Exercise 2.1

a. For a matrix $A \in \mathbb{R}^{m * n}$, it can be decomposed into $A = UDV^T$

   Now, for $A^TA = (UDV^T)^T * (UDV^T)$
   $= (U^TD^TV) * (UDV^T)$
   $= VD^2V^T$ $[D^T = D \ \& \ U^TU = I]$

   Given, D contains the eigenvalues of A^TA, so from $A^TA = VD^2V^T$, we can come to the conclusion that the singular values of A are the square root of the eigenvalues of A^TA.
b. The answers are: (additional pdf attached for handwritten answer)
   $\lambda\_1 = 2$
   $\lambda\_2 = 8$
   $\lambda\_3 = 9$
   Eigenvectors:
   Corresponding to λ_1: [0, -3/2, 1]
   Corresponding to λ_2: [0, 0, 1]
   Corresponding to λ_3: [7/69, 5/69, 1]
c. Relationship between **B** and $\mathbf{B^{-1}}$**:** (handwritten proof in attached pdf)

   The eigenvalues of B and B^{-1} are reciprocals.
   The eigenvectors of B and B^{-1} are identical.

   Between **B** and $\mathbf{B^T}$:

   Eigenvalues and eigenvectors are identical (since **B** is symmetric).