COM S 476/576 Homework 2

Problem 3.1 (5 points) Define a semi-algebraic model that removes a triangular "nose" from the region shown in Figure 3.4.

Problem 3.4 (5 points) An alternative to the yaw-pitch-roll formulation from Section 3.2.3 is considered here. Consider the following Euler angle representation of rotation (there are many other variants). The first rotation is $R_z(\gamma)$, which is just (3.39) with α replaced by γ. The next two rotations are identical to the yaw-pitch-roll formulation: $R_y(\beta)$ is applied, followed by $R_z(\alpha)$. This yields $R_{euler}(\alpha, \beta, \gamma) = R_z(\alpha)R_y(\beta)R_z(\gamma)$.

(a) Determine the matrix R_{euler}.

(b) Show that $R_{euler}(\alpha, \beta, \gamma) = R_{euler}(\alpha - \pi, -\beta, \gamma - \pi)$.

Problem 3.7 (5 points) Consider the articulated chain of bodies shown in Figure 3.29. There are three identical rectangular bars in the plane, called $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$. Each bar has width 2 and length 12. The distance between the two points of attachment is 10. The first bar, $\mathcal{A}_1$, is attached to the origin. The second bar, $\mathcal{A}_2$, is attached to $\mathcal{A}_1$, and $\mathcal{A}_3$ is attached to $\mathcal{A}_2$. Each bar is allowed to rotate about its point of attachment. The configuration of the chain can be expressed with three angles, $(\theta_1, \theta_2, \theta_3)$. The first angle, θ_1, represents the angle between the segment drawn between the two points of attachment of $\mathcal{A}_1$ and the x-axis. The second angle, θ_2, represents the angle between $\mathcal{A}_2$ and $\mathcal{A}_1$ ($\theta_2 = 0$ when they are parallel). The third angle, θ_3, represents the angle between $\mathcal{A}_3$ and $\mathcal{A}_2$. Suppose the configuration is $(\pi/4, \pi/2, -\pi/4)$.

(a) Use the homogeneous transformation matrices to determine the locations of points a, b, and c.

(b) Characterize the set of all configurations for which the final point of attachment (near the end of $\mathcal{A}_3$) is at $(0, 0)$ (you should be able to figure this out without using the matrices).

Problem 3.14 (5 points) Develop and implement a kinematic model for a 2D kinematic chain $\mathcal{A}_1, \ldots, \mathcal{A}_m$. Each link has the following properties:

- W: The width of each link.
- L: The length of each link.
- D: The distance between the two points of attachment.

Write a script named `chain.py` that takes above properties and the configuration of the chain and display the arrangement of links in the plane. The script should be executed using the following command:

```
chain.py -W <width> -L <length> -D <distance> <theta_1, ..., theta_m>
```

where <width> $= W$, <length> $= L$, <distance> $= D$, and <theta_1, ..., theta_m> specifies the angles that define the chain configuration.