Chinese Remainder Theorem

1.

(a) Given that,

$$x \equiv 1 \pmod 3,\ x \equiv 2 \pmod 5,\ x \equiv 3 \pmod 7$$

(i) Combine mod 3 and 5.

let, $x = 1+3t$

$1+3t \equiv 2 \pmod 5$

$\Rightarrow 3t \equiv 1 \pmod 5$

Since $3^{-1} \equiv 2 \pmod 5$, $t=2$

So, $x \equiv 1+3\cdot 2 = 7 \pmod{15}$.

(ii) combine $x \equiv 7 \pmod{15}$ with $x \equiv 3 \pmod 7$

let, $x = 7+15s$

$7+15s \equiv 3 \pmod 7 \Rightarrow 15s \equiv -4 \equiv 3 \pmod 7$

But, $15 \equiv 1 \pmod 7$ So, $s \equiv 3$

Then $x = 7+15\cdot 3 = 52$

So, $x \equiv 52 \pmod{105}$

(Ans)

(b) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$.

(i) Combine mod 11 and 29. Let $x = 5 + 11t$

$5 + 11t \equiv 14 \pmod{29} \Rightarrow 11t \equiv 9 \pmod{29}$

Inverse $11^{-1} \equiv 8 \pmod{29}$, so, $t \equiv 8 \cdot 9 = 14$

Thus $x \equiv 5 + 11 \cdot 14 = 159 \pmod{319}$

(ii) Combine $x \equiv 159 \pmod{319}$ with $x \equiv 15$ (mod

Let $x = 159 + 319s$

$159 + 319s \equiv 15 \pmod{31} \Rightarrow 319s \equiv 15 - 159 = -14$

$\equiv 11$ (mod

$319 \equiv 9 \pmod{31}$, so, $9s \equiv 11 \pmod{31}$.

Inverse $9^{-1} \equiv 7 \pmod{31}$, so, $s \equiv 7 \cdot 11 \equiv 15$

Then $x = 159 + 319 \cdot 15 = 4944$

So, $x \equiv 4944 \pmod{9889}$

(c) $x \equiv 5 \pmod 6$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$.

(i) Combine mod 6 and 11. Let $x = 5+6t$

$5+6t \equiv 4 \pmod{11} \Rightarrow 6t \equiv -1 \equiv 10 \pmod{11}$. $6^{-1} \equiv 2 \pmod{11}$

So, $t \equiv 2 \cdot 11 \equiv 9$

Thus $x \equiv 5 + 6 \cdot 9 = 59 \pmod{66}$

(ii) combine $x \equiv 59 \pmod{66}$ with $x \equiv 3 \pmod{17}$

Let, $x = 59 + 66u$

$59 + 66u \equiv 3 \pmod{17} \Rightarrow 66u \equiv 3 - 59 = -56 \equiv 12$ mod 17

$66 \equiv 15 \pmod{17}$, so $15u \equiv 12 \pmod{17}$. $15^{-1} \equiv 8 \pmod{17}$

So, $u = 8 \cdot 12 \equiv 11$

Then $x = 59 + 66 \cdot 11 = 785$

So, $x \equiv 785 \pmod{1122}$

(Ans)