# Chapter 12 Oscillations

1. Simple Harmonic Motion (SHM)
2. Expression Methods of SHM
3. Energy in SHM
4. Pendulums
5. Resonance
6. Superposition of Oscillations

## What is the difference between Oscillation and Vibration?

**An oscillation is periodic motion in which the cycle involves a back-and-forth motion about a given position (motions that repeat themselves). The "given position" is called the equilibrium point; it's where the object would normally rest when it is not oscillating. Examples can include:**

- **the pendulum of a grandfather clock;**
- **a block suspended from the end of a spring;**
- **alternating current.**

**We can further distinguish the above three examples.**

**The first two (pendulum and block) involve the periodic motion of matter and could be further classified as vibrations. When matter is required for a given motion, we tend to use the word mechanical to describe the motion.**

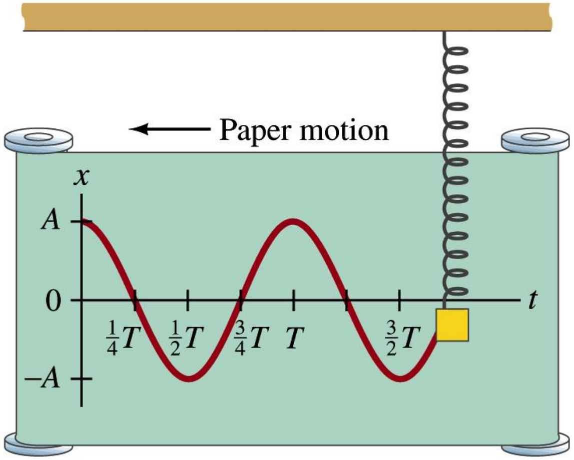
**The third does not require the motion of matter and is therefore an oscillation but not a vibration.**

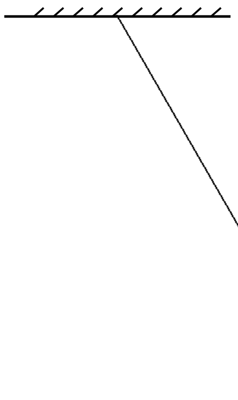
**An oscillation is a back-and-forth, periodic motion.**

**A vibration is a mechanical oscillation.**

# 12-1 Simple Harmonic Motion (P299)

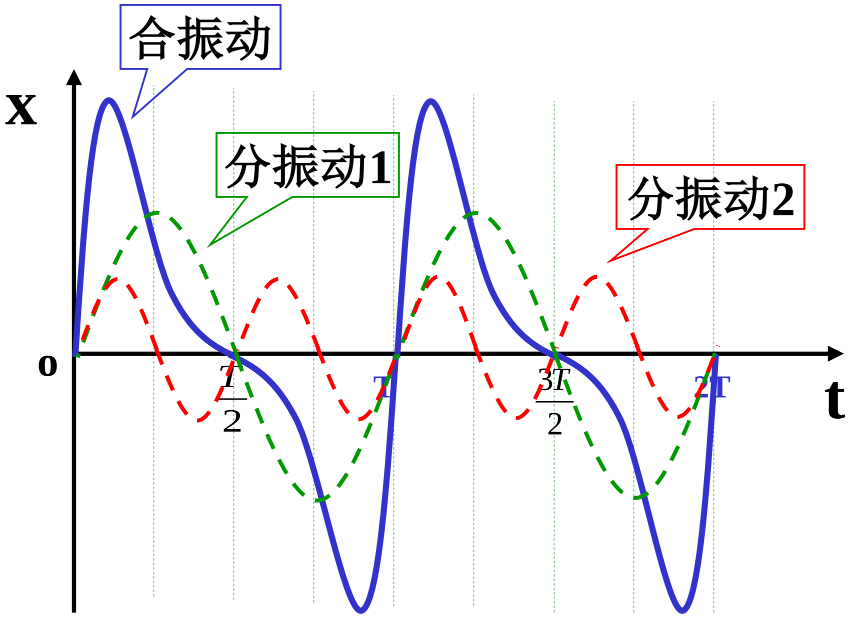
**We limit our considerations largely to 1D systems —— motion can be described by one single variable. In each system, the displacement varies sinusoidally with time. The sine and cosine functions are called harmonic functions, and this type of motion is referred to as Simple Harmonic Motion (SHM) or Simple Harmonic Oscillation (SHO).**



**The simple harmonic motion (SHM) is the most simple and basic vibration. Every complicated vibration can be considered as composing of several SHM.**

(简谐振动是一种最简单最基本的振动, 一切复杂的振动都可以看作是若干简谐振动的合成的结果)

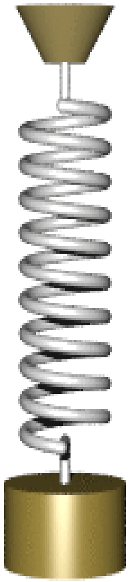


# 1. Features of SHM:

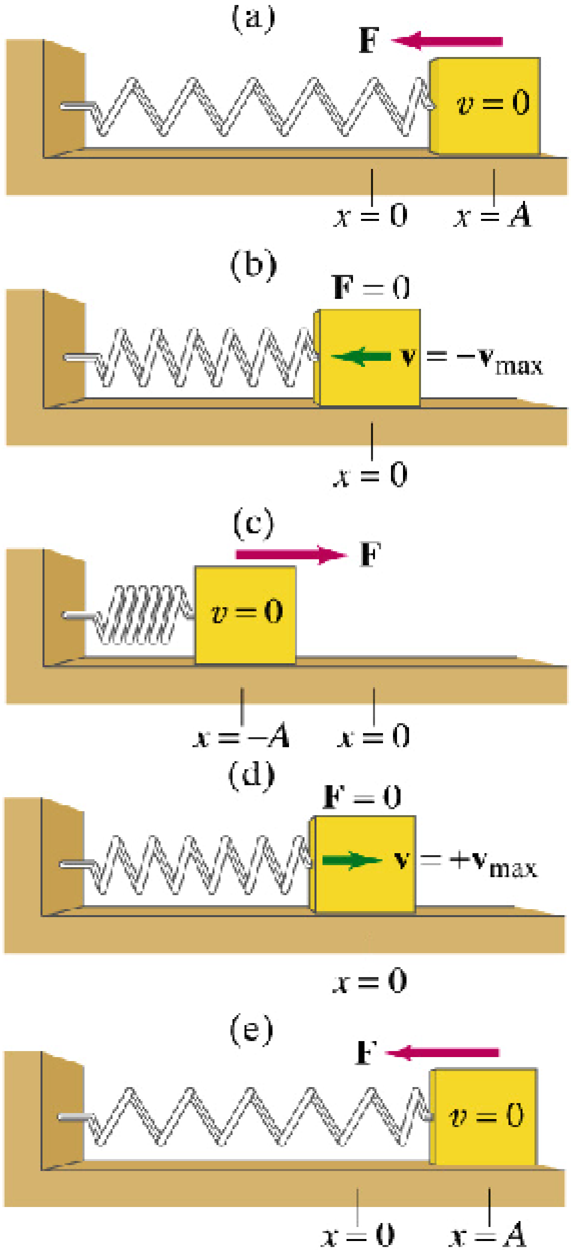
- **It can be represented by a simple sine or cosine function.**

- **A restoring force must act on the body (Body must have acceleration in a direction opposite to the displacement).**

- **The acceleration must be directly proportional to displacement.**



# 2. Equation of SHM (P299):




**The small-amplitude vibration (小幅振动) for the spring,**

**Object:** **block-spring system;**

**Origin:** **equilibrium position**

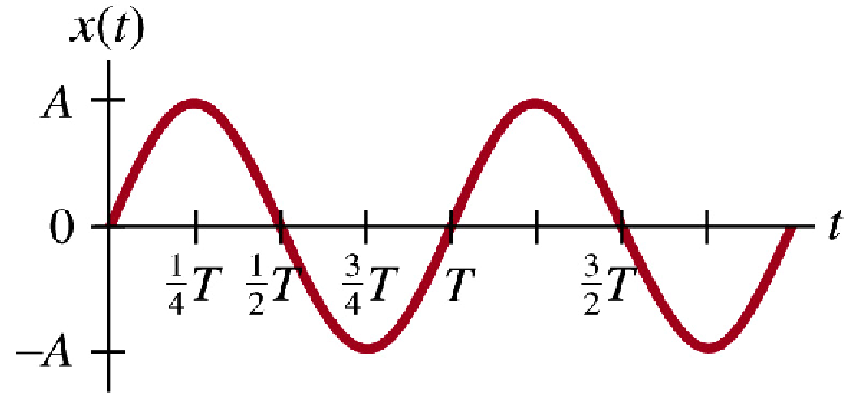
**From Hooke's Law:** $F = -kx$

**Applying N-II law:**

$$\frac{d^2x}{dt^2} = \frac{F}{m} = -\frac{k}{m}x$$

Let $\dfrac{k}{m} = \omega^2$

$$\boxed{\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \omega^2 x = 0}$$




**The solution of this differential equation is:**

$$x = A\cos(\omega t + \phi)$$

**SHM振动方程**

**Features of SHM:**

$$\boxed{x = A\cos(\omega t + \phi)} \qquad \boxed{F = -kx} \qquad \boxed{\frac{d^2 x}{dt^2} + \omega^2 x = 0}$$

**Three criterions of SHM (简谐振动三判据)**

# 3. Characteristic Quantities of SHM (P301)

**For SHM,**

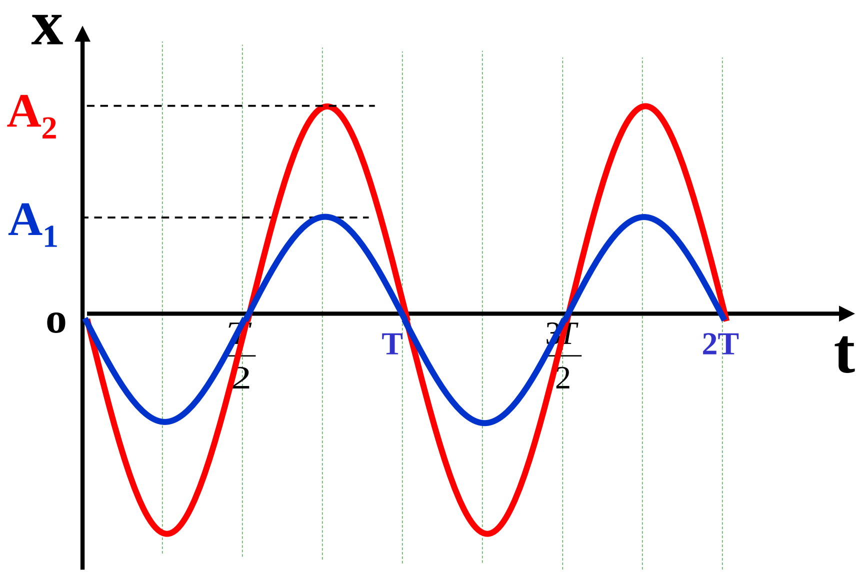
$$x(t) = \underbrace{A}_{\text{Amplitude}} \cos(\underbrace{\omega}_{\text{Angular Frequency}} t + \underbrace{\phi}_{\text{Phase Angle (Initial Phase)}})$$

$\omega t + \phi$: **Phase**

**Where $A$, $\omega$, and $\phi$ are constants. They are characteristic quantities of SHM (描述简谐振动的特征量).**

## i. Amplitude (振幅) $A$ :

$$x = A\cos(\omega t + \phi) \qquad (A > 0)$$




The range of $x$

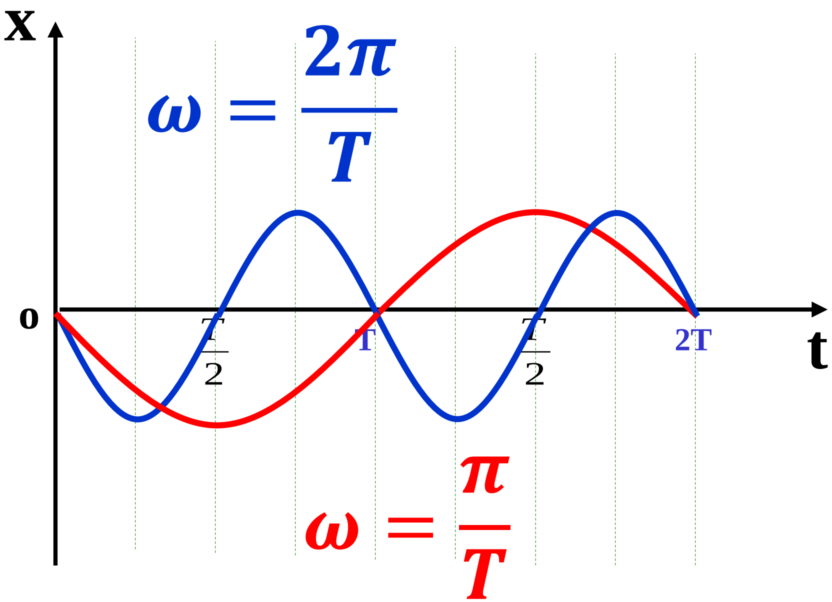
$$-A \le x \le A$$

反映了振动的幅度

The distance $x$ from the equilibrium point at any moment is called the displacement, and the maximum displacement is called the amplitude.

# ii. Angular Frequency (角频率) $\omega$：

$$x = A\cos(\omega t + \phi)$$




**Period $T$**

**Frequency $f$**

**Angular Frequency $\omega$**

反映了振动的快慢

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

**Where *frequency, f,* is the number of complete cycles per second. Its SI unit is the hertz (Hz)**

**1 hertz = 1 Hz = 1 cycle per second = 1 $s^{-1}$**

**Related to the frequency is the period $T = 1/f$ of the motion —— time for one complete cycle;**

## For Spring Oscillator (弹簧振子):

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

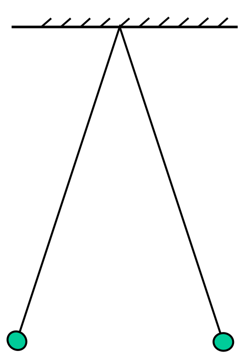
***T* and *f* is also called intrinsic period or frequency (固有周期和固有频率) —— which are determined by system property.**

## iii. Phase & Initial Phase (初相位) $\phi$ :

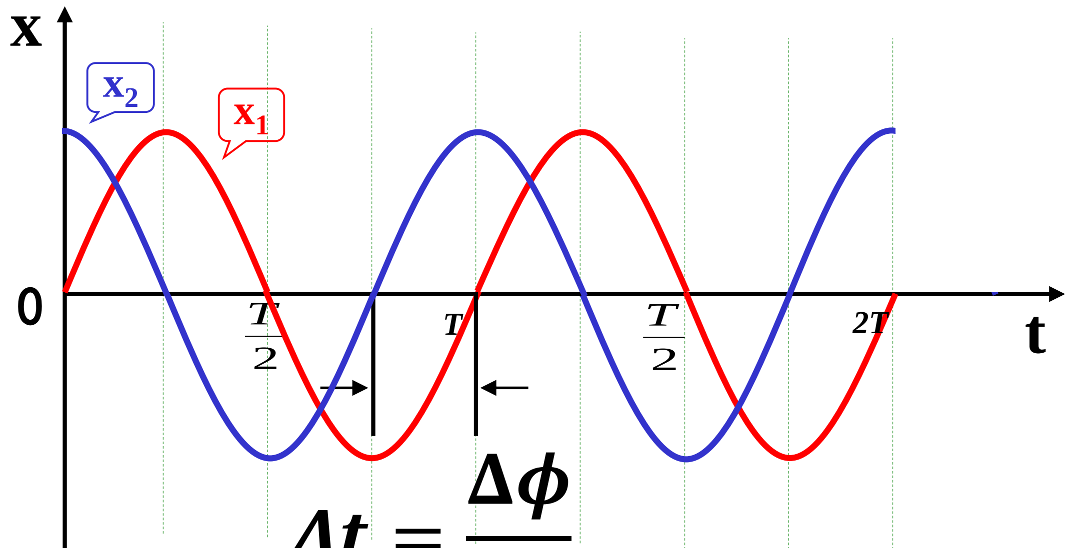
$$x = A\cos(\omega t + \phi)$$

**For a certain oscillatory object, $A$ and $\omega$ are fixed. At any time, its motion state ($x$ & $v$) will be determined by ($\omega t + \phi$). ($\omega t + \phi$) is called phase. ($\phi$ is the phase at $t$ = 0 —— initial phase)**

**Different phase corresponding to different state.**



# Phase difference (相位差)




$$\Delta t = \frac{\Delta\phi}{\omega}$$

$x_1 = A\cos(\omega t + \phi_1)$ $\qquad$ $x_2 = A\cos(\omega t + \phi_2)$

对两同频率的简谐振动，相位差等于初相差.

$$\Delta\phi = \phi_2 - \phi_1$$

$\Delta\phi > 0$ **indicates:**

**Oscillation 2 is ahead of oscillation 1** $\Delta\phi$

**or oscillation 1 is behind of oscillation 2** $\Delta\phi$

**The concept of phase：**

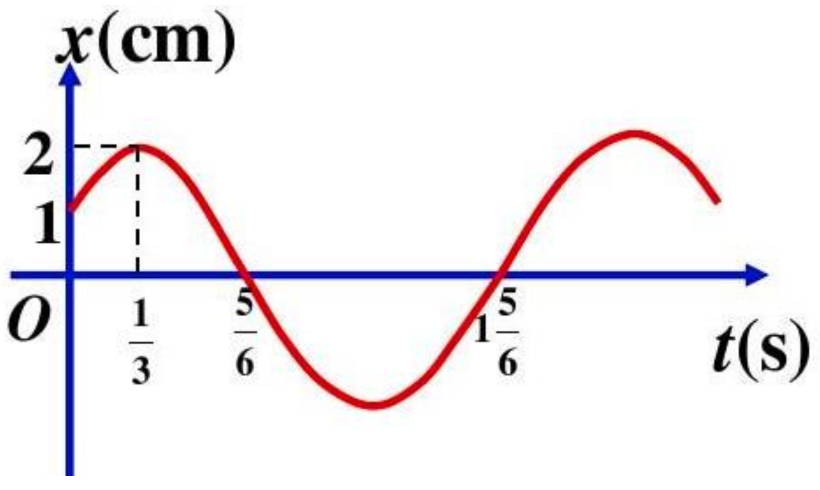
**(1) Describing the states of motion;**

**(2) Comparing the difference in steps of two oscillations (比较两个振动在步调上的差异).**

领先, 落后以<π的相位角(或<*T*/2的时间间隔)来判断。

## Example:

**The figure shows a curve of *x*-*t* of SHM. Which one correctly describes its displacement?**




A. $x = \cos\left(\pi t - \frac{\pi}{3}\right) cm$

B. $x = 2\cos\left(\pi t - \frac{\pi}{3}\right) cm$

C. $x = 2\cos\left(\pi t + \frac{\pi}{3}\right) cm$

D. $x = 2\cos\left(2\pi t - \frac{\pi}{3}\right) cm$

# 4. The Velocity of SHM:

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi)]$$

$$\boxed{v(t) = -\omega A\sin(\omega t + \phi) = \omega A\cos(\omega t + \phi + \frac{\pi}{2})}$$

- **The velocity of a simple harmonic oscillator also varies sinusoidally and varies in the range of [-*ωA*, *ωA*].**
- **$v(t)$ is ahead of $x(t)$ by $\pi/2$.**

## 5. The Acceleration of SHM:

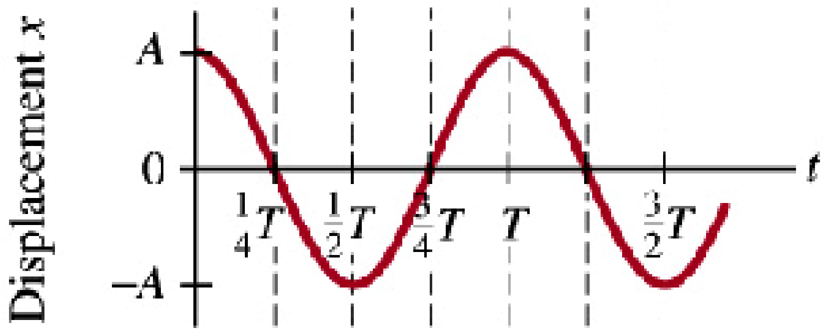
$$a(t) = \frac{dv(t)}{dt}$$

$$\boxed{a(t) = -\omega^2 A\cos(\omega t + \phi) = \omega^2 A\cos(\omega t + \phi + \pi)}$$

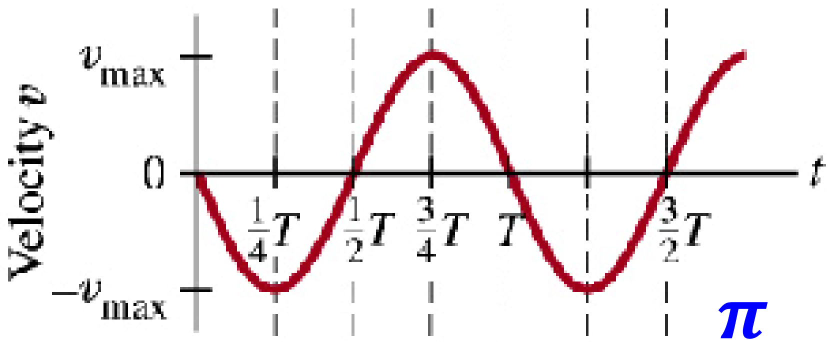
- The acceleration of a simple harmonic oscillator also varies sinusoidally and varies in the range of [$-\omega^2 A$, $\omega^2 A$].

- $a(t)$ is ahead of $x(t)$ by $\pi$.

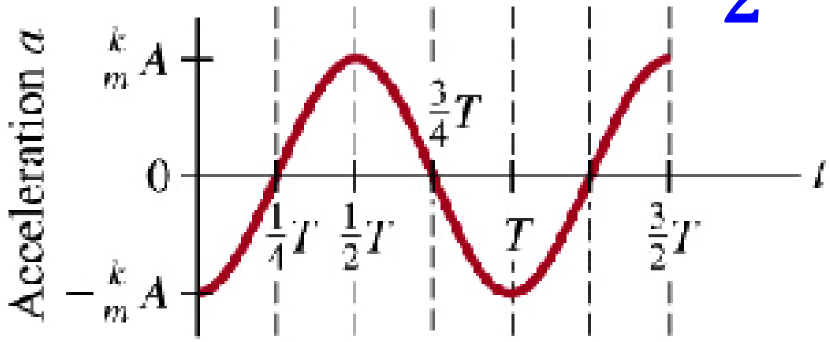
$\phi = 0$




$$x = A\cos\omega t$$




$$v = \omega A\cos(\omega t + \frac{\pi}{2})$$




$$a = -\omega^2 A\cos\omega t = \omega^2 A\cos(\omega t + \pi)$$

- **The curve of $v(t)$ is shifted (to the left) from the curve of $x(t)$ by $T/4$.**
- **The curve of $a(t)$ is always opposite to the direction of $x(t)$ (反相). Phase difference is $\pi$.**

**When** $t = 0$**,** $$\begin{cases} x_0 = A\cos\phi \\ v_0 = -\omega A\sin\phi = -v_{max}\sin\phi \\ a_0 = -\omega^2 A\cos\phi = -a_{max}\cos\phi \end{cases}$$

**—— Initial condition of oscillation**

**For Spring Oscillator:**

$$v_{max} = \omega A = \sqrt{\frac{k}{m}}A$$

$$a_{max} = \omega^2 A = \frac{k}{m}A$$

# 12-2 Expression Methods of SHM

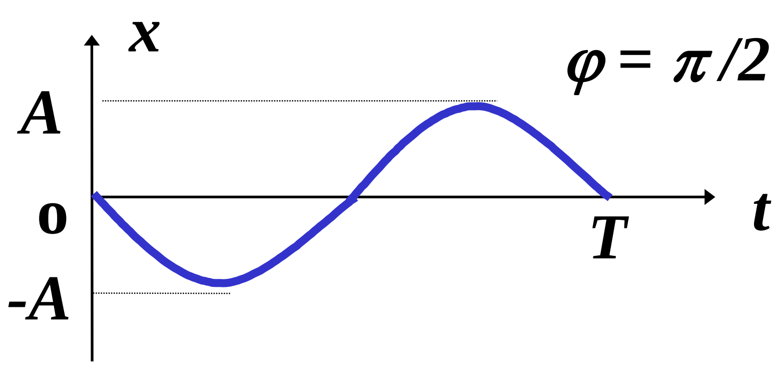
## 1. Analytical Method:

From $x = A\cos(\omega t + \phi)$

Given expression $\Rightarrow$ $A, \omega (T, f), \phi$

Given $A, \omega (T, f), \phi$ $\Rightarrow$ expression

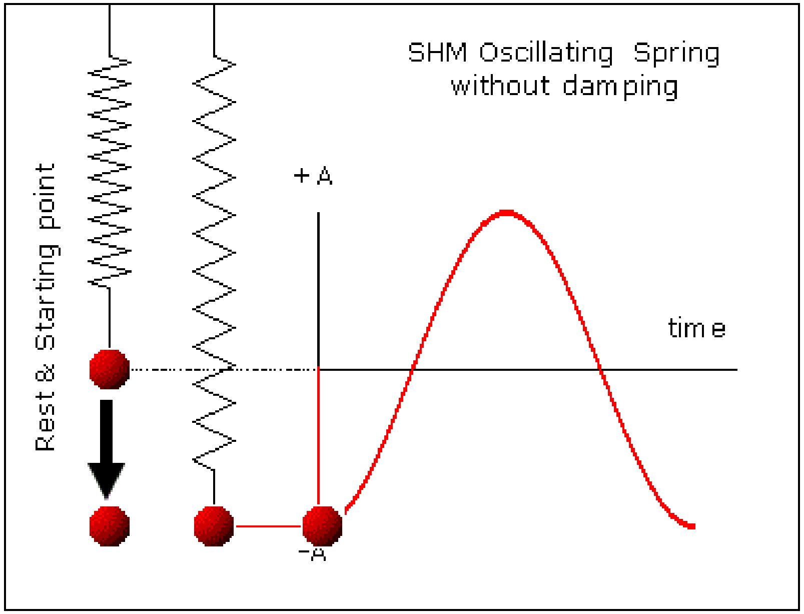
## 2. Curve Method:

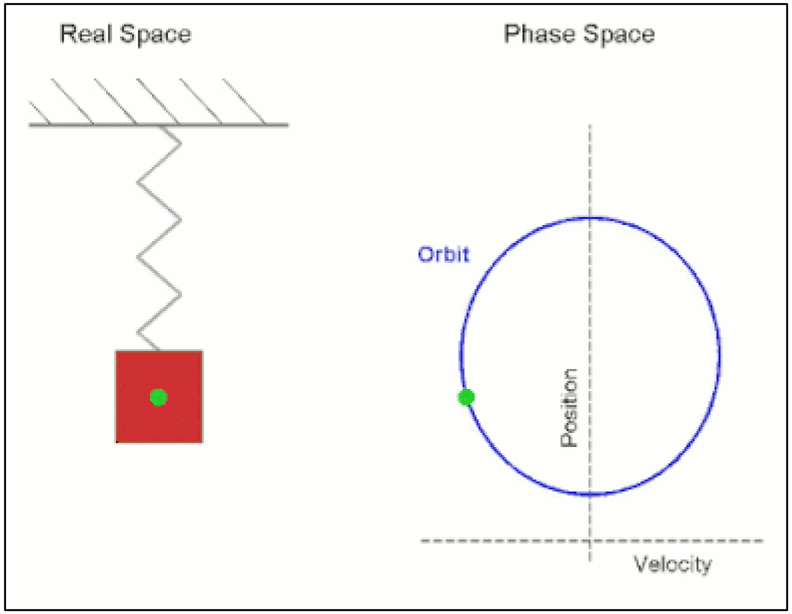



Given curve $\Rightarrow$ $A, \omega (T, f), \phi$

Given $A, \omega (T, f), \phi$ $\Rightarrow$ curve

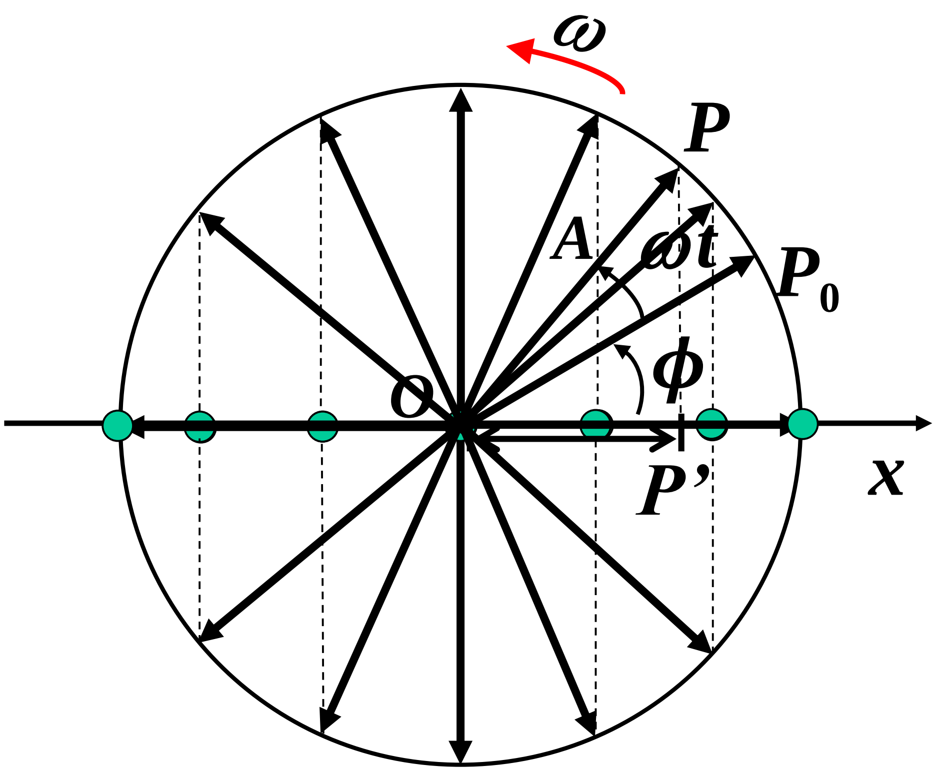
# 3. Uniform Circular Motion: (P306)



## 旋转矢量法:

**Simple harmonic motion is the projection of uniform circular motion onto a diameter of a circle.**




$$x = A\cos(\omega t + \phi)$$

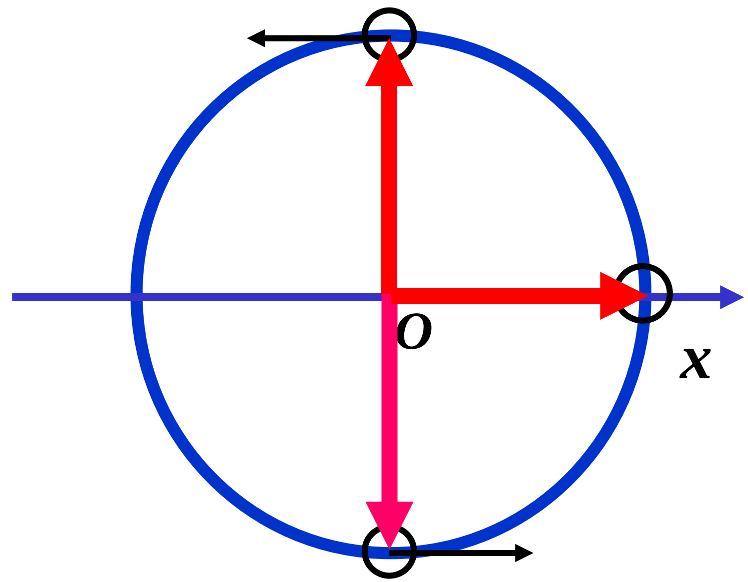
作匀速转动的矢量 $\vec{A}$，其端点*P*在*x*轴上的投影*P'*的运动是SHM.

(规定为逆时针)

# EXAMPLE:

## Initial State 1:

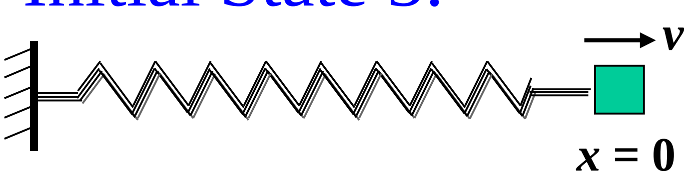

$$\phi = 0$$




## Initial State 2:


$$\phi = \frac{\pi}{2}$$

## Initial State 3:

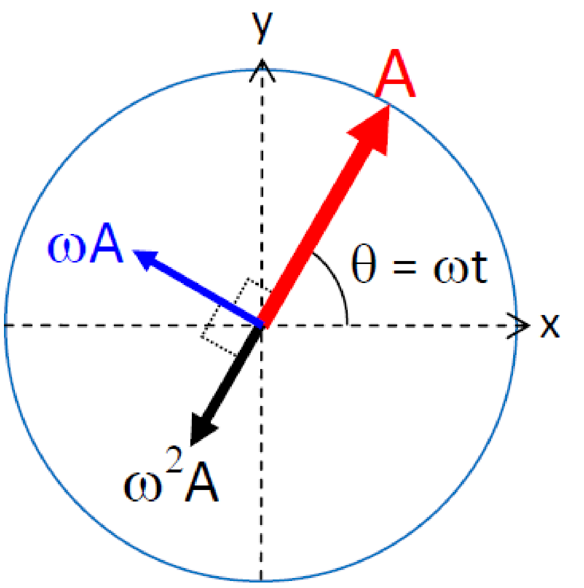



$$\phi = \frac{3\pi}{2} \text{ or } -\frac{\pi}{2}$$

$$x = A\cos\omega t$$

$$v = \omega A\cos(\omega t + \frac{\pi}{2})$$

$$a = \omega^2 A\cos(\omega t + \pi)$$




**EXAMPLE:**

**A body ($m$=10g) is in SHM along $x$ axis, $A$=20cm, $T$=4s. When $t$ = 0, $x_o$= -10cm, and $v$ < 0 (moves along negative $x$ direction). Find: (1) $x(t=1\text{s})$=?; (2) At what time will the body pass the position of $x$=10cm first time (何时物体第一次通过); (3) How long does the body need to pass $x$=10cm second time (再经多少时间物体第二次运动到$x$=10cm处)?**

**Solution:**

$$\omega = \frac{2\pi}{T} = \frac{\pi}{2} \quad \text{and} \quad \phi = \frac{2\pi}{3}$$

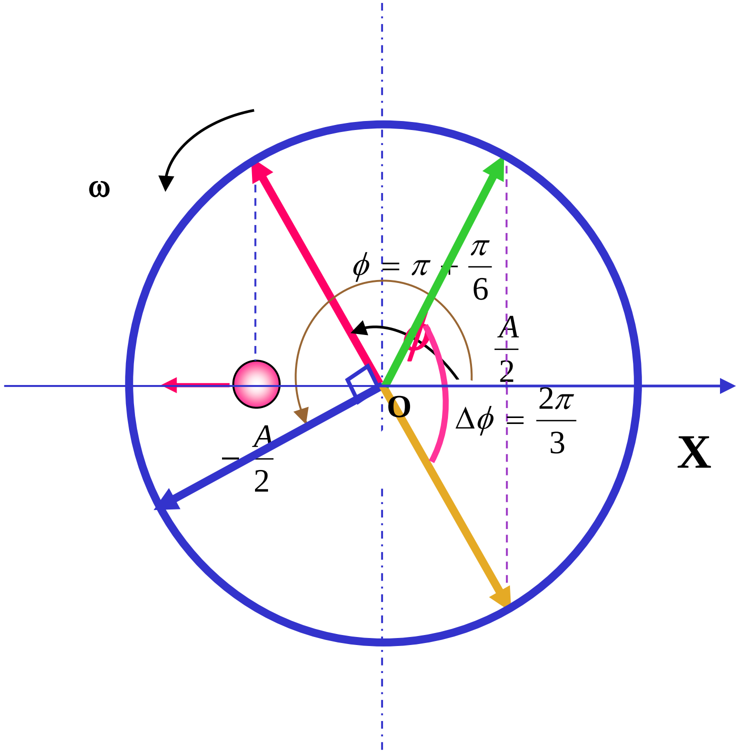
$$x = 0.2\cos(\frac{\pi}{2}t + \frac{2\pi}{3})m$$

**(1)** When $t = 1$s,

$$x = 0.2\cos(\frac{\pi}{2} + \frac{2\pi}{3}) = -0.173m$$

**(2)** $$t = t - t_o = \frac{\Delta\phi}{\omega} = 2s$$

**(3)** $$\Delta t = \frac{\Delta\phi}{\omega} = \frac{4}{3}s$$



# 12-3 Energy in SHM (P304)

**Take a spring oscillator as an example,**

**Potential Energy:**

$$U=\frac{1}{2}kx^2=\frac{1}{2}kA^2\cos^2(\omega t+\phi)$$

$$\because\ \frac{k}{m}=\omega^2$$

$$=\frac{1}{2}m\omega^2A^2\cos^2(\omega t+\phi)$$

**Kinetic Energy:**

$$K=\frac{1}{2}mv^2=\frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)$$

$$=\frac{1}{2}kA^2\sin^2(\omega t+\phi)$$

$$E=K+U=\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$$

简谐振动是无阻尼自由振动，系统无外界输入能量，也不受任何阻力作用，系统机械能守恒。

From $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ we find that

$$v(x) = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

**The ± sign means that at a given value of *x* the body can be moving in either direction.**

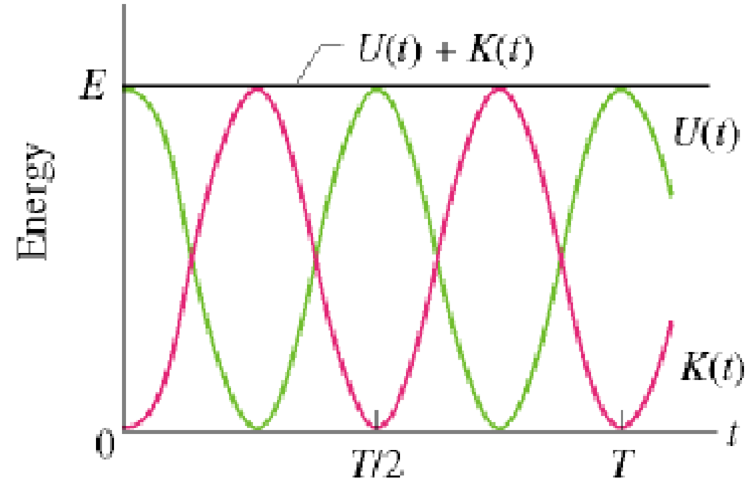
**The maximum speed $v_{max}$ occurs at $x=0$,**

$$v_{max} = \sqrt{\frac{k}{m}}A = \omega A$$

**This agrees with what we obtained in Slide 21.**

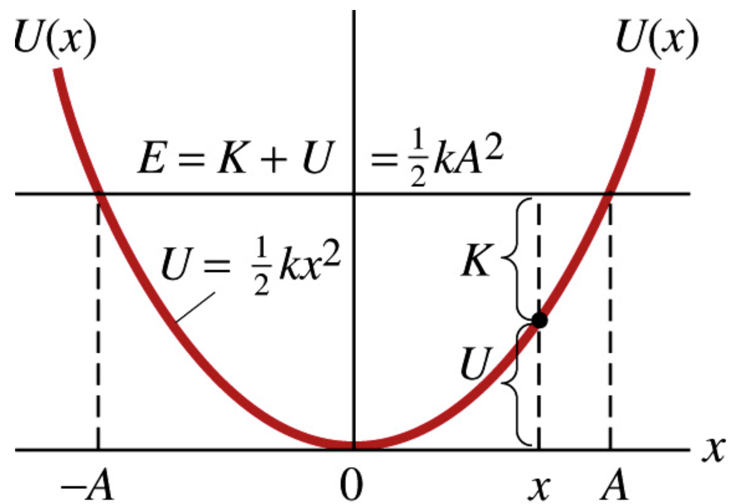
# Conclusion:

(1) In an oscillatory system, both $K$ and $U$ are periodic functions of time $t$. When $U$=0, $K$ is its max and vice versa. The $\Delta\phi$ between them is π/2.




(2) Total energy of oscillatory system is a constant.

$$\boxed{E = \frac{1}{2}kA^2 \propto A^2}$$

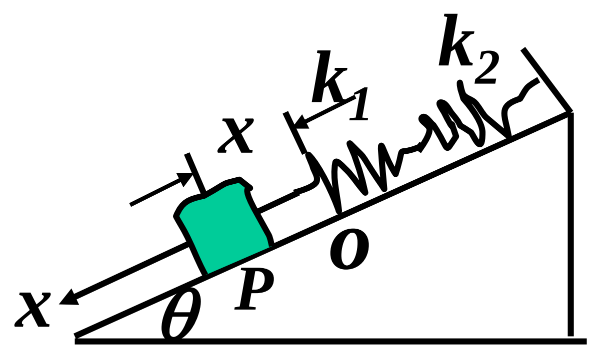



振幅不仅给出简谐振动运动的范围，而且还反映振动系统总能量的大小及振动强度。

## Example:

**A mass $m$ connected to two springs of $k_1$ and $k_2$ oscillates on a frictionless ramp. (a) Proof its motion is SHM; (b) find its frequency and (c) maximum speed if the maximum displacement is $x_m$.**

## Solution:




**When the block is at *O*, it is at the equilibrium point.**

$$mg\sin\theta = k_1 x_1 = k_2 x_2$$

**When the block is at *P*, the force is**

$$F = mg\sin\theta - k_1(x_1 + x'_1) = mg\,sin\,\theta - k_2(x_2 + x'_2)$$

$$F = -k_1 x'_1 = -k_2 x'_2 \quad (1)$$

**and the displacement is**

$$x = x'_1 + x'_2 \quad (2)$$

**(a) From (1) & (2), we have**

$$F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right) x = -k_{eff} x$$

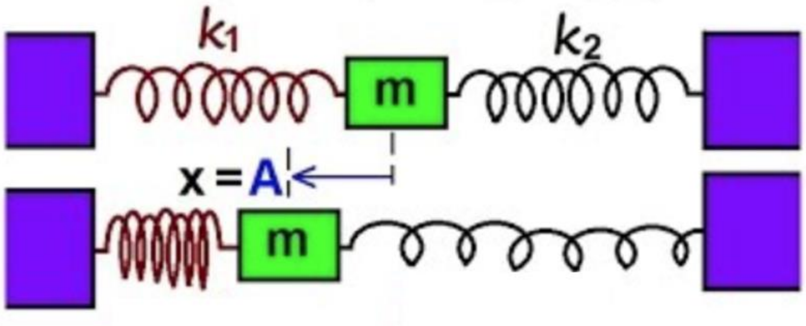
**It is a SHM !**

**(b)** $$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}$$

**(c)** $$v_{max} = \sqrt{\frac{k_{eff}}{m}}\, x_m = \sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}\, x_m$$

**Example:**

**A mass $m$ is connected to two springs of spring constants $k_1$ and $k_2$ as in Figure. When $t=0s$, $x=A$. Calculate: (a) angular frequency $\omega$; (b) $v(x)$; (c) $a(x)$; (d) $x(t)$.**




**Solution:**

**(a) For both springs, the displacement is the same, so**

$$F_1 = -k_1 x \quad \text{and} \quad F_2 = -k_2 x$$

$$F = F_1 + F_2 = -k_1 x - k_2 x = -(k_1 + k_2)x$$
$$= -k_{eff} x$$

$$k_{eff} = k_1 + k_2 \qquad \textbf{So} \quad w = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$$

**(b1)** $$E = \frac{1}{2}k_1A^2 + \frac{1}{2}k_2A^2 = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 + \frac{1}{2}mv^2$$

**Thus** $$\frac{1}{2}mv^2 = \frac{1}{2}k_1A^2 + \frac{1}{2}k_2A^2 - \frac{1}{2}k_1x^2 - \frac{1}{2}k_2x^2$$

$$v(x) = \pm\sqrt{\frac{k_1 + k_2}{m}(A^2 - x^2)}$$

**(b2)** $$v(x) = \pm\sqrt{\frac{k_{eff}}{m}(A^2 - x^2)} = \pm\sqrt{\frac{k_1 + k_2}{m}(A^2 - x^2)}$$

**(c)** $$F = F_1 + F_2 = -k_1 x - k_2 x = ma$$

$$a(x) = -\frac{(k_1 + k_2)x}{m}$$

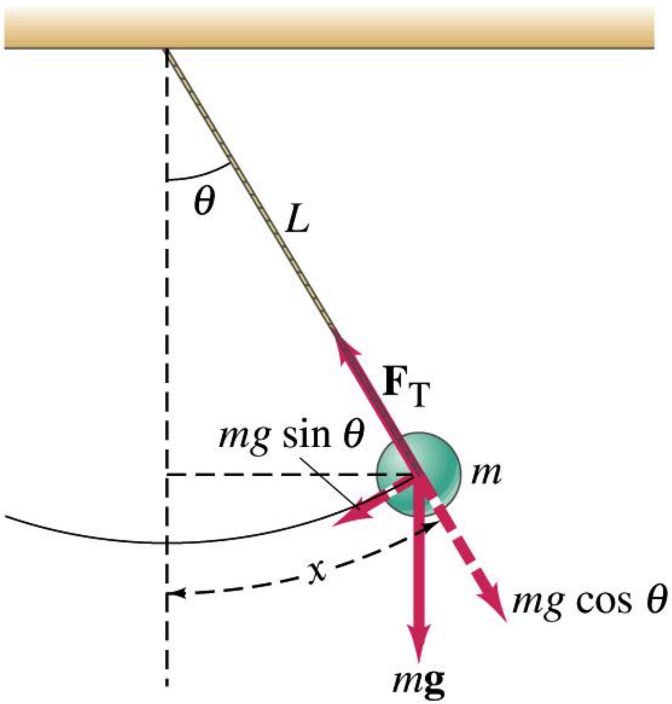
**(d)** $$x(t) = Acoswt = Acos\left(\sqrt{\frac{k_1 + k_2}{m}}\,t\right)$$

# 12-4 Pendulums

## 1. The Simple Pendulum (单摆) (P307)

**The restoring force** $F = -mg sin\theta$

**"-" means the force is in the direction opposite to θ**




**For small angles** $sin\theta \approx \theta$

$$F \approx -mg\theta = -\frac{mg}{L}x$$

**For small displacement, the motion of simple pendulum is SHM.**

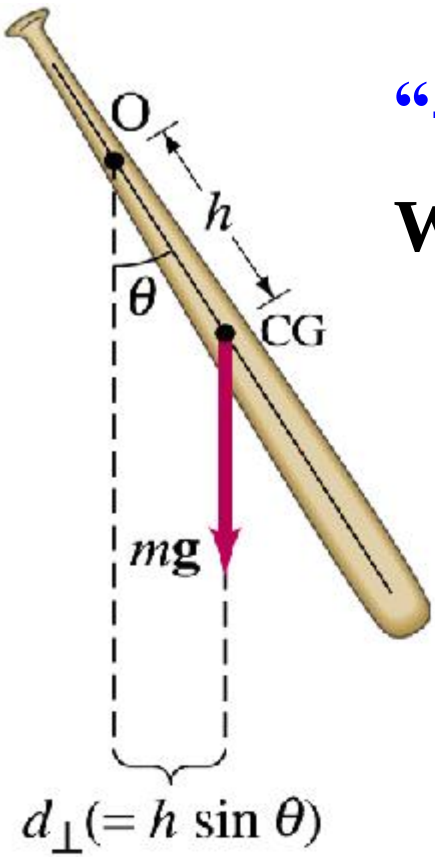
$$k = \frac{mg}{L}\ ,\ w = \sqrt{\frac{g}{L}}\ ,\ T = 2\pi\sqrt{\frac{L}{g}}$$

**Geologists use this instrument to measure *g* !**

## 2. The Physical Pendulum (复摆) (P308)

The torque of physical pendulum to axis *O*,

$$\tau = -mgh\sin\theta$$




"-" indicates that the $\tau$ acts to reduce $\theta$

When $\theta$ is small, $\sin\theta \approx \theta$ $\quad \tau = -mgh\,\theta$

Based on rotational law, $\tau = I\alpha$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{mgh}{I}\theta = -\omega^2\theta \qquad \text{Let } \omega = \sqrt{\frac{mgh}{I}},$$

then $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \omega^2\theta = 0,$ $\qquad \theta = \theta_{\max}\cos(\omega t + \phi)$

It is a SHM. Its period $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{I}{mgh}}$

Read the statements & examples in P307~309!

# 12-5 Resonance (共振)

## 1. Damped (阻尼) Oscillations (P310)

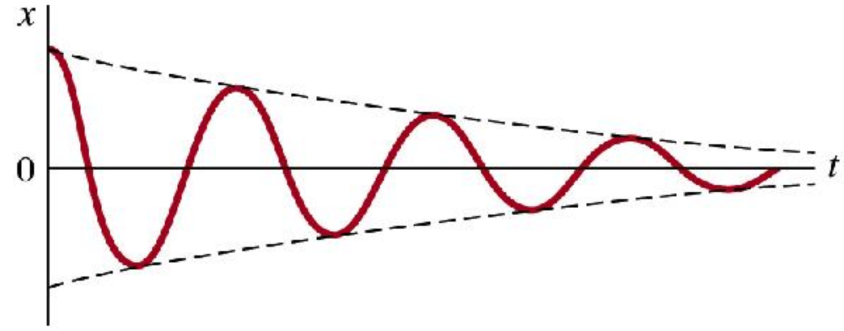
When the motion of an oscillator is **damped by a frictional force**, its **amplitude decreases** with time. The motion is called **damped oscillation (or damped harmonic motion)**.

The component of **damping force** $\vec{F}_d$ along $x$ is,

$$F_d = -bv$$ "-": $\vec{F}_d$ opposes the motion.

**$b$: damping constant**

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b\frac{\mathrm{d}x}{\mathrm{d}t} + kx = 0$$



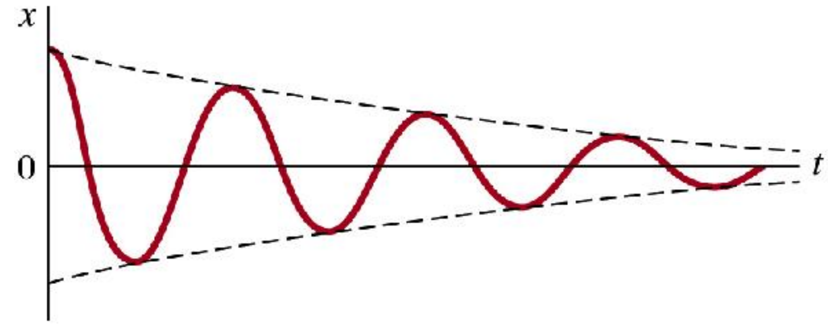

$$x = Ae^{-bt/2m}\cos(\omega' t + \phi) \qquad (b^2 < 4mk)$$ 阻尼较小

$$x = Ae^{-bt/2m}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \qquad f = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

**The frequency is lower, and the period longer than for undamped SHM.**

$$\alpha = \frac{b}{2m}$$

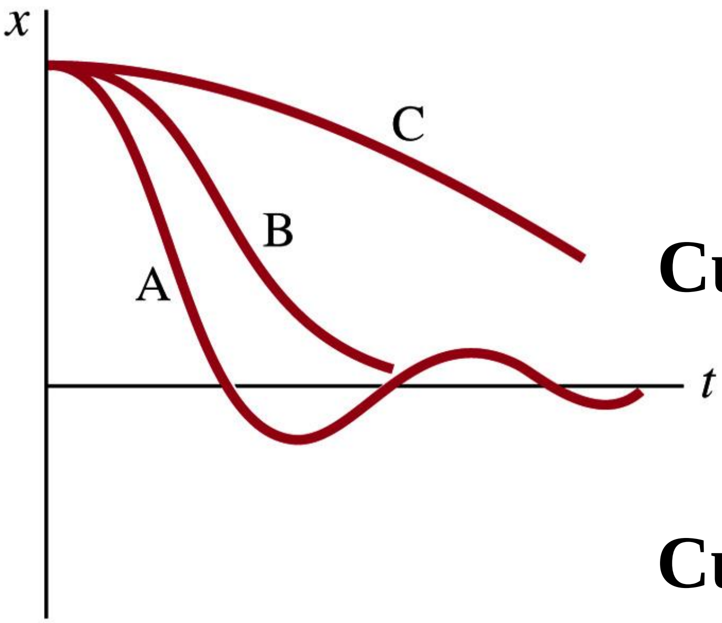



**The constant $\alpha = b/2m$ is a measure of how quickly the oscillations decrease toward zero.**

**The time $t_L = 2m/b$ is the time taken for the oscillations to drop to $1/e$ of the original amplitude.**

**$t_L$ is called the "mean lifetime" of the oscillations.**

**Three common cases of heavily damped system are shown based on relation of $b^2$ to $4mk$ in below Fig.**




**Curve A — underdamping 欠阻尼:**

$$b^2 < 4mk$$

**Curve B — critical damping 临界阻尼:**

$$b^2 = 4mk$$

**Curve C — overdamping 过阻尼:**

$$b^2 > 4mk$$

**If the oscillator is damped, the mechanical energy is not constant but decreases with time.**

## 2. Forced Oscillations and Resonance (P313)

**A damped oscillator will eventually stop moving. But we can maintain a constant-amplitude oscillation by applying a force that varies with time in a periodic way.**

**We call this external force driving force**

$$F_{\text{ext}} = F_0 \cos \omega_d t$$

**The equation of motion with damping and driving force is**

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b\frac{\mathrm{d}x}{\mathrm{d}t} + kx = F_0 \cos \omega_d t$$

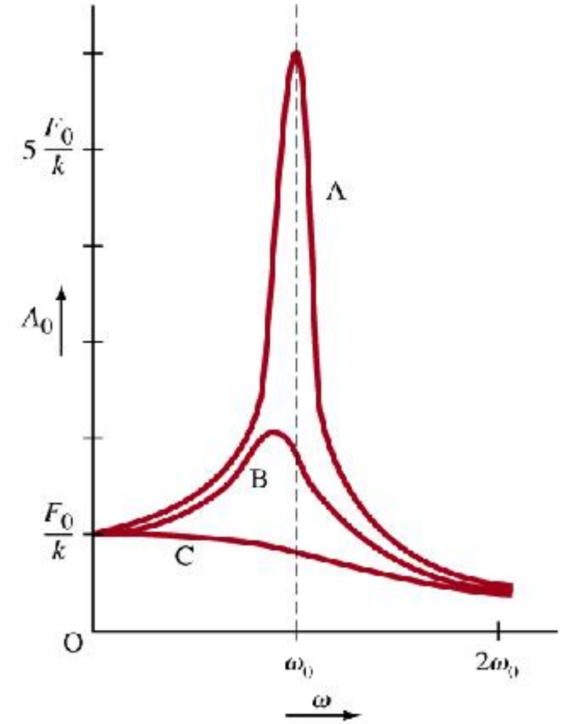
$$x = A_0 e^{-bt/2m}\cos(\omega_0 t + \phi_0) + A\cos(\omega_d t + \phi)$$

**The first term approaches zero in time, we need to be concerned only with**

$$x = A\cos(\omega_d t + \phi)$$

$$A = \frac{F_0}{m[(\omega_0{}^2 - \omega_d{}^2)^2 + b^2\omega_d{}^2/m^2]^{1/2}}$$

$$\phi = \tan^{-1}\frac{\omega_0{}^2 - \omega_d{}^2}{\omega_d(b/m)}$$




**The amplitude can become large when the driving frequency $\omega_d$ is near the natural frequency, $\omega_d \approx \omega_0$ as long as the damping is not too large.**

$$\omega_d \approx \omega_0 \text{ (resonance)}$$

**The natural vibrating frequency of a system is its resonant frequency.**



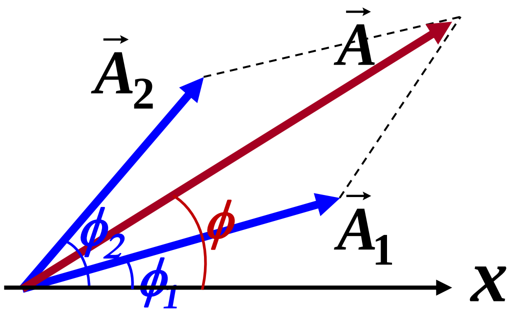
**Large-amplitude oscillation of the Tacoma Narrows Bridge, due to gusty winds, led to its collapse (1940).**

# 12-6 Superposition of Oscillations

**1. Superposition of two SHM in same direction with same frequency (同方向、同频率两简谐运动的合成):**

$$x_1 = A_1\cos(\omega t + \phi_1)$$

$$x_2 = A_2\cos(\omega t + \phi_2)$$



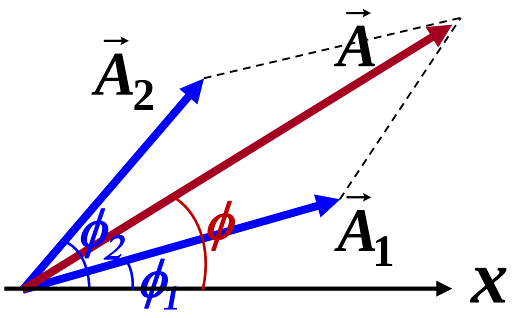

$x = x_1 + x_2$ **That is,** $x = A\cos(\omega t + \phi)$

**The resultant oscillation is also SHM, and its frequency is also $\omega$,**

**where** $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2 - \phi_1)}$ **Law of cosines**

$$\tan\phi = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$$

The resultant oscillation is related to the initial phase difference $\phi_2$-$\phi_1$ .




**Two special cases:**

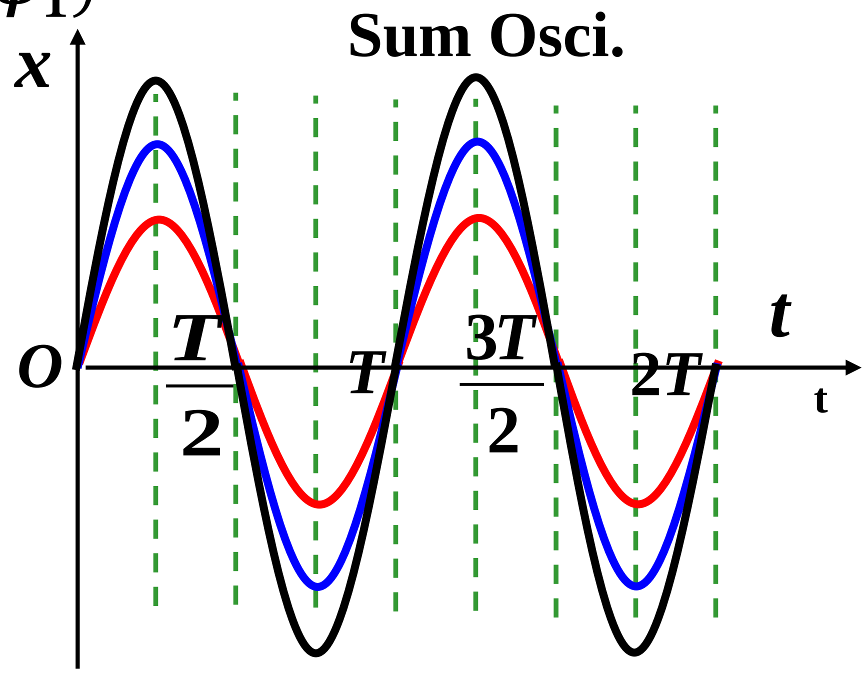
- If $\phi_2 - \phi_1 = \pm 2k\pi,\ (k = 0,1,2,\cdots)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2 - \phi_1)}$$

$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2}$$

$$A = A_1 + A_2$$

——maximum amplitude (两分振动同相, 相互加强).

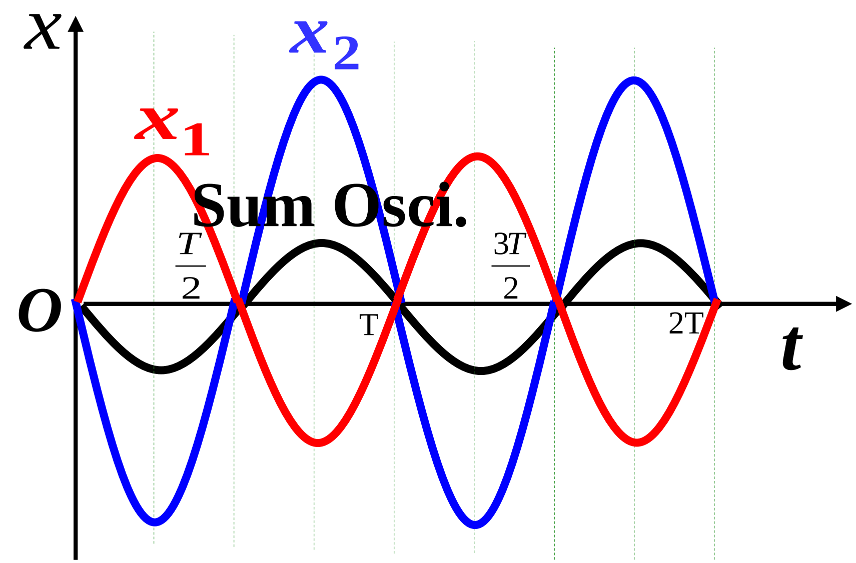


•• If $\phi_2 - \phi_1 = \pm(2k+1)\pi \quad (k = 0,1,2...)$

$A=|A_1-A_2|$

——minimum amplitude

(两分振动反相, 相互减弱)



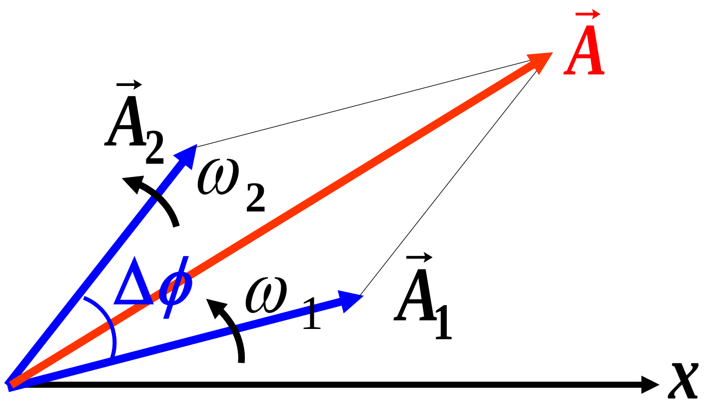

If $A_1=A_2$, then $A=0$ !

••• In general, $|A_1 - A_2| < A < (A_1 + A_2)$

The above results shows us that phase difference of two SHM plays an important role to superposition!

In similar way, we may get the results for the super-position of multi-oscillations.

## 2. Superposition of two SHM in same direction with different frequencies (同方向、不同频率两简谐运动合成):




If $\omega_1 = \omega_2, \Delta\phi = C,$

if $\omega_1 \neq \omega_2, \Delta\phi$ will vary,

resultant oscillation isn't SHM.

$$x_1 = A_1 \cos(\omega_1 t + \phi_1)$$

$$x_2 = A_2 \cos(\omega_2 t + \phi_2)$$

Assume $A_1 = A_2 = A$, $\phi_1 = \phi_2 = \phi$

$$x = x_1 + x_2 = 2A\cos\left(\frac{\omega_2 - \omega_1}{2}t\right)\cos\left(\frac{\omega_2 + \omega_1}{2}t + \phi\right)$$

**A special case:** when $\omega_1$ and $\omega_2$ are slightly different, i.e. $|\omega_2 - \omega_1| << \omega_2 + \omega_1$

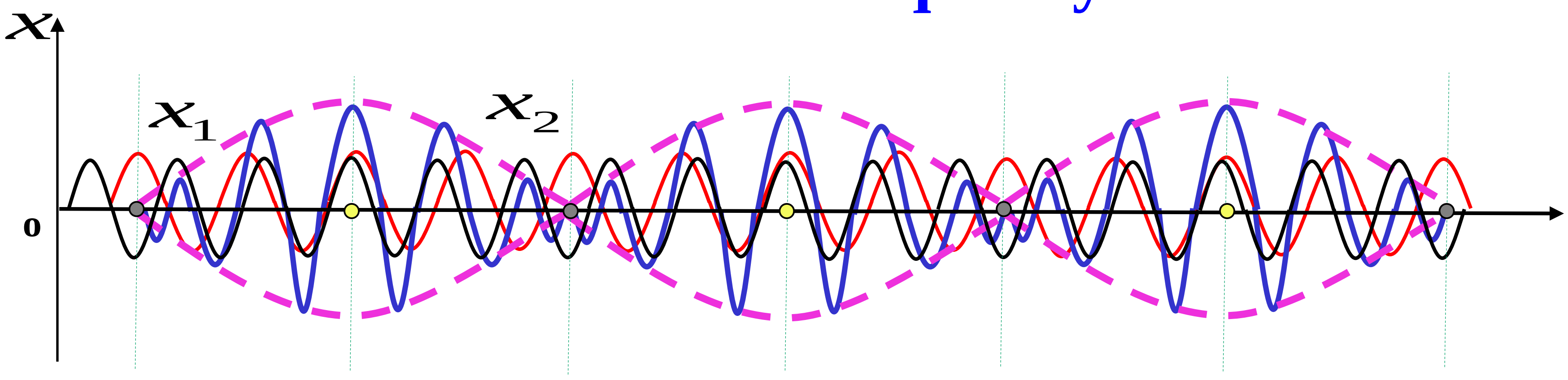
$$x = x_1 + x_2 = \boxed{2A\cos(\frac{\omega_2 - \omega_1}{2}t)}\boxed{\cos(\frac{\omega_2 + \omega_1}{2}t + \phi)}$$

$$x = A(t)cos(\bar{\omega}t + \phi)$$ where

$$A(t) = 2A\cos(\frac{\omega_2 - \omega_1}{2}t)$$ slowly varies with $t$

$$cos(\bar{\omega}t + \phi) = cos(\frac{\omega_2 + \omega_1}{2}t + \phi)$$

quickly varies with $t$




合振动可看作振幅缓变的简谐振动

## Beat (拍) phenomenon:

**The phenomenon that resultant oscillation increases or decreases slowly (合振动忽强忽弱的现象).**

$$T_{beat} = \frac{2\pi}{|\omega_2 - \omega_1|/2}/2 = \frac{2\pi}{|\omega_2 - \omega_1|}$$

**Beat frequency (合振幅在单位时间内加强或减弱的次数):**

$$f_{beat} = \frac{|\omega_2 - \omega_1|}{2\pi} = |f_2 - f_1|$$

**Beat phenomenon is a very important physical phenomenon. For example, Musician use it in tuning their instruments, such as oboe, piano …**

## Homework for Chap. 12

**P316** **3*, 5*, 6*, 10*, 16*, 19***

**P317-319** **4, 6, 7, 11, 12, 13, 19, 20, 23, 27, 35**

**P322** **70** **P324** **87***

**P369** **23**

***Optional problems (选做)**