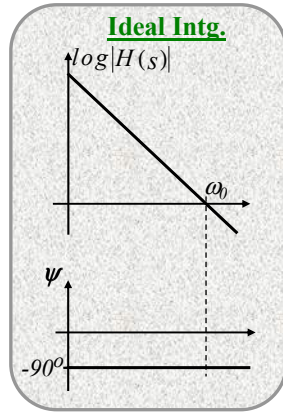# EE247
Lecture 5

- Filters
  - Effect of integrator non-idealities on filter behavior
    - Integrator quality factor and its influence on filter frequency characteristics
    - Filter dynamic range limitations due to limited integrator linearity
      - Measures of linearity: Harmonic distortion, intermodulation distortion, intercept point
    - Effect of integrator component variations and mismatch on filter response
  - Various integrator topologies utilized in monolithic filters
    - Resistor + C based filters
    - Transconductance (gm) + C based filters
    - Switched-capacitor filters
  - Continuous-time filter considerations
    - Facts about monolithic Rs, gms, & Cs and its effect on integrated filter characteristics
  - Other continuous-time filter topologies
    - Opamp MOSFET-C filters
    - Opamp MOSFET-RC filters
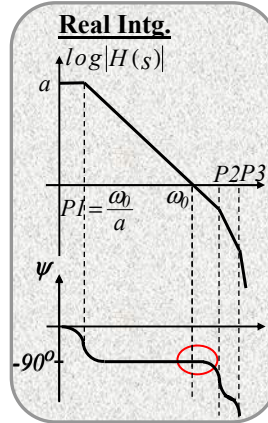   - Gm-C filters

Summary of Lecture 4

- Ladder type RLC filters converted to integrator based active filters
  - All pole ladder type filters
    - Convert RLC ladder filters to integrator based form
    - Example: 5th order Butterworth filter
  - High order ladder type filters incorporating zeros
    - 7th order elliptic filter in the form of ladder RLC with zeros
      - Sensitivity to component mismatch
      - Compare with cascade of biquads
    - → *Doubly terminated LC ladder filters* ⇨ *Lowest sensitivity to component variations*
    - Convert to integrator based form utilizing SFG techniques
    - Example: Differential high order filter implementation
  - Effect of integrator non-idealities on continuous-time filter behavior
    - Effect of integrator finite DC gain & non-dominant poles on filter frequency response (continued today)
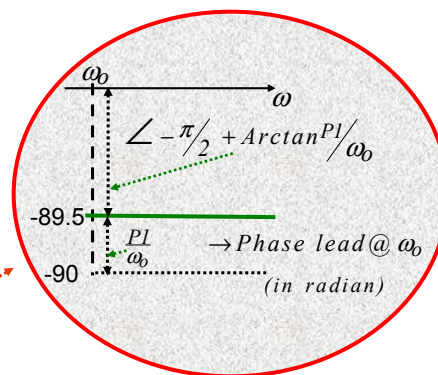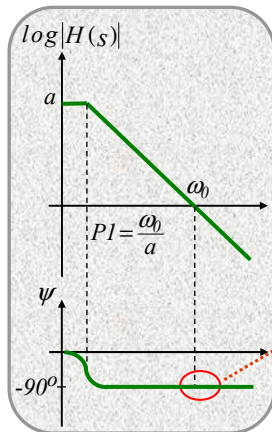
Real Integrator Non-Idealities

Ideal Intg.

$log|H(s)|$

ω_0

ψ

-90°

$$H(s) = \frac{-\omega_0}{s}$$

Real Intg.

$log|H(s)|$

a

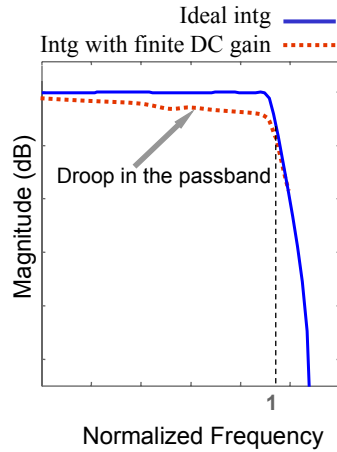$P1 = \frac{\omega_0}{a}$ ω_0 $P2P3$

ψ

-90°

$$H(s) \approx \frac{-a}{\left(1+s\frac{a}{\omega_0}\right)\left(1+\frac{s}{p2}\right)\left(1+\frac{s}{p3}\right)\cdots}$$
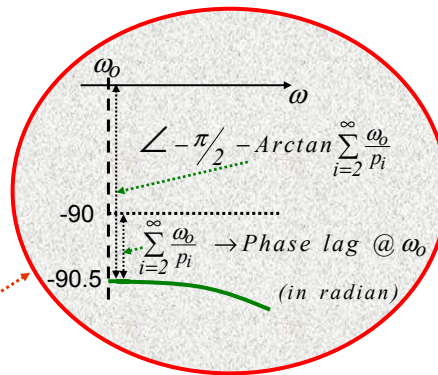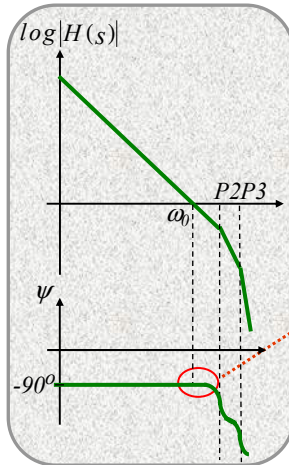

Effect of Integrator Finite DC Gain on Q

$log|H(s)|$

a

ω_0

$P1 = \frac{\omega_0}{a}$

ψ

-90°

ω_0

ω

$\angle -\frac{\pi}{2} + Arctan\frac{P1}{\omega_0}$

-89.5

$\frac{P1}{\omega_0}$ $\rightarrow Phase\ lead\ @\ \omega_0$

-90 $(in\ radian)$

Example: $a=100 \rightarrow P1/\omega_0 = 1/100$

$\rightarrow$ phase error $\cong +0.5 degree$

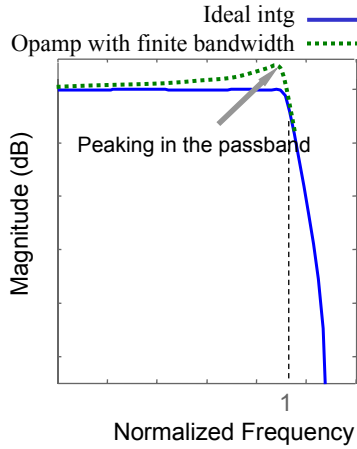## Effect of Integrator Finite DC Gain on Q
Example: Lowpass Filter

Ideal intg ——
Intg with finite DC gain ········

Magnitude (dB)

Droop in the passband

1

Normalized Frequency

• Finite opamp DC gain

→ Phase lead @ ω_0

→ Droop in the passband

Effect of Integrator Non-Dominant Poles

$log|H(s)|$

$P2P3$

ω_0

ψ

-90°

ω_0

ω

$\angle -\pi/2 - Arctan \sum_{i=2}^{\infty} \frac{\omega_o}{p_i}$

-90

$\sum_{i=2}^{\infty} \frac{\omega_o}{p_i} \rightarrow Phase\ lag\ @\ \omega_o$

-90.5

$(in\ radian)$

Example: $\omega_0/P2 = 1/100$

→ phase error $\cong -0.5degree$

Effect of Integrator Non-Dominant Poles
Example: Lowpass Filter

Ideal intg ——
Opamp with finite bandwidth ·······

Magnitude (dB)

Peaking in the passband

1

Normalized Frequency

- Additional poles due to opamp poles:

  → Phase lag @ ω_0

  → Peaking in the passband
  In extreme cases could
  result in oscillation!

Effect of Integrator Non-Dominant Poles &
Finite DC Gain on Q

$log|H(s)|$

a

$P2P3$

$P1 = \frac{\omega_0}{a}$ ω_0

ψ

-90°

ω_0

$\angle -\pi/2 + Arctan P1/\omega_0$

$- Arctan \sum\limits_{i=2}^{\infty} \frac{\omega_0}{p_i}$

-90

$P1/\omega_0 - \sum\limits_{i=2}^{\infty} \frac{\omega_0}{p_i}$
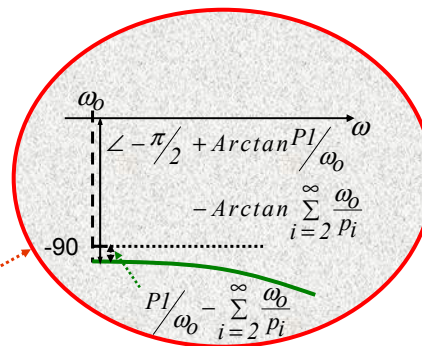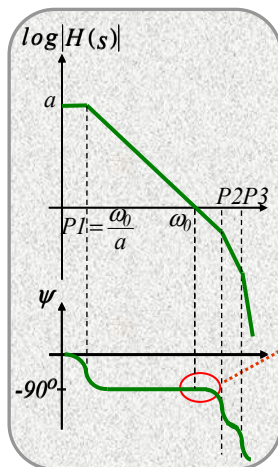
**Note that the two terms have different signs
→ Can cancel each other's effect!**

Integrator Quality Factor

Real intg. transfer function: $H(s) \approx \dfrac{-a}{\left(1 + s\frac{a}{\omega_o}\right)\left(1 + \frac{s}{p2}\right)\left(1 + \frac{s}{p3}\right)\cdots}$

Based on the definition of Q and assuming that:

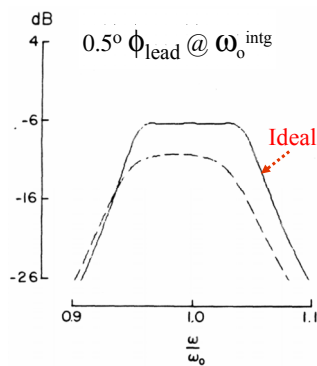$$\dfrac{\omega_o}{p2,3,\ldots} << 1 \qquad \& \qquad a >> 1$$

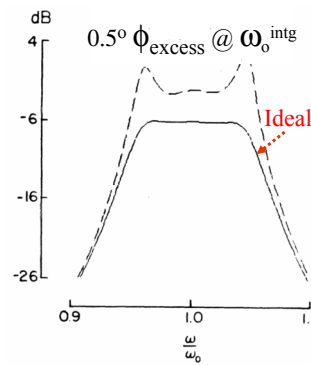It can be shown that <u>in the vicinity</u> of unity-gain-frequency:

$$Q_{real}^{intg.} \approx \dfrac{1}{\dfrac{1}{a} - \omega_0 \sum\limits_{i=2}^{\infty} \dfrac{1}{p_i}}$$

Phase lead @ ω_o   *Phase lag @ ω_0*

Example:
Effect of Integrator Finite Q on Bandpass Filter Behavior

$0.5^o \ \phi_{lead} @ \omega_o^{intg}$

Ideal

Integrator DC gain=*100*

$0.5^o \ \phi_{excess} @ \omega_o^{intg}$

Ideal

Integrator *P2 @ 100.ω_o*

Example:
Effect of Integrator Q on Filter Behavior

$$(\, 0.5^{\mathrm{o}} \, \phi_{\mathrm{lead}} - 0.5^{\mathrm{o}} \, \phi_{\mathrm{excess}}) \; @ \; \omega_o^{\,\mathrm{intg}}$$

$$\rightarrow \phi_{\mathrm{error}} \, @ \; \omega_o^{\,\mathrm{intg}} \; \sim 0$$
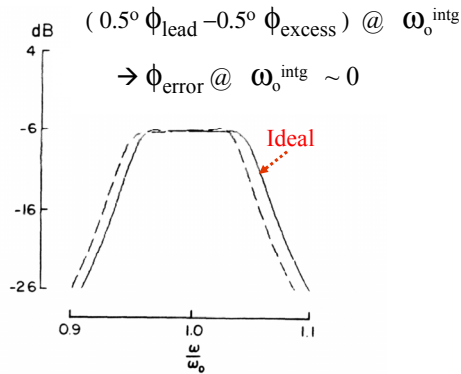
Integrator DC gain=*100 & P2 @ 100.* ω_o

Summary
Effect of Integrator Non-Idealities on Q

$$Q_{ideal}^{intg.} = \infty$$

$$Q_{real}^{intg.} \approx \frac{1}{\frac{1}{a} - \omega_o \sum_{i=2}^{\infty} \frac{1}{p_i}}$$

- Amplifier DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements
- Amplifier poles located above integrator unity-gain frequency enhance the *Q*!
    - If non-dominant poles close to unity-gain freq. → Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!
- Overall quality factor of the integrator has to be much higher compared to the filter's highest pole Q

Effect of Integrator Non-Linearities on Overall Integrator-Based Filter Performance

- Dynamic range of a filter is determined by the ratio of maximum signal output with acceptable performance over total noise

- Maximum signal handling capability of a filter is determined by the non-linearities associated with its building blocks

- Integrator linearity function of opamp/R/C (or any other component used to build the integrator) linearity-

- Linearity specifications for active filters typically given in terms of :
  - Maximum allowable harmonic distortion @ the output
  - Maximum tolerable intermodulation distortion
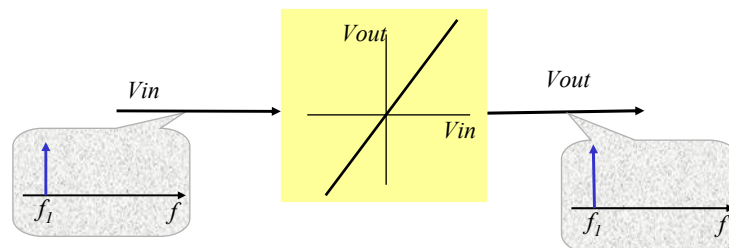 - Intercept points & compression point referred to output or input

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Component Linearity versus Overall Filter Performance
1- Ideal Components

Ideal DC transfer characteristics:

Perfectly linear output versus input tranfer function with no clipping

$Vout = \alpha \, Vin$ for $-\infty \le Vin \le \infty$

$\text{If } Vin = A\sin(\omega_1 t) \rightarrow Vout = \alpha \; A\sin(\omega_1 t)$

Component Linearity versus Overall Filter Performance
2- Semi-Ideal Components

Semi-ideal DC transfer characteristics:

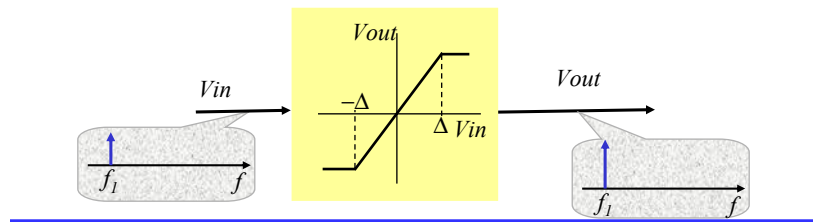Perfectly linear output versus input transfer function with clipping

$Vout = \alpha\, Vin$ for $-\Delta \leq Vin \leq +\Delta$

$Vout = -\Delta\alpha$ for $Vin \leq -\Delta$

$Vout = \Delta\alpha$ for $Vin \geq \Delta$

If $Vin = A\sin(\omega_1 t) \rightarrow Vout = \alpha\, A\sin(\omega_1 t)$ for $-\Delta \leq Vin \leq +\Delta$

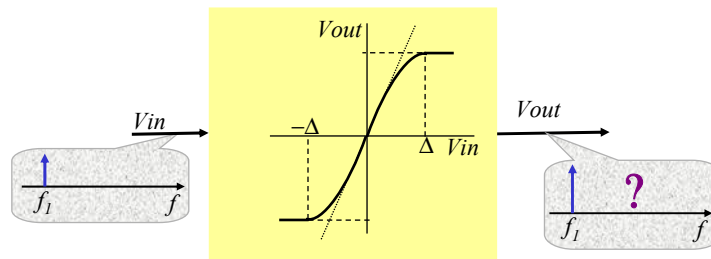Clipped & distorted otherwise

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Effect of Component Non-Linearities on Overall Filter Linearity
Real Components including Non-Linearities

Real DC transfer characteristics: Both soft non-linearities & hard (clipping)

$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$ for $-\Delta \leq Vin \leq \Delta$
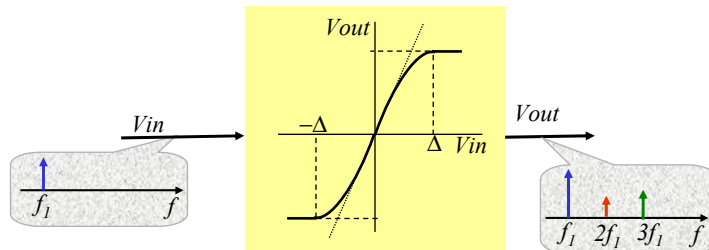
Clipped otherwise

If $Vin = A\sin(\omega_1 t)$

Real DC transfer characteristics:

$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ... \quad If \ Vin = A\sin(\omega_1 t) \ \& \ A < \Delta$$

Then:

$$\rightarrow Vout = \alpha_1 A\sin(\omega_1 t) + \alpha_2 A^2 \sin(\omega_1 t)^2 + \alpha_3 A^3 \sin(\omega_1 t)^3 + ...$$

$$or \ Vout = \alpha_1 A\sin(\omega_1 t) + \frac{\alpha_2 A^2}{2}(1 - \cos(2\omega_1 t)) + \frac{\alpha_3 A^3}{4}(3\sin(\omega_1 t) - \sin(3\omega_1 t)) + ...$$

$$Vout = \alpha_1 A\sin(\omega t) + \frac{\alpha_2 A^2}{2}(1 - \cos(2\omega t))$$

$$+ \frac{\alpha_3 A^3}{4}(3\sin(\omega t) - \sin(3\omega t)) + ...$$

$$HD2 = \frac{amplitude \, 2^{nd} \, harmonic \, distortion \, component}{amplitude \, fundamental}$$

$$HD3 = \frac{amplitude \, 3^{rd} \, harmonic \, distortion \, component}{amplitude \, fundamental}$$

$$\rightarrow HD2 = \frac{1}{2} \times \frac{\alpha_2}{\alpha 1} A \ , \qquad HD3 = \frac{1}{4} \times \frac{\alpha_3}{\alpha 1} A^2$$

Example: Significance of Filter Harmonic Distortion in Voice-Band CODECs

- Voice-band CODEC filter (CODEC stands for coder-decoder, telephone circuitry includes CODECs with extensive amount of integrated active filters)
- Specifications includes limits associated with maximum allowable harmonic distortion at the output (< typically < 1% → -40dB)

CODEC Filter including Output/Input transfer characteristic non-linearity's
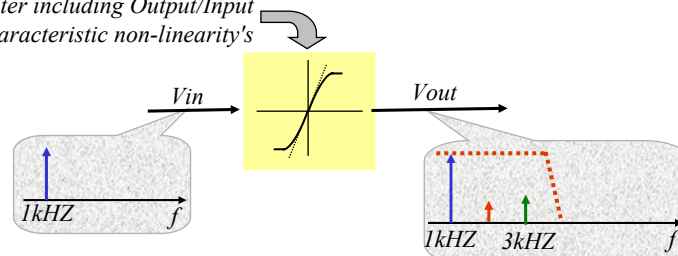
Vin $Vout$

1kHZ f *1kHZ*   *3kHZ* f

Effect of Component Non-Linearities on Overall Filter Linearity
Intermodulation Distortion

DC transfer characteristics including nonlinear terms, input 2 sinusoidal waveforms:

$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$$

$$If \ Vin = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$

Then Vout will have the following components:

$$\alpha_1 Vin \rightarrow \alpha_1 A_1 \sin(\omega_1 t) + \alpha_1 A_2 \sin(\omega_2 t)$$

$$\alpha_2 Vin^2 \rightarrow \alpha_2 A_1^2 \sin(\omega_1 t)^2 + \alpha_2 A_2^2 \sin(\omega_2 t)^2 + 2\alpha_2 A_1 A_2 \ \sin(\omega_1 t)\sin(\omega_2 t) \ + ...$$

$$\rightarrow \frac{\alpha_2 A_1^2}{2}(1 - \cos(2\omega_1 t)) + \frac{\alpha_2 A_2^2}{2}(1 - \cos(2\omega_2 t))$$

$$+ \alpha_2 A_1 A_2 \left[\cos((\omega_1 - \omega_2)t) - \cos((\omega_1 + \omega_2)t)\right]$$

$$\alpha_3 Vin^3 \rightarrow +\alpha_3 A_1^3 \sin(\omega_1 t)^3 + \alpha_3 A_2^3 \sin(\omega_2 t)^3$$

$$+ 3\alpha_3 A_1^2 A_2 \ \sin(\omega_1 t)^2 \sin(\omega_2 t) \ + 3\alpha_3 A_2^2 A_1 \ \sin(\omega_2 t)^2 \sin(\omega_1 t)$$
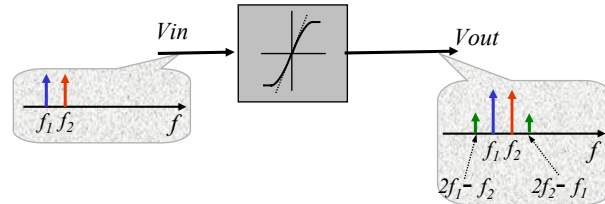
$$+ \frac{3\alpha_3 A_1^2 A_2}{4}\left[\sin(2\omega_1 + \omega_2)t - \sin(2\omega_1 - \omega_2)t\right]$$

Effect of Component Non-Linearities on Overall Filter Linearity
Intermodulation Distortion

Real DC transfer characteristics, input 2 sin waves:

$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$$

$$If \quad Vin = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$

For f_1 & f_2 close in frequency →Components associated with $(2f_1\text{-}f_2)$ & $(2f_2\text{-}f_1)$ are the closest to the fundamental signals on the frequency axis and thus most harmful

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Effect of Component Non-Linearities on Overall Filter Linearity
Intermodulation Distortion

Intermodulation distortion is measured in terms of IM2 and IM3:

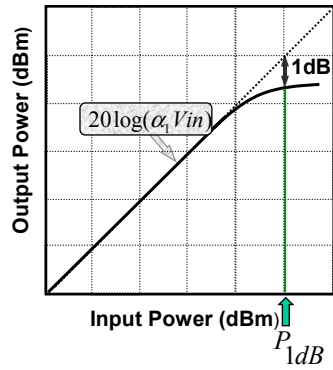Typically for input two sinusoids with equal amplitude ($A1 = A2 = A$)

$$IM2 = \frac{amplitude\,2^{nd}\,IM\,\,component}{amplitude\,fundamental}$$

$$IM3 = \frac{amplitude\,3^{rd}\,IM\,\,component}{amplitude\,fundamental}$$

$$IM2 = \frac{\alpha_2}{\alpha_1}A \ + ... \qquad IM3 = \frac{3}{4}\frac{\alpha_3}{\alpha_1}A^2 + \frac{25}{8}\frac{\alpha_5}{\alpha_1}A^4 + ...$$
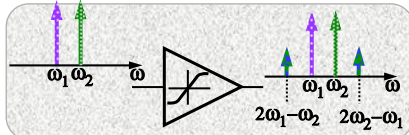
Wireless Communications Measure of Linearity

1dB Compression Point

$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$$

Wireless Communications Measure of Linearity Third Order Intercept Point

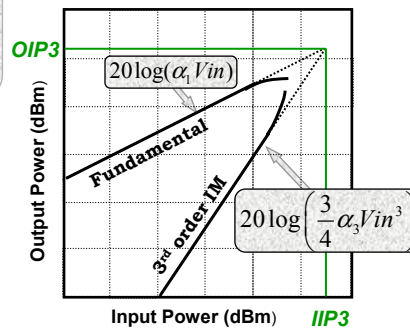$$Vout = \alpha_1 Vin + \alpha_2 Vin^2 + \alpha_3 Vin^3 + ...$$

$$IM_3 = \frac{3rd}{1st}$$

$$= \frac{3}{4}\frac{\alpha_3}{\alpha_1} Vin^2 + \frac{25}{8}\frac{\alpha_5}{\alpha_1} Vin^4 + ...$$

$$= 1 @ IIP3$$

Typically:
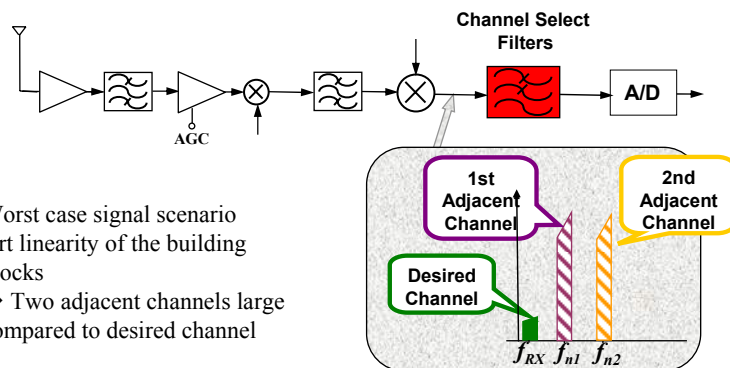
$$IIP_3 - P_{1dB} = 9.6dB$$

Most common measure of linearity for wireless circuits:
→ **OIP3 & IIP3, Third order output/input intercept point**

Example: Significance of Filter Intermodulation Distortion in Wireless Systems

- Typical wireless receiver architecture

Worst case signal scenario wrt linearity of the building blocks
→ Two adjacent channels large compared to desired channel

Example: Significance of Filter Intermodulation Distortion in Wireless Systems
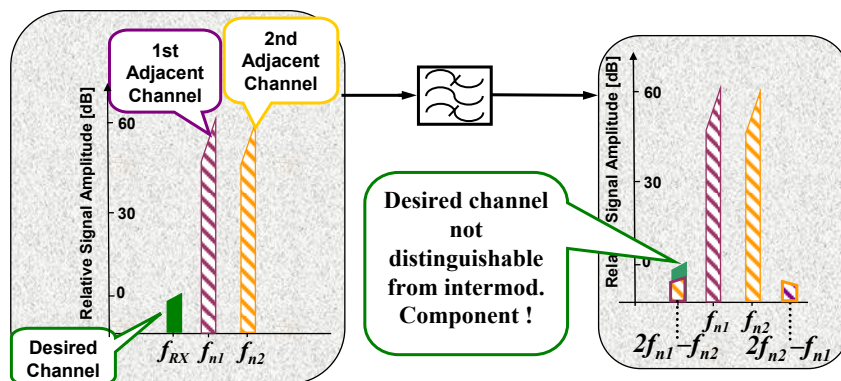
- Adjacent channels can be as much as 60dB higher compared to the desired RX signal!
- Notice that in this example, 3rd order intermodulation component associated with the two adjacent channel, falls on the desired channel signal!

Filter Linearity
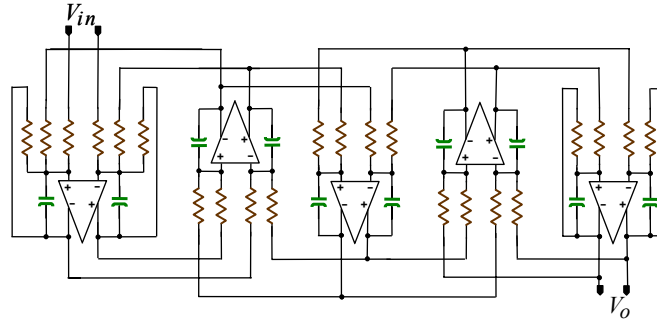
- Maximum signal handling capability is usually determined by the specifications wrt harmonic distortion and /or intermodulation distortion Distortion in a filter is a function of linearity of the components
- Example: In the above circuit linearity of the filter is mainly a function of linearity of the *opamp* voltage transfer characteristics

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# Various Types of Integrator Based Filter

- Continuous Time
  - Resistive element based
    - Opamp-RC
    - Opamp-MOSFET-C
    - Opamp-MOSFET-RC
  - Transconductance (Gm) based
    - Gm-C
    - Opamp-Gm-C
- Sampled Data
  - Switched-capacitor Integrator

Continuous-Time Resistive Element Type Integrators
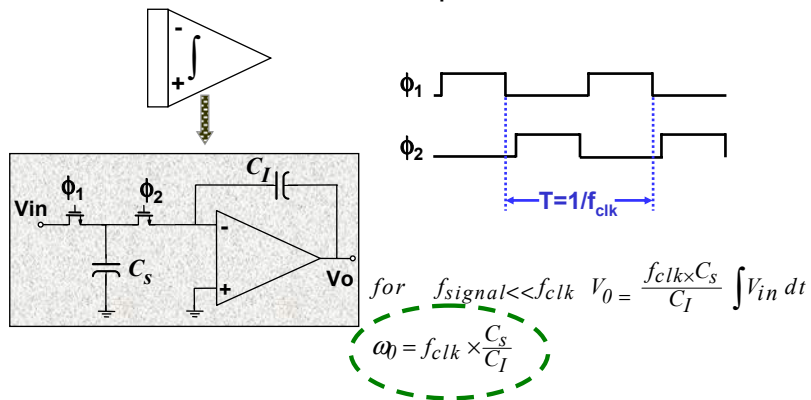## Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC

Opamp-*RC*

Opamp-*MOSFET-C*

Opamp-*MOSFET-RC*
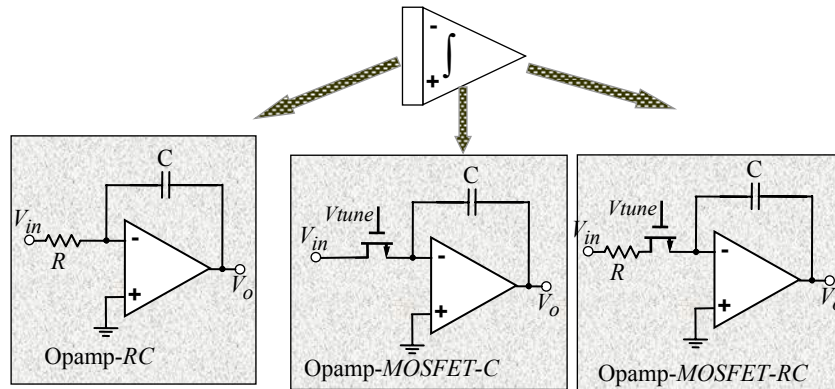
Ideal transfer function: $\dfrac{V_o}{V_{in}} = \dfrac{-\omega_o}{s}$ where $\omega_o = \dfrac{1}{R_{eq}C}$

Continuous-Time Transconductance Type Integrator
Gm-C & Opamp-Gm-C

GmC Intg.

GmC-OTA Intg.
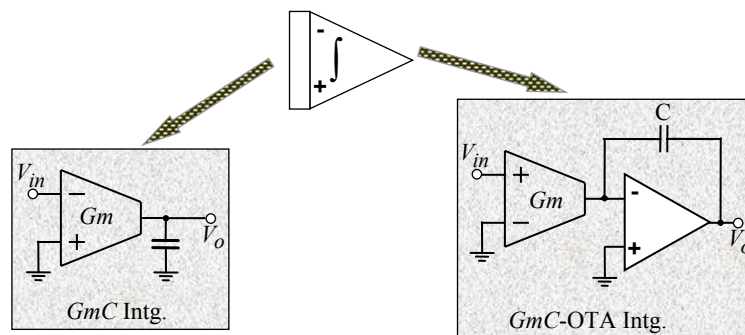
Ideal transfer function: $\dfrac{V_o}{V_{in}} = \dfrac{-\omega_o}{s}$ where $\omega_o = \dfrac{G_m}{C}$

Integrator Implementation
Switched-Capacitor

$$for \quad f_{signal} << f_{clk} \quad V_0 = \frac{f_{clk} \times C_S}{C_I} \int V_{in} \, dt$$

$$\omega_0 = f_{clk} \times \frac{C_s}{C_I}$$

Main advantage: Critical frequency function of *ratio* of caps & clock freq.
→ Critical filter frequencies (e.g. LPF -3dB freq.) very accurate

---

Few Facts About Monolithic *Rs & Cs & Gms*

- Monolithic continuous-time filter critical frequency set by *RxC* or C/*Gm*
- Absolute value of integrated *Rs & Cs & Gms* are quite variable
  - *Rs* vary due to doping and etching non-uniformities
    - Could vary by as much as ~+-*20 to 40%* due to process & temperature variations
  - *Cs* vary because of oxide thickness variations and etching inaccuracies
    - Could vary ~ +-*10 to15%*
  - *Gms* typically function of mobility, oxide thickness, current, device geometry …
    - Could vary > ~ +- *40%* or more with process & temp. & supply voltage
    - →Continuous-time filter critical frequency could vary by over +-50%

Few Facts About Monolithic Rs & Cs

- While absolute value of monolithic *Rs & Cs* and *gms* are quite variable, with special attention paid to layout, *C & R & gms* quite well-matched
  - *Ratios* very accurate and stable over processing, temperature, and time

- With special attention to layout (e.g. interleaving, use of dummy devices, common-centroid geometries…):
  - Capacitor mismatch << 0.1%
  - Resistor mismatch < 0.1%
  - Gm mismatch < 0.5%

Impact of Component Variations on Filter Characteristics
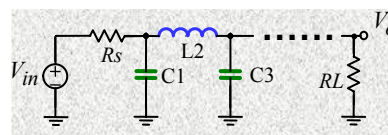
RLC Filters

Facts about RLC filters

- ω_{-3dB} determined by absolute value of *Ls & Cs*

$$C_1^{RLC} = C_r \times C_1^{Norm} = \frac{C_1^{Norm}}{R^* \times \omega_{-3dB}}$$

- Shape of filter depends on *ratios* of normalized *Ls & Cs*

$$L_2^{RLC} = L_r \times L_2^{Norm} = \frac{L_2^{Norm} \times R^*}{\omega_{-3dB}}$$

Effect of Monolithic R & C Variations on Filter Characteristics

- Filter shape (whether Elliptic with 0.1dB Rpass or Butterworth..etc) is a function of *ratio* of normalized *Ls & Cs* in RLC filters

- Critical frequency (e.g. ω_{-3dB}) function of absolute value of *Ls xCs*

- Absolute value of integrated *Rs & Cs & Gms* are quite variable

- *Ratios* very accurate and stable over time and temperature

    → What is the effect of on-chip component variations on monolithic filter frequency characteristics?

Impact of Process Variations on Filter Characteristics

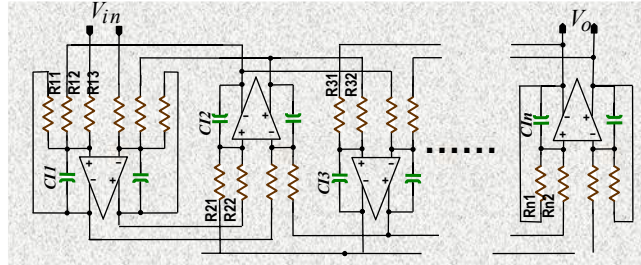RLC Filters   Integrator Based Filters
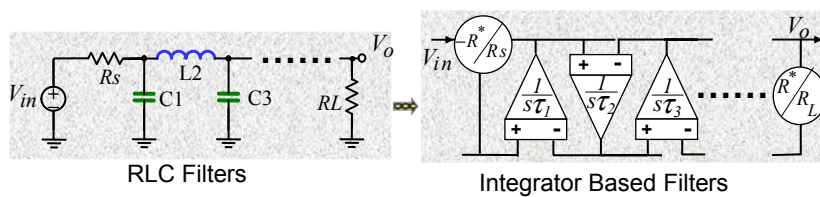
$$\tau_1 = C_1^{RLC}.R^* = \frac{C_1^{Norm}}{\omega_{-3dB}}$$

$$\tau_2 = \frac{L_2^{RLC}}{R^*} = \frac{L_2^{Norm}}{\omega_{-3dB}}$$

$$\frac{\tau_1}{\tau_2} = \frac{C_1^{Norm}}{L_2^{Norm}}$$

## Impact of Process Variations on Filter Characteristics

$$\tau_1^{int\,g} = C_{I1}.R_1 = \frac{C_1^{Norm}}{\omega_{-3dB}}$$

$$\tau_2^{int\,g} = C_{I2}.R_2 = \frac{L_2^{Norm}}{\omega_{-3dB}}$$

Variation in absolute value of integrated Rs & Cs → change in critical freq. (ω_{-3dB})

$$\frac{\tau_1^{int\,g}}{\tau_2^{int\,g}} = \frac{C_{I1}.R_1}{C_{I2}.R_2} = \frac{C_1^{Norm}}{L_2^{Norm}}$$

Since *Ratios* of Rs & Cs very accurate

→ Continuous-time monolithic filters retain their shape due to good component matching even with variability in absolute component values


Example: LPF Worst Case Corner Frequency Variations

Detailed passband
(note shape is well-retained)

Nominal Bandwidth

Worst case bandwidth variation
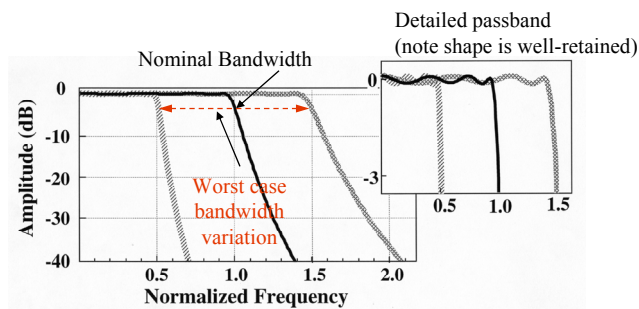
- While absolute value of on-chip RC (gm-C) time-constants could vary by as much as 100% (process & temp.)
- With proper precautions, excellent component matching can be achieved:
    → Well-preserved relative amplitude & phase vs freq. charactersitics
    → Need to only adjust (tune) continuous-time filter critical frequencies

Tunable Opamp-RC Filters
Example

- 1st order Opamp-RC filter is designed to have a corner frequency of *1.6MHz*
- Assuming process variations of:
  - *C* varies by *+-10%*
  - *R* varies by *+-25%*
- Build the filter in such a way that the corner frequency can be adjusted post-manufacturing.
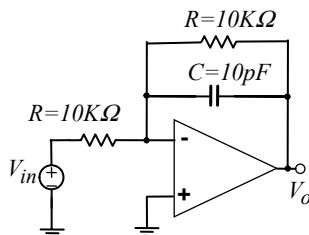
*Nominal R & C values
for 1.6MHz corner frequency*

Filter Corner Frequency Variations

- Assuming expected process variations of:

  - Maximum C variations by +-10%
    Cnom=10pF → Cmin=9pF, Cmax=11pF

  - Maximum R variations by +-25%
    Rnom=10K→ Rmin=7.5K, Rmax=12.5K

  - Corner frequency ranges from
    → 2.357MHz to 1.157MHz

    →Corner frequency varies by +48% & -27%

Variable Resistor or Capacitor

- Make provisions for either R or C to be adjustable (this example→ adjustable R)
- Monolithic Rs can only be made adjustable in discrete steps (not continuous)

$$\frac{R_{nom}^{max}}{R_{nom}} = \frac{f_{max}}{f_{nom}} = 1.48$$
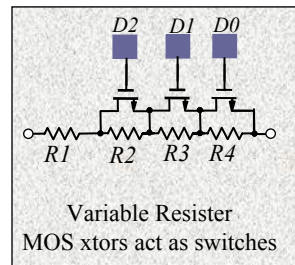
$$\rightarrow R_{nom}^{max} = 14.8k\Omega$$

$$\frac{R_{nom}^{min}}{R_{nom}} = \frac{f_{min}}{f_{nom}} = 0.72$$

$$\rightarrow R_{nom}^{min} = 7.2k\Omega$$

Variable Resister
MOS xtors act as switches

Tunable Resistor

- Maximum C variations by +-10%→ Cmin=9pF, Cmax=11pF
- Maximum R variations by +-25%→ Rmin=7.5K, Rmax=12.5K
     →Corner frequency varies by +48% & -27.%
- Assuming $n = 3$bit (0 or 1) control signal for adjustment
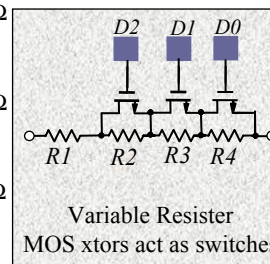
$$R_1 = R_{nom}^{min} = 7.2k\Omega$$

$$R_2 = \left(R_{nom}^{max} - R_{nom}^{min}\right) \times \frac{2^{n-1}}{2^n - 1} = (14.8k - 7.2k)\frac{4}{7} = 4.34k\Omega$$

$$R_3 = \left(R_{nom}^{max} - R_{nom}^{min}\right) \times \frac{2^{n-2}}{2^n - 1} = (14.8k - 7.2k)\frac{2}{7} = 2.17k\Omega$$

$$R_4 = \left(R_{nom}^{max} - R_{nom}^{min}\right) \times \frac{2^{n-3}}{2^n - 1} = (14.8k - 7.2k)\frac{1}{7} = 1.08k\Omega$$

Tuning resolution $\approx 1.08k/10k \approx 10\%$

Variable Resister
MOS xtors act as switches

Tunable Opamp-RC Filter

| D2	D1	D0	Rnom
1	1	1	7.2K
1	1	0	8.28K
1	0	1	9.37K
⋮	⋮	⋮	
⋮	⋮	⋮	
⋮	⋮	⋮	
0	0	0	14.8K

Post manufacturing:
- Set all Dx to 100 (mid point)
- Measure -3dB frequency
  - If frequency too high decrement D to D-1
  - If frequency too low increment D to D+1
  - If frequency within 10% of the desired corner frequency → stop
  - else

For higher order filters, all filter integrators tuned simultaneously

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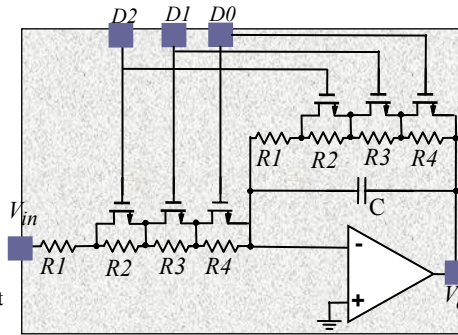
Tunable Opamp-RC Filters
Summary

MOS devices used as switches
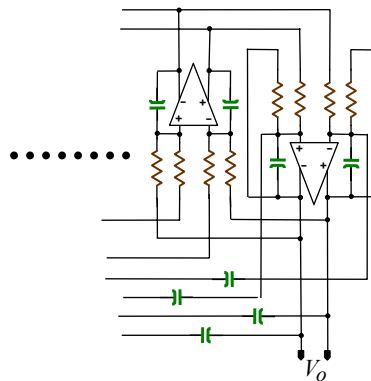
Tunable Opamp RC Integrator
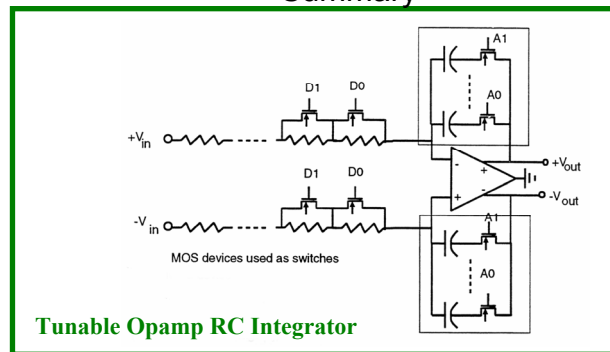
- Program Cs and/or Rs to freq. tune the filter
- All filter integrators tuned simultaneously
- Tuning in discrete steps & not continuous
- Tuning resolution limited
- Switch parasitic C & series R can affect the freq. response of the filter

Example: Tunable Low-Pass Opamp-RC Filter
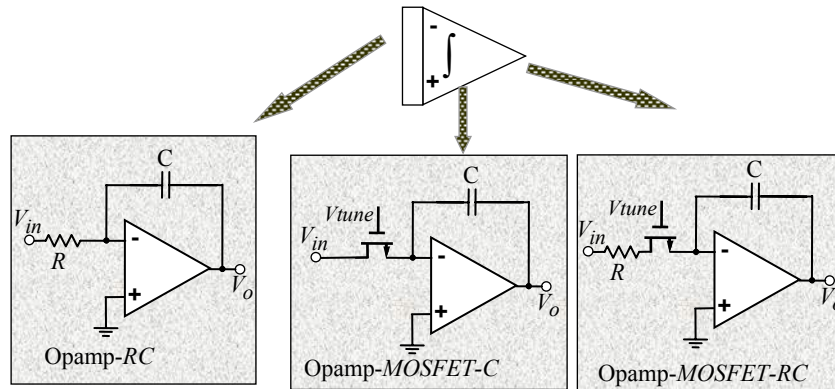## Adjustable Capacitors

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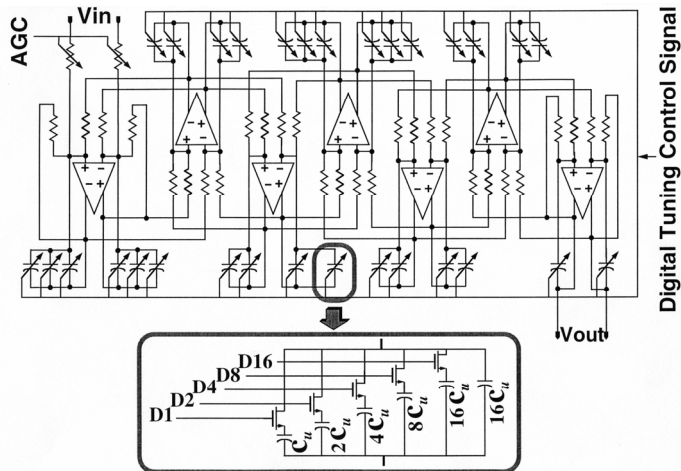
Opamp RC Filters

- Advantages
  - Since resistors are quite linear, linearity only a function of opamp linearity
    - → good linearity

- Disadvantages
  - Opamps have to drive resistive load, low output impedance is required
    - → High power consumption
  - Continuous tuning not possible- tuning only in discrete steps
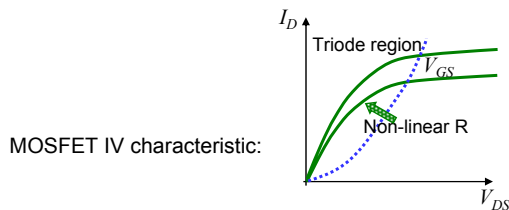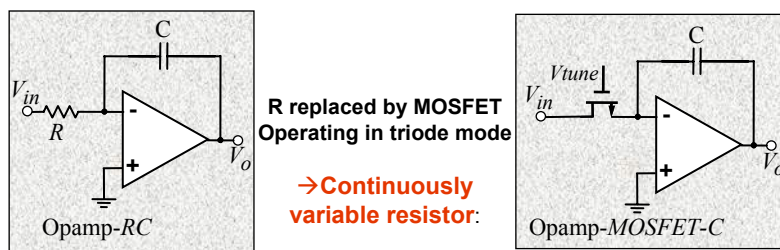 - Tuning requires programmable Rs and/or Cs

V_o

Integrator Implementation
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC

Opamp-*RC*

Opamp-*MOSFET-C*

Opamp-*MOSFET-RC*

$$\frac{V_o}{V_{in}} = \frac{-\omega_o}{s} \quad \text{where} \quad \omega_o = \frac{1}{R_{eq}C}$$

Use of MOSFETs as Variable Resistors

Opamp-*RC*

R replaced by MOSFET Operating in triode mode

→**Continuously variable resistor**:

Opamp-*MOSFET-C*

I_D

Triode region

V_{GS}

Non-linear R
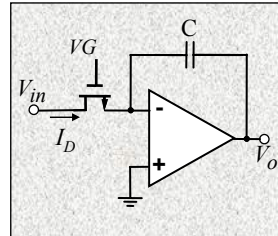
MOSFET IV characteristic:

V_{DS}

Opamp MOSFET-C Integrator
Single-Ended Integrator

$$I_D = \mu C_{ox} \frac{W}{L} \left[(V_{gs} - V_{th}) V_{ds} - \frac{V_{ds}^2}{2} \right]$$

$$I_D = \mu C_{ox} \frac{W}{L} \left[(V_{gs} - V_{th}) V_i - \frac{V_i^2}{2} \right]$$

$$G = \frac{\partial I_D}{\partial V_i} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th} - V_i)$$

→**Tunable by varying *VG*:**

By varying *VG* effective admittance is tuned
→ Tunable integrator

Problem: Single-ended MOSFET-C Integrator→ Effective R non-linear
Note that the non-linearity is mainly 2nd order type

Use of MOSFETs as Resistors
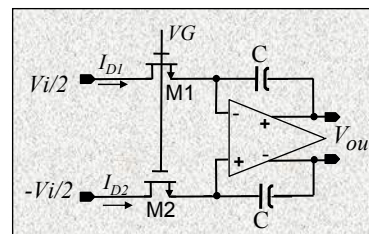## Differential Integrator

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} - \frac{V_{ds}}{2} \right) V_{ds}$$

$$I_{D1} = \mu C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} - \frac{V_i}{4} \right) \frac{V_i}{2}$$

$$I_{D2} = -\mu C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} + \frac{V_i}{4} \right) \frac{V_i}{2}$$

$$I_{D1} - I_{D2} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th}) V_i$$

$$G = \frac{\partial (I_{D1} - I_{D2})}{\partial V_i} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th})$$
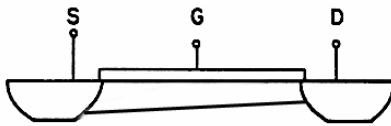

Opamp-*MOSFET-C*

• **Non-linear term is of even order & cancelled!**
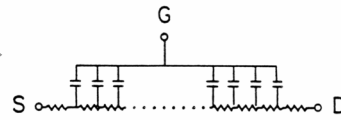• **Admittance independent of Vi**

Problem: Threshold voltage dependence

Use of MOSFET as Resistor Issues

**MOS xtor operating in triode region
Cross section view**

**Distributed channel resistance &
gate capacitance**

• Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles:

> → Excess phase @ the unity-gain frequency of the integrator
> → Enhanced integrator Q
> → Enhanced filter Q,
> →Peaking in the filter passband

Use of MOSFET as Resistor Issues

**MOS xtor operating in triode region
Cross section view**
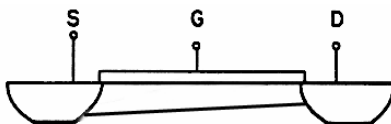
**Distributed channel resistance &
gate capacitance**
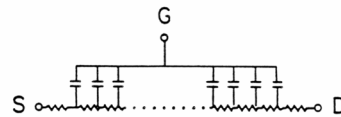
• Tradeoffs affecting the choice of device channel length:
  – Filter performance mandates well-matched MOSFETs → long channel devices desirable
  – Excess phase increases with L^2 → Q enhancement and potential for oscillation!
    →Tradeoff between device matching and integrator Q
    →This type of filter limited to low frequencies

Example:
Opamp MOSFET-C Filter

- Suitable for low frequency applications

- Issues with linearity

- Linearity achieved ~40-50dB

- Needs tuning
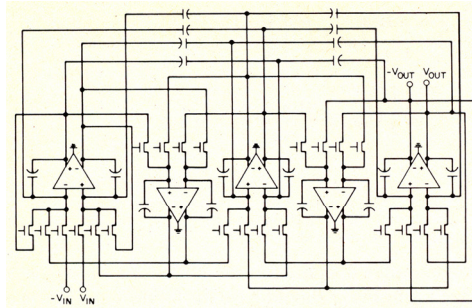
5th Order Elliptic MOSFET-C LPF
with 4kHz Bandwidth

Ref: Y. Tsividis, M.Banu, and J. Khoury, "Continuous-Time MOSFET-C Filters in VLSI", *IEEE Journal of Solid State Circuits* Vol. SC-21, No.1 Feb. 1986, pp. 15-30

Improved MOSFET-C Integrator

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{gs} - V_{th} - \frac{V_{ds}}{2}\right)V_{ds}$$

$$I_{D1} = \mu C_{ox} \frac{W}{L}\left(V_{gs1} - V_{th} - \frac{V_i}{4}\right)\frac{V_i}{2}$$

$$I_{D3} = -\mu C_{ox} \frac{W}{L}\left(V_{gs3} - V_{th} + \frac{V_i}{4}\right)\frac{V_i}{2}$$

$$I_{X1} = I_{D1} + I_{D3}$$

$$= \mu C_{ox} \frac{W}{L}\left(V_{gs1} - V_{gs3} - \frac{V_i}{2}\right)\frac{V_i}{2}$$

$$I_{X2} = \mu C_{ox} \frac{W}{L}\left(V_{gs3} - V_{gs1} - \frac{V_i}{2}\right)\frac{V_i}{2}$$

$$I_{X1} - I_{X2} = \mu C_{ox} \frac{W}{L}\left(V_{gs1} - V_{gs3}\right)V_i$$

$$G = \frac{\partial\left(I_{X1} - I_{X2}\right)}{\partial V_i} = \mu C_{ox} \frac{W}{L}\left(V_{gs1} - V_{gs3}\right)$$

No threshold dependence

M1,2,3,4 → equal *W/L*

Linearity achieved in the order of 50-70dB

Ref: Z. Czarnul, "Modification of the Banu-Tsividis Continuous-Time Integrator Structure," *IEEE Transactions on Circuits and Systems,* Vol. CAS-33, No. 7, pp. 714-716, July 1986.

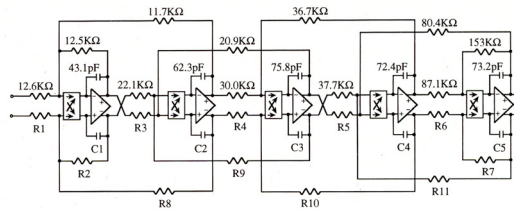# R-MOSFET-C Integrator

- •Improvement over MOSFET-C by adding resistor in series with MOSFET
- •Voltage drop primarily across fixed resistor → small MOSFET Vds → improved linearity & reduced tuning range
- •Generally low frequency applications

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits,* Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

R-MOSFET-C Lossy Integrator

Negative feedback around the non-linear MOSFETs improves linearity but compromises frequency response accuracy

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits,* Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

Example:
Opamp MOSFET-RC Filter

**5th Order Bessel MOSFET-RC LPF 22kHz bandwidth**
THD →-90dB for 4Vp-p , 2kHz input signal

- Suitable for low frequency, low Q applications
- Significant improvement in linearity compared to MOSFET-C
- Needs tuning

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits,* Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.
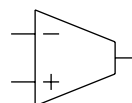

Operational Amplifiers (Opamps) versus
Operational Transconductance Amplifiers (OTA)

Opamp **Voltage controlled** **voltage source**	**OTA** **Voltage controlled** **current source**
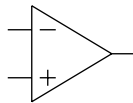
|	
• Output in the form of voltage	• Output in the form of current
• Low output impedance	• High output impedance
• Can drive R-loads	• In the context of filter design called *gm-cells*
• Good for RC filters, OK for SC filters	• Cannot drive R-loads
	• Good for SC & gm-C filters
• Extra buffer adds complexity, power dissipation	• Typically, less complex compared to opamp→ higher freq. potential
	• Typically lower power

Integrator Implementation
Transconductance-C & Opamp-Transconductance-C
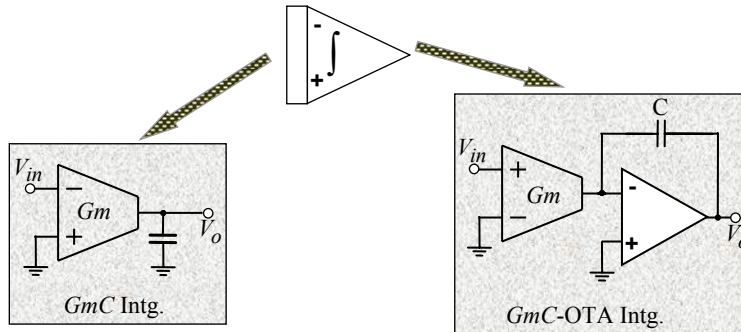
$$\frac{V_O}{V_{in}} = \frac{-\omega_o}{s} \quad \text{where} \quad \omega_o = \frac{G_m}{C}$$

Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

- Transconductance element formed by the source-coupled pair *M1 & M2*

- All MOSFETs operating in saturation region

- Current in *M1 & M2* can be varied by changing $V_{control}$

  → Transconductance of *M1 & M2* varied through $V_{control}$
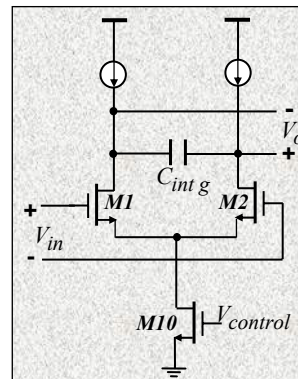
Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," IEEE Journal of Solid-State Circuits, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.