \mathbb{D}(level) = \{zero, plus, max\}$*. Causal dependencies* P *include the inflow, which causes the water level to rise,* $I^{+}(flow_{in}, level)$*, while the outflow causes it to fall,* $I^{-}(flow_{out}, level)$*. Additionally, the outflow is proportional to the water level,* $P^{+}(level, flow_{out})$*, and directly corresponds to it,* $C(level, flow_{out})$*. Figure 2 depicts a state at a time point* t_1 *where* $val(flow_{in}, t_1) = val(flow_{out}, t_1) = plus$*, yet* $val(flow_{in}, t_1) >_p val(flow_{out}, t_1)$ *and thus* $\delta(level, t_1) = +$*.*

[2] for details on QSM components, see [Bredeweg *et al.*, 2009]

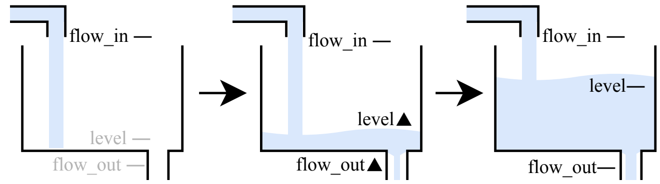

Figure 1: Water fills a container from a faucet and drains out below. The water level indicates volume, with equilibrium when inflow equals outflow.

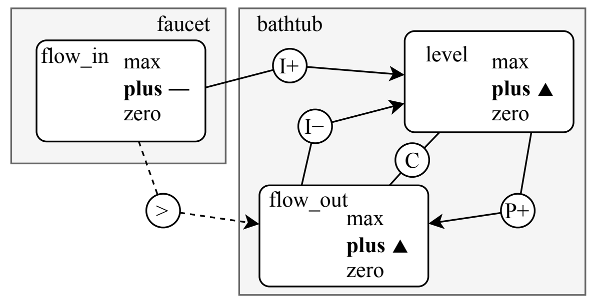

Figure 2: Graphical representation of a qualitative simulation model explaining the behavior shown in Figure 1, depicting magnitudes and derivatives from the middle panel.

Constraints on State Transitions

In the context of this paper, we view a QSM as a formalism that constrains transitions on a system's state space.

Definition 1 (State Transitions)**.** *The set of all possible state-transitions of a system is a set* $\mathcal{T}_Q \subseteq \Sigma_Q \times \Sigma_Q$*, where:*

- Σ_Q *denotes states i.e. a system's set of configurations.*
- *All* $(\sigma_i, \sigma_j) \in \mathcal{T}_Q$ *have to adhere to the continuity of quantity spaces of* Q *and conceptual neighborhood of relations [Freksa, 1991], i.e., quantities may only change to neighboring values.*

By constraining the set of transitions, a QSM determines which transitions define the behavior of a system. In other words, a QSM $\mathcal{M} = (Q, P, C)$ enables and forbids certain transitions of $\mathcal{T}_Q$ based on the dynamic and static constraints, forming a *state diagram*[3].

Definition 2 (State Diagram)**.** *For a QSM* $\mathcal{M}$*, a state diagram, also called an envisioning graph [Kleer and Brown, 1982], is a directed graph Env*$(\mathcal{M}) = \langle \Sigma_Q, \mathcal{T}_{\mathcal{M}} \rangle$*, with:*

- *A finite set of states* Σ_Q

[3] A state diagram can also be viewed as a transition-system without actions.

- *A finite set of transitions $\mathcal{T}_{\mathcal{M}} \subseteq \mathcal{T}_Q$ consistent with $\mathcal{M}$.*

We abbreviate a sequence of consecutive transitions $\{(\sigma_0, \sigma_1), (\sigma_1, \sigma_2), \ldots, (\sigma_{n-1}, \sigma_n)\} \in \mathcal{T}^*_{\mathcal{M}}$ as $(\sigma_0, \ldots, \sigma_n)$.

Observations

In most system, not all variables are directly observable, even though they may influence behavior. To connect qualitative simulation models (QSMs) with data, we define how observations are derived from underlying system states.

Definition 3 (Partially Observable System)**.** *A partially observable system modeled by a QSM is a tuple $(Env(\mathcal{M}), \Omega, Obs)$ where:*

- *$Env(\mathcal{M})$ is the state diagram (envisioning graph) of the system*
- *Ω is the set of observables, a partial mapping of quantities to qualitative magnitudes*
- *$Obs : \Sigma_Q \to 2^{\Omega}$ assigns to each qualitative state $\sigma \in \Sigma_Q$ the set of perceivable observations*

Transitions between states generates observable changes:

Definition 4 (Observed Transition)**.** *Given a system $\mathcal{S} = (Env(\mathcal{M}), \Omega, Obs)$, an* observed transition *is a pair (μ, μ') such that for some state transition $(\sigma, \ldots, \sigma')$:*

$$(\sigma, \ldots, \sigma') \models (\mu, \mu') \quad \Leftrightarrow \quad Obs(\sigma) = \mu \text{ and } Obs(\sigma') = \mu'.$$

Definition 5 (Observation)**.** *An* observation *θ is a sequence of observed transitions:*

$$\theta = \{(\mu_0, \mu_1), (\mu_1, \mu_2), \ldots, (\mu_{n-1}, \mu_n)\}.$$

We write $\mathcal{M} \models \theta$ if there exists a trace in the QSM's state diagram that includes transitions justifying all pairs in θ. A set of observations Θ may contain several observations that may be generated from unrelated traces of a system.

Observations may vary in specificity. We say $\theta \models \theta'$ if all traces satisfying θ also satisfy θ'. Due to the inherent non-determinism of QSMs, we typically cannot infer $\theta \models \neg\theta'$. For instance, observing an object *not* passing through a wall does not rule out the hypothetical scenario that it *could* pass under different conditions.

Example 2 (Hidden Variables)**.** *Figure 3 shows a modified QSM where outflow depends on a hidden variable: bottom_pressure, which increases with water level. This extension explains scenarios (e.g., pressure-induced outflow) that the original model cannot explain, highlighting how unobservable factors may justify observed behavior.*

2.2 Belief Base Revision

Belief revision is the study of how an agent updates its beliefs in light of new information. Typically, this involves making a minimal change to the current belief state. Such belief changes are often formalized as operations governed by the well-known AGM framework [Alchourrón *et al.*, 1985] we summarize below.

Let $\mathcal{L}$ be a propositional language with a finite set of propositional variables $\mathcal{P}$ and the usual connectives. An interpretation ω of $\mathcal{P}$ is an assignment to truth values for each variable. An interpretation ω is also called a *possible world* and set of all possible worlds is $\mathcal{W}$. A *fact* is a formula $\Phi \in \mathcal{L}$, where $[\Phi]$ denotes the set of worlds that satisfy Φ, i.e., $[\Phi] = \{\omega \in \mathcal{W} \mid \omega \models \Phi\}$. A set $K \subseteq \mathcal{L}$ is called a belief set when it is deductively closed. Logical consequences are denoted by $Cn(B)$, thus $K = Cn(K)$ representing an agent's beliefs. Belief revision based on belief sets, as characterized by the AGM postulates, has been widely studied within the context of classical logic. A revision operator ($\star$) is introduced that represents belief updates. Revising a belief set K by a sentence Φ results in a transition of belief set as $K \star \Phi$.

Example 3. *A container is being filled with water. Let $\mathcal{P} = \{f, o\}$ where f means "there is inflow" and w means "the container eventually overflows". An agent beliefs $Cn(K)$ where $K = \{f, f \to o\}$, but now observes $\Phi = \neg o$. Which is inconsistent with K. A minimal change is made to the beliefs i.e. $K \star \Phi = \{f, \neg o\}$, giving up the implication $f \to o$ in for $\neg o$.*

In the field of Artificial Intelligence, belief sets are often impractical due to their potentially large size [Nebel, 1998]. By contrast, syntax-based approaches focus on revising specific formulas rather than entire belief sets. These methods operate on a more compact representation known as a *belief base* [Hansson, 1999b].

A belief base $B \subseteq \mathcal{L}$ is a finite collection of beliefs that is not necessarily deductively closed. Its logical consequences are denoted by $Cn(B)$, with $B \neq Cn(B)$ in general. For two logically equivalent belief bases A and B, such that $Cn(A) = Cn(B)$ revising $Cn(A)$ with Φ will not be the same as revising $Cn(B)$ with Φ in general since A and B can differ significantly despite the fact that $Cn(A) = Cn(B)$ holds. Thus, there is a strong sensitivity to syntax. This distinction allows for a more compact representation of beliefs and enables derived beliefs to be discarded when their supporting justifications are removed during revision [Peppas, 2008]. As a result, belief bases are particularly well-suited for contexts that require attention to the explanatory structure of beliefs or diagnostic reasoning. Specifically, belief bases ease understanding *why* something is believed.

Rationality Postulates

A belief base change operation is denoted by an operation $\circ : B \times \mathcal{L} \to B$. Revising ($\star$) a belief base B by a sentence

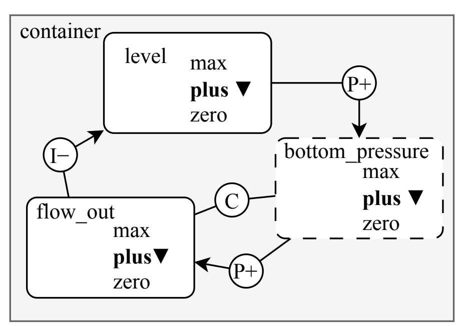

Figure 3: Qualitative Simulation Model with a hidden variable (`bottom_pressure`).

φ results in a transition of belief base as $B \star \varphi$. Belief base revision operators are considered to be *rational* if they satisfy the following postulates given by Hansson [Hansson, 1999a]. Let B and B' be consistent belief bases, $B \nvdash \bot$, and μ and φ be propositional formulas:

$$B \star \mu \vdash \mu \qquad \text{(Success)}$$

$$B \star \mu \subseteq B \cup \{\mu\} \qquad \text{(Inclusion)}$$

$$\text{If } \mu \nvdash \bot \text{, then } B \star \mu \nvdash \bot \qquad \text{(Consistency)}$$

$$\text{If } B \cup \{\mu\} \nvdash \bot \text{, then } B \star \mu = B \cup \{\mu\} \qquad \text{(Vacuity)}$$

$$\begin{aligned} &\text{If } \varphi \in B \text{ and } \varphi \notin B \star \mu, \text{ then } \exists B' \\ &\quad \text{such that } B \star \mu \subseteq B' \subseteq B \cup \{\mu\}, \\ &\quad B' \nvdash \bot \text{ and } B' \cup \{\varphi\} \vdash \bot \qquad \text{(Relevance)} \end{aligned}$$

$$\begin{aligned} &\text{If } \forall B' \subseteq B, \\ &\quad B' \cup \{\mu\} \vdash \bot \Leftrightarrow B' \cup \{\varphi\} \vdash \bot \text{, then} \\ &\quad B \setminus (B \star \mu) = B \setminus (B \star \varphi) \qquad \text{(Uniformity)} \end{aligned}$$

These postulates ensure that revision accepts new facts (Success), avoids inconsistency when possible (Consistency), and stays within the bounds of the original base expanded by the new information (Inclusion). When a new fact is consistent with the belief base B, revision simply adds it to the belief base (Vacuity).

Naturally, a belief is only removed if keeping it would cause inconsistency with newly received facts (Relevance). Finally, if facts cause exactly the same contradictions, then they should lead to identical changes in what is believed (Uniformity).

3 Revising QSMs from Observations

There are several motivations for revising Qualitative Simulation Models (QSMs). First, revision is helpful for repair and debugging of preexisting models, as it restores consistency when contradictions or errors are detected. Second, since QSMs aim to represent conceptual models, revision can serve as a way to align abstract or intangible models—for example, aligning a QSM with a human's mental model, where observations correspond to that person's verbal or behavioral inputs. Finally, in an incremental learning context, revision provides an automated process for hypothesis generation and model refinement, allowing QSMs to evolve and improve as new data or observations become available.

We will start with an example of how revising QSM can be used go build upon a existing understanding of the world.

Example 4 (Bathtub II)**.** *An agent has previously observed a bathtub filling with water while the drain was open. The agent's beliefs in the QSM are consistent with this observation. Now, the faucet is closed. According to the agent's current QSM, the water level should grow or remain the same. However, the agent observes that the water level is decreasing, which contradicts its current beliefs (see Figure 4). To accommodate this new observation, the agent must revise its beliefs. Specifically, it must include $I^-(flow_{out}, level)$ to justify the decreasing water level.*

Recall Example 3, where propositions are treated as epistemic inputs. The standard AGM framework is not typically concerned with selecting hypotheses or explaining observations. However, in the context of common-sense or scientific reasoning, belief updates must be grounded in an underlying model of the world (as illustrated in Example 4).

A QSM can explain new observations when incorporating model components as *hypotheses*. The construction of these explanations is inherently inductive. To formalize this, we draw from the notion of a *hypothesis space* in relational learning [De Raedt, 2008]. We distinguish between raw observational data and *induced beliefs*, those formed by selecting a QSM from a space of possible models, where each model defines a language of plausible observations derived from its structure.

3.1 Rational QSM Revision

We now adapt the postulates for belief base revision to qualitative simulation models and observations.

Given a language of QSM components $\mathcal{L}_{\text{QSM}}$ (hypothesis) and observations $\mathcal{L}_{\text{OBS}}$ (examples), we define a cover relation $c : 2^{\mathcal{L}_{\text{QSM}}} \times \mathcal{L}_{\text{OBS}}$, where $c(\mathcal{M}, \theta) \iff \mathcal{M} \models \theta$. Relating to belief base revision, this could also be phrased as the observation being justified as a consequences of a belief in a certain model $Cn(\mathcal{M}) \models \theta$. However, unlike relational learning, QSMs do not exhibit a general-to-specific ordering of hypotheses. Adding model components will not necessarily result in less examples covered (non-monotonic). We split belief bases into two parts such that $\Psi = (\mathcal{M}, \text{OBS})$ where $\mathcal{M}$ is a QSM representing the induced beliefs and OBS is the set of integrity constraints, for example past observations. Revising ($\star$) a belief base Ψ of a QSM $\mathcal{M}$ and the retained observations OBS by a set of *new* observations Θ results in a transition to $\Psi \star \Theta$.

Adapted Rationality Postulates

Let $\mathcal{M}, \mathcal{M}' \in 2^{\mathcal{L}_{\text{QSM}}}$ be QSMs, $\Theta, \Theta', \Theta'' \in 2^{\mathcal{L}_{\text{OBS}}}$ be sets of observations and $\mu, \varphi \in \mathcal{L}_{\text{QSM}}$ model components that may be added or removed from a QSM $\mathcal{M} = \langle Q, P, C \rangle$.

The notion of consistency can be adapted to QSMs: Intuitively a system is consistent with a set of observations if it can justify an outcome by its model components. Furthermore, since models are nondeterministic, consistency relies on the stepwise satisfaction of the observation given as sequence of state transitions. For brevity we define $\mathcal{M} \models \Theta \Leftrightarrow \forall \theta \in \Theta : c(\mathcal{M}, \theta)$. We write $\Psi \models \Theta$ iff $\mathcal{M} \models \Theta$.

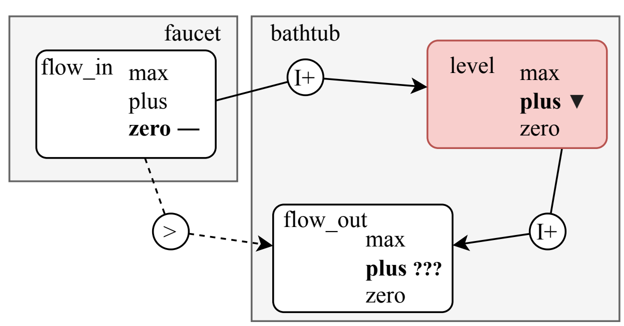

Figure 4: Version of a QSM modelling a Bathtub. The Model fails to justify the represented state and is thus inconsistent. A minimal revision is the addition of $I^-(flow_{out}, level)$.

Definition 6 (QSM Consistency)**.** *A QSM belief base* $\Psi = (\mathcal{M}, OBS)$ *is consistent if it can justify every retained observation. In other words,* Ψ *is consistent iff* $\mathcal{M} \models OBS$*, otherwise* $\Psi \models \bot$*.*

Let $\Psi = (\mathcal{M}, \text{OBS})$ be consistent and

let $\Psi \star \Theta = (\mathcal{M}', \text{OBS}')$ and $\Psi \star \Theta' = (\mathcal{M}'', \text{OBS}'')$:

$$\Psi \star \Theta \models \Theta \qquad \text{(Success*)}$$

$$\text{If } \Theta \not\models \bot \text{, then } \Psi \star \Theta \text{ is consistent} \qquad \text{(Consistency*)}$$

$$\text{If } \Psi \models \Theta \text{ , then } (\mathcal{M}, \text{OBS}) \star \Theta = (\mathcal{M}, \text{OBS}') \qquad \text{(Vacuity*)}$$

$$\begin{aligned}&\text{If } \varphi \in \mathcal{M} \text{ and } \varphi \notin \mathcal{M}' \text{ then}\\ &\exists \Psi_\Delta = (\mathcal{M}_\Delta, \text{OBS}_\Delta) \text{ where } \mathcal{M}' \subseteq \mathcal{M}_\Delta, \text{OBS}_\Delta \subseteq \text{OBS}',\\ &\text{and } \Psi_\Delta \text{ is consistent, but } (\mathcal{M}_\Delta \cup \{\varphi\}, \text{OBS}_\Delta) \text{ is not}\end{aligned} \qquad \text{(Relevance*)}$$

$$\begin{aligned}&\text{If } \forall \mathcal{M}_\Delta \subseteq \mathcal{M} \text{ and } \forall \text{OBS}_\Delta \subseteq \text{OBS}\\ &(\mathcal{M}_\Delta, \text{OBS}_\Delta \cup \Theta) \models \bot \Leftrightarrow (\mathcal{M}_\Delta, \text{OBS}_\Delta \cup \Theta') \models \bot\\ &\text{then } \mathcal{M} \setminus \mathcal{M}' = \mathcal{M} \setminus \mathcal{M}''\end{aligned} \qquad \text{(Uniformity*)}$$

For these postulates the notion of consistency has changed as it is determined by violation of *integrity constraints*, i.e., observations. Consistency, success and vacuity are defined in a similar way to their counterparts for standard belief base revision.

Most notably, inclusion cannot be transfered to QSMs since the generation of *hypotheses* in the form of model components, will inevitably supersede the initial model. Consequently, relevance and uniformity remain similar in intuition, yet require an adjustment in order to the handle the distinction of model components and observations.

Construction

Typically, belief revision considers classical logics that are propositional and monotonic. Revision operators can then be constructed using contraction and expansion operations on the belief base, called Levi identity [Gärdenfors, 1981].

However, with respect to observations classical negation does not apply, and a QSM may need to add a dynamic constraint to repair inconsistencies, which is non-monotonic. In this section we describe an operator that accomplishes to satisfy the above postulates.

Revising a QSM in response to an unexplainable observation is closely related to *abductive diagnosis* [Console and Torasso, 1991], the key difference being that diagnoses determines deviations of the current model. Generally, we can implement revision as a logic-based abduction problem[4]. Given a logical theory T, a set of observations O, and a set of hypotheses H, an *abductive explanation* is a minimal subset $S \subseteq H$ such that: $T \cup S \models O$ and $T \cup S$ is consistent. Intuitively, the subset S here corresponds to a minimal modification to the underlying QSM as a theory.

With QSMs we aim to capture the natural causal structure and explanations as they are perceived by humans. In order to represent the process of changing these underlying models we define a QSM revision operator using minimal revisions using a distance function.

Definition 7 (Pseudo-Distance [Schlechta *et al.*, 1996])**.** *A function* $d : \mathcal{W} \times \mathcal{W} \to \mathbb{R}$ *is called a* pseudo-distance *if for all* $\omega, \omega' \in \mathcal{W}$ *it satisfies the following properties:*

$$d(\omega, \omega') \geq 0 \qquad \text{(Non-negativity)}$$

$$d(\omega, \omega') = 0 \iff \omega = \omega' \qquad \text{(Identity)}$$

$$d(\omega, \omega') = d(\omega', \omega) \qquad \text{(Symmetry)}$$

For QSMs $\mathcal{M} = \langle Q, P, C \rangle$ and $\mathcal{M}' = \langle Q', P', C' \rangle$ we define a distance using the symmetric difference of components.

$$d_\mathcal{M}(\mathcal{M}, \mathcal{M}') = |Q \,\Delta\, Q'| + |P \,\Delta\, P'| + |C \,\Delta\, C'| \qquad (1)$$

Intuitively, QSM distance $d_\mathcal{M}$ captures change in the QSM's set of components, and clearly satisfies non-negativity, symmetry and identity, and is thus a pseudo-distance. A revision $\star_\mathcal{M}$ minimal with respect to this distance maintains as much of the original model as possible, while the QSM itself acts as the supporting justification for a new observations.

Definition 8 (Abductive QSM Revision)**.** *Let* $\Psi = (\mathcal{M}, OBS)$ *and* $\Psi' = (\mathcal{M}', OBS')$ *be belief bases and* Θ *an observation. An* abductive QSM revision *is defined as* $\Psi \star_\mathcal{M} \Theta = \Psi'$ *such that:*

$$OBS' = OBS \cup \Theta$$

$$\underset{\mathcal{M}' \subseteq \mathcal{L}_\mathcal{M}}{\operatorname{argmin}} \{ d_\mathcal{M}(\mathcal{M}, \mathcal{M}') \,|\Psi' \textit{ is consistent}\}$$

In other words, a QSM that is similar in terms of model components to the current believed model is preferred over drastic changes.

In QSMs, adding components to explain new observations can interact nontrivially with existing structure, resulting in inconsistencies with past observations. To address this, the revision operator is designed to be *strongly data-retentive* [Baltag *et al.*, 2019], it explicitly retains past observations as integrity constraints, ensuring that new model hypotheses do not invalidate previously accepted evidence.

Formal Analysis

Finally we show that the operator defined in Definition 8 is rational w.r.t. the adapted postulates.

Proposition 1. *The* $\star_\mathcal{M}$ *operator using* $d_\mathcal{M}$ *as distance measure verifies **(Consistency*)**.*

Proof. $\Psi \star_\mathcal{M} \Theta$ beeing consistent follows from Def. 8. □

Proposition 2. *The* $\star_\mathcal{M}$ *operator using* $d_\mathcal{M}$ *as distance measure verifies **(Success*)**.*

Proof. Let $\Psi = (\mathcal{M}, \text{OBS})$, from Def. 8 have $\Psi \star_\mathcal{M} \Theta = (\mathcal{M}', \text{OBS} \cup \Theta)$. From Proposition 3.1 (Consistency*) holds, hence $\Psi \star_\mathcal{M} \Theta \models \text{OBS} \cup \Theta$, which subsumes $\Psi \star_\mathcal{M} \models \Theta$. Thus $\star_\mathcal{M}$ achievs **(Success*)**. □

Proposition 3. *The* $\star_\mathcal{M}$ *operator using* $d_\mathcal{M}$ *as distance measure verifies **(Vacuity*)**.*

[4] logic-based abduction in general is a problem that also falls into a high complexity class of Σ_2^P [Eiter and Gottlob, 1995]

Proof. Have $(\mathcal{M}, \text{OBS}) \star_{\mathcal{M}} \Theta = (\mathcal{M}', \text{OBS} \cup \Theta)$, if $\Psi \models \Theta$. From Def 8 and (Consistency*) have $\text{argmin}_{\mathcal{M}' \subseteq \mathcal{L}_{\mathcal{M}}} \{d_{\mathcal{M}}(\mathcal{M}, \mathcal{M}') \,|\mathcal{M}' \models \text{OBS} \cup \Theta\}$ and because of (Identity) and (Non-Negativity) have $d_{\mathcal{M}}(\mathcal{M}', \mathcal{M}) = 0$ where $\mathcal{M} = \mathcal{M}'$ thus $(\mathcal{M}, \text{OBS}) \star \Theta = (\mathcal{M}, \text{OBS} \cup \Theta)$ verifying (Vacuity*). ☐

Proposition 4. *The $\star_{\mathcal{M}}$ operator verifies **(Relevance*)**.*

Proof. We show this by relating to Consistency-Based Diagnosis [Reiter, 1987], where an inconsistency is caused by a model component φ, and subsequently resolved by removing φ or supplementing it with other model components.

Let $\mathcal{M}$ be the QSM, and $\mathcal{M}'$ the QSM after revision by Θ. If $\mathcal{M}$ is inconsistent with Θ, the conservation of model components conflicts with (Consistency*). If a model component φ is retracted, then this change is only made if that component is the cause of the inconsistency. Since, due to the symmetric difference, we have $d_{\mathcal{M}}(\mathcal{M}, \mathcal{M}') \geq 1$, (Relevance*) is verified. ☐

Proposition 5. *The $\star_{\mathcal{M}}$ operator verifies **(Uniformity*)**.*

Proof. Assume for $\Psi \star \Theta' = (\mathcal{M}', \text{OBS}')$ and $\Psi \star \Theta' = (\mathcal{M}'', \text{OBS}'')$, for all $\mathcal{M}_\Delta \subseteq \mathcal{M}, \text{OBS}_\Delta \subseteq \text{OBS}, (\mathcal{M}_\Delta, \text{OBS}_\Delta \cup \Theta') \models \bot \Leftrightarrow (\mathcal{M}_\Delta, \text{OBS}_\Delta \cup \Theta'') \models \bot$. Since $\mathcal{M}$ revised by Θ' or Θ'' is behaviorally equivalent w.r.t. consistency, their model components share the same dependency w.r.t. the conflict resolution. Since (Relevace*) holds, the minimal set of retracted components to resolve the conflicts following Def 8 are also identical, such that $\mathcal{M} \setminus \mathcal{M}' = \mathcal{M} \setminus \mathcal{M}''$, which verifies (Uniformity*). ☐

Furthermore, strong data retention does not allow for correction in the face of noisy data. However, there could be an option to discredit certain observations or to perform a separate type of belief revision on observations only to account for noise.

4 Learning Capabilities

Belief revision is concerned with the dynamics of how a rational agent would change its beliefs to maintain a consistent representation of the world. However, when considering the formation of explanatory and predictive models, ideally, an agent would have the ability to learn and converge on a *correct* model. In this section, we examine the interaction between belief revision and the learning process in QSMs.

Identification in the limit is the process by which a learning agent, given data about an unknown formal system (e.g., a language), eventually converges on a correct hypothesis after a finite amount of time, despite not knowing when this convergence occurs [Gold, 1967].

Viewing the revision of $\Psi = (\mathcal{M}, \text{OBS})$ as $\Psi \star_{\mathcal{M}} \Theta$ as a learning process, where Θ is a sequence of facts. Since QSMs are distinguished by their syntax, they may justify the same observations using different model components.

Observation 1. *Operator $\star_{\mathcal{M}}$, as a learning process, cannot identify QSMs in the limit.*

We can however investigate the convergence towards QSM with identical state diagrams.

Proposition 6. *Assuming all negative and positive observations can be provided, operator $\star_{\mathcal{M}}$, as a learning process, will determine a QSM that is extensionally equivalent to ground truth.*

Proof Sketch. If there exists a QSM $\mathcal{M}'$, with a corresponding state diagram $\text{Env}(\mathcal{M}')$. All positive examples are observations consistent with $\mathcal{M}'$ and negative are those inconsistent with $\mathcal{M}'$. When revising $\Psi \star_{\mathcal{M}} \Theta$, we condition on Θ thus only models consistent with the observations remain, eventually converging such that $\text{Env}(\mathcal{M}) = \text{Env}(\mathcal{M}')$. ☐

However, in the real world, there is no such thing as *negative observations*. Even so, since the resulting models may be syntactically different, an agent may have extensionally the same beliefs as another agent, yet both may have entirely different explanations for the observed phenomena.

When considering only positive observations, revising the model will not identify constraints such as *correspondences*, for example, since a revision cannot be triggered without inconsistency (Vacuity*). For instance, consider the QSM in Figure 2, but without the correspondence $C(flow_{out}, level)$. Regardless of which (positive) observation is presented, a revision using $\star_{\mathcal{M}}$ will not favor the more specific model depicted in Figure 2 over the already consistent model that lacks $C(flow_{out}, level)$.

4.1 Discussion

There is a clear distinction between rationality in belief revision and in hypothesis formation using abduction, particularly when observations are involved [Pagnucco and Rajaratnam, 2005]. This raises an question: Do we need quality criteria beyond parsimony (i.e., Occam's Razor) to evaluate hypotheses that aim to converge towards ground truth? Scientific inquiry often favors simplicity and elegance when selecting candidate theories, yet elegance lacks a quantifiable and elegant measure. Moreover, how can we design or guide reasoners toward intrinsically correct models? An agent that holds true beliefs for the wrong reasons still harbors misconceptions, which can hinder generalization and impair the effective application of knowledge in new contexts. Thus, how can we condition an agent to revise its beliefs in a way that promotes accurate conceptualization and steers away from persistent misconceptions?

5 Related Work

There exists a range of learning techniques that relate to QSM model learning and belief revision.

Inductive Programming: Symbolic machine learning, in particular Inductive Logic Programming (ILP), intersects naturally with belief revision. Logical and relational learning combines the subfields of machine learning and knowledge representation. The learning process typically involves a search through various generalizations of the examples [De Raedt, 2008]. During learning, ILP constructs explicit rules to approximate given training examples. These rules can also be seen as explanations. While standard ILP constructs theories from data and background knowledge, **Theory Revision**, a subfield of ILP, updates existing theories to align with new evidence. Abduction plays a key role in theory revision by proposing explanations for observed discrepancies, guiding how the theory should change. However, the changes to theories are goal oriented, rationality of individual updates are not considered. Intermediate steps of incremental ILP learning algorithms can thus be regarded as distinct belief states that reflect the portion of training examples that agree with the theory constructed so far [Ourston and Mooney, 1994; Adé *et al.*, 1994].

Qualitative Model Learning: Inverting the task of qualitative simulation, i.e., extracting a model of a system from the available observational data, is referred to as *Qualitative Model Learning* (QML) [Pang and Coghill, 2010][5]. QML tasks have been explored in the past by work in inductive and abductive logic programming (ILP/ALP) [Bratko *et al.*, 1991; Richards *et al.*, 1992; Coghill *et al.*, 2008], but the have only been based on QSMs in context of Kuiper's Qualitative Differential Equation (QDE) models [Kuipers, 1994]. Compared to QSM frameworks based on Qualitative Process Theory (QP theory) [Forbus, 1984], such as QPE [Forbus, 1990] and Garp3 [Bredeweg *et al.*, 2009], QDE models compile causal dependencies such as influences into static constraints during construction, thus causal dependencies are not directly represented [Crawford *et al.*, 1990].

Conceptual change, including theory revision, has been modeled by INTHELEX [Esposito *et al.*, 2000], an incremental theory revision program using ILP. The revision of computational models in the context of emulating changes in human mental models using abductive reasoning has been explored using TIMBER [Friedman *et al.*, 2018], where model revisions occur in the form of *model fragments*. In [Falkenhainer and Forbus, 1991], predefined compositions of QP theory model components are organized in a *domain theory* [Friedman and Forbus, 2011].

Learning as Belief Change: Several works have further explored the connection between learning and belief revision. For example, the foundational ILP technique inverse resolution [Muggleton and Buntine, 1988] can also be modeled using belief change operators [Pagnucco and Rajaratnam, 2005].

The explicit use of belief revision operators allows a strong inductive bias to be modeled. Put differently, designing belief revision operators allows us to control which model will be preferred over another during learning. This is key to learning reasonable models from few informative examples. Such an approach differs from standard *theory revision* (e.g., [Ourston and Mooney, 1994; Adé *et al.*, 1994]) by focusing on navigating the space of potential explanations for training examples that is consistent with the rationale of AGM belief revision. Belief revision can be seen as navigation in the *epistemic space* that is shaped by the sequence of sentences received for revision [Baltag *et al.*, 2019].

6 Conclusion

We have presented a principled approach to revising qualitative simulation models from observations by adapting AGM belief revision to accommodate the non-monotonic nature of qualitative simulation. Our main contributions are: (1) adapted belief base revision postulates for QSMs, (2) an abductive revision operator that minimizes structural changes while maintaining consistency, and (3) formal proofs of the operator's rationality. The revisions enable automated hypothesis generation and model refinement by treating model components as inductively formed hypotheses. While syntactic limitations prevent unique model identification from observations alone, the proposed revision operation is data-retentive and ensures consistency with past evidence. This work provides a foundation for adaptive agents that can refine their causal understanding through experience, with applications in model debugging, explanation generation and common-sense reasoning systems.

[5] QML specifically refers to **Q**DE **M**odel **L**earning

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