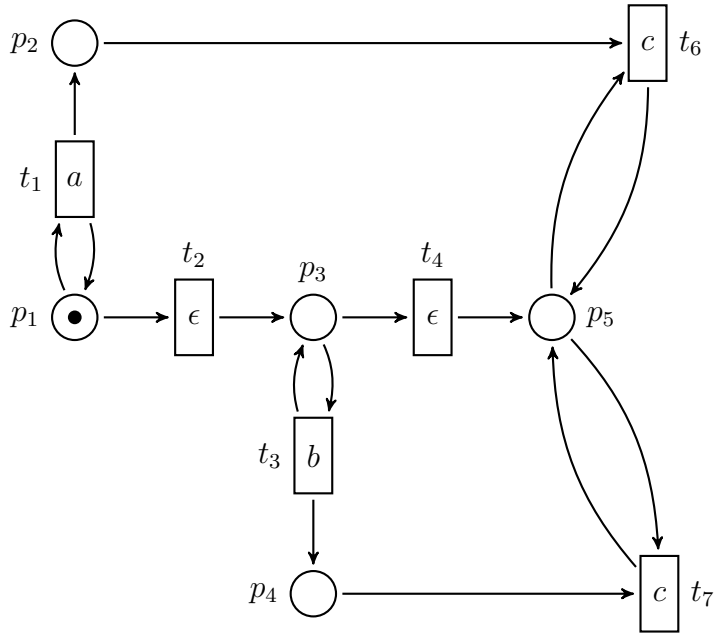
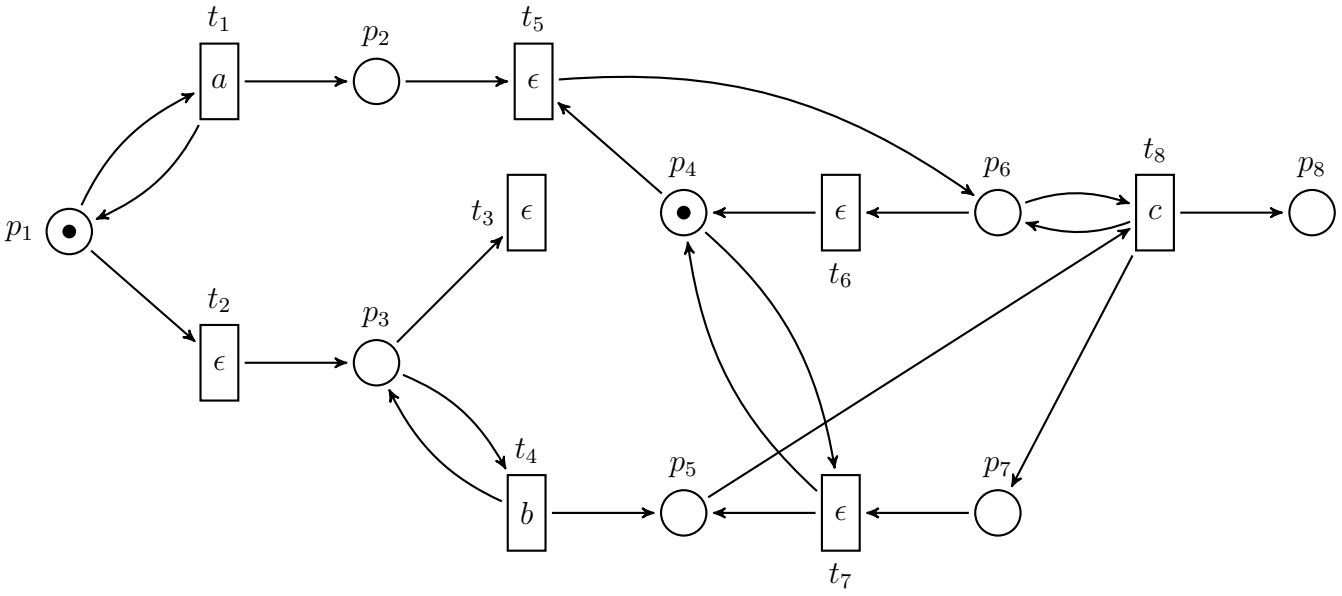
ComS 331 **Spring 2025** **Name:** Mobina Amrollahi

HW 8 OPTIONAL BONUS HW Due: April 4^{th} 2025

1. We know that NPDA can sum: $L = \{a^i b^j c^{i+j} : i, j \in \mathbb{N}\}$ is a CFL. Petri nets can perform the same operation: show a T-type unrestricted labeling Petri net for the same language.
Answer.



2. We know that NPDA cannot multiply: $\{a^r b^s c^t : r \cdot s = t\}$ is not a CFL. Indeed they cannot even perform a *weak* type of multiplication: $L = \{a^r b^s c^t : t \leq r \cdot s\}$ is not a CFL either. However, Petri nets can perform a weak multiplication: show a T-type unrestricted labeling Petri net for L.
Answer.



3. Consider the language $L = \{w \in \{a, b, c\}^* : |w|_a = |w|_b = |w|_c\}$.

(a) Prove that L is not a context-free language.

Answer.

Prove by contradiction: Assume the contrary that L is context-free. Let p be the given pumping length. Now, consider the string $s = a^p b^p c^p$. Since $|s|_a = |s|_b = |s|_c$, $s \in L$. Additionally, $| \ s \ | \geq p$, and based on the pumping lemma, s may be divided into five pieces $s = uvxyz$ satisfying the conditions:

i. for each $i \geq 0$, $uv^i xy^i z \in L$,

ii. $| \ vy \ | > 0$, and

iii. $| \ vxy \ | \leq p$.

There will be two cases on how v and y will span out:

i. When both v and y contain only one type of alphabet symbol, v does not contain both a's and b's or both b's and c's, and the same holds for y. Now that because v and y contain only one type of alphabet symbol, one of the symbols a, b, or c doesn't appear in v or y. We further subdivide this case into three sub-cases according to which symbol does not appear.

   a. The a's do not appear. Then we try pumping down to obtain the string $uv^0xy^0z = uxz$. That contains the same number of a's as s does, but it contains fewer b's or fewer c's. Therefore, it is not a member of L, and a contradiction occurs.

   b. The b's do not appear. Then either a's or c's must appear in v or y because both can't be the empty string. If a's appear, the string uv^2xy^2z contains more a's than b's, so it is not in L. If c's appear, the string uv^0xy^0z contains more b's than c's, so it is not in L. Either way, a contradiction occurs.

   c. The c's do not appear. Then the string uv^2xy^2z contains more a's or more b's than c's, so it is not in L, and a contradiction occurs.

ii. When either v or y contains more than one type of symbol, uv^2xy^2z will not contain the symbols in the correct order. Hence it cannot be a member of C, and a contradiction occurs.

Thus we have shown that s cannot be pumped in violation of the pumping lemma and that C is not context free.

(b) Prove that L is an L-type non-ϵ labeling Petri net language.

Answer.

By setting $F = \{[p_0]\}$

(c) Prove that L is a T-type unrestricted labeling Petri net language.

Answer.

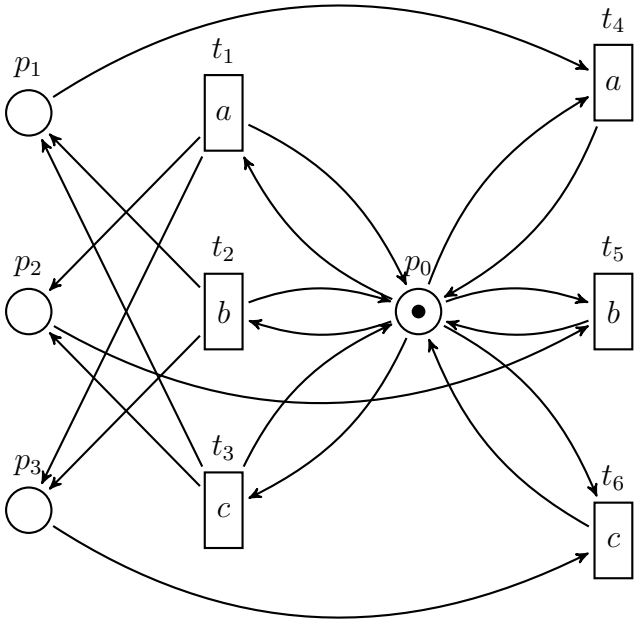


Figure 1: Question 3 - b

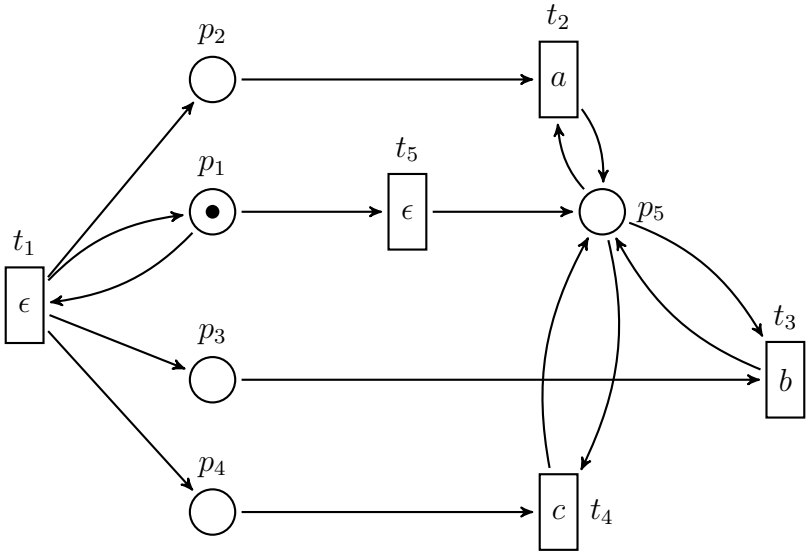


Figure 2: Question 3 - c