ComS 331 **Spring 2025** **Name:** Mobina Amrollahi

HW 10 Due: April 11^{th} 2025

1. Prove that the Turing-acceptable languages are closed under union, intersection, and reversal. For each property, give a detailed sketch of the proof, by saying how you would build a Turing machine that accepts the resulting language, given the Turing machine(s) that accept the original language(s).
Answer.

(a) Turing-recognizable languages are closed under union. For any two Turing-recognizable languages L_1 and L_2, let M_1 and M_2 be Turing machines that recognize them such that $L_1 = L(M_1)$ and $L_2 = L(M_2)$. We construct a Turing machine M that recognizes the union $L_1 \cup L_2$:

$$L(M) = \{w \mid w \in L(M_1) \vee w \in L(M_2)\}$$

Algorithm 1 Turing machine accepting the union of two Turing-recognizable languages

- **Input:** $w \in \Sigma^*$
- Simulate M_1 and M_2 on w concurrently through dovetailing
- **if** at least one of the computations halts and accepts **then**
  - Halt and accept

(b) Turing-acceptable languages are closed under intersection. For any two Turing-recognizable languages L_1 and L_2, let M_1 and M_2 be Turing machines that recognize them such that $L_1 = L(M_1)$ and $L_2 = L(M_2)$. We construct a Turing machine M that recognizes the intersection $L_1 \cap L_2$:

$$L(M) = \{w \mid w \in L(M_1) \wedge w \in L(M_2)\}$$

Algorithm 2 Turing machine accepting the intersection of two Turing-recognizable languages

- **Input:** $w \in \Sigma^*$
- Simulate M_1 on w
- **if** M_1 halts and accepts **then**
  - Simulate M_2 on w
  - **if** M_2 halts and accepts **then**
    - Halt and accept

(c) Turing-acceptable languages are closed under reversal. For any Turing-recognizable language such as L let M be the Turing machine that recognize it such that $L = L(M)$. We construct a Turing machine M' that recognizes the reversal of L:

$$L(M') = \{w \mid w^R \in L(M)\}$$

Algorithm 3 Turing machine accepting the reversal of a Turing-recognizable language

Input: $w \in \Sigma^*$
$w^R =$ **reverse of** w
Simulate M on w^R
if M halts and accepts **then**
    Halt and accept

2. Prove or disprove that the Turing-acceptable languages are closed under concatenation. For any two Turing-recognizable languages L_1 and L_2, let M_1 and M_2 be Turing machines that recognize them, such that $L_1 = L(M_1)$ and $L_2 = L(M_2)$. We construct a Turing machine M that recognizes the concatenation of L_1 and L_2, defined as:

$$L(M) = \{x \mid x = yz, y \in L(M_1), z \in L(M_2)\}$$

Algorithm 4 Turing Machine Accepting the Concatenation of Two Turing-Recognizable Languages

Input: $x \in \Sigma^*$
Simulate all pairs where $x = x_i y_i$; $(0 \leq i \leq n)$ by running M_1 on x_i and M_2 on y_i concurrently through dovtailing
if M_1 halts and accepts x_i **then**
    if M_2 halts and accepts y_i **then**
        Halt and accept