ComS 331 **Spring 2025** **Name: Mobina Amrollahi**

HW 6 Due: March 7^{th} 2025

1. The truth value of a logical expression is defined recursively as:

   - The truth value of t is t.
   - The truth value of f is f.
   - The truth value of $(x_1 \wedge x_2)$ is t if both x_1 and x_2 have truth value t, it is f otherwise.
   - The truth value of $(x_1 \vee x_2)$ is f if both x_1 and x_2 have truth value f, it is t otherwise.
   - The truth value of $\neg(x)$ is f if x has truth value t, it is t otherwise.

   Define a CFG that generates the following language over $\{t, f, \wedge, \vee, \neg, (,), =\}$:

   $$L = \{w = x : w \text{ is a logical expression over } \{t, f\}, x \in \{t, f\}, \text{ and } x \text{ is the truth value of } w\}$$

   Thus, "$t = t$", "$((t \wedge f) \vee f) = f$", and "$\neg(((t \wedge f) \vee f)) = t$" are in L, but "$((t \wedge f) \vee f) = t$" and "$(t \wedge f) \vee f = f$" are not: the former because $((t \wedge f) \vee f)$ is false and not true, the latter because the expression lacks the outermost set of parentheses.

   Answer:

   Consider the grammar $G = (\{S, T, F, N\}, \{t, f, \wedge, \vee, \neg, (,), =\}, R, S)$ where the set of rules, R, is

   $$\begin{aligned} S &\to T = t \mid F = f \\ T &\to \neg(F) \mid (T \wedge T) \mid (T \vee N) \mid (N \vee T) \mid t \\ F &\to \neg(T) \mid (F \vee F) \mid (F \wedge N) \mid (N \wedge F) \mid f \\ N &\to T \mid F. \end{aligned}$$

2. Define a context-free grammar for the language $L = \{0^n 1^m 0^m 1^n : n, m \in \mathbb{N}\}$.

   Answer:

   Consider the grammar $G = (\{S, W\}, \{0, 1\}, R, S)$ where the set of rules, R, is

   $$\begin{aligned} S &\to 0S1 \mid W \\ W &\to 1W0 \mid \epsilon. \end{aligned}$$

3. Define a context-free grammar for the language $L = \{uv \in \{0, 1\}^* : |u| = |v| \wedge u \neq v^R\}$.

   Answer:

   Consider the grammar $G = (\{S, W\}, \{0, 1\}, R, S)$ where the set of rules, R, is

   $$\begin{aligned} S &\to 0S0 \mid 1S1 \mid 0W1 \mid 1W0 \\ W &\to 0W0 \mid 1W1 \mid 0W1 \mid 1W0 \mid \epsilon. \end{aligned}$$

4. Define a context-free grammar for the language $L = \{a^n b^m : n \leq 3m\}$.

   Answer:

   Consider the grammar $G = (\{S, A\}, \{a, b\}, R, S)$ where the set of rules, R, is

   $$\begin{aligned} S &\to AAASb \mid \epsilon \\ A &\to a \mid \epsilon. \end{aligned}$$

5. Find a CFG for the language $L = \{a^n b^m : n, m \in \mathbb{N}, n \neq m\}$.

   Answer:

   Consider the grammar $G = (\{S, W, T\}, \{a, b\}, R, S)$ where the set of rules, R, is

$$S \to aSb \mid T \mid W$$
$$T \to aT \mid a$$
$$W \to Wb \mid b.$$