Modern Control Systems
Ball and Beam System

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1. Introduction

Magnetic levitation is a method by which, using only a magnetic field, one can overcome gravity and cause an object to float. In fact, the magnetic field generates a magnetic force that counteracts the effects of gravity and makes the object suspended. A practical example of this system is the **maglev train**.
Maglev trains, due to the absence of contact with the surface and floating in the air, experience very little friction. As a result, they can reach very high speeds. The system of these trains is such that the magnetic field in the rail is varied in a specific manner so as to propel the train forward or backward. An example of these trains is shown in Figure 1.

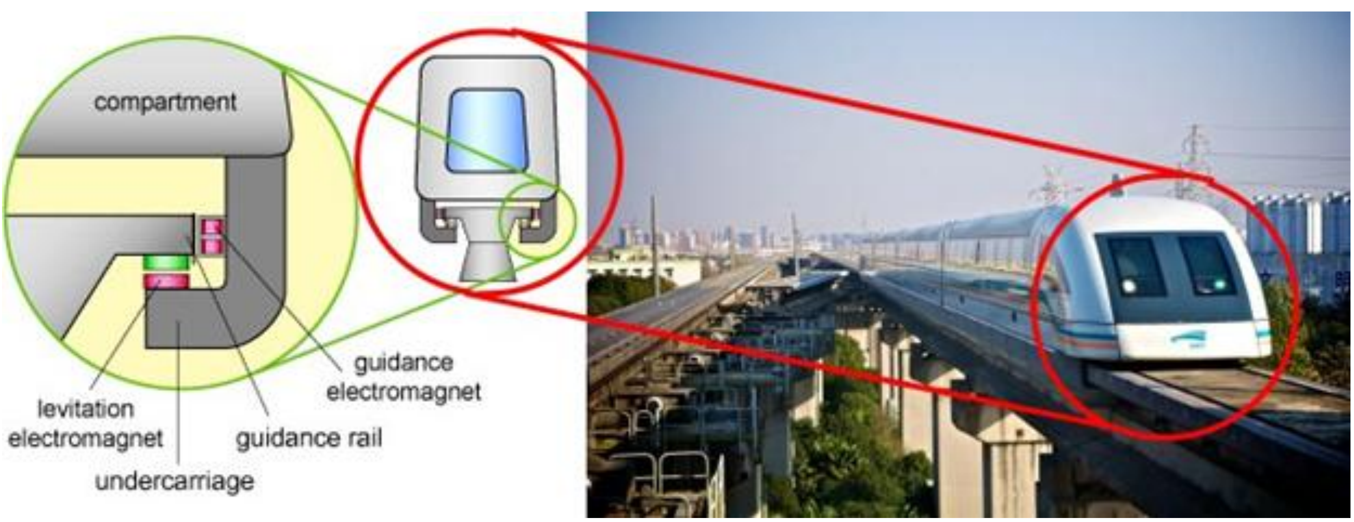

Figure 1

The foundation of a maglev train is the magnetic levitation system. This electromechanical system, due to properties such as nonlinearity and inherent instability (caused by the Earth's gravity), can serve as a suitable system for analysis and design.

2. Modeling

In this system, illustrated in Figure 2, there exists a coil around a magnetic core and an iron ball. By applying voltage to the coil and letting current flow, the iron ball is attracted upwards. The objective of this system is to control the height of the iron ball using the input voltage.

Figure 2: Example of a Magnetic Levitation System

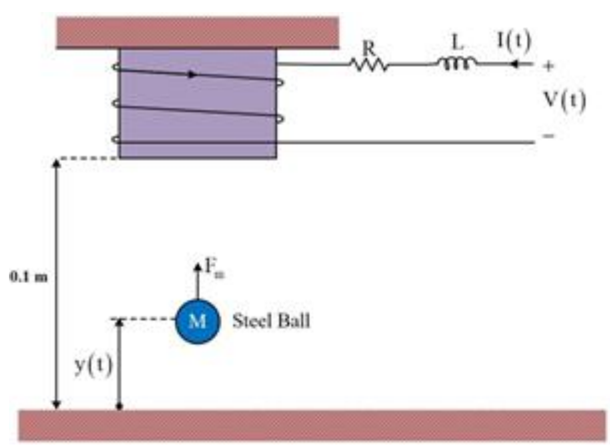

Figure 3: Magnetic Levitation System under Study

States and Inputs of the System

This system has 3 states, 1 control input, and 1 output as follows:

x_1 : position of the iron ball,
x_2 : velocity of the iron ball,
x_3 : coil current,
u : input voltage,
output : position of the iron ball

System State Equations

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -g + \dfrac{c}{M}\dfrac{{x_3}^2}{0.1 - x_1} - \dfrac{f_v x_2}{M} \\ \dot{x}_3 = \dfrac{1}{L}(-Rx_3 + u) \end{cases} \qquad \{y = x_1$$

Parameters

R	Total resistance of circuit	50Ω
L	Coil inductance	0.2H
g	Gravitational acceleration	$9.8\,\dfrac{m}{s^2}$
M	Mass of iron ball	0.425 kg
c	Electromagnetic force constant	$0.3\,\dfrac{N.m}{A^2}$
f_v	Air friction	$0.04\,\dfrac{N.s}{m}$
y^*	Output operating point	0.06m

3. Tasks & Objectives (Phase 1)

1. Find the equilibrium points of the system considering the given operating point.
2. Linearize the system around the equilibrium points and obtain the state-space equations.
3. Analyze the stability of the equilibrium points using the eigenvalues of the linearized matrix.
4. Examine controllability, observability, and minimality of the obtained state-space system. If the system is not minimal, provide a minimal realization and use it in subsequent tasks.
5. Compute the state transition matrix of this system.
6. Derive the transfer function of the linearized state-space system. Report the poles and zeros.
7. Design a **PID controller** to stabilize the linearized system. Choose the desired performance index freely. What issues does the designed controller have?
8. Using the designed controller, plot the state responses to a step input.
9. Connect the designed controller to the nonlinear system as well, and plot the state responses to a step input. For convenience, the given dynamic system is available in the **Levitation Magnetic subsystem** of the Simulink file vrmaglev_sys.slx, where you can observe the results in 3D. You just need to define and assign parameter values in the workspace.

4. Tasks and Objectives (Phase 2)

1. Consider two sets of poles: fast and slow. For each set, design a state-feedback controller. Compare the responses of each system via simulation. One performance metric is speed. Show based on the simulation what cost is paid to quicken poles.
2. Repeat the previous task in the presence of a disturbance signal added to the states, and report the results. (Use a bounded, time-varying disturbance).
3. Investigate the controllability of the system with state-feedback and integral control. If controllable, design such a system and show, through simulation, the robustness of the system and controller against disturbance. Compare with the PID controller from Phase 1.
4. Design a **full-order Luenberger observer** for the system and repeat task (1) in its presence.
5. Design and simulate a controller with **reduced-order observer** for the system.